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The Manufacture and Sale of Saint Einstein

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10 "SPACE-TIME" OR IS IT "TIME-SPACE"?

The ancients expressed "space-time" theories thousands of years ago. Albert Einstein did not introduce the idea of space-time into the theory of relativity, rather it was Henri Poincaré who first propounded the special theory of relativity in its modern four-dimensional form. When Minkowski adopted Poincaré's quadri-dimensional theory, Einstein opposed the idea, and did not adopt it until much later.

> "As I've already said, it is not possible to conceive of more than three *dimensions*. However, a brilliant wit with whom I am acquainted considers duration a fourth *dimension*, and that the product of time multiplied by solidity would, in some sense, be a product of four *dimensions*."—D'ALEMBERT

> "This rigid four-dimensional space of the special theory of relativity is to some extent a four-dimensional analogue of H. A. Lorentz's rigid three-dimensional æther."—ALBERT EINSTEIN [2414]

10.1 Introduction

Popular myth has it that Albert Einstein originated the concept of "space-time". However, not only did Einstein not originate the idea of "space-time", he vigorously opposed it for quite some space of time.[2415] In fact, space-time theories have been quite common in folk-lore, philosophy, mathematics, religion,[2416] science, science fiction,[2417] psychology,[2418] and are even inherent in some languages.[2419]

Space-time theories which antedate Einstein's entrance into the arena include those of: the ancient Eleatic philosophers,[2420] Ocellus Lucanus,[2421] Plato,[2422] Aristotle,[2423] Critolaus of Phaselis, Jesus,[2424] Philo Judæus,[2425] Taurus,[2426] St. Augustine,[2427] Julius Firmicus Maternus,[2428] Proclus,[2429] *Zohar*,[2430] Bruno,[2431] More,[2432] Locke,[2433] Newton,[2434] Clarke,[2435] Leibnitz,[2436] Berkeley,[2437] Hartley,[2438] Boscovich,[2439] Lagrange,[2440] Kant,[2441] Schopenhauer,[2442] Hegel, Herbart,[2443] Fechner,[2444] Poe,[2445] Stallo,[2446] Hamilton,[2447] Spencer,[2448] Mach,[2449] Baumann,[2450] Dühring,[2451] Lange,[2452] Green,[2453] Hinton,[2454] Venn,[2455] Teichmüller,[2456] "S.",[2457] Mewes,[2458] Voigt,[2459] Shand,[2460] Bergson,[2461] Bradley,[2462] Guyau and Fouillée,[2463] Wells,[2464] Palágyi,[2465] Fullerton,[2466] Ziegler,[2467] Smith,[2468] Poincaré,[2469] Mehmke,[2470] Marcolongo,[2471] Hargreaves,[2472] Welby,[2473] McTaggart[2474] and Minkowski.[2475] Secondary literature expressly referring to such theories before Einstein adopted the view includes that of: D'Alembert,[2476] Klügel,[2477] Cranz[2478] and Wölffing.[2479]

10.2 The Ancients and "Space-Time"

The relational image of time to space and motion is an ancient conception. Consider Anaximander's philosophy (ca. 611-546 B.C.), which speaks of the absolute world of "space-time", and hints at "Mach's principle",

> "Anaximander, then, was the hearer of Thales. Anaximander was son of Praxiadas, and a native of Miletus. This man said that the originating principle of existing things is a certain constitution of the Infinite, out of which the heavens are generated, and the worlds therein; and that this principle is eternal and undecaying, and comprising all the worlds. And he speaks of time as something of limited generation, and subsistence, and destruction. This person declared the Infinite to be an originating principle and element of existing things, being the first to employ such a denomination of the originating principle. But, moreover, he asserted that there is an eternal motion, by the agency of which it happens that the heavens [Or, 'men.'] are generated; but that the earth is poised aloft, upheld by nothing, continuing (so) on account of its equal distance from all (the heavenly bodies)".[2480]

As John Elof Boodin,[2481] Karl Popper and Dean Turner[2482] noted, "space-time", as a concept, as a quadri-dimensional statue, harkens back to the ancients, to Parmenides and the Eleatics,

> "For what is different from being does not exist, so that it necessarily follows, according to the argument of Parmenides, that all things that are are one and this is being."[2483]

Paul Carus had already noted in 1912, that:

> "Many who have watched the origin and rise of the new movement are startled at the paradoxical statements which some prominent physicists have made, and it is remarkable that the most materialistic sciences, mechanics and physics, seem to surround us with a mist of mysticism. The old self-contradictory statements of the Eleatic school revive in a modernized form, and common sense is baffled in its attempt to understand how the same thing may be longer and shorter at the same time, how a clock will strike the hour later or sooner according to the point of view from which it is watched; and the answer of this most recent conception of physics to the question, How is this all possible? is based on the principle of the relativity of time and space."[2484]

Popper wrote,

> "At the same time I realized that such myths may be developed, and become testable; that historically speaking all — or very nearly all — scientific theories originate from myths, and that a myth may contain important anticipations of scientific theories. Examples are Empedocles' theory of evolution by trial and error, or Parmenides' myth of the unchanging block universe in which nothing ever happens and which, if we add another

dimension, becomes Einstein's block universe (in which, too, nothing ever happens, since everything is, four-dimensionally speaking, determined and laid down from the beginning)."[2485]

When Minkowski, in 1908, uttered the infamous words,

"Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a union of the two will preserve an independent reality,"[2486]

his words were not only unoriginal, they were trite, and more archaic, than arcane.

Anton Reiser (Rudolf Kayser) proclaimed,

"The universe becomes a four-dimensional continuum in the time-space sense of Minkowski. Physical occurrences are now represented by three spatial coördinates as well as by one time coördinate, or in other words, there is no Becoming, only Being."[2487]

One is left to wonder how "the universe Becomes a four-dimensional continuum", if "there is no Becoming, only Being."

Hermann Weyl stated,

"The great advance in our knowledge described in this chapter consists in recognising that the scene of action of reality is not a three-dimensional Euclidean space but rather a **four-dimensional world, in which space and time are linked together indissolubly.** However deep the chasm may be that separates the intuitive nature of space from that of time in our experience, nothing of this qualitative difference enters into the objective world which physics endeavours to crystallise out of direct experience. It is a four-dimensional continuum, which is neither 'time' nor 'space'. Only the consciousness that passes on in one portion of this world experiences the detached piece which comes to meet it and passes behind it, as **history,** that is, as a process that is going forward in time and takes place in space."[2488]

and

"The objective world simply *is*, it does not *happen*. Only to the gaze of my consciousness, crawling upward along the lifeline of my body, does a section of the world come to life as a fleeting image in space which continuously changes in time."[2489]

Ebenezer Cunningham wrote,

"With Minkowski space and time become particular aspects of a single four-dimensional concept; the distinction between them as separate modes of correlating and ordering phenomena is lost, and the motion of a point in time is represented as a stationary curve in four-dimensional space. The whole history of a physical system is laid out as a changeless whole."[2490]

and,

> "**1.** The main objections urged against the Principle of Relativity are [***] (*iii*) that time and space are such immediate objects of perception that the artificial view which it adopts of them cannot in any sense correspond to reality.
>
> **2.** In respect of the last difficulty little can be said to meet the natural shrinking which the observer of natural phenomena feels from such a calculus as Minkowski's, in which we seem to lose sight of the most obvious distinction between time and space as essentially different modes of ordering events.
>
> It must be remarked, however, that an essential part in the practice of the calculus is the final process of interpreting the analytical result in terms of the ordinary modes of thought. There is perhaps an analogy to be drawn between the analysis which lays out the whole history of phenomena as a single whole, and the things in themselves, the natural phenomena apart from the human intelligence, for which consciousness of time and space does not exist, the laws of which, when expressed for instance by means of a principle of least action, consist in a relation between the whole aggregate of configurations which their history contains; in which, so far as they are mechanically determinate, the past and the future are interchangeable. Such a view of the universe is inseparable from a mechanical determinism in which the future is unalterably determined by the past and in which the past can be uniquely inferred from the present state of the universe. It is the view of an intelligence which could comprehend at one glance the whole of time and space.
>
> But the limitations of the human mind resolve this changeless whole into its temporal and spatial aspects, and the past and future of the physical world is the past and future of the intelligence perceiving it. Only to a being outside the physical universe, free from participation in its phenomena, is time a meaningless term. The human consciousness and the physical universe are inseparably parts of a greater whole. They run parallel to one another, and the brain cannot do otherwise than order physical and external events relative to the internal sequences of its own consciousness.
>
> It is by such a process of correlation that any analytical scheme of relations is constructed for the description of natural processes. When this has been carried out, it is claimed for it that it, at any rate approximately, contains within it the whole history of those processes for the mind to grasp as one whole. Thus the very act of formulating a set of equations which make the present state of the system to contain implicitly within it the whole history, past and to be, is one step, and that the largest, towards eliminating

> the peculiar characteristic of time as a product of the inner consciousness from its place in physical relations. It is but a small step further to the timeless universe of Minkowski.
>
> It is in fact the sole aim of theoretical physics to distinguish between and disentangle one from the other those factors in perceived events which are dependent upon human consciousness and those which are completely independent of it. The achievements of the past in this direction are quite sufficient to warrant further and continuous effort. That the mind should be able to conceive such a daring project and to progressively realize it, seems almost in itself sufficient to indicate that the resolution of its own workings into a chain of physically determinate processes is one incapable of complete realization."[2491]

Milič Čapek opposed this mystical "myth of the frozen passage."[2492]

It was a great injustice to attribute priority for this Eleatic stance to Minkowski. Charles Howard Hinton justified the classical principle of relativity in four-dimensions in 1880. It is irrational to assert that the principle of relativity compels invariant light speed, on the same grounds that it is irrational to assert that the principle of relativity requires that if I rest in inertial system *A*, I also rest in inertial system *B*, which is in motion relative to inertial system *A*.

In 1882, Gustav Teichmüller presented an Eleatic space-time theory—H. N. Gardiner explained in 1902,

> "The most precise elucidation, and perhaps the most original development of the subjectivistic doctrine of time since Kant, may probably be ascribed to Teichmüller (*Met*., 192 ff.). Teichmüller conceives time as entirely a perspective order given to objects by a timeless, substantial ego, and duration as a mere immanent measuring of that order. According to this, if we abstract from the perspective nature of consciousness and the comparison, through memory and expectation, of part of its ideal content with other parts, all chronological arrangement and temporal duration disappear. The bare concept of time, he says, has in it nothing of magnitude, just as the concept 'mammal' has in it nothing of the specific nature of tiger, sheep, and elephant. Further, the determination of magnitude in the realm of time is purely relative. Hence the duration of the world has no absolute magnitude, nor has any given time-interval, a day or a second. The objective time-order is a perspective view, like every other. It is the product of scientific thinking, based on comparison of individual consciousnesses and aided by language. It is the order of history, and this order is true, but also, like every other content of scientific truth, timeless. A real order of actual activities corresponds to the perspective order, but this is to be ultimately conceived as a technical system. As all determination of duration is relative, we cannot say that the future is separated by any time-interval actually given from the present or the past. Indeed, taken absolutely, the whole series of the world's phenomena must be regarded as being all together at once. But only an

absolute consciousness could so intuite it.

The standing objection to the doctrine thus or similarly expressed is that it denies the metaphysical reality of change. This objection is urged in various forms. It is said, for example, that if time is merely a form of intuition or a perspective ordering of phenomena, then the world is really a changeless unity, and consequently not only is all effort on our part to determine in any degree the course of things illusory, but past and future are contemporaneous—Nero is still burning Rome and the unborn babe now lives—which is absurd. Again, it is urged, positively, that change, and therefore time, which is the form of change, is real. For at least, it is argued, the succession of ideas is real, since it is only as ideas that phenomena can properly be said to exist at all. If, however, the succession of ideas is held to be phenomenal, the reply is that while this may be true if 'ideas' are taken as 'objects,' yet it is not true of the necessarily successive series of synthetic acts whereby their succession is presented. But not only, the argument continues, is change real in the subject, it is also real in external things; for the specific changes and the specific order of change appearing in objects, as they are certainly not due to a mere *a priori* form of the subject, imply a real succession in things themselves. Some writers appeal directly to the 'trans-subjective' nature of consciousness[1]. Much of this criticism, however loses its force when it is pointed out that the form of change, as such, is not time at all. Aristotle already distinguished between motion and time as number of motion. Time is a certain arrangement and measure of motion, a further determination of the content. It would be quite possible, therefore, to hold to any amount of real change and yet to regard the temporal view of such change as subjective. But the conception of a subject indifferently related to series of changes which it arranges in temporal order cannot, of course, be ultimate."[2493]

In the years 1884-1894, Rudolf Mewes worked on the laws of causality based on nature and matter in "space-time". Palágyi added the German nomenclature, and more precise mathematical formalism; and he also iterated the principle of relativity as a quadri-dimensional Eleatic ideal of a motionless, spaceless and timeless world, in 1901, stating, *inter alia*,

"However, it would also be, in reality, a spaceless conception of the world, since all points of this four-dimensional space would be given to us at the same time and it would not take up any length of time to grasp this four-dimensional world in all its parts. The four-dimensional conception of space would accordingly actually signify the complete removal of the spatiotemporalness of the world."

"Es wäre aber im Grunde genommen auch ein raumloses Auffassen der Welt, da alle Punkte dieses vierdimensionalen Raumes uns gleichzeitig gegeben wären und es keine Zeitdauer in Anspruch nehmen dürfte, diese vierdimensionale Welt in allen ihren Teilen zu überblicken. Die vierdimensionale Raumvorstellung würde sonach eigentlich die völlige Aufhebung der Raumzeitlichkeit der Welt bedeuten."[2494]

This belief system is truly archaic. Ueberweg, writing about the ancient Eleatics, penned these words before Einstein was born:

"§ 18. Xenophanes, of Colophon, in Asia Minor (born 569 B. C.), who removed later to Elea, in Lower Italy, combats in his poems the anthropomorphitic and anthropopathic representations of God presented by Homer and Hesiod, and enounces the doctrine of the one, all-controlling God-head. God is all eye, all ear, all intellect; untroubled, he moves and directs all things by the power of his thought. [***] That the God of Xenophanes is the unity of the world is a supposition that was early current. We do not find this doctrine expressed in the fragments which have come down to us, and it remains questionable whether Xenophanes pronounced himself positively in this sense, in speaking of the relation of God to the world, or whether such a conception was not rather thought to be implied in his teachings by other thinkers, who then expressed it in the phraseology given above. In the (Platonic?) dialogue, *Sophistes* (p. 242), the leading interlocutor, a visitor from Elea, says: 'The Eleatic race among us, from Xenophanes and even from still earlier times, assume in their philosophical discourses that what is usually called All, is One [***]. The 'still earlier' philosophers are probably certain Orphists, who glorified Zeus as the all-ruling power, as beginning, middle, and end of all things. Aristotle says, Metaph., I. 5: 'Xenophanes, the first who professed the doctrine of unity—Parmenides is called his disciple—has not expressed himself clearly concerning the nature of the One, so that it is not plain whether he has in mind an ideal unity (like Parmenides, his successor) or a material one (like Melissus); he seems not to have been at all conscious of this distinction, but, with his regard fixed on the whole universe, he says only that God is the One.' [***]

§ 19. Parmenides of Elea, born about 515—510 B. C. (so that his youth falls in the time of the old age of Xenophanes), is the most important of the Eleatic philosophers. He founds the doctrine of unity on the conception of being. He teaches: Only being is, non-being is not; there is no becoming. That which truly is exists in the form of a single and eternal sphere, whose space it fills continuously. Plurality and change are an empty semblance. The existent alone is thinkable, and only the thinkable is real. Of the one true existence, convincing knowledge is attainable by thought; but the deceptions of the senses seduce men into mere opinion and into the deceitful, rhetorical display of discourse respecting the things, which are supposed to be manifold and changing.—In his (hypothetical) explanation of the world of appearance, Parmenides sets out from two opposed principles, which bear to each other, within the sphere of appearance, a relation similar to that which exists between being and non-being. These principles are light and night, with which the antithesis of fire and earth corresponds. [***] Truth consists in the knowledge that being is, and non-being can not be; deception lies in the belief that non-being also is and must be. [***] The predicate being belongs to thought itself; that I think something and that this, which I think, *is* (in my thought), are identical assertions; non-being—that which is not—can not be thought, can, so to speak, not be reached, since every thing, when it is thought, exists as thought; no thought can be non-existent or without being, for there is nothing to which the predicate being does not belong, or which exists outside of the sphere of being.—In this argumentation Parmenides mistakes the distinction between the subjective being of thought and an objective realm of being to which thought is directed, by directing his attention only to the fact that both are subjects of the predicate being. [***] Not the senses, which picture to us plurality and change, conduct to truth, but only thought, which recognizes the being of that which is, as necessary, and the existence of that which is not, as impossible. [***] Much severer still than his condemnation of the naïve confidence of the mass of men in the illusory reports of the senses, is that with which Parmenides visits a philosophical doctrine which, as he assumes, makes of this very illusion (not, indeed, as illusion, in which sense Parmenides himself proposes a theory of the sensible, but as supposed truth) the basis of a theory that falsifies thought, in that it declares non-being identical with being. It is very probable that the Heraclitean doctrine is the one on which Parmenides thus animadverts, however indignantly Heraclitus might have resented this association of his doctrine with the prejudice of the masses, who do not rise above the false appearances of the senses; [***] Parmenides (in a passage of some length, given by Simpl., *Ad Phys*., fol. 31 a b) ascribes to the truly existent all the predicates which are implied in the abstract conception of *being*, and then proceeds further to characterize it as a continuous sphere, extending uniformly from the center in all directions—a description which we are scarcely authorized in interpreting as merely symbolical, in the conscious intention of Parmenides. That which truly is, is without origin and indestructible, a unique whole, only-begotten, immovable, and eternal; it was not and will not be, but *is*, and forms a continuum. [***] For what origin should it have? How could it grow? It can neither have arisen from the non-existent, since this has no existence, nor from the existent, since it is itself the existent. There is, therefore, no becoming, and no decay [***]. The truly existent is indivisible, everywhere like itself, and ever identical with itself. It exists independently, in and for itself [***], thinking, and comprehending in itself all thought; it exists in the form of a well-rounded sphere [***].The Parmenidean doctrine of the *apparent* world is a cosmogony, suggesting, on the one hand, Anaximander's doctrine of the warm and the cold as the first-developed contraries and the Heraclitean doctrine of the transformations of fire, and, on the other, the Pythagorean opposition of 'limit' and 'the unlimited' [***], and the Pythagorean doctrine of contraries generally. It is founded on the hypothesis of a universal mixture of warm and cold, light and dark. The warm and light is ethereal fire, which, as the positive and efficient principle, represents within the sphere of appearance the place of being; the cold and dark is air and its product, by condensation [***], earth. The combining or 'mixing' of the contraries is effected by the all-controlling Deity [***] at whose will Eros came into existence as first, in time, of the gods [***]. That which fills space and that which thinks, are the same; how a man shall think, depends on the 'mixture' of his bodily organs; a dead body perceives cold and silence [***]. If the verse in the long fragment,[***], could be amended (as is done by Gladisch, who seeks in it an analogue to the Maja of the Hindus) so as to read: [***], Parmenides would appear as having explained the plurality and change attested by the senses, as a dream of the one true existence. But this conjecture is arbitrary; and the words cited in the *Soph*., p. 242: [***], as also the doctrine of the Megarians concerning the many names of the One, which alone really exists, confirm the reading [***] of the MSS. The sense of the passage is therefore: 'All the manifold and changing world, which mortals suppose to be real, and which they call the sum of things, *is* in reality only the One, which alone truly is.' In the philosophy of Parmenides no distinction is reached between appearance, or semblance, and phenomenon. The terms being and appearance remain with him philosophically unreconciled; the existence of a realm of mere appearance is incompatible with the fundamental principle of Parmenides.

§ 20. Zeno of Elea (born about 490—485 B. C.) defended the doctrine of Parmenides by an indirect demonstration, in which he sought to show that the supposition of the real existence of things manifold and changing, leads to contradictions. In particular, he opposed to the reality of motion four arguments: 1. Motion can not begin, because a body in motion can not arrive at another place until it has passed through an unlimited number of intermediate places. 2. Achilles can not overtake the tortoise, because as often as he reaches the place occupied by the tortoise at a previous moment, the latter has already left it. 3. The flying arrow is at rest; for it is at every moment only in one place. 4. The half of a division of time is equal to the whole; for the same point, moving with the same velocity, traverses an equal distance (*i.e*., when compared, in the one case, with a point at rest, in the other, with a point in motion) in the one case, in half of a given time, in the other, in the whole of that time. [***] In the (Platonic?) dialogue *Parmenides*, a prose writing [***] of Zeno is mentioned, which was distributed into several series of argumentations [***], in each of which a number of hypotheses [***] were laid down with a view to their *reductio in absurdum*, and so to the indirect demonstration of the truth of the doctrine that Being is One. It is probably on account of this (indirect) method of demonstration from hypotheses, that Aristotle [***] called Zeno the inventor of dialectic [***]. If the manifold exists, argues Zeno [***], it must be at the same time infinitely small and infinitely great; the former, because its last divisions are without magnitude, the latter, on account of the infinite number of these divisions. (In this argument Zeno leaves out of consideration the inverse ratio constantly maintained between magnitude and number of parts, as the division advances, whereby the same product is constantly maintained, and he isolates the notions of smallness and number, opposing the one to the other.) In a similar manner Zeno shows that the manifold, if it exists must be at the same time numerically limited and unlimited. Zeno argues, further [***], against the reality of space. If all that exists were in a given space, this space must be in another space, and so on *in infinitum*. Against the veracity of sensuous perception, Zeno directed [***] the following argument: If a measure of millet-grains in falling produce a sound, each single grain and each smallest fraction of a grain must also produce a sound ; but if the latter is not the case, then the whole measure of grains, whose effect is but the sum of the effects of its parts, can also produce no sound. (The method of argumentation here employed is similar to that in the first argument against plurality.) The arguments of Zeno against the reality of motion [***] have had no insignificant influence on the development of metaphysics in earlier and later times. Aristotle answers the two first [***] with the observation [***] that the divisions of time and space are the same and equal [***] for both time and space are continuous [***]; that a distance divisible *in infinitum* can therefore certainly be traversed in a finite time, since the latter is also in like manner divisible *in infinitum*, and the divisions of time correspond with the divisions of space; the infinite in division [***] is to be distinguished from the infinite in extent [***]; his reply to the third argument [***] is, that time does not consist of single indivisible points (conceived as discontinuous) or of ‘nows’ [***]. In the fourth argument he points out what Zeno, as it seems, had but poorly concealed, viz., the change of the standard of comparison [***]. It can be questioned whether the Aristotelian answers are fully satisfactory for the first three arguments (for in the fourth the paralogism is obvious). Bayle has attacked [***]. Hegel [***] defends Aristotle against Bayle. Yet Hegel himself also sees in motion a contradiction; nevertheless, he regards motion as a real fact. Herbart denies the reality of motion on account of the contradiction which, in his opinion, it involves. [***]

§ 21. Melissus of Samos attempts by a direct demonstration to establish the truth of the fundamental thought of the Eleatic philosophy, that only the One is. By unity, however, he understands rather the continuity of substance than the notional identity of being. That which is, the truly existent, is eternal, infinite, one, in all points the same or ‘like itself,’ unmoved and passionless. [***] If nothing were, argues Melissus, how were it then even possible to speak of it, as of something being? But if any thing is, then it has either become or is eternal. In the former case, it must have arisen either from being or from non-being. But nothing can come from non-being; and being can not have arisen from being, for then there must have been being, before being came to be (became). Hence being did not become; hence it is eternal. It will also not perish; for being can not become non-being, and if being change to being, it has not perished. Therefore it always was and always will be. As without genesis, and indestructible, being has no beginning and no end; it is, therefore, infinite. (It is easy to perceive here the leap in argumentation from temporal infinity to the infinity of space, which very likely contributed essentially to draw on Melissus Aristotle's reproach of feebleness of thought.) As infinite, being is One; for if it were dual or plural, its members would mutually limit each other, and so would not be infinite. As one, being is unchangeable; for change would pluralize it. More particularly, it is unmoved; for there exists no empty space in which it can move, since such a space, if it existed, would be an existing nothing; and being can not move within itself for then the One would become a *divisum*, hence manifold. Notwithstanding the infinite extension which Melissus attributes to being, he will not have it called material, since whatever is material has parts, and so can not be a unity."[2495]

Ocellus Lucanus also had a space-time theory thousands of years before Einstein:

"OCELLUS LUCANUS

ON THE UNIVERSE.

CHAP. I.

OCELLUS LUCANUS has written what follows concerning the Nature of the Universe; having learnt some things through clear arguments from Nature herself, *but others from opinion*, *in conjunction with reason* [*Footnote:* See Additional Notes, (A.)], it being his intention [in this work] to derive what is probable from intellectual perception.

It appears, therefore, to me, that the Universe is indestructible and unbegotten, since it always was, and always will be; for if it had a temporal beginning, it would not have always existed: thus, therefore, the universe is unbegotten and indestructible; for if some one should opine that it was once generated, he would not be able to find anything into which it can be corrupted and dissolved, since that from which it was generated would be the first part of the universe; and again, that into which it would be dissolved would be the last part of it.

But if the universe was generated, it was generated together with all things; and if it should be corrupted, it would be corrupted together with all things. This, however, is impossible [*Footnote:* The universe could not be generated together with all things, for the principle of it must be unbegotten; since everything that is generated, is generated from a cause; and if this cause was also generated, there must be a progression of causes ad infinitum, unless the unbegotten is admitted to be the principle of the universe. Neither, therefore, can the universe be corrupted together with all things; for the principle of it being unbegotten is also incorruptible; that only being corruptible, which was once generated.]. The universe, therefore, is without a beginning, and without an end; nor is it possible that it can have any other mode of subsistence.

To which may be added, that everything which has received a beginning of generation, and which ought also to participate of dissolution, receives two mutations; one of which, indeed, proceeds from the less to the greater, and from the worse to the better; and that from which it begins to change is denominated generation, but that at which it at length arrives, is called acme. The other mutation, however, proceeds from the greater to the less, and from the better to the worse: but the termination of this mutation is denominated corruption and dissolution.

If, therefore, the whole and the universe were generated, and are corruptible, they must, when generated, have been changed from the less to the greater, and from the worse to the better; but when corrupted, they must be changed from the greater to the less, and from the better to the worse. Hence, if the world was generated, it would receive increase, and would arrive at its acme; and again, it would afterwards receive decrease and an end. For every nature which has a progression, possesses three boundaries and two intervals. The three boundaries, therefore, are generation, acme, and end; but the intervals are, the progression from generation to acme, and from acme to the end.

The whole, however, and the universe, affords, as from itself, no indication of a thing of this kind; for neither do we perceive it rising into existence, or becoming to be, nor changing to the better and the greater, nor becoming at a certain time worse or less; but it always continues to subsist in the same and a similar manner, and is itself perpetually equal and similar to itself.

Of the truth of this, the orders of things, their symmetry, figurations, positions, intervals, powers, swiftness and slowness with respect to each other; and, besides these, their numbers and temporal periods, are clear signs and indications. For all such things as these receive mutation and diminution, conformably to the course of a generated nature: for things that are greater and better acquire acme through power, but those that are less and worse are corrupted through imbecility of nature.

I denominate, however, the whole and the universe, the whole world; for, in consequence of being adorned with all things, it has obtained this appellation; since it is from itself a consummate and perfect system of the nature of all things; for there is nothing external to the universe, since whatever exists is contained in the universe, and the universe subsists together with this, comprehending in itself all things, some as parts, but others as supervenient.

Those things, therefore, which are comprehended in the world, have a congruity with the world; but the world has no concinnity with anything else, but is itself co-harmonized with itself. For all other things have not a consummate or self-perfect subsistence, but require congruity with things external to themselves. Thus animals require a conjunction with air for the purpose of respiration, but sight with light, in order to see; and the other senses with something else, in order to perceive their peculiar sensible object. A conjunction with the earth also is necessary to the germination of plants. The sun and moon, the planets, and the fixed stars, have likewise a coalescence with the world, as being parts of its common arrangement. The world, however, has not a conjunction with anything else than itself.

Further still [*Footnote:* Critolaus, the Peripatetic, employs nearly the same arguments as those contained in this paragraph, in proof of the perpetuity of the world, as is evident from the following passage, preserved by Philo, in his Treatise Π ι Αφθα σιας Κοσμου, "On the Incorruptibility of the World": το αιτιον αυτ του υγιαιν ιν, ανοσον στι αλλα αι το αιτιον αυτ του αγ υπν ιν, αγ υπνον στιν. ι δ τουτο, αι το αιτιον αυτ του υπα χ ιν, αϊδιον στιν. αιτιος δ ο οσμος αυτ του υπα χ ιν, ιγ αι τοις αλλοις απασιν. αϊδιος ο οσμος στιν. i. e. "That which is the cause to itself of good health, is without disease. But, also, that which is the cause to itself of a vigilant energy, is sleepless. But if this be the case, that also which is the cause to itself of existence, is perpetual. The world, however, is the cause to itself of existence, since it is the cause of existence to all other things. The world, therefore, is perpetual." Everything divine, according to the philosophy of Pythagoras and Plato, being a self-perfect essence, begins its own energy from itself, and is therefore primarily the cause to itself of that which it imparts to others. Hence, since the world, being a divine and self-subsistent essence, imparts to itself existence, it must be without non-existence, and therefore must be perpetual.], what has been said will be easily known to be true from the following considerations. Fire, which imparts heat to another thing, is itself from itself hot; and honey, which is sweet to the taste, is itself from itself sweet. The principles likewise of demonstrations, which are indicative of things unapparent, are themselves from themselves manifest and known. Thus, also, that which becomes to other things the cause of self-perfection, is itself from itself perfect; and that which becomes to other things the cause of preservation and permanency, is itself from itself preserved and permanent. That, likewise, which becomes to other things the cause of concinnity, is itself from itself co-harmonized; but the world is to other things the cause of their existence, preservation, and self-perfection. The world, therefore, is from itself perpetual and self-perfect, has an everlasting duration, and on this very account becomes the cause of the permanency of the whole of things.

In short, if the universe should be dissolved, it would either be dissolved into that which has an existence, or into nonentity. But it is impossible that it should be dissolved into that which exists, for there will not be a corruption of the universe if it should be dissolved into that which has a being; for being is either the universe, or a certain part of the universe. Nor can it be dissolved into nonentity, since it is impossible for being either to be produced from non-beings, or to be dissolved into nonentity. The universe, therefore, is incorruptible, and can never be destroyed.

If, nevertheless, some one should think that it may be corrupted, it must either be corrupted from something external to, or contained in the universe, but it cannot be corrupted by anything external to it; for there is not anything external to the universe, since all other things are comprehended in the universe, and the world is *the whole* and *the all*. Nor can it be corrupted by the things which it contains, for in this case it will be requisite that these should be greater and more powerful than the universe. This, however, is not true [*Footnote:* i. e. It is not true that the universe can contain anything greater and more powerful than itself.], for all things are led and governed by the universe, and conformably to this are preserved and co-adapted, and possess life and soul. But if the universe can neither be corrupted by anything external to it, nor by anything contained within it, the world must therefore be incorruptible and indestructible; for we consider the world to be the same with the universe [*Footnote:* Philo Judæus, in his before-mentioned Treatise Π ι Αφθα σιας Κοσμου, has adopted the arguments of Ocellus in this paragraph, but not with the conciseness of his original.].

Further still, the whole of nature surveyed through the whole of itself, will be found to derive continuity from the first and most honourable of bodies, attenuating this continuity proportionally, introducing it to everything mortal, and receiving the progression of its peculiar subsistence; for the first [and most honourable] bodies in the universe, revolve according to the same, and after a similar manner. The progression, however, of the whole of nature, is not successive and continued, nor yet local, but subsists according to mutation.

Fire, indeed, when it is congregated into one thing, generates air, but air generates water, and water earth. From earth, also, there is the same circuit of mutation, as far as to fire, from whence it began to be changed. But fruits, and most plants that derive their origin from a root, receive the beginning of their generation from seeds. When, however, they bear fruit and arrive at maturity, again they are resolved into seed, nature producing a complete circulation from the same to the same.

But men and other animals, in a subordinate degree, change the universal boundary of nature; for in these there is no periodical return to the first age, nor is there an antiperistasis of mutation into each other, as there is in fire and air, water and earth; but the mutations of their ages being accomplished in a four-fold circle [*Footnote:* This four-fold mutation of ages in the human race, consists of the infant, the lad, the man, and the old man, as is well observed by Theo of Smyrna. See my Theoretic Arithmetic, p. 189.], they are dissolved, and again return to existence; these, therefore, are the signs and indications that the universe, which comprehends [all things], will always endure and be preserved, but that its parts, and such things in it as are supervenient, are corrupted and dissolved.

Further still, it is credible that the universe is without a beginning, and without an end, from its figure, from motion, from time, and its essence; and, therefore, it may be concluded that the world is unbegotten and incorruptible: for the form of its figure is circular; but a circle is on all sides similar and equal, and is therefore without a beginning, and without an end. The motion also of the universe is circular, but this motion is stable and without transition. Time, likewise, in which motion exists is infinite, for this neither had a beginning, nor will have an end of its circulation. The essence, too, of the universe, is without egression [into any other place], and is immutable, because it is not naturally adapted to be changed, either from the worse to the better, or from the better to the worse. From all these arguments, therefore, it is obviously credible, that the world is unbegotten and incorruptible. And thus much concerning the whole and the universe.

CHAP. II.

SINCE, however, in the universe, one thing is generation, but another the cause of generation; and generation indeed takes place where there is a mutation and an egression from things which rank as subjects; but the cause of generation then subsists where the subject matter remains the same: this being the case, it is evident that the cause of generation possesses both an effective and motive power, but that the recipient of generation is adapted to passivity, and to be moved.

But the Fates themselves distinguish and separate the impassive part of the world from that which is perpetually moved [or mutuable] [*Footnote:* In the original, το τ απαθ ς μ ος του οσμου αι το α ινητον, which is obviously erroneous. Nogarola, in his note on this passage, says, “Melius arbitror si legatur το τ α ιπαθ ς μ ος, αι α ι ινητον, ut sit sensus, semper patibilem, et semper mobilem partem distinguunt ac separant.” But though he is right in reading α ι ινητον for α ινητον, he is wrong in substituting α ιπαθ ς for απαθ ς; for Ocellus is here speaking of the distinction between the celestial and sublunary region, the former of which is *impassive*, because not subject to generation and corruption, but the latter being subject to both these is *perpetually mutable*.]. For the course of the moon is the isthmus of immortality and generation. The region, indeed, above the moon, and also that which the moon occupies, contain the genus of the gods; but the place beneath the moon is the abode of strife and nature; for in this place there is a mutation of things that are generated, and a regeneration of things which have perished.

In that part of the world, however, in which nature and generation predominate, it is necessary that the three following things [*Footnote:* Aristotle, in his treatise on Generation and Corruption, has borrowed what Ocellus here says about the three things necessary to generation. See my translation of that work.] should be present. In the first place, the body which yields to the touch, and which is the subject of all generated natures. But this will be an universal recipient, and a signature of generation itself having the same *relation* to the things that are generated from it, as water to taste, *silence to sound* [*Footnote:* In the original, αι ψοφος π ος σιγην, instead of which it is necessary to read αι σιγη π ος ψοφον, conformably to the above translation. See the Notes to my translation of the First Book of Aristotle’s Physics, p. 73, &c., in which the reader will find a treasury of information from Simplicius concerning matter. But as matter is devoid of all quality, and is a privation of all form, the necessity of the above emendation is immediately obvious.], darkness to light, and the matter of artificial forms to the forms themselves. For water is tasteless and devoid of quality, yet is capable of receiving the sweet and the bitter, the sharp and the salt. Air, also, which is formless with respect to sound, is the recipient of words and melody. And darkness, which is without colour, and without form, becomes the recipient of splendour, and of the yellow colour and the white; but whiteness pertains to the statuary’s art, and to the art which fashions figures from wax. Matter, however, has a relation in a different manner to the statuary’s art; for in matter all things prior to generation are in capacity, but they exist in perfection when they are generated and receive their proper nature. Hence matter [or a universal recipient] is necessary to the existence of generation.

The second thing which is necessary, is the existence of contrarieties, in order that mutations and changes in quality may be effected, matter for this purpose receiving passive qualities, and an aptitude to the participation of forms. Contrariety is also necessary, in order that powers, which are naturally mutually repugnant, may not finally vanquish, or be vanquished by, each other. But these powers are the hot and the cold, the dry and the moist.

Essences rank in the third place; and these are fire and water, air and earth, of which the hot and the cold, the dry and the moist, are powers. But essences differ from powers; for essences are locally corrupted by each other, but powers are neither corrupted nor generated, for the reasons [or forms] of them are incorporeal.

Of these four powers, however, the hot and the cold subsist as causes and things of an effective nature, but the dry and the moist rank as matter and things that are passive [*Footnote:* Thus also Aristotle, in his Treatise on Generation and Corruption, μον δ αι ψυχ ον, αι γ ον, τα μ ν τ ποιητι α ιναι, τα δ τ παθητι α λ γ ται. i. e. “With respect to heat and cold, dryness and moisture, the two former of these are said to be effective, but the two latter passive powers.”]; but matter is the first recipient of all things, for it is that which is in common spread under all things. Hence, the body, which is the object of sense in capacity, and ranks as a principle, is the first thing; but contrarieties, such as heat and cold, moisture and dryness, form the second thing; and fire and water, earth and air, have an arrangement in the third place. For these change into each other; but things of a contrary nature are without change.

But the differences of bodies are two: for some of them indeed are primary, but others originate from these: for the hot and the cold, the moist and the dry, rank as primary differences; but the heavy and the light, the dense and the rare, have the relation of things which are produced from the primary differences. All of them, however, are in number sixteen, viz, the hot and the cold, the moist and the dry, the heavy and the light, the rare and the dense, the smooth and the rough, the hard and the soft, the thin and the thick, the acute and the obtuse. But of all these, the touch has a knowledge, and forms a judgement; hence, also, the first body in which these differences exist in capacity, may be sensibly apprehended by the touch.

The hot and the dry, therefore, the rare and the sharp, are the powers of fire; but those of water are, the cold and the moist, the dense and the obtuse; those of air are, the soft, the smooth, the light, and the attenuated; and those of earth are, the hard and the rough, the heavy and the thick.

Of these four bodies, however, fire and earth are the transcendencies and summits [or extremities] of contraries. Fire, therefore, is the transcendency of heat, in the same manner as ice is of cold: hence, if ice is a concretion of moisture and frigidity, fire will be the fervour of dryness and heat. On which account, nothing is generated from ice, nor from fire [*Footnote:* The substance of nearly the whole of what Ocellus here says, and also of the two following paragraphs, is given by Aristotle, in his Treatise on Generation and Corruption.].

Fire and earth, therefore, are the extremities of the elements, but water and air are the media, for they have a mixed corporeal nature. Nor is it possible that there could be only one of the extremes, but it is necessary that there should be a contrary to it. Nor could there be two only, for it is necessary that there should be a medium, since media are opposite to the extremes.

Fire, therefore, is hot and dry, but air is hot and moist; water is moist and cold, but earth is cold and dry. Hence, heat is common to air and fire; cold is common to water and earth; dryness to earth and fire; and moisture to water and air. But with respect to the peculiarities of each, heat is the peculiarity of fire, dryness of earth, moisture of air, and frigidity of water. The essences, therefore, of these remain permanent, through the possession of common properties; but they change through such as are peculiar, when one contrary vanquishes another.

Hence, when the moisture in air vanquishes the dryness in fire, but the frigidity in water, the heat in air, and the dryness in earth, the moisture in water, and vice versa, when the moisture in water vanquishes the dryness in earth, the heat in air, the coldness in water, and the dryness in fire, the moisture in air, then the mutations and generations of the elements from each other into each other are effected.

The body, however, which is the subject and recipient of mutations, is a universal receptacle, and is in capacity the first tangible substance.

But the mutations of the elements are effected, either from a change of earth into fire, or from fire into air, or from air into water, or from water into earth. Mutation is also effected in the third place, when that which is contrary in each element is corrupted, but that which is of a kindred nature, and connascent, is preserved. Generation, therefore, is effected, when one contrariety is corrupted. For fire, indeed, is hot and dry, but air is hot and moist, and heat is common to both; but the peculiarity of fire is dryness, and of air moisture. Hence, when the moisture in air vanquishes the dryness in fire, then fire is changed into air.

Again, since water is moist and cold, but air is moist and hot, moisture is common to both. The peculiarity however of water is coldness, but of air heat. When, therefore, the coldness in water vanquishes the heat in air, the mutation from air into water is effected.

Further still, earth is cold and dry, but water is cold and moist, and coldness is common to both; but the peculiarity of earth is dryness, and of water moisture. When, therefore, the dryness in earth vanquishes the moisture in water, a mutation takes place from water into earth.

The mutation, however, from earth, in an ascending progression, is performed in a contrary way; but an alternate mutation is effected when one whole vanquishes another, and two contrary powers are corrupted, nothing at the same time being common to them. For since fire is hot and dry, but water is cold and moist; when the moisture in water vanquishes the dryness in fire, and the coldness in water the heat in fire, then a mutation is effected from fire into water.

Again, earth is cold and dry, but air is hot and moist. When, therefore, the coldness in earth vanquishes the heat in air, and the dryness in earth, the moisture in air, then a mutation from air into earth is effected.

But when the moisture of air corrupts the heat of fire, from both of them fire will be generated; for the heat of air and the dryness of fire will still remain. And fire is hot and dry.

When, however, the coldness of earth is corrupted, and the moisture of water, from both of them earth will be generated. For the dryness of earth, indeed, will be left, and the coldness of water. And earth is cold and dry.

But when the heat of air, and the heat of fire are corrupted, no element will be generated; for the contraries in both these will remain, viz, the moisture of air and the dryness of fire. Moisture, however, is contrary to dryness.

And again, when the coldness of earth, and in a similar manner of water, are corrupted, neither thus will there be any generation; for the dryness of earth and the moisture of water will remain. But dryness is contrary to moisture. And thus, we have briefly discussed the generation of the first bodies, and have shown how and from what subjects it is effected.

Since, however, the world is indestructible and unbegotten, and neither received a beginning of generation, nor will ever have an end, it is necessary that the nature which produces generation in another thing, and also that which generates in itself, should be present with each other. And that, indeed, which produces generation in another thing, is the whole of the region above the moon; but the more proximate cause is the sun, who, by his accessions and recessions, continually changes the air, so as to cause it to be at one time cold, and at another hot; the consequence of which is, that the earth is changed, and everything which the earth contains.

The obliquity of the zodiac, also, is well posited with respect to the motion of the sun, for it likewise is the cause of generation. And universally this is accomplished by the proper order of the universe; so that one thing in it is that which makes, but another that which is passive. Hence, that which generates in another thing, exists above the moon; but that which generates in itself, has a subsistence beneath the moon; and that which consists of both these, viz, of an ever-running divine body, and of an ever-mutable generated nature, is the world.

CHAP. III.

THE origin, however, of the generation of man was not derived from the earth, nor that of other animals, nor of plants; but the proper order of the world being perpetual, it is also necessary that the natures which exist in it, and are aptly arranged, should, together with it, have a never-failing subsistence. For the world primarily always existing, it is necessary that its parts should be co-existent with it: but I mean by its parts, the heavens, the earth, and that which subsists between these; which is placed on high, and is denominated aerial; for the world does not exist without, but together with, and from these.

The parts of the world, however, being consubsistent, it is also necessary that the natures, comprehended in these parts, should be co-existent with them; with the heavens, indeed, the sun and moon, the fixed stars, and the planets; but with the earth, animals and plants, gold and silver; with the place on high, and the aerial region, pneumatic substances and wind, a mutation to that which is more hot, and a mutation to that which is more cold; for it is the property of the heavens to subsist in conjunction with the natures which it comprehends; of the earth to support the plants and animals which originate from it; and of the place on high, and the aerial region, to be consubsistent with all the natures that are generated in it.

Since, therefore, in each division of the world, a certain genus of animals is arranged, which surpasses the rest contained in that division; in the heavens, indeed, the genus of the gods, but in the earth men, and in the region on high demons;— this being the case, it is necessary that the race of men should be perpetual, since reason truly induces us to believe, that not only the [great] parts of the world are consubsistent with the world, but also the natures comprehended in these parts.

Violent corruptions, however, and mutations, take place in the parts of the earth; at one time, indeed, the sea overflowing into another part of the earth; but at another, the earth itself becoming dilated and divulsed, through wind or water latently entering into it. But an entire corruption of the arrangement of the whole earth never did happen, nor ever will.

Hence the assertion, that the Grecian history derived its beginning from the Argive Inachus, must not be admitted as if it commenced from a certain first principle, but that it originated from some mutation which happened in Greece; for Greece has frequently been, and will again be, barbarous, not only from the migration of foreigners into it, but from nature herself, which, though she does not become greater or less, yet is always younger, and with reference to us, receives a beginning.

And thus much has been sufficiently said by me respecting *the whole* and *the universe*; and further still, concerning the generation and corruption of the natures which are generated in it, and the manner in which they subsist, and will for ever subsist; one part of the universe consisting of a nature which is perpetually moved, but another part of a nature which is always passive; and the former of these always governing, but the latter being always governed.

CHAP. IV.

CONCERNING the generation of men, however, from each other, after what manner, and from what particulars, it may be most properly effected, law, and temperance and piety at the same time cooperating, will be, I think, as follows. In the first place, indeed, this must be admitted,—that we should not be connected with women for the sake of pleasure, but for the sake of begetting children.

For those powers and instruments, and appetites, which are subservient to copulation, were imparted to men by Divinity, not for the sake of voluptuousness, but for the sake of the perpetual duration of the human race. For since it was impossible that man, who is born mortal, should participate of a divine life, if the immortality of his genus was corrupted; Divinity gave completion to this immortality through individuals, and made this generation of mankind to be unceasing and continued. This, therefore, is one of the first things which it is necessary to survey,—that copulation should not be undertaken for the sake of voluptuous delight.

In the next place, the co-ordination itself of man should be considered with reference to the whole, viz, that he is a part of a house and a city, and (which is the greatest thing of all) that each of the progeny of the human species ought to give completion to the world [*Footnote* In the original, π ιτα δ αι την αυτην τ ανθ ωπ συνταξιν π ος το λον, τι μ ος πα χων οι ου τ αι πολ ως, αι το μ γιστον οσμου, συμπλη ουν οφ ιλ ι το απογ νομ νον τουτων αστον, . τ. λ. Here, for αι το μ γιστον οσμου, συμπλη ουν, . τ. λ., it is requisite to read, conformably to the above translation, αι το μ γιστον, οσμου συμπλη ουν, . τ. λ. Nogarola, in his version, from not perceiving the necessity of this emendation, has made Ocellus say that man is the greatest part of the universe; for his translation is as follows: “Mox eandem hominis constitutionem ad universam referendam, quippe qui non solum domûs et civitatis, verum etiam mundi maxima habetur pars," &c.], if it does not intend to be a deserter either of the domestic, or political, or divine Vestal hearth.

For those who are not entirely connected with each other for the sake of begetting children, injure the most honourable system of convention. But if persons of this description procreate with libidinous insolence and intemperance, their offspring will be miserable and flagitious, and will be execrated by gods and demons, and by men, and families, and cities.

Those, therefore, who deliberately consider these things, ought not, in a way similar to irrational animals, to engage in venereal connections, but should think copulation to be a necessary good. For it is the opinion of worthy men, that it is necessary and beautiful, not only to fill houses with large families, and also the greater part of the earth [*Footnote:* This observation applies only to well regulated cities, but in London and other large cities, where the population is not restricted to a definite number, this abundant propagation of the species is, to the greater part of the community, attended with extreme misery and want. Plato and Aristotle, who rank among the wisest men that ever lived, were decidedly of opinion, that the population of a city should be limited. Hence, the former of these philosophers says, "that in a city where the inhabitants do not know each other, there is no light, but profound darkness;" and the latter, "that as 10,000 inhabitants are too few for a city, so 100,000 are too many."], (for man is the most mild and the best of all animals,) but, as a thing of the greatest consequence, to cause them to abound with the most excellent men.

For on this account men inhabit cities governed by the best laws, rightly manage their domestic affairs, and [if they are able] impart to their friends such political employments as are conformable to the polities in which they live, since they not only provide for the multitude at large, but [especially] for worthy men.

Hence, many err, who enter into the connubial state without regarding the magnitude of [the power of] fortune, or public utility, but direct their attention to wealth, or dignity of birth. For in consequence of this, instead of uniting with females who are young and in the flower of their age, they become connected with extremely old women; and instead of having wives with a disposition according with, and most similar to their own, they marry those who are of an illustrious family, or are extremely rich. On this account, they procure for themselves discord instead of concord; and instead of unanimity, dissention; contending with each other for the mastery. For the wife who surpasses her husband in wealth, in birth, and in friends, is desirous of ruling over him, contrary to the law of nature. But the husband justly resisting this desire of superiority in his wife, and wishing not to be the second, but the first in domestic sway, is unable, in the management of his family, to take the lead.

This being the case, it happens that not only families, but cities, become miserable. For families are parts of cities, but the composition of the whole and the universe derives its subsistence from parts [*Footnote:* For *whole*, according to the philosophy of Pythagoras and Plato, has a triple subsistence; since it is either prior to parts, or consists of parts, or exists in each of the parts of a thing. But a *whole*, prior to parts, contains in itself parts causally. The universe is a whole of wholes, the wholes which it comprehends in itself (viz, the inerratic sphere, and the spheres of the planets and elements) being its parts. And in the whole which is in each part of a thing, every part according to participation becomes a whole, i. e. a partial whole.]. It is reasonable, therefore, to admit, that such as are the parts, such likewise will be the whole and the all which consists of things of this kind.

And as in fabrics of a primary nature the first structures co-operate greatly to the good or bad completion of the whole work; as, for instance, the manner in which the foundation is laid in building a house, the structure of the keel in building a ship, and in musical modulation the extension and remission of the voice; so the concordant condition of families greatly contributes to the well or ill establishment of a polity.

Those, therefore, who direct their attention to the propagation of the human species, ought to guard against everything which is dissimilar and imperfect; for neither plants nor animals, when imperfect, are prolific, but to their fructification a certain portion of time is necessary, in order that when the bodies are strong and perfect, they may produce seeds and fruits.

Hence, it is necessary that boys, and girls also while they are virgins, should be trained up in exercises and proper endurance, and that they should be nourished with that kind of food, which is adapted to a laborious, temperate, and patient life.

Moreover, there are many things in human life of such a kind, that it is better for the knowledge of them to be deferred for a certain time. Hence, it is requisite that a boy should be so tutored, as not to seek after venereal pleasures before he is twenty years of age, and then should rarely engage in them. This, however, will take place, if he conceives that a good habit of body, and continence, are beautiful and honourable.

It is likewise requisite that such legal institutes as the following should be taught in Grecian cities, viz. that connection with a mother, or a daughter, or a sister, should not be permitted either in temples, or in a public place; for it is beautiful and advantageous that numerous impediments to this energy should be employed.

And universally, it is requisite that all preternatural generations should be prevented, and those which are attended with wanton insolence. But such as are conformable to nature should be admitted, and which are effected with temperance, for the purpose of producing a temperate and legitimate offspring.

Again, it is necessary that those who intend to beget children, should providentially attend to the welfare of their future offspring. A temperate and salutary diet, therefore, is the first and greatest thing which should be attended to by him who wishes to beget children; so that he should neither be filled with unseasonable food, nor become intoxicated, nor subject himself to any other perturbation, from which the habits of the body may become worse. But, above all things, it is requisite to be careful that the mind, in the act of copulation, should be in a tranquil state: for, from depraved, discordant, and turbulent habits, bad seed is produced.

It is requisite, therefore, to endeavour, with all possible earnestness and attention, that children may be born elegant and graceful, and that when born, they should be well educated. For neither is it just that those who rear horses, or birds, or dogs, should, with the utmost diligence, endeavour that the breed may be such as is proper, and from such things as are proper, and when it is proper [*Footnote:* In the original, ς δ ι, αι ξ ν δ ι, αι τ δ ι, a mode of diction which frequently occurs in Aristotle, and from him in Platonic writers.]; and likewise consider how they ought to be disposed when they copulate with each other, in order that the offspring may not be a casual production; —but that men should pay no attention to their progeny, but should beget them casually; and when begotten, should neglect both their nutriment and their education: for these being disregarded, the causes of all vice and depravity are produced, since those that are thus born will resemble cattle, and will be ignoble and vile.

OCELLUS LUCANUS ON LAWS.

A FRAGMENT PRESERVED BY STOBÆUS, ECLOG. PHYS.
LIB. 1. CAP. 16.

LIFE, connectedly—contains in itself bodies; but of this, soul is the cause. Harmony comprehends, connectedly, the world; but of this, God is the cause. Concord binds together families and cities; and of this, law is the cause. Hence, there is a certain cause and nature which perpetually adapts the parts of the world to each other, and never suffers them to be disorderly and without connection. Cities, however, and families, continue only for a short time; the progeny of which, and the mortal nature of the matter of which they consist, contain in themselves the cause of dissolution; for they derive their subsistence from a mutable and perpetually passive nature. For the destruction

[*Footnote:* In the original, απογ ν σις; but the true reading is doubtless απωλ ια, and Vizzanus has in his version *interitus*. What is here said by Ocellus is in perfect conformity with the following beautiful lines of our admirable philosophic poet, Pope, in his Essay on Man:

“All forms that perish other forms supply;
By turns they catch the vital breath and die;
Like bubbles on the sea of matter born,

They rise, they break, and to that sea return."]

of things which are generated, is the salvation of the matter from which they are generated. That nature, however, which is perpetually moved [*Footnote:* i. e. The celestial region.] governs, but that which is always passive [*Footnote:* i. e. The sublunary region.] is governed; and the one is in capacity prior, but the other posterior. The one also is divine, and possesses reason and intellect, but the other is generated, and is irrational and mutable."[2496]

10.3 Einstein and "Space-Time"

Albert Einstein stated,

> "This rigid four-dimensional space of the special theory of relativity is to some extent a four-dimensional analogue of H. A. Lorentz's rigid three-dimensional æther."[2497]

and,

> "I think, that the ether of the general theory of relativity is the outcome of the Lorentzian ether, through relativation."[2498]

Henri Poincaré provided the "four-dimensional analogue"[2499] to Lorentz' æther in 1905 and relativized the "Lorentzian ether" in 1895, long before Hermann Minkowski or Albert Einstein manipulated credit for his work. The Einsteins' 1905 paper contains no four-dimensional analogue, and is, therefore, a theory of the "unrelativized Lorentzian æther", *per se*. Though Einstein credited Minkowski with the quadri-dimensional analogue,

> "And now let me say just a few words about the highly interesting mathematical elaboration that the theory has undergone, thanks, mainly, to the sadly so prematurely deceased mathematician Minkowski,"[2500]

in fact, Minkowski was well aware of Poincaré's earlier work, before Minkowski recited it in 1907, as if it were his own.[2501] Max Born recounts that,

> "My first encounter with the difficulties of this orthodox creed happened in 1905, the year which we celebrate today, in a seminar on the theory of electrons, held not by a physicist but by a mathematician, HERMANN MINKOWSKI. My memory of these long bygone days is of course blurred, but I am sure that in this seminar we discussed what was known at this period about the electrodynamics and optics of moving systems. We studied papers by HERTZ, FITZGERALD, LARMOR, LORENTZ, POINCARÉ, and others but also got an inkling of MINKOWSKI's own ideas which were published only two years later."[2502]

and,

> "[In 1905] I was a student in Göttingen and attended a seminar conducted by the mathematicians David Hilbert and Hermann Minkowsky. They dealt with the electrodynamics and optics of moving bodies — the subject that was Einstein's point of departure for the theory of relativity. We studied papers by H. A. Lorentz, Henri Poincaré, G. F. Fitzgerald, Larmor and others, but Einstein was not mentioned. [***] When I mentioned Minkowsky's contributions to the seminars in Gottingen, which already contained the germ of his four-dimensional representation of the electromagnetic field, published in 1907-8, Reiche and Loria told me about Einstein's paper and suggested that I should study it."[2503]

and,

> "The result was that in the same year (I have forgotten whether simultaneously or in consecutive terms) two advanced seminars were held on mathematical physics: one by Klein and Runge on elasticity, the other by Hilbert and Minkowski on electromagnetic theory. It was the latter which fascinated me. We studied the papers of Lorentz, Poincaré and others on the difficulties which the theories of the electromagnetic ether had run into as a result of Michelson's celebrated experiment. [***] One day Reiche asked me whether I knew a paper by a man named Einstein on the principle of relativity. He said Planck considered it most important. I had not heard of it, but when I learned that it had something to do with the fundamental principles of electrodynamics and optics which years ago had fascinated me in Hilbert's and Minkowski's seminar, I agreed at once to join Reiche in studying it."[2504]

The nature of these lectures at the Göttingen Academy and their historical importance is treated by Jules Leveugle, Reid and Pyenson.[2505] Both Hilbert and Minkowski failed to give Lorentz and Poincaré due credit for their contributions to the development of the theory of relativity, and the contributions of Hilbert and Minkowski have likewise since been underrated or forgotten by others.

Roberto Marcolongo,[2506] also, in 1906, published a four-dimensional analysis of the Poincaré-Lorentz theory of relativity, before Minkowski. Einstein's brief evaluation exclusively cites work which was accomplished by Poincaré before Minkowski copied it, but Einstein nowhere mentions Poincaré or Marcolongo. Mehmke's work is significant and preceded Poincaré's.[2507] Richard Hargreaves[2508] and Harry Bateman[2509] also deserve mention, for their development of the special and the general theories of relativity. In this same lecture, followed by a discussion which is on record,[2510] Einstein shamelessly parroted Poincaré's enquiries into the nature of simultaneity[2511] and his clock synchronization procedures, without citing Poincaré; and Einstein failed to correct those who credited Einstein with the ideas he repeated, which he knew were not his own.

Harry Bateman wrote of Hargreaves contributions,

> "§ 2. In the year 1908 two very important papers on electromagnetic theory were published. One of these was Minkowski's paper on the electrodynamical equations for moving bodies,[1] a paper which soon influenced mathematical thought very considerably and received world wide attention. The other paper was by Mr. Richard Hargreaves, of Southport, England, and was entitled 'Integral forms and their connection with physical equations.' This paper which is perhaps the more important of the two, contains two new presentations of the principles of electromagnetism in terms of space-time integrals. This at once places the time coördinate on the same level as the other coördinates and suggests the idea of space-time vectors just as in Minkowski's work. The chief importance of Mr. Hargreaves' work lies, however, in the fact that it throws light at once upon the nature of the solutions of the electromagnetic equations and that the principles are presented in a form which is independent of the choice of the space and time coördinates. The last circumstance enables one to obtain the transformations of the theory of relativity in a simple and natural manner and makes it easy to obtain the invariants by a simple application of the methods of the absolute calculus of Ricci and Levi Civita.[3] "[2512]

Consider the psychological import of the attitude of some later writers toward those who actually originated the ideas compared to their attitude toward the *heroes* "Einstein" and "Minkowski", who merely parroted what others had pioneered,

> "All the main ideas, of course, are due to Einstein and Minkowski. [***] It may be mentioned that the historical order of appearance of the ideas of our subject, as so often happens, has been quite different from the order which seems natural and in which we have presented them. First the formulas of transformation involving space coordinates and time were introduced by Lorentz without, however, giving to them the meaning they now have. In Lorentz's theory there exists one universal time t, and other times t' play only an auxiliary part. The credit for taking the decisive step recognizing the fact that all these variables are on the same footing is due to Einstein (1905). The four-dimensional point of view, after some preliminary work had been done by Poincaré and Marcolongo, was introduced most emphatically by Minkowski in 1908."[2513]

One must wonder how Minkowski "introduced" in 1908, that which was already extant in Poincaré's work of 1905, and in Marcolongo's work of 1906. It was Poincaré who first attacked Lorentz' and Larmor's distinction between local time and time, beginning in 1898, and eliminated said distinction long before 1905—which distinction was not even present in Voigt's formulation of 1887.

Olivier Darrigol stated in 1996,

"The physicist-historian and the philosopher-historian usually argue that Einstein's new kinematics was an extremely important innovation that overthrew previous physical and philosophical concepts of time; and they tend to interpret Poincaré's, Lorentz's, and others' fidelity to the ether as a failure to understand Einstein's superior point of view. On the contrary, the social historian would argue that in 1905 Einstein's relativity had no stabilized meaning, that it could be read and used in various manners depending on the receiving local culture, and that it acquired a precise meaning only at the end of a complex, social structuring process."[2514]

There was no novelty in asserting time as a fourth dimension in 1908. In 1907,Victoria Welby wrote,

"Or we may, if we like, compare our 'present' to the sweep of our outlook from horizon to horizon, and the great mind's area of vision to the broad land- or sea-scape from a high mountain. But then the present moment must be seen as dimensional. It must give us the cube, the volume, the solid. It must be the true analogue of what from the highest vantage point attainable is the range and content of our bodily vision. The Future, then, to begin with, becomes that which is yet below a given horizon; if you will, the antipodes to the Present whereon we stand. But see what follows. For the Past, that is the world already explored by Man on his great journey through the life-country, has thus sunk below the horizon behind us; the Future is the world waiting for him, ready for the Columbus of the race, *the Copernicus of Time*. When that Time-Explorer appears he will know how to set forth on his voyage of exploration, and will bring us evidence that his discoveries are not conjectural nor fantastic. He will show that the prophet actually sees and gives us here and now, what the ordinary man merely predicts, foretells and guesses at, as far away; and that if we will learn to use his means and use them with his energy, we too may go forth into 'new continents' of Time and colonise the 'future' at our will."[2515]

In 1906, Cassius J. Keyser wrote,

"Herewith is immediately suggested the generic concept of dimensionality: *if an assemblage of elements of any given kind whatsoever, geometric or analytic or neither, as points, lines, circles, triangles, numbers, notions, sentiments, hues, tones, be such that, in order to distinguish every element of the assemblage from all the others, it is necessary and sufficient to know exactly n independent facts about the element, then the assemblage is said to be n-dimensional in the elements of the given kind*. It appears, therefore, that the notion of dimensionality is by no means exclusively associated with that of space but on the contrary may often be attached to the far more generic concept of assemblage, aggregate or manifold. For example, duration, the total aggregate of time-points, or instants, is a simple or one-fold assemblage.

[*Emphasis found in the original.*]”[2516]

In 1902, Walter Smith observed,

> “The first thing to be noticed in regard to time is its spatial character. This statement is not a mere paradox. When a succession of events is thought of, the events are ranged in spatial order. We speak of time as long or short; we speak of the distant past and the near future, or of the receding past and the coming years; we ‘look before and after.’ These expressions are not simply figures of speech; they indicate what forms are present in consciousness when a temporal succession is referred to. Nor does this spatial form of the temporal series mean merely that images originally intuited in space are reproduced with this spatial character. If the images simply arise and dissolve in what seems to be one space, there is little if any perception of time; when the sense of time is present, the images of the past recede into the distance. It is very important to note this feature of the time-concept. It has received too little attention from students of the mind. Kant speaks of time as a line; and psychologists are learning to regard time as a projection at right angles to the plane of the present. But that this spatiality is essential to the time-concept has not been, in general, recognized. To F. A. Lange[1] belongs the credit of having given it due emphasis.”[2517]

With respect to psychologists and their equating of time with space, G. F. Stout stated in 1902,

> “Psychologists generally hold the same type of theory for the two cases of space and time cognition, and the indications of individual views given under Extension (q. v.) hold largely also for time.”[2518]

Herbert Spencer wrote extensively on space and time in his works on psychology. Henry Longueville Mansel wrote in the “Psychology” section of his *Metaphysics*, “Much of what has been said of space is applicable of time also.”[2519]

Neither Minkowski, nor the Einsteins, nor Poincaré, hold priority on the concept of four-dimensional space-time. In 1894, H. G. Wells wrote about it in a popular novel, *The Time Machine*, long before Minkowski claimed priority,

> “‘Can a cube that does not last for any time at all, have a real existence?’ Filby became pensive. ‘Clearly,’ the Time Traveller proceeded, ‘any real body must have extension in *four* directions: it must have Length, Breadth, Thickness, and—Duration. But through a natural infirmity of the flesh, which I will explain to you in a moment, we incline to overlook this fact. There are really four dimensions, three which we call the three planes of Space, and a fourth, Time. There is, however, a tendency to draw an unreal distinction between the former three dimensions and the latter, because it happens that our consciousness moves intermittently in one direction along the latter from

the beginning to the end of our lives.'"

An article by "S." had appeared in *Nature*, Volume 31, Number 804, (26 March 1885), p. 481, titled, "Four-Dimensional Space", which presented the concepts of "time-space", "four-dimensional solid" ("sur-solid", after Des Cartes), "time area", and "time-line"; which later became "space-time"[2520] ("*Zeit-Raum*" is a confusing pun in German with the word "*Zeitraum*". It was used for quite some time in the theory of relativity but has largely died out. Rudolf Mewes was using the term "Space-Time" at least as early as 1889[2521] and Palágyi used the "*Raumzeit*" combination in 1901.[2522]), "absolute world", and "world-line". Here is the work of 1885, which appeared some 23 years before Minkowski's derivative lecture on the same subject:

"**Four-Dimensional Space**

POSSIBLY the question, What is the fourth dimension? may admit of an indefinite number of answers. I prefer, therefore, in proposing to consider Time as a fourth dimension of our existence, to speak of it as *a* fourth dimension rather than *the* fourth dimension. Since this fourth dimension cannot be introduced into space, as commonly understood, we require a new kind of space for its existence, which we may call time-space. There is then no difficulty in conceiving the analogues in this new kind of space, of the things in ordinary space which are known as lines, areas, and solids. A straight line, by moving in any direction not in its own length, generates an area; if this area moves in any direction not in its own plane it generates a solid; but if this solid moves in any direction, it still generates a solid, and nothing more. The reason of this is that we have not supposed it to move in the fourth dimension. If the straight line moves in its own direction, it describes only a straight line; if the area moves in its own plane, it describes only an area; in each case, motion in the dimensions in which the thing exists, gives us only a thing of the same dimensions; and, in order to get a thing of higher dimensions, we must have motion in a new dimension. But, as the idea of motion is only applicable in space of three dimensions, we must replace it by another which is applicable in our fourth dimension of time. Such an idea is that of successive existence. We must, therefore, conceive that there is a new three-dimensional space for each successive instant of time; and, by picturing to ourselves the aggregate formed by the successive positions in time-space of a given solid during a given time, we shall get the idea of a four-dimensional solid, which may be called a sur-solid. It will assist us to get a clearer idea, if we consider a solid which is in a constant state of change, both of magnitude and position; and an example of a solid which satisfies this condition sufficiently well, is afforded by the body of each of us. Let any man picture to himself the aggregate of his own bodily forms from birth to the present time, and he will have a clear idea of a sur-solid in time-space.

Let us now consider the sur-solid formed by the movement, or rather, the

> successive existence, of a cube in time-space. We are to conceive of the cube, and the whole of the three-dimensional space in which it is situated, as floating away in time-space for a given time; the cube will then have an initial and a final position, and these will be the end boundaries of the sur-solid. It will therefore have sixteen points, namely, the eight points belonging to the initial cube, and the eight belonging to the final cube. The successive positions (in time-space) of each of the eight points of the cube, will form what may be called a time-line; and adding to these the twenty-four edges of the initial and final cubes, we see that the sur-solid has thirty-two lines. The successive positions (in time-space) of each of the twelve edges of the cube, will form what may be called a time area; and, adding these to the twelve faces of the initial and final cubes, we see that the sur-solid has twenty-four areas. Lastly, the successive positions (in time-space) of each of the six faces of the cube, will form what may be called a time-solid; and, adding these to the initial and final cubes, we see that the sur-solid is bounded by eight solids. These results agree with the statements in your article. But it is not permissible to speak of the sur-solid as resting in 'space,' we must rather say that the section of it by any time is a cube resting (or moving) in 'space.' S. March 16"[2523]

This article, "Four-Dimensional Space", was probably a reaction to an earlier one, "Scientific Romances", *Nature*, Volume 31, Number 802, (March12, 1885), p. 431; which discusses Hinton's question, "What is the Fourth Dimension?" and Edwin A. Abbott's book *Flatland: A Romance of Many Dimensions*.[2524] *Nature* and *Mind* published numerous articles, which discussed time and space.

The author of "Four-Dimensional Space" is named as "S.", who may have been Simon Newcomb. Wells' *The Time Machine* includes the following passage,

> "'It is simply this. That Space, as our mathematicians have it, is spoken of as having three dimensions, which one may call Length, Breadth, and Thickness, and is always definable by reference to three planes, each at right angles to the others. But some philosophical people have been asking why *three* dimensions particularly—why not another direction at right angles to the other three?—and have even tried to construct a Four-Dimension geometry. Professor Simon Newcomb was expounding this to the New York Mathematical Society only a month or so ago. You know how on a flat surface, which has only two dimensions, we can represent a figure of a three-dimensional solid, and similarly they think that by models of thee dimensions they could represent one of four—if they could master the perspective of the thing. See?'"

A bibliography of Newcomb's works on the fourth-dimension is to be found in the endnote.[2525] However, Newcomb does not seem to be a believer in *time* as a fourth dimension—so the mysterious "S." may well have been someone else, "S." Tolver Preston, perhaps? James E. Beichler believes that James Joseph Sylvester was the mysterious "S."[2526]

Before Wells, in 1881, in a work which reminds one of Camille Flammarion's *Lumen*, John Venn wrote,

> "These requirements seem reducible to the two following—regard being had to the nature of our faculties and the general conditions under which we have to employ them: power to move about as freely as we may wish in space or time, and power to enlarge space and time to any extent we may need. [***] Let us begin with the former, *viz*., our power of locomotion (the reader will observe that we are obliged to use, in many cases, space-words for time-ideas, and *vice versâ*, from inadequacy in ordinary terminology). What our powers in this respect as regards space, every one knows. Within very small limits we can move ourselves, or the objects with which we are concerned, up and down and about, in three dimensions, as we please. Within wider limits, *viz*., that of the surface of the globe, we are restricted to two dimensions. Beyond that again we are hampered still further by being confined to one dimension only, our motion along that even being quite beyond our own control. [***] Now this state of powerlessness represents almost exactly our relation to events in respect of time. We are bound, as we all know, to go steadily forwards: we have no power to stand still, go sideways or backwards. [***] What we want is the power to stop still and to go backwards whenever we please. [***] What we want in fact is a microscope with a double set of stage-screws; one set to move the stage about as is now done, in respect of space, and the other to move it about in a similar way in respect of time. [***] Physical speculators have not unfrequently indulged in fanciful modes of attaining the equivalent of such a power as that just indicated. Since light travels with finite velocity, we are at liberty to conceive an object moving so fast as to outstrip it. Suppose a human eye receding from our system into space with a velocity greater than that of light, and occasionally pausing for a moment so as to permit the rays from the objects which it was leaving behind to overtake it and record their impression. We should then invert, so far as that eye was concerned, the relative course of events, and this would be, so far as all visual considerations are applied, precisely that regression into past time which is desired."[2527]

Charles Howard Hinton queried as to what might be the fourth dimension in 1880, and argued that time constitutes a fourth dimension resulting in an Eleatic universal state of being, without cause or effect,

> "And in the first place, a being in four dimensions would have to us exactly the appearance of a being in space. A being in a plane would only know solid objects as two dimensional figures—the shapes namely in which they intersected his plane. So if there were four-dimensional objects, we should only know them as solids—the solids, namely, in which they intersect our space. Why, then, should not the four-dimensional beings be ourselves, and

our successive states the passing of them through the three-dimensional space to which our consciousness is confined?

Let us consider the question in more detail. And for the sake of simplicity transfer the problem to the case of three and two dimensions instead of four and three.

Suppose a thread to be passed through a table cloth. It can be passed through in two ways. Either it can be pulled through, or it can be held at both ends, and moved downwards as a whole. Suppose a thread to be grasped at both ends, and the hands to be moved downwards perpendicularly to the tablecloth. If the thread happens to be perpendicular to the tablecloth it simply passes through it, but if the thread be held, stretched slanting wise to the tablecloth, and the hands are moved perpendicularly downwards, the thread will, if it be strong enough, make a slit in the tablecloth.

If now the tablecloth were to have the faculty of closing up behind the thread, what would appear in the cloth would be a moving hole.

Suppose that instead of a tablecloth and a thread, there were a straight line and a plane. If the straight line was placed slanting wise in reference to the plane and moved downwards, it would always cut the plane in a point, but that point of section would move on. If the plane were of such a nature as to close up behind the line, if it were of the nature of a fluid, what would be observed would be a moving point. If now there were a whole system of lines sloping in different directions, but all connected together, and held absolutely still by one framework, and if this framework with its system of lines were as a whole to pass slowly through the fluid plane at right angles to it, there would then be the appearance of a multitude of moving points in the plane, equal in number to the number of straight lines in the system. The lines in the framework will all be moving at the same rate—namely, at the rate of the framework in which they are fixed. But the points in the plane will have different velocities. They will move slower or faster according as the lines which give rise to them are more or less inclined to the plane. A straight line perpendicular to the plane will, on passing through, give rise to a stationary point. A straight line that slopes very much inclined to the plane will give rise to a point moving with great swiftness. The motions and paths of the points, would be determined by the arrangement of the lines in the system. It is obvious that if two straight lines were placed lying across one another like the letter X, and if this figure were to be stood upright and passed through the plane, what would appear would be at first two points. These two points would approach one another. When the part where the two strokes of the X meet came into the plane, the two points would become one. As the upper part of the figure passed through, the two points would recede from one another.

If the lines be supposed to be affixed to all parts of the framework, and to loop over one another, and support one another, [*figure deleted*] it is obvious that they could assume all sorts of figures, and that the points on the plane would move in very complicated paths. The annexed figure represents a section of such a framework. Two lines X X and Y Y are shown, but there must be supposed to be a great number of others sloping backwards and forwards as well as sideways.

Let us now assume that instead of lines, very thin threads were attached to the framework: they on passing through the fluid plane would give rise to very small spots. Let us call the spots, atoms, and regard them as constituting a material system in the plane. There are four conditions which must be satisfied by these spots if they are to be admitted as forming a material system such as ours. For the ultimate properties of matter (if we eliminate attractive and repulsive forces, which may be caused by the motions of the smallest particles), are—1, Permanence; 2, Impenetrability; 3, Inertia; 4, Conservation of energy.

According to the first condition, or that of permanence, no one of these spots must suddenly cease to exist. That is, the thread which by sharing in the general motion of the system gives rise to the moving point, must not break off before the rest of them. If all the lines suddenly ended this would correspond to a ceasing of matter.

2. Impenetrability.—One spot must not pass through another. This condition is obviously satisfied. If the threads do not coincide at any point, the moving spots they give rise to cannot.

3. Inertia.—A spot must not cease to move or cease to remain at rest without coming into collision with another point. This condition gives the obvious condition with regard to the threads, that they, between the points where they come into contact with one another, must be straight. A thread which was curved would, passing through the plane, give rise to a point which altered in velocity spontaneously. This the particles of matter never do.

4. Conservation of energy.—The energy of a material system is never lost, it is only transferred from one form to another, however it may seem to cease. If we suppose each of the moving spots on the plane to be the unit of mass, the principle of the conservation of energy demands that when any two meet, the sum of the squares of their several velocities before meeting shall be the same as the sum of the squares of their velocities after meeting. Now we have seen that any statement about the velocities of the spots in the plane is really a statement about the inclinations of the threads to the plane. Thus the principle of the conservation of energy gives a condition which must be satisfied by the inclinations of the threads of the plane. Translating this statement, we get in mathematical language the assertion that the sum of the squares of the tangents of the angles the threads make with the normal to the plane remains constant.

Hence, all complexities and changes of a material system made up of similar atoms in a plane could result from the uniform motion as a whole of a system of threads.

We can imagine these threads as weaving together to form connected shapes, each, complete in itself, and these shapes as they pass through the fluid plane give rise to a series of moving points. Yet, inasmuch as the threads are supposed to form consistent shapes, the motion of the points would not be wholly random, but numbers of them would present the semblance of moving figures. Suppose, for instance, a number of threads to be so grouped as to form a cylinder for some distance, but after a while to be pulled apart by other threads with which they interlink. While the cylinder was passing through the plane we should have in the plane a number of points in a circle. When the part where the threads deviated came to the plane, the circle would break up by the points moving away. These moving figures in the plane are but the traces of the shapes of threads as those shapes pass on. These moving figures may be conceived to have a life and a consciousness of their own.

Or if it be irrational to suppose them to have a consciousness when the shapes of which they are momentary traces have none, we may well suppose that the shapes of threads have consciousness, and that the moving figures share this consciousness, only that in their case it is limited to those parts of the shapes that simultaneously pass through the plane. In the plane, then, we may conceive bodies with all the properties of a material system moving and changing, possessing consciousness. After a while it may well be that one of them becomes so disassociated that it appears no longer as a unit, and its consciousness as such may be lost. But the threads of existence of such a figure are not broken, nor is the shape which gave it origin altered in any way. It has simply passed on to a distance from the plane. Thus nothing which existed in the conscious life on the plane would cease. There would in such an existence be no cause and effect, but simply the gradual realisation in a superficies of an already existent whole. There would be no progress, unless we were to suppose the threads as they pass to interweave themselves in more complex shapes.

Can a representation such as the preceding be applied to the case of the existence in space with which we have to do? Is it possible to suppose that the movements and changes of material objects are the intersections with a three-dimensional space of a four-dimensional existence? Can our consciousness be supposed to deal with a spatial profile of some higher actuality?

It is needless to say that all the considerations that have been brought forward in regard to the possibility of the production of a system satisfying the conditions of materiality by the passing of threads through a fluid plane, holds good with regard to a four-dimensional existence passing through a three-dimensional space. Each part of the ampler existence which passed through our space would seem perfectly limited to us. We should have no indication of the permanence of its existence. Were such a thought adopted, we should have to imagine some stupendous whole, wherein all that has ever come into being or will come co-exists, which, passing slowly on, leaves in this flickering consciousness of ours, limited to a narrow space and a single moment, a tumultuous record of changes and vicissitudes that are but to us.

Change and movement seem as if they were all that existed. But the appearance of them would be due merely to the momentary passing through our consciousness of ever existing realities.

In thinking of these matters it is hard to divest ourselves of the habit of visual or tangible illustration. If we think of a man as existing in four dimensions, it is hard to prevent ourselves from conceiving him as prolonged in an already known dimension. The image we form resembles somewhat those solemn Egyptian statues which in front represent well enough some dignified sitting figure, but which are immersed to their ears in a smooth mass of stone which fits their contour exactly.

No material image will serve. Organised beings seem to us so complete that any addition to them would deface their beauty. Yet were we creatures confined to a plane, the outline of a Corinthian column would probably seem to be of a beauty unimprovable in its kind. We should be unable to conceive any addition to it, simply for the reason that any addition we could conceive would be of the nature of affixing an unsightly extension to some part of the contour. Yet, moving, as we do in space of three dimensions, we see that the beauty of the stately column far surpasses that of any single outline. So all that we can do is to deny our faculty of judging of the ideal completeness of shapes in three dimensions.

Our conception of existence in four dimensions need not be confined to any particular supposition. There is no reason why a being existing in four dimensions should not be conceived to be as completely limited in all four directions as we are in three. All that we can say in regard to the possibility of such beings is, that we have no experience of motion in four directions. The powers of such beings and their experience would be ampler, but there would be no fundamental difference in the laws of force and motion.

Such a being would be able to make but a part of himself visible to us. He would suddenly appear as a complete and finite body, and as suddenly disappear, leaving no trace of himself, in space. There would be no barrier, no confinement of our devising that would not be perfectly open to him. He would come and go at pleasure; he would be able to perform feats of the most surprising kind. It would be possible by an infinite plane extending in all directions to divide our space into two portions absolutely separated from one another; but a four-dimensional being would slip round this plane with the greatest ease.

With regard to the possibility of the application of any test to discover whether a fourth dimension does exist or not, all that can be said is that no such test has succeeded. And, indeed, before searching for tests a theoretical point of the utmost importance has to be settled. In discussing the geometrical properties of straight lines and planes, we suppose them to be respectively of one and two dimensions, and by so doing deny them any real existence. A plane and a line are mere abstractions. Every portion of matter is of three dimensions. If we consider beings on a plane not as mere idealities, we must suppose them to be of some thickness. If their experience is to be limited to a plane this thickness must be very small compared to their other dimensions. If, then, we suppose a fourth dimension to exist, either our consciousness itself must consist in a limitation of the knowledge of existence to three instead of four dimensions, or we must be very small in the fourth dimension as compared to others. In such a case it would probably be in the phenomena of the ultimate particles of matter, where the dimensions in all four directions would be comparable, that any indication of the new direction would have to be sought.

It is evident that these speculations present no point of direct contact with fact. But this is no reason why they should be abandoned. The course of knowledge is like the flow of some mighty river, which, passing through the rich lowlands, gathers into itself the contributions from every valley. Such a river may well be joined by a mountain stream, which, passing with difficulty along the barren highlands, flings itself into the greater river down some precipitous descent, exhibiting at the moment of its union the spectacle of the utmost beauty of which the river system is capable. And such a stream is no inapt symbol of a line of mathematical thought, which, passing through difficult and abstract regions, sacrifices for the sake of its crystalline clearness the richness that comes to the more concrete studies. Such a course may end fruitlessly, for it may never join the main course of observation and experiment. But if it gains its way to the great stream of knowledge, it affords at the moment of its union the spectacle of the greatest intellectual beauty, and adds somewhat of force and mysterious capability to the onward current.”[2528]

Hinton’s and Abbott’s works are highly derivative of another *Nature* article by G. F. Rodwell, “On Space of Four Dimensions”, *Nature*, Volume 8, Number 183, (May 1, 1873), pp. 8-9. This same volume of *Nature* contains Clifford’s translation of Riemann’s, “On the Hypotheses which Lie at the Bases of Geometry”.[2529]

Long before Hinton, Abbott, Rodwell, and even Riemann, was Stallo, who expressed the fundamental “space-time” concept in 1847,

“THE Spiritual, the absolute primitive movement within itself, can be real and substantial only in stating itself exteriorly; and we have repeatedly seen that this statement is absolute multiplicity. That the result of the statement, the Exterior, is BUT *a statement*, and the statement of an internal *movement*, implies its transience; the statement is from its very nature transient. This transience must exhibit itself, therefore, in the stated Exterior, wherever we take it; it must appear throughout, for the Exterior is *inherently* transient. Otherwise expressed: the Exterior is but a transience in position; a position *in One* of existence and non-existence,—or a *position* and a *negation* in one. The Exterior can therefore first be taken *as such*, and then it is SPACE, in which the transience, dependency, shows itself as absolute *relativity;* secondly, as *the bearer of its vivifying movement*, and thus it is TIME. Or, the Exterior as an *existence*, as positive, fixed, is *space;* as a *negation*, *non-*

existence, it is *time*. Logically, the first two exteriorations of the Spiritual are therefore *space* and *time*. They are both *abstractions*, i.e. they *are* only, inasmuch as the understanding forcibly keeps them asunder, though their truth is their *being in one*, their inseparability in spite of their distinctness. [***] Time and space, whose *first* reality is their difference, will therefore further state their identity as *real unity;* and this statement is real MOTION. Real motion is the union of space and time. The motion under consideration here, namely, the *primitive motion in the sphere of the Exterior*, is not motion in *any given*, *definite direction;* it is motion IN ALL DIRECTIONS, to which we have no observable analogon. *It is the pure movement of abstract statement and annulment*. [***] The so-called *dimensions* of space present no difficulty in their deduction, and depend, like all deductions, upon the inherent *references* of space. Space, the absolute extension, as OPPOSED to the Spiritual, is *spatial infinitude*, *unbounded* (mathematical) *solidity;* as opposed TO THE SPIRITUAL, to the absolute intensity, it is *a point*, —in space, and yet spaceless; as the unity of the two, it is the *line*, —extended intensity or punctuality. If we seek for a spatial analogon of time, it must be the line, for it has been seen that time is the Extensive considered in its ideal bearing, the mediating unity therefore between extension and intensity. Now the absolutely Extensive, the Solid, is from its nature limited, —it contains the limit; and this limit of solidity is the *surface*. Thus punctuality, solidity, surface, and linearity are inherent in the idea of space; we are logically compelled to see space under this fourfold aspect. The mathematical statement, that the motion of a point generates a line, that of a line a surface, and that of a surface a solid, is true only in the following sense:—Spatiality, extension as such, is the absolute reference to the without, *beyond itself*, absolute relativity. If, then, we ideally isolate a point, we are at the same moment compelled to refer it to ideal adjacent points, and thus the idea of the line starts up in the mind spontaneously. The same takes place with the line and with the surface. The ideas of point, line, surface, &c., from their nature, give birth to each other. The movement of a point, &c., however, *as something real, to which the motion accedes*, is a false assumption. [*Notation in the original:* Already Hegel has pointed this out. See my exposition of his philosophy of nature.]"[2530]

Before Stallo, Gustav Theodor Fechner presented a four-dimensional theory of space-time in 1846 under the pseudonym "Dr. Mises".[2531] Fechner stated, *inter alia*, in 1846,

> "Jedoch, um mein Möglichstes zu tun, sehe ich wieder bei dem Farbenmännchen in zwei Dimensionen nach; weiß ich erst in zwei Dimensionen die dritte zu packen, so muß es ja dann um so leichter sein, in dreien die vierte zu packen. Auch ist dies nur eine besondere Anwendung der von jeher mit Frucht angewandten Methode, das, was man in drei Dimensionen nicht realiter finden kann, in zwei Dimensionen, d. h. auf dem

> Papier zu suchen und zu finden. Und siehe da, es gelingt.
>
> Zur Sache: ich nehme die Fläche, worin mein Scheinmännchen sich befindet, und führe sie durch die dritte Dimension hindurch, so erfährt das Scheinmännchen alles, was in dieser dritten Dimension ist; es wird sogar, indem es in andere Lichträume kommt, wo sich die Strahlen anders ordnen und färben, selbst sich hiermit ändern und vielleicht zu Ende des Weges bleich und runzlig aussehen, während es zu Anfange des Weges rot und glatt aussah. Freilich hat das Männchen niemals ein Stück der dritten Dimension auf einmal und glaubt also in jedem Augenblicke immer noch bloß in seinen zwei Dimensionen zu sein; es faßt von der ganzen Bewegung bloß das zeitliche Element und die vor sich gehende Änderung auf. Aber faktisch durchmißt es doch die dritte Dimension und Alles, was darin ist. Demgemäß sagt das Männchen: es gibt eine Zeit und in der Zeit ändert sich Alles, auch ich selbst.
>
> Nun, wir sagen auch: es gibt eine Zeit und in der Zeit ändert sich alles, auch wir selbst. Was liegt dem also zu Grunde? Die Bewegung unsers Raums von drei Dimensionen durch die vierte, von welcher Bewegung wir aber auch nur das zeitliche Element und die Veränderung, welche erfolgt, wahrnehmen.
>
> Nichts ist auch im Grunde einfacher und natürlicher: unsere Welt von drei Dimensionen ist eine ungeheure Kugel, die in eine Menge einzelner Kugeln zerfällt. Jede von diesen läuft; also wird die große Urkugel wohl auch laufen; aber wo sollte sie hinlaufen, wenn es nicht eine vierte Dimension gäbe? Indem sie aber selbst durch diese vierte Dimension läuft, laufen natürlich auch alle Kugeln in ihr, und alles was auf diesen Kugeln lebt und webt, durch die vierte Dimension mit durch.”[2532]

Boscovich stated, centuries ago,

> “Hence, the number of other points of space is an infinity of the third order; & thus the probability is infinitely greater with an infinity of the third order, when we are concerned with any other particular instant of time.”[2533]

Joseph Larmor, in 1900, raised space-time’s significance to relativity theory and expressly called it a “*continuum*”, long before Minkowski. Larmor is perhaps guilty of pun, using “continuum” with both its mathematical and metaphysical meanings,

> “At the same time all that is known (or perhaps need be known) of the aether itself may be formulated as a scheme of differential equations defining the properties of a *continuum* in space, which it would be gratuitous to further explain by any complication of structure; though we can with great advantage employ our stock of ordinary dynamical concepts in describing the succession of different states thereby defined.”[2534]

Note the absolutism implicit in the term “continuum”, which Minkowski dubbed the “absolute world”. The “continuum” is Newton’s unchanging God—his myth that the human Self does not change during a lifetime, and, therefore, neither can God—absolute "space-time".

Eugen Karl Dühring[2535] published a space-time theory in 1873, which inspired Rudolf Mewes' space-time theory of 1889.[2536] Inspired by Johann Julius Baumann,[2537] Friedrich Albert Lange[2538] presented a theory of the space-time manifold in 1877. In 1882, Gustav Teichmüller[2539] published a lengthy treatise enunciating an Eleatic space-time theory free from paradoxes, in which he recognized the abstract nature, and absolute relativity, of space and time, and created a space-time with three space dimensions and three time dimensions. E. H. Synge argued that Sir William Rowan Hamilton's space-time theory anticipated Minkowski's theory by sixty-five years.[2540] Menyhért (Melchior) Palágyi, in 1901, published *Neue Theorie des Raumes und der Zeit* (*New Theory of Space and of Time*), in which he argued for an Eleatic quadri-dimensional space-time, and in which he justified the principle of relativity in four-dimensions.[2541] Before Palágyi, was Rudolf Mewes, who, in 1889-1894, developed a relativistic space-time theory, declaring in 1889, "*Und doch beruht die ganze Wirklichkeit allein auf der Vereinigung von Raum und Zeit.*"[2542] Johann Heinrich Ziegler lectured in Switzerland in 1902 on the unity of space, time and force and the significance of light in empty space, doing away with the æther hypothesis.[2543] Poincaré established the Palágyi-style four-dimensional analysis of the "Lorentz Transformation", before Minkowski, or Einstein. Roberto Marcolongo[2544] presented his four-dimensional view of the "Lorentz Transformation", before Minkowski.

Henri Bergson wrote in 1888 in his lengthy and detailed theory of space and time,

> "in a word, we create for them a fourth dimension of space, which we call homogenous time, and which enables the movement of the pendulum, although taking place at one spot, to be continually set in juxtaposition to itself."[2545]

Prior to Bergson, Ernst Mach discussed quadri-dimensional position in 1866,

> "Now, I think that we can go still farther in the scale of presentations of space and thus attain to presentations whose totality I will call *physical space.*
>
> It cannot be my intention here to criticize our conceptions of matter, whose insufficiency is, indeed, generally felt. I will merely make my thoughts clear. Let us imagine, then, a something behind (*unter*) matter in which different states can occur; say, for simplicity, a pressure in it, which can become greater or smaller.
>
> Physics has long been busied in expressing the mutual action, the mutual attraction (opposite accelerations, opposite pressures) of two material particles as a function of their distance from each other—therefore of a spatial relation. Forces are functions of the distance. But now, the spatial relations of material particles can, indeed, only be recognized by the forces

which they exert one on another.

Physics, then, does not strive, in the first place, after the discovery of the fundamental relations of the various pieces of matter, but after the derivation of relations from other, already given, ones. Now, it seems to me that the fundamental law of force in nature need not contain the spatial relations of the pieces of matter, but must only state a dependence between the states of the pieces of matter.

If the positions in space of the material parts of the whole universe and their forces as functions of these positions were once known, mechanics could give their motions completely, that is to say, it could make all the positions discoverable at any time, or put down all positions as functions of time.

But, what does time mean when we consider the universe? This or that 'is a function of time' means that it depends on the position of the vibrating pendulum, on the position of the rotating earth, and so on. Thus, 'All positions are functions of time' means, for the universe, that all positions depend upon one another.

But since the positions in space of the material parts can be recognized only by their states, we can also say that all the states of the material parts *depend upon one another.*

The physical space which I have in mind—and which, at the same time, contains time in itself—is thus nothing other than *dependence of phenomena on one another*. A complete physics, which would know this fundamental dependence, would have no more need of special considerations of space and time, for these latter considerations would already be included in the former knowledge."[2546]

I confine the discussion to quadri-dimensional hyperspace in which the fourth dimension signifies time or spiritual motion of some kind in a fourth dimension, whatever that should be interpreted to mean as ghosts retreating into a "fourth dimension" to undo tri-dimensional knots, leaving from one position in our world to return in another; but there was a tremendous body of work involving hyperspace beyond this restriction, with a long history pre-dating the special and the general theories of relativity.

For example, Stewart and Tait, in their widely read *Unseen Universe*, averred, in the then fairly recent tradition of the transcendental geometers,

> "Just as points are the terminations of lines, lines the boundaries of surfaces, and surfaces the boundaries of portions of space of three dimensions:—so we may suppose our (essentially three-dimensional) matter to be the mere skin or boundary of an Unseen whose matter has *four* dimensions."[2547]

The history of four-dimensional spaces is aptly recorded in Henry Parker Manning's *Geometry of Four Dimensions*, Macmillan, (1914), republished by Dover, (1956). Bibliographies appear in Manning's *The Fourth Dimension Simply*

Explained, Dover, (1960), pp. 40-41; and in Duncan M'Laren Young Sommerville, *Bibliography of Non-Euclidean Geometry*, Harrison & Sons, London, (1911); reprinted Chelsea Pub. Co., New York, (1970); and George Bruce Halsted, "Bibliography of Hyper-Space and Non-Euclidean Geometry", *American Journal of Mathematics*, Volume 1, (1878), pp. 261-276, 384-385. The development of non-Euclidean geometry is outlined by Oswald Veblen, "The Foundations of Geometry", *Popular Science Monthly*, Volume 68, Number 1, (January, 1906), pp. 21-28. Other important works include Roland Weitzenböck's *Der vierdimensionale Raum*, F. Vieweg & Sohn, Braunschweig, (1929); reprinted Birkhäuser, Basel, (1956); and E. Wölffing, "Die vierte Dimension", *Die Umschau*, Volume 1, Number 18, (1 May 1897), pp. 309-314. A good overview with an emphasis on the religious and spiritualistic aspects of hyperspace theories is found in Carl Cranz' popular "Gemeinverständliches über die sogenannte vierte Dimension", *Sammlung gemeinverständlicher wissenschaftlicher Vorträge*, New Series, Volume 5, Number 112/113, (1890), pp. 567-636.

Returning to the concept of *time* as the fourth dimension, Edgar Allen Poe wrote in 1848,

> "A rational cause for the phænomenon, I maintain that Astronomy has palpably failed to assign: — but the considerations through which, in this Essay, we have proceeded step by step, enable us clearly and immediately to perceive that Space and Duration are one."[2548]

Poe was under the spell of Alexander von Humboldt (and opium). Humboldt stated "Mach's Principle" long before Mach, but long before Humboldt, Boscovich stated it. Humboldt's influence on Stallo, Poe and the general intellectual community toward relativism cannot be emphasized strongly enough!

Immanuel Kant stated in his inaugural dissertation of 1770,

> "*Simultaneous facts* are not such for the reason that they do not succeed each other. Removing succession, to be sure, a conjunction is withdrawn which existed by the time-series. Yet thence does not originate *another* true relation, the conjunction of all things in the same moment. For simultaneous things are joined in the same moment of time exactly as successive things are joined in different moments. Hence, though time is of but one dimension, still the *ubiquity* of time, to speak with Newton, by which all things sensuously thinkable are *some time*, adds to the quantity of actual things another dimension, inasmuch as they hang, so to speak, on the same point of time. For designating time by a straight line produced infinitely, and the simultaneous things at any point of time whatever by lines applied in succession, the surface thus generated will represent the *phenomenal world*, both as to substance and accidents."[2549]

D'Alembert let us in on a secret back in 1754,

> "As I've already said, it is not possible to conceive of more than three *dimensions*. However, a brilliant wit with whom I am acquainted considers duration a fourth *dimension*, and that the product of time multiplied by solidity would, in some sense, be a product of four *dimensions*. This idea is perhaps contestable, but it appears to me to be of some merit, even if it is only that of novelty."[2550]

Lagrange worked out a new mechanics with time as the fourth dimension, ca. 1788,

> "We will apply the theory of functions to mechanics. Here, the functions absolutely correspond to time, which we will always designate with *t*, and, since the position of a point in space depends upon the three rectilinear coordinates *x*, *y*, *z*, these coordinates, in the problems of mechanics, will be assumed to be functions of *t*. In this way, we can look upon mechanics as a geometry of four dimensions, and the analysis of mechanics like an extension of the analysis of geometry."

> "Nous allons employer la théorie des fonctions dans la Mécanique. Ici les fonctions se rapportent essentiellement au temps, que nous désignerons toujours par *t*, et, comme la position d'un point dans l'espace dépend de trois coordonnées rectangulaires *x*, *y*, *z*, ces coordonnées, dans les problèmes de Mécanique, seront censées être des fonctions de *t*. Ainsi, on peut regarder la Mécanique comme une Géométrie à quatre dimensions et l'Analyse mécanique comme une extension de l'Analyse géométrique."[2551]

John Locke asserted, ca. 1689,

> "To conclude: expansion and duration do mutually embrace and comprehend each other; every part of space being in every part of duration, and every part of duration in every part of expansion. Such a combination of two distinct ideas is, I suppose, scarce to be found in all that great variety we do or can conceive, and may afford matter to further speculation."[2552]

In 1671, Henry More argued that spirits inhabit four dimensions.[2553] The fourth dimension as a "realm of spirits" became a popular topic, and it often appears in the literature.[2554] Samuel Clarke, of the Newton-Leibnitz dispute fame, wrote an Eleatic treatise on the universal nature of God, which certainly qualifies as a space-time theory: *A Demonstration of the Being and Attributes of God*. The clerical inspired the profane, and four dimensional fantasies become a common theme in popular fiction.[2555]

The spiritualistic belief was pursued by astrophysicist Johann Karl Friedrich Zöllner, in the 1870's,[2556] and Bernhard Riemann,[2557] who used the spiritual concept to explain gravitation; which spiritualistic four-dimensional views were questioned by physicist Ernst Mach,[2558] but embraced by physicist A. E. Dolbear[2559] and by T.

Proctor Hall,[2560] who was criticized by Edmund C. Sanford.[2561] Hall noted, in 1892, that the fourth dimension is useful; in that,

> "the theologian could use it for the world of spirits; the physicist for forces [***] 'All are but parts of one stupendous whole, Whose body Nature is, and God the soul.' [Alexander Pope, *Essay on Man*] If four-fold space exists, it is evident that it must contain an infinite variety of three-fold spaces, of which we know only one. It must also be everywhere possible for a four-fold being to step out of our space at any point and re-enter it at any other point; for his relation to our space is nearly the same as our relation to a plane. If ghosts are four-fold beings, the erratic nature of their movements may become more comprehensible in the course of time. An ordinary knot could in four-fold space be readily untied by carrying one loop out of our space and bringing it back in a different place. In fact, a knot in our space would be simply a loop or coil in four-fold space. A flexible closed shell could be turned inside out as easily as a thin hoop can with us; and many other apparent impossibilities become mere child's play."

Hermann Schubert attacked Zöllner and the Spiritualists, and their fourth dimension,

> "The high eminence on which the knowledge and civilization of humanity now stands was not reached by the thoughtless employment of fanciful ideas, nor by recourse to a four-dimensional world, but by hard, serious labor, and slow, unceasing research. Let all men of science, therefore, band themselves together and oppose a solid front to methods that explain everything that is now mysterious to us by the interference of independent spirits. For these methods, owing to the fact that they can explain everything, explain nothing, and thus oppose dangerous obstacles to the progress of real research, to which we owe the beautiful temple of modern knowledge."[2562]

Zöllner could not even find respite in *The Journal of Speculative Philosophy*, where George Stuart Fullerton attacked him.[2563] In 1878, P. G. Tait published a polemic against Zöllner, and his fourth dimension, in the journal *Nature*, which evinces the emerging prejudice against Metaphysics, generated by Bacon,[2564] and later by the positivists,

> "He is, as Helmholtz long ago said, a genuine Metaphysician, and (as such) is a curiosity really worthy of study:—not of course merely because he is a Metaphysician, but because in this nineteenth century he attempts to bring his metaphysics into pure physical science. [***] In conclusion, though I cannot make pretensions to any minute acquaintance with the German language, I think I may venture to suggest to Prof. Zöllner, for his next edition, a title which shall at least more accurately describe the contents of his work than does his present one. I cannot allow that the title 'Scientific Papers' is at all

correctly descriptive. But I think that something like the following would suit his book well

Patriotische
METAPHYSIK DER PHYSIK,
für moderne deutsche Verhältnisse.

Mit speciellem Bezug auf die vierte Dimension und den Socialdemokratismus bearbeitet.

With this little hint, which I hope will be taken, as it is meant, in good part, I heartily wish him and his work farewell. P. G. TAIT"[2565]

It is ironic that what was considered metaphysical in the Nineteenth Century, with its belief in an observable reality; is today, with the scientific method turned on its head,[2566] considered scientific; *i. e.* unobservable and purely abstract "space-time" is today considered the absolute world, and questioning this internally contradictory ontological "nonsense" is today incorrectly, pejoratively and hypocritically referred to as "Metaphysics".

Just as quadri-dimensional speculation and non-Euclidean geometry have a long and continuing history, so, too, does opposition to it.[2567] Eugen Karl Dühring (a Socialist who was attacked by Friedrich Engels[2568] and alternatively praised and mocked by Rudolf Mewes, Ernst Mach, Alexander Moszkowski and Albert Einstein[2569]) lampooned the transcendental mysticism of Helmholtz, Gauss and Riemann. Johann Bernhard Stallo wrote much against hyperspace, concluding,

"If Riemann's argument were fundamentally valid, it could be presented in very succinct and simple form. It would be nothing more than a suggestion that, because algebraic quantities of the first, second, and third degrees denote geometrical magnitudes of one, two, and three dimensions respectively, there must be geometrical magnitudes of four, five, six, etc., dimensions corresponding to algebraic quantities of the fourth, fifth, sixth, etc., degree. [*Stallo notes:* It is not unworthy of remark, here, that the practice of reading x^2 and x^3 as x square and x cube, instead of x of the second order or third power, is founded upon the silent or express assumption that an algebraic quantity has an inherent geometric import. The practice is, therefore, misleading, and ought to be disused. *Principiis obsta!*]

It is hardly necessary to say, after all this, that the analytical argument in favor of the existence, or possibility, of transcendental space is another flagrant instance of the reification of concepts."[2570]

Stallo's and Schubert's foreboding is profound, given the absolutist ontology of the special theory of relativity, which soon followed their admonitions to us. James H. Hyslop wrote in 1896

"THE FOURTH DIMENSION OF SPACE.

MR. SCHILLER'S summary of the discussion on this subject in the March number of this REVIEW indicates very clearly that the advocates of a fourth dimension latterly show a decided tendency to withdraw from some of their original claims, but it omits to notice a matter of very considerable importance in the problem which has received very scant attention on the part of the defenders of the doctrine, and has not been developed by its opponents, whose arguments often imply it. I allude to the purely logical principles at the basis of the matter. That these must first be satisfied, I think, is shown by several facts: (*a*) the tendency to abandon certain arguments in the case; (*b*) the absence of all deductive proof for a fourth dimension; (*c*) the want of data in experience to make the claim inductively rational; (d) the dependence upon analogies and symbolic conceptions as evidence.

But I shall waive all proof of the claim here made and allow the discussion itself to show its truth. The first step is to consider the general grounds upon which the doctrine is supposed to rest, as stated by some of its ablest advocates. They are: (*a*) the empirical nature of the Euclidean axioms; (*b*) the relativity of knowledge in general, shutting out a dogmatic denial of the hypothesis; (*c*) the Kantian doctrine of space, which, though it may prove the inconceivability (non-imaginable nature) of a fourth dimension, supports its possibility beyond the limits of experience; (*d*) the necessities of non-Euclidean geometry, especially for pseudo-spherical surfaces.

The first thing to be said regarding these arguments is that, if the laws of logic have first been respected, they may be entitled to some weight, but if these laws have been violated, the arguments can count for nothing. Hence I wish to call attention to certain irrelevancies in them, in order to show how the prior conditions of all intelligible discussion in this problem are certain logical principles that reveal very clearly where the confusion originates in the controversy. This irrelevancy is that which connects the question with the problems about empiricism, intuitionism, transcendentalism, realism, idealism, etc. These, in fact, have nothing to do with the matter until after we know the logical terms of the problem. In all cases we have to do with certain conceptions which carry with them the same implications *logically*, whether we choose to regard them as real or ideal, objective or subjective, empirical or intuitive. What I have to consider, therefore, is the logical use made of the conceptions ‘space,’ ‘property,’ ‘dimension,’ ‘mathematics,’ etc., in the attempt to prove a fourth dimension.

Now I shall first state a few simple logical principles upon which I shall proceed, and which determine the limits of legitimate reasoning in this problem. They are perfectly familiar laws to the logician, but seem to be wholly ignored by mathematicians. They are summarized in this one proposition: *The transfer of predicates and implications from one conception to another is limited to a qualitative identity between them.* This can be clearly illustrated by reference to the relation between certain conceptions and certain tendencies in the growth of knowledge.

Concepts express certain definite relations between genus and species, and between different species. We may express this generally by the formula that their extension varies inversely with their intension. In common parlance, this is only to say that the number of individuals denoted by the genus is greater than the number denoted by the species, while the number of qualities denoted by the species is greater than that denoted by the genus. It is not necessary here to assert or defend the *absolute* universality of this rule, but only that it is unquestionable in a certain class of conceptions, and these are the conceptions with which we have to deal in our present discussion. Now the plain simple rule here is that we can never transfer the differential predicates of the species to the genus, and also that general formulas have to be modified to suit the differentia of the species. For example, I cannot transfer the differential quality expressed by ‘Caucasian’ over to the concept ‘man,’ and I cannot express the meaning of ‘Caucasian’ by stopping with the predicates of the term ‘man.’ These are simple truisms, but they get great importance in connection with discussions that violate them, owing to the additions made to knowledge by intellectual progress.

The development of knowledge involves two different changes in conceptions. They may be widened or they may be narrowed in their import. These two processes are known to the logician as generalization and specialization. Until the new meaning becomes the only and fixed import of the term, it gives rise to equivocation. In this way an interchange of predicates and implications will occur, and often unconsciously. But this is the illusion for which intelligent men are required to be on the alert. This difficulty, however, is greatly increased by the several ways in which concepts may grow in denotation and meaning. First, concepts may increase or decrease in nothing but quantitative import. Secondly, they may increase or decrease only in qualitative import. Thirdly, quantitative and qualitative import may vary in an inverse ratio with each other. Thus the first of these processes occurs when a new individual or species is added to the genus, or an old one withdrawn, without affecting the conferentia (common qualities) expressed by it. Here the change does not affect the transfer of predicates. It is purely quantitative, and this is the peculiarity of all purely mathematical concepts. In the second process the change occurs when a new quality is added, or an old one withdrawn from a concept, without changing its quantitative import or extension. This change also does not affect the truth or universality of old propositions, and a transfer of predicates will not take place. No equivocation, however, will occur. But it is the third form that causes all the trouble. In this the extension may increase at the expense of the intension and *vice versa*. This occurs when a new species is added to a genus so as to decrease the intension, or a species withdrawn so as to increase the intension. In such cases the transfer of predicates cannot take place. Or, to summarize the discussion, when conceptions change quantitatively, but not qualitatively, the transfer of predicates can be made with perfect logical impunity. When they change qualitatively, but not quantitatively, new predicates are added which are differentially distinct from the old ones, but there is no occasion for a transfer. But when quantitative and qualitative import vary inversely, a transfer of predicates cannot be assumed without proof. Now, since mathematics is limited to the quantitative concepts or qualities, and logic extends to both quantitative and qualitative meanings of terms, it is apparent how they come into relation with each other, and how a habit contracted in the quantitative determinations of mathematics may pass over to cases where the changes are qualitative as well. In mathematics we either do not deal at all with genus and species, but with whole and part, which are qualitatively identical; or, if we call the broader and narrower concepts 'genus' and 'species,' they are still qualitatively of the same import. But in logic, besides whole and part we deal with genus and species, which are qualitatively different from each other. The consequences of this may be brought out by illustration.

The instance is taken from the fluctuations in the conception 'metal.' In physics and chemistry brass and bronze are not metals; in common parlance they are. Now in scientific usage I can say, 'All metals are elements'; in common parlance I cannot say it, because brass and bronze are compounds. Here, with the extension of the term 'metal,' I cannot carry the predicate of its narrower import with me. With this increase of extension, 'element' becomes the differentia of a species. Hence in any case where we undertook to define the differential quality of brass and bronze, we should have to call it non-elemental, not having any right to use the term 'element' to describe it, unless it also be generalized. On the other hand, the same process is illustrated by another interesting generalization of the same term. At one time it was assumed that a specific gravity greater than water was an essential property of metals. It was conceived as essential to a metal that it sink in water. This conception excluded at least three of the alkali metals, potassium, sodium, and lithium. But the discovery that these substances possessed metallic lustre and probably other metallic properties, resulted in extending the class 'metals' to include them while diminishing the conferentia, and this in spite of the fact that their specific gravity is *less* than water. Now we have here a generalization of the term 'metal' in which we cannot carry with us the old proposition, 'All metals sink in water.' This relation now becomes the differentia of a species, and is no longer a conferentia. If the reverse process had taken place, it would have been necessary to have added a new predicate to the species.

The value of these principles will be apparent in the examination of the argument for a fourth dimension, most especially as it appears in Helmholtz' celebrated articles in *Mind*,[*Footnote:* Vol. I, p. 301; vol. III, p. 212.] which have done more than anything else to make philosophers take the subject seriously. The first illusion of which he and mathematicians generally have been the victims, is not one which comes under the principles just enunciated, but is nevertheless an important weakness in their argument. It is the transference to the conception of space of assumptions and conceptions that are true of material substance. Now the mathematician tells us that geometry deals with the properties of space. Dimension is said to be one of these properties, if not the only one, and as there are admittedly three of these dimensions, the limitations of our empirical knowledge at once suggest the possibility of more of them. The only problem is to produce the facts which will either prove their real existence, or show that they are thinkable and possible. The fact that we know of no limits to the properties of matter, and that discovery constantly shows additions to our knowledge of new properties, forces, or modes of action (the Röntgen rays, for example), or at least new phenomena, stands in good stead to shut off dogmatic denials of other than the known dimensions of space. But it is precisely here that the illusion occurs. The mathematician permits himself to be fooled by words, and pays no attention to their real import. He assumes without criticism that the relation between space and its dimensions is the same as that between matter or a metaphysical substance and its properties. This assumption may be absolutely denied, and I certainly deny the right to make it. The illusion arises first from the language about the ‘properties’ of space, and secondly from identifying ‘properties’ with dimensions, while distinguishing tacitly between space and its ‘properties’ on the one hand, and space and its dimensions on the other. Metaphysical realities, subjects or substances, like matter, spirit, ether, etc., may have any number of properties, known and unknown. But we have no *a priori* right to carry this possibility over to space, because no one entertains for a moment the supposition that it is a metaphysical substance like matter or other reality. It is qualitatively distinguished from such conceptions. It may be that space possesses an indefinite number of properties, but we can neither assume the fact or possibility from what we hold to be true of matter, mind, and other subjects or substances, nor assume that we can treat the conception of space in the same way. We have to prove on other grounds that the conception of space is subject to the same treatment. What I contend for is, that we cannot logically pass, as the mathematicians do, from one of these conceptions to the other, and that propositions in the two cases, notwithstanding their formal resemblances, do not have the same meaning and implication unless proved on other grounds than this formal identity; so that the very first step in the argument for a fourth dimension is vitiated by presumptions which have no right to exist.

The whole problem of the advocates of a fourth dimension is to find a basis for non-Euclidean geometry. Euclidean geometry is admittedly based upon the three dimensions, and they assume that this new kind of geometry requires a new differential principle. They are at least formally correct, according to the principles established regarding the relation between genus and species or between different species. But we must examine what difference they assume to exist between the two kinds of geometry. If the two are the same, the demand for a fourth dimension would be absurd, according to their own admission. If they are different, if non-Euclidean geometry is different from Euclidean, the difference must be either quantitative, or qualitative, or both. If it be merely quantitative, the qualitative principle or condition is the same as the Euclidean; if it be qualitatively different, then the new principle must be a new quality, a new property of space, as the fourth dimension is supposed to be. If the difference be both quantitative and qualitative, then the distinction between Euclidean and non-Euclidean geometry is not absolute, but they interpenetrate in the dimensions determining Euclidean geometry. After ascertaining the alternatives between which we are placed, the only question that remains to determine concerns the conceptions of the problem entertained by non-Euclidean mathematicians. The second alternative is the one maintained; and this with its qualitative distinction between the two kinds of geometry, implies that the fourth dimension must be a new quality or property of space, or qualitatively different from the other dimensions. The first alternative is fatal because it limits the difference to quantity, the qualitative principle remaining the same, so that but one rational course is open to the mathematician, which is to affirm a difference of kind. We start, then, with the assumption that non-Euclidean geometry requires a principle for its basis qualitatively distinct from that of Euclidean geometry. What is the consequence of this step?

The basis of geometry is said to be the 'properties of space.' We may ask what is meant by the 'properties' of space, and this question proposes the problem of determining whether 'space' is synonymous with its 'dimensions,' or may include other 'properties' than dimension, and whether its 'properties' are the same as its dimensions. This problem ought first to be solved by the non-Euclidean geometer before he takes any other step. But I know of no attempt to do this. He has two alternatives. He may limit the intension of space to the dimensions, or he may extend it to include other properties than dimension, such as penetrability and divisibility or indivisibility. (I hold that space is absolutely indivisible, though it is usually spoken of as divisible. In reality it is body that is divisible.) Now if space denote or imply other properties than dimension, we may ask what evidence is there that the so-called 'fourth dimension' is a dimension at all. The non-Euclideans agree that their geometry is based upon the 'properties of space.' This limits them to two alternative conceptions, assuming that the two geometries must be distinguished. Either 'space' denotes other properties than dimension, or in being limited to dimension we must suppose, as they do, that the fourth dimension is qualitatively different from the other three. The supposition that the 'fourth dimension' is different in kind from the other three, and at the same time that space denotes only the three dimensions, would imply that non-Euclidean geometry is non-spatial; that is, not based upon space at all, which is contrary to the original assumption. But, taking the two conceptions just mentioned, it should be noticed that the first may justify us in selecting some other property than dimension for the basis of non-Euclidean geometry. What reason have the non-Euclideans for distinguishing between the fourth dimension and some other property not a dimension at all, especially as they admit that this new 'dimension' cannot be pictured or represented in experience? Taking the second alternative, we find that a generalization either of the term 'space' or of the term 'dimension' has been made. If of the term 'space,' the 'fourth dimension' either becomes a non-dimensional property, or the basis of geometry has been altered in its conception, which might enable us to take any quality of anything as the principle of non-Euclidean geometry.

Let me make the case clearer by another form of statement. If we assume the qualitative difference between Euclidean and non-Euclidean geometry, there are four conceptions of space to be considered, three of them absolutely necessary to satisfy this assumption: (I) Space = three dimensions ; (2) space = three plus the fourth dimension or *n* dimensions; (3) space = three dimensions plus other properties ; (4) space = four or *n* dimensions plus other properties.

Taking space in the first of these three conceptions, the fourth dimension must make non-Euclidean geometry nonspatial, which is contrary to the supposition. On the third conception, the principle of non-Euclidean geometry is not a dimension, but some other property. Assuming the fourth conception, the non-Euclidean geometer must show the distinction to be made between the fourth dimension and other properties, especially that this dimension is qualitatively different from the other three. If not qualitatively different, non-Euclidean geometry falls to the ground as anything more than a modification of Euclidean geometry. This leaves, as the only alternative for the non-Euclidean, the second, which is the conception, and the only conception, of space that can present even a plausible claim in favor of a fourth dimension for the principle of non-Euclidean geometry.

Now, in regard to this second conception of space, the first remark is that it is an extension of the meaning involved in the first. But passing this by as unimportant, though necessary to non-Euclidean geometry, the second remark is that the term 'dimension' is either generalized in its import qualitatively, or it is a name to denote a non-dimensional property. The only other alternative is to hold that the three dimensions and the fourth are not different from each other. I want, therefore, to show the logical consequences to the doctrine from each one of these alternatives.

The assumption is that the fourth dimension is qualitatively different from the other three dimensions. It is, therefore, a species in contradistinction to them as other species. Now, when the term 'dimension' includes all of them, it denotes a common property, the conferentia, or genus; and cannot be used to denote the species. This would be in violation of the principle of logical division, which is that the same conception cannot denominate both genus and species. Assuming that it denotes only the genus, or common quality of all the dimensions, we find that both Euclidean and non-Euclidean geometry are based upon the same quality of space, which is contrary to the supposition. On the other hand, if it denote only a species, it must be limited either to the three dimensions or to the fourth, if a qualitative distinction between them is to be maintained. If limited to the three, then it is not legitimate to call the 'fourth dimension' a dimension at all, and non-Euclidean geometry would be based upon a non-dimensional property, say penetrability or indivisibility, which is contrary to the original supposition.

If it be limited to the fourth, then the other three are not 'dimensions' properly considered, and Euclidean geometry would be non-dimensional, which is also contrary to the supposition. The only alternative left is to apply the term equally to all four dimensions. But this identifies them qualitatively and breaks down the distinction between Euclidean and non-Euclidean geometry, which again is contrary to the supposition, unless we go outside of space altogether for the basis of the latter, which again contradicts the first assumption. Such a fatal set of dilemmas could hardly have been suspected on a first glance at the controversy; but they are there as long as we use the word 'dimension' in the case, and distinguish qualitatively between Euclidean and non-Euclidean geometry.

The fundamental fault of the mathematicians has been in extending the meaning of the term 'dimension' by adding a new species and calling it by the same name as the old. This mistake never occurs in the natural sciences. When a new species is discovered, increasing the extension of the genus, a new name must be adopted expressing the differentia by which this species is distinguished from the others. If the fourth dimension be a new species qualitatively different from the others, it should either not be called a dimension at all, or something should be indicated to determine the differentia by which it is presumably differentiated from the others. We may generalize the term 'dimension' if we choose, but we must not carry with it the differentia which separates the species; and we are equally forbidden to employ the same term for the species. The reply to this criticism would be that the differentia is expressed in the number of the dimension, and this reply is formally legitimate. But it is fatal in two respects to the hypothesis of a new dimension qualitatively determined. First, if number be the differentia of the species, it is purely quantitative, and the basis of non-Euclidean geometry is not qualitatively distinguished from the Euclidean. Secondly, if the conception 'fourth,' *i.e.*, number, determines a qualitative differentia, then the first, second, and third dimensions should be qualitatively different from each other, which is contrary to the supposition of Euclidean geometry. They are assumed to express the same commensurable quality, while their supposed differences are only relations of direction from a given point.

The language easily lends itself to an illusion, because it is formally the same as that in which qualitative differences are actually expressed or implied. But in mathematics our first duty is to remember that our conceptions are primarily quantitative, and that when we go beyond purely quantitative distinctions we are transcending mathematics altogether.

What I have said here about the illusory nature of the language in the case is beautifully illustrated in the expression, 'Space has dimension.' This proposition resembles the ordinary intensive judgment (such as 'Man is wise,' where it is possible to have other predicates in the same subject) only when we conceive the subject, space, as possibly having other properties than dimension; but when the term 'space' is made convertible with 'dimension,' as is usually or always the case in mathematics, we should either not assume that ‘Space *has* dimension,’ or when using the phrase we should recognize logically its true import, namely, that ‘Space *is* dimension.’ For geometry, space and dimension are the same, and hence in reality to assert the existence of a fourth dimension is equivalent to saying that the three dimensions have a fourth or *n* dimension, or that the three dimensions are four or *n* dimensions. The absurdity of this is apparent, but it is concealed by the formal correctness of the proposition, ‘Space has properties,’ or, ‘Space has dimension.’ But the moment we see that, for geometry, space and its dimensions are the same, we are forced to recognize that the fourth dimension becomes a predicate of the other three dimensions, which is contrary to the supposition of non-Euclidean geometry.

We are now prepared to examine some concrete fallacies and illusions of the same kind committed by Helmholtz in the celebrated articles in *Mind* already referred to, on the ‘Origin and Meaning of Geometrical Axioms.’ His argument here is to prove the empirical nature of geometrical axioms, and thus to avail himself of the inference, which the limitations of empiricism justify, that there are possibly other data in existence than the three known dimensions. In order to establish this empiricism, he undertakes to show that the axioms do not have the universal and necessary application which they are supposed to have. In this procedure he is half conscious of the principle that I have here laid down about the impossibility of transferring differential predicates when an increase in the extension of our concepts takes place, and the force of his argument derives all its influence from the truth of this principle. But he immediately violates the principle by equivocations which are due to specializing terms without reckoning with the logical consequences of the act. Let us examine his procedure briefly.

He calls attention to the assumed universality of the axiom about a straight line being the shortest path between two points, only to show that it is not true to a being living on a curved surface, to whom a *curved* line is the shortest distance between two points. This fact is supposed to set aside the universality of the Euclidean axiom. But there is a curious illusion in this claim which can be dispelled in two ways. In the first place, there is an equivocation in the word ‘shortest.’ Mathematically speaking, the Euclidean axiom still remains true to any being living on a spherical surface, though it may not be *physically* true. Even if it be assumed that such a being could not move directly at all from one point to the other, the distance physically and temporally the shortest to him would be a curved line, but this truth has nothing in it to contradict or modify the Euclidean axiom which still remains true mathematically where we have to do with *pure* space relations and not with qualities other than the spatial. Secondly, if the being living on the sphere *knew* that this surface was curved, it would recognize the Euclidean axiom, and, if influenced by any economic motives prevalent about walking on the diagonals of street corners, would sigh for the *physical* capacity to conform to mathematical principles. But if it did not know that the surface was a curved one, *it could not draw any distinction between a straight and a curved line*. Its mathematical and physical conceptions of 'shortest' would coincide, so that *straight* and *curved* would mean the same thing, and the Euclidean axiom would still remain. But Helmholtz happens to know the difference between mathematical space and physical body, and by an equivocation in the use of 'shortest' can obtain an apparent limitation to this axiom, when applying it from the standpoint of his own assumed knowledge compared with that of a being supposed to be ignorant of his point of view. But the equivocation does not help the matter, and the ignorance of the other being does not interfere with the truth of the Euclidean axiom.

A long examination of another instance by Helmholtz, impeaching the universality of the proposition that the sum of the angles of a triangle is equal to two right angles, might be given, but it is sufficient to take note of two omissions in order to vitiate the conclusion that he wishes to draw from his result. In the first place, he confuses two different degrees of extension in the use of the term 'triangle,' one limited to plane and the other including spherical triangles, which shows only that the universality of a proposition is never intended to extend beyond its subject. The proposition about the sum of the angles remains forever true within these limits, and Helmholtz forgets that the language, while it may include spherical triangles, is *conceived* by the mathematician concretely to mean plane triangles. He can also obtain a universal proposition for both. Secondly, Helmholtz fails to see that, although a modification of the formula or principle in this proposition is required to meet the conditions of a new species, this modification is purely *quantitative*, not qualitative, and hence the analogy lends no support to the qualitative difference implied or asserted in the fourth dimension as the basis of the relations in pseudo-spherical surfaces. There is an illusion also in assuming or insinuating that pseudo-spherical surfaces are more than quantitatively different from plane and spherical surfaces, so far as commensurable quality is concerned.

The effect of the equivocation in the use of the word 'dimension' is apparent in another way, to which attention must be called. If there is anything upon which mathematicians and mankind generally are agreed, it is that space has at least three dimensions, Euclidean geometers and most others holding that it has *only* three dimensions. But I think both can be denied, without favoring the contention of non-Euclidean mathematicians that there is a fourth dimension in any sense in which they are understood to affirm it. In denying the existence of three dimensions, we have two alternative affirmative propositions, both of which may be true if we assume two meanings for the term 'dimension.' They are: (1) that space has only *one* dimension; (2) that it has an indefinite or infinite number of dimensions. This claim is borne out by the fact that, when we speak of space as having 'dimension,' we express a single quality which is divided up into 'three dimensions,' without implying that the species are qualitatively different from their base, but are only relations of the same quality to different points of view. In fact the 'three dimensions' are properly defined and reducible to *commensurable quality* in which the units are always the same in each dimension. The three dimensions, therefore, cannot qualitatively differ from this without losing their commensurable nature. Why, then, are they called 'dimensions,' as if they were species of a genus? The answer to this question must be, either that the term is illegitimate altogether, or that it expresses only certain quantitative relations having mathematical convenience in the mensuration of bodies. Both alternatives are fatal to the supposition of a fourth 'dimension' in a qualitative sense without either going outside the meaning of dimension as denoting commensurable quality, or going outside the conception of space, which are both contrary to the supposition of non-Euclideans.

The supposition that there are three dimensions instead of one, or that there are only three dimensions, is purely arbitrary, though convenient for certain practical purposes. Here the supposition expresses only differences of relation; that is, *differences of direction from an assumed point*. Thus, what would be said to lie in a plane in one relation, would lie in the third dimension in another. There is no way to determine absolutely what is the first, second, or third dimension. If the plane horizontal to the sensorium be called plane dimension, the plane vertical to it will be called solid, or the third dimension, but a change of position will change the names of these dimensions without involving the slightest qualitative change or difference in meaning. Moreover, we usually select three lines or planes terminating vertically at the same point, the lines connecting the three surfaces of a cube with the same point, as the representatives of what is meant by three dimensions, and reduce all other lines and planes to these. But interesting facts are observable here. (I) If the vertical relation between two lines be necessary for defining a 'dimension,' then all other lines than the specified ones are either not in any dimension at all, or they are outside the three given dimensions. This is denied by all parties, which only shows that a vertical relation to other lines is not necessary to the determination of a dimension. (2) If lines outside the three vertically intersecting lines still lie in dimension, or are reducible to the other dimensions, they may lie in more than one dimension at the same time, which after all is a fact. This only shows that qualitatively all three dimensions are the same, and that any line outside of another can only represent a dimension in the sense of *direction* from a given point or line, and we are entitled to assume as many dimensions as we please, all within the 'three dimensions.'

This mode of treatment shows the source of the illusion about the 'fourth dimension.' The term in its generic import denotes commensurable quality and denotes only one such quality, so that the property supposed to determine non-Euclidean geometry must be qualitatively different from this, if its figures involve the necessary qualitative differentiation from Euclidean mathematics. But this would shut out the idea of 'dimension' as its basis, which is contrary to the supposition. On the other hand, the term has a specific meaning, which, as different qualitatively from the generic, excludes the right to use the generic term to describe them differentially, but if used only quantitatively, that is, to express *direction*, as it in fact does in these cases, involves the admission of the *actual*, not a supposititious, existence of the fourth dimension, which again is contrary to the supposition of non-Euclidean geometry. Stated briefly, dimension as commensurable quality makes the existence of a fourth dimension a transcendental problem, but as mere direction an empirical problem, and the last conception satisfies all the requirements of the case, because it conforms to the purely quantitative differences which exist between Euclidean and non-Euclidean geometry, as the very language about 'surfaces,' 'triangles,' etc., in spite of the prefix 'pseudo,' necessarily implies. If the difference be made qualitative, neither the conception of direction will satisfy the case, because this is quantitative, nor that of dimension, because the fourth dimension would have to be *non*-dimensional. The simple illusion of Helmholtz lies in the confusion of dimension, now denoting commensurable quality, with direction, now denoting certain quantitative relations, and he merely carries this confusion over to the 'fourth dimension,' with the implications of transcendentalism in its qualitative differentiation from the others.

Why Helmholtz should have been guilty of this confusion it is hard to say, when we remember his own conception of the basis of geometry. In the very article above referred to, he says: 'In conclusion, I would again urge that the axioms of geometry are not propositions pertaining only to the pure doctrine of space. As I said before, they are concerned with quantity.' If geometry can be based upon the notion of quantity as well as space quality, he ought to have seen at once that his 'fourth dimension' did not require to be a new quality, but only a new quantitative relation of the one quality of space, which it in reality is. Distinguish between 'dimension' as commensurable quality and the use of the term to denote directional relations, and the problem is solved. The fourth and even *n* 'dimensions' can be admitted as empirical *facts*, and there will be no necessity for showing the empirical nature of geometrical axioms, in order to obtain an *a priori* presumption, from the limitations and indefinite capacities of experience, in favor of a possible existence for transcendental properties of space.

There is one more illusion growing out of this confusion of 'dimension' with direction. It relates to the movements of points, lines, and figures, assumed by mathematicians in representing the various relations expressed by Euclidean space. The motion of a point is said to produce a line in one dimension; the motion of a line about one end produces a plane, and the motion of a plane about one of its sides will produce a solid, or the third dimension. The 'fourth dimension' is demanded for a certain motion of a solid! But we may say first that, in mathematical parlance, a point cannot be made to move, nor can a line or a plane. Only bodies can move. This may be admitted to be quibbling, but it calls attention to the fact that, if mechanical motion is to determine the matter of dimension, the motion of a 'point,' or ‘atom,’ must be in more than one ‘dimension’ at a time. A solid, being in three dimensions, will move in them, and, if it gets out of them, will either not be a solid at all, or, if it is in the ‘fourth dimension,’ we should require a transcendental physics as the basis of non-Euclidean geometry, and this is not in the contract of the mathematician, but only a new property of space. But to dismiss quibbling, if we accept the fact that the dimensions can be constructively represented as described, why assume that a point can move only in one dimension, a line in two, and a plane in the third? From what has been said about the relative and interchangeable nature of the dimensions, any one being the other according to point of view, and from the fact that the motion of a point must pass *through* what is called the third dimension and also exists in a plane at the same time, it is evident that even a moving point must imply all three dimensions. It cannot move in all three *directions* at the same time, but the whole commensurable quality of space is implied by the existence of a point, a line, and a plane, as well as a solid. Hence geometry, constructive and symbolic, is based, not upon dimensions as commensurable quality, but upon dimensions as directions, and in this way creates no presumptions in favor of any new commensurable quality. To argue for it is simply one of those equivocations which ought not to deceive a common schoolboy, not to say anything of men with the reputation of Helmholtz and Riemann.

Several other similar illusions might be pointed out, such as Helmholtz’ language about *flat space* and *curved space*, but I shall not discuss them here. They are either a confusion of the abstract with the concrete, or of quantitative with qualitative logic; and after our lengthy exposure of this latter all-pervading fallacy, it is not necessary to do more than to reiterate the one important rule that qualitative differences can never be expressed by the same term, so that all this discussion about a fourth dimension is simply an extended mass of equivocations turning upon the various meanings of the term ‘dimension.’ This, when once discovered, either makes the controversy ridiculous or the claim for non-Euclidean properties a mere truism, but effectually explodes the logical claim for a new dimensional quality for space, as a piece of mere jugglery in which the juggler is as badly deceived as his spectators. It simply forces mathematics to transcend its own functions as defined and limited by its own advocates, and to assume the prerogatives of metaphysics. With the non-Euelideans it would become a science of quality as well as, or instead of, quantity, and would hardly stop with Helmholtz’ empiricism for an argument in favor of its transcendental ‘dimension.’

I have intended this exhaustive logical criticism as a precaution against a great deal of crazy metaphysics which might support itself upon the authority of men like Helmholtz and Riemann. Occultism simply revels in the doctrine of a fourth dimension, and is absolved from the duty of proving it *in se* by the authority of presumably sane scientific men; and while it may be sufficient simply to laugh at the pretensions of the occultist, and while it only dignifies his speculations seriously to consider them, there are some at least quasi-genuine phenomena which throw the reins to madhouse theories, when both parties soberly discuss the claims for a fourth dimension and remain wholly ignorant of the logical principles, which not only vitiate the argument for the existence, or even possibility, of this 'dimension,' but make the talk about it mere child's play. In taking this position, however, it is not necessary to deny the fact of other than the known properties of existence, nor to deny that there is more than is dreamt of in any of our philosophies, but only that the logical terms of the problem take us wholly beyond the limits of geometry and mathematics for our 'metadimension.' Not only must we distort and change our conception of space, but we require equally to modify that of geometry and mathematics, so that they cease to deal with mere quantity and are made to share the precarious fortunes of metaphysics. We may take this course if we like, but our science would lose its much boasted certitude by the change, and would very soon turn into a fool's paradise. We cannot limit mathematics by definition to the consideration of pure quantity, and then introduce into our data qualitative differentials which bear no quantitative import but the name. If we do this, the futility of our procedure is only concealed by one of the simplest of illusions, unless it is our distinct purpose to base mathematics upon a system of metaphysics which is as fanciful as wonderland. An equivocation is a poor compass, when we set out on Kant's shoreless ocean in search of a harbor, and, if we discover its character before we make the venture, we shall be all the wiser for it. But without equivocation we can in no case accomplish any more than the man in Mother Goose, who 'ran fourteen miles in fifteen days and never looked behind him,' only to find in the end that he was just where he had started."[2571]

Edward H. Cutler succinctly stated in 1909,

"The fourth dimension has no real existence in the sense in which the external world that we know by means of our senses has real existence. It is a philosophical and metaphysical conception, whose actual existence cannot be demonstrated by observation or by logical reasoning."[2572]

Manning and Whitrow cite Michael Stifel, in 1553, and John Wallis, in 1685, as stigmatizing the conjecture of a fourth or higher dimension, as being *unnatural*, an expression with religious implications in those times.[2573]

Aristotle, in contrast to Stewart and Tait, argued for a limitation of three, his favorite number, dimensions,

"The line has magnitude in one way, the plane in two ways, and the solid in three ways, and beyond these there is no other magnitude because three are all [***] There is no transfer from length to area and from area to a solid."[2574]

And then there was Ptolemy,

> "The admirable Ptolemy in his book *On Distance* well proved that there are not more than three distances, because of the necessity that distances should be taken along perpendicular lines, and because it is possible to take only three lines that are mutually perpendicular, two by which the plane is defined and a third measuring depth; so that if there were any other distance after the third it would be entirely without measure and without definition. Thus Aristotle seemed to conclude from induction that there is no transfer into another magnitude, but Ptolemy proved it."[2575]

Galileo questioned on what basis Aristotle drew his conclusion, but did not really dispute it.

Not only did Albert Einstein not originate the idea of space-time, he initially strongly opposed it. Einstein, together with Jakob Laub, denounced Minkowski's recitation of Poincaré's four-dimensional interpretation of the Lorentzian æther, in 1908, in a paper fraught with mistakes.[2576] It wasn't until it was made clear to Einstein that Poincaré's quadri-dimensional interpretation of Lorentz' quasi-rigid æther could be exploited to arrive at Paul Gerber's 1898 formulation of gravitation, that Einstein ended his attack on it, and instead copied it in the general theory of relativity of 1915—though, predictably, Einstein failed to cite either Poincaré or Gerber.[2577]

In 1930, Einstein effectively admitted that he did not originate the special theory of relativity, though he wrongly attributes the theory's basis to an undeserving Minkowski. Einstein stated,

> "The next step in the development of the concept of space is that of the special theory of relativity. The law of the transmission of light in empty space in connection with the principle of relativity with reference to uniform movement led necessarily to the conclusion that space and time had to be combined in a unified four-dimensional continuum. For it was recognized that nothing real corresponded to the inclusive concept of all simultaneous events. As MINKOWSKI was the first to see clearly, this four-dimensional space had to be regarded as possessing a Euclidean metric which was quite analogous to the metric of the three-dimensional space of Euclidean geometry with the use of an imaginary time-coordinate."[2578]

Einstein, by his own definitions, did not achieve the special theory of relativity in 1905, and instead, when first made aware of it, he opposed it! Poincaré created the theory, and Einstein repeatedly stole credit for it and wrongfully gave Minkowski credit for many of Poincaré's ideas. Each element of Einstein's argument as to what constitutes the uniqueness of the special theory of relativity was stated by Poincaré before Einstein and Minkowski.

Minkowski noted that Lorentz *and Einstein* believed in absolute space, "Neither Einstein nor Lorentz made any attack on the concept of space[.]"[2579]

Einstein's admission that the æther of relativity theory is analogous to Lorentz' æther is an admission that Lorentz holds priority on the formalism of the theory, and, further, that Einstein felt forced to switch camps from that of Lorentz to that of Poincaré, in 1916, much after the 1905 paper appeared, to a theory which Einstein, himself, together with Jakob Laub, had denounced in 1908, only to admit in 1920 that this "absolute world" of Minkowski "space-time" resulted again in Lorentz' æther. As Einstein stated,

> "According to the general theory of relativity space without ether is unthinkable; for in such space there not only would be no propagation of light, but also no possibility of existence for standards of space and time (measuring-rods and clocks), nor therefore any space-time intervals in the physical sense."[2580]

Relativists would counter this citation by pointing out that Einstein's æther differs from that of Lorentz in that it is ultimately vague, a word without meaning, and no supposition is made as to its fundamental properties, such as the assertion that the æther may be an ideal fluid of particles immersed in a void of empty space.[2581] Einstein denied it the property of "motion", an assertion made many decades earlier by Philipp Spiller in a much read work.[2582] However, this argument over semantics is one made against a straw man, for Lorentz stated as early as 1895,

> "It does not suit my purpose to examine more thoroughly such speculations, or to express presumptions about the nature of the æther. I merely wish, as far as possible, to free myself of all preconceived notions regarding this substance and not to ascribe to it, for example, any of the qualities of ordinary liquids and gasses. Should it be shown, that a description of the phenomena is best arrived at through the assumption of absolute permeability, then one must surely in the meantime adopt this sort of hypothesis, and leave it to further research, if possible, to open up a deeper understanding to us."

> "Es liegt nicht in meiner Absicht, auf derartige Speculationen näher einzugehen oder Vermuthungen über die Natur des Aethers auszusprechen. Ich wünsche nur, mich von vorgefassten Meinungen über diesen Stoff möglichst frei zu halten und demselben z. B. keine von den Eigenschaften der gewöhnlichen Flüssigkeiten und Gase zuzuschreiben. Sollte es sich ergeben, dass eine Darstellung der Erscheinungen am besten unter der Voraussetzung absoluter Durchdringlichkeit gelänge, dann müsste man sich zu einer solchen Annahme einstweilen schon verstehen und es der weiteren Forschung überlassen, uns, womöglich, ein tieferes Verständniss zu erschliessen."[2583]

Compare this with Schubert's views,

> "In mathematics, in fact, the extension of any notion is admissible, provided such extension does not lead to contradictions with itself or with results which are well established. Whether such extensions are necessary, justifiable, or important for the advancement of science is a different question. It must be admitted, therefore, that the mathematician is justified in the extension of the notion of space as a point-aggregate of three dimensions, and in the introduction of space or point-aggregates of more than three dimensions, and in the employment of them as means of research. Other sciences also operate with things which they do not know exist, and which, though they are sufficiently defined, cannot be perceived by our senses. For example, the physicist employs the ether as a means of investigation, though he can have no sensory knowledge of it. The ether is nothing more than a means which enables us to comprehend mechanically the effects known as action at a distance and to bring them within the range of a common point of view. Without the assumption of a material which penetrates everything, and by means of whose undulations impulses are transmitted to the remotest parts of space, the phenomena of light, of heat, of gravitation, and of electricity would be a jumble of isolated and unconnected mysteries. The assumption of an ether, however, comprises in a systematic scheme all these isolated events, facilitates our mental control of the phenomena of nature, and enables us to produce these phenomena at will. But it must not be forgotten in such reflexions that the ether itself is even a greater problem for man, and that the ether-hypothesis does not solve the difficulties of phenomena, but only puts them in a unitary conceptual shape. Notwithstanding all this, physicists have never had the least hesitation in employing the ether as a means of investigation. And as little do reasons exist why the mathematicians should hesitate to investigate the properties of a four-dimensioned point-aggregate, with the view of acquiring thus a convenient means of research."[2584]

Though Schubert allowed for mathematical speculation—useful fictions, he opposed pretending that such four-dimensional fantasies be taken to signify a reflection of physical reality,

> "The high eminence on which the knowledge and civilization of humanity now stands was not reached by the thoughtless employment of fanciful ideas, nor by recourse to a four-dimensional world, but by hard, serious labor, and slow, unceasing research. Let all men of science, therefore, band themselves together and oppose a solid front to methods that explain everything that is now mysterious to us by the interference of independent spirits. For these methods, owing to the fact that they can explain everything, explain nothing, and thus oppose dangerous obstacles to the progress of real research, to which we owe the beautiful temple of modern knowledge."[2585]

Wölffing wrote in 1897,

> "It has also been suggested that the vainly sought after fourth dimension is to be found in *time*, whereby Kinematics (Kinetics) transforms into a four-dimensional Geometry. This is incorrect because time has nothing in common with and (pursuant to this viewpoint) interchangeable with the remaining dimensions; nevertheless, time can be used to advantage to produce four-dimensional bodies from three-dimensional ones."

> "Man hat auch in der *Zeit* die vergeblich gesuchte vierte Dimension zu finden geglaubt, wodurch sich die Kinematik (Bewegungslehre) in eine vierdimensionale Geometrie verwandelt. Richtig ist dies deshalb nicht, weil die Zeit nichts mit den übrigen Dimension gleichartiges und (je nach dem Standpunkt) vertauschbares ist; immerhin kann die Zeit bei der Erzeugung der vierdimensionalen Körper durch dreidimensionale mit Vorteil Verwendung finden."[2586]

Archbishop Tillotson preached that,

> "Others say, God sees and knows future Things by the presentiality and co-existence of all Things in Eternity; For they say, that future Things are actually present and existing to God, though not *in mensura propria*, yet *in mensura aliena*. The Schoolmen have much more of this Jargon and canting Language. I envy no Man the understanding these Phrases: But to me they seem to signify nothing, but to have been Words invented by idle and conceited Men; which a great many ever since, lest they should seem to be ignorant, would seem to understand. But I wonder most, that Men, when they have amused and puzzled themselves and others with hard Words, should call this *Explaining* Things."[2587]

Both Hendrik Antoon Lorentz and Albert Einstein maintained a tri-dimensional privileged frame of "physical space" or "æther", which is the same *physical* hypothesis given two different names. The appellation "æther", which more clearly maintains the concept of a physical entity, is the more fitting title. It was Poincaré, Marcolongo and Minkowski, who incorporated Stallo's quadri-dimensional æther into the theory of relativity, not Albert Einstein. Stallo stated in 1847 in the explicit context of four-dimensional "space-time",

> "The abstract totality of extension in itself is devoid of all internal difference and distinction. It is, from its ideal origin and nature, absolutely *moving*; *but this motion is yet perfectly the same as absolute repose*. For there are no distinct particles as yet successively occupying distinct spaces; in every respect there is thorough homogenousness. We have absolute multiplicity, but a multiplicity intimately and completely blended in extensive continuous unity. It is indifferent to me whether this primitive matter be called *ether*, or

any other name be given it; the only thing important is, to keep this absence of further material differentiation in view."

"Simplified Theory of Electrical and Optical Phenomena in Moving Bodies", *Proceedings of the Section of Sciences, Koninklijke Akademie van Wetenschappen te Amsterdam*, Volume 1, (1899), pp. 427–442; reprinted in K. F. Schaffner, *Nineteenth Century Aether Theories*, Pergamon Press, New York, Oxford, (1972), pp. 255-273.

2410. P. Drude, *Lehrbuch der Optik*, S. Hirzel, Leipzig, (1900); translated into English *The Theory of Optics*, Longmans, Green and Co., London, New York, Toronto, (1902), *see especially* pp. 457-482. On Einstein's ownership of this work, *see: The Collected Papers of Albert Einstein*, Volume 2, Princeton University Press, (1989), pp. 135-136, footnote 13.

2411. E. Cohn, "Zur Elektrodynamik bewegter Systeme", *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften zu Berlin, Sitzung der physikalisch-mathematischen Classe*, (November, 1904), pp. 1294-1303, at 1295. Einstein cited Cohn's paper in his *Jahrbuch* review article of 1907, and a copy of Cohn's 1904 paper is in his preserved collection. *See: The Collected Papers of Albert Einstein*, Volume 2, Note 128, Hardcover, p. 272. Cohn cites the Dutch version of Lorentz' work, "Electromagnetische Verschijnselen in een Stelsel dat zich met Willekeurige Snelheid, Kleiner dan die van het Licht, Beweegt." *Verslagen van de Gewone Vergaderingen der Wis- en Natuurkundige Afdeeling, Koninklijke Akademie van Wetenschappen te Amsterdam* , Volume 12, (23 April 1904), pp. 986-1009. Einstein cites Cohn in the direct context of Lorentz' 1904 paper in:"Über das Relativitätsprinzip und die aus demselben gezogenen Folgerung"*, Jahrbuch der Radioaktivität und Elektronik*, Volume 4, (1907), pp. 411-462, at 413.

2412. *Das Relativitätsprinzip*, Note 2, B. G. Teubner, Leipzig, Berlin, (1913), p. 27.

2413. A. Einstein, "Das Prinzip von der Erhaltung der Schwerpunktsbewegung und die Trägheit der Energie", *Annalen der Physik*, Series 4, Volume 20, (1906), pp. 627-633, at 627. Einstein cites Poincaré's "La Théorie de Lorentz et le Principe de la Réaction", *Archives Néerlandaises des Sciences Exactes et Naturelles*, Series 2, Volume 5, (1900), pp. 252-278, which gives the clock synchronization method at pp. 272-273.

2414. A. Einstein, *Relativity, The Special and the General Theory*, Crown Publishers, Inc., New York, (1961), pp. 150-151.

2415. A. Einstein and J. Laub, "Über die elektromagnetischen Grundgleichungen für bewegte Körper", *Annalen der Physik*, Series 4, Volume 26, (1908), pp. 532-540.

2416. The Eleatic Philosophers. ***See also:*** Ocellus Lucanus, Taurus, Julius Firmicus Maternus, Proclus, in: T. Taylor, *Ocellus Lucanus. On the Nature of the Universe. Taurus, the Platonic Philosoher, on the Eternity of the World. Julius Firmicus Maternus of the Thema Mundi; in Which the Positions of the Stars at the Commencement of the Several Mundane Periods Is Given. Select Theorems on the Perpetuity of Time, by Proclus*, T. Taylor, London, (1831). Numerous Reprints. ***See also:*** Philo Judæus, "On the Eternity of the World", *The Works of Philo*, Hendrickson Publishers, U.S.A., (1993), pp. 707-724. ***See also:*** Moses Maimonides, *The Guide for the Perplexed*, Part 2, Chapters 13 and 14, Dover, New York, (1956), pp. 171-176. ***See also:*** Spinoza, "Concerning God", *Ethics*, Part 1, (1677), Multiple Editions. ***See also:*** I. Newton, *Principia*, Book I, Definition VIII, Scholium; **and** Book III, General Scholium. *Opticks*, Query 31. ***See also:*** S. Clarke, *A Demonstration of the Being and Attributes of God : More Particularly in Answer to Mr. Hobbs, Spinoza and Their Followers*, W. Botham, for J. Knapton, London, (1705); **and** G. W. Leibnitz, S. Clarke, *Collection of PAPERS, Which passed between the late Learned Mr. LEIBNITZ, and Dr. CLARKE, In the Years 1715 and 1716. Relating to the PRINCIPLES OF Natural Philosophy and Religion*, James Knapton, London, (1717). ***Confer:*** Thomas Reid, *Essays on the Intellectual Powers of Man*, Essay III, Of Memory, CHAPTER III, OF DURATION, (1785); in *The Works of Thomas Reid, D.D. F.R.S. Edinburgh. Late Professor of Moral Philosophy in the University of Glasgow. With an Account of His Life and Writings*, Edited by D.

Stewart, Volume 2, E. Duyckinck, Collins and Hannay, and R. and W. A. Bartow, New York, (1822), pp. 132-134. ***See also:*** D. Hartley, "Of the Being and Attributes of God, and of Natural Religion", *Observations on Man, His Frame, His Duty, and His Expectations in Two Parts*, Volume 2, Chapter 1, Printed by S. Richardson for James Leake and Wm. Frederick, booksellers in Bath and sold by Charles Hitch and Stephen Austen, booksellers in London, London, (1749), pp. 5-70. ***See also:*** P. G. Tait and B. Stewart, *The Unseen Universe; or Physical Speculations on a Future State*, Macmillan, London, (1875). ***See also:*** T. L. Nichols, *The Spiritualist Newspaper: A Record of the Progress of the Science and Ethics of Spiritualism*, London, (12 April 1878), p. 175; **and** (19 April 1878), p. 189. ***See also:*** M. Wirth, *Herrn Professor Zöllner's Hypothese intelligenter vierdimensionaler Wesen und seine Experimente mit dem amerikanischen Medium Herrn Slade. Ein Vortrag, gehalten am 25. Oct. und 1. Nov., 1878, im Akademisch-Philosophischen Verein zu Leipzig und als Aufruf zur Parteiergreifung an die deutschen Studenten*, O. Mutze, Leipzig, (1878). ***See also:*** F. Michelis, *Ist die Annahme eines Raumes mit mehr als drei Dimensionen wissenschaftlich berechtigt? Eine an die Addresse des Herrn Professor Dr. Zöllner zu Leipzig gerichtete Frage*, Fr. Wagner'schen Buchh., Freiburg in Baden, (1879). ***See also:*** O. Simony, *Über spiritistische Manifestationen vom naturwissenschaftlichen Standpunkte*, Hartleben, Wien, (1884). ***See also:*** Charles Garner, under the pseudonym Stuart C. Cumberland, *Besucher aus dem Jenseits*, Breslau, (1885). ***See also:*** C. Cranz, "Gemeinverständliches über die sogenannte vierte Dimension", *Sammlung gemeinverständlicher wissenschaftlicher Vorträge*, New Series, Volume 5, Number 112/113, (1890), pp. 567-636; **and** "Die vierte Dimension in der Astronomie", *Himmel und Erde*, Volume 4, (1891), pp. 55-73. ***See also:*** E. Carpenter, *From Adam's Peak to Elephanta: Sketches in Ceylon and India*, MacMillan and S. Sonnenschein, New York, (1892). ***See also:*** A. Willink, *The World of the Unseen; An Essay on the Relation of Higher Space to Things Eternal*, Macmillan, New York, (1893). ***See also:*** F. Podmore, *Studies in Psychical Research*, Putnam, New York, (1897); reproduced by Arno Press, New York, (1975); **and** *Modern Spiritualism: A History and a Criticism*, Methuen, London, (1902). ***See also:*** Schrey von Kalgen, *Dimensionen. Eine neue Weltanschauung. Die Beweis der Zöllnerschen Theorie*, O. Mutze, Leipzig, (1901). ***See also:*** R. J. Campbell, *The New Theology*, Macmillan, New York, (1907). This work drew sharp criticism, and several books were published in an effort to refute it. ***See also:*** W. W. Smith, *A Theory of the Mechanism of Survival: The Fourth Dimension and its Applications*, Kegan Paul, London, (1920).

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2418. With respect to psychologists and their equating of time with space, W. Smith stated in 1902, “Kant speaks of time as a line; **and** psychologists are learning to regard time as a projection at right angles to the plane of the present. But that this spatiality is essential to the time-concept has not been, in general, recognized.” in “The Metaphysics of Time”, *The Philosphical Review*, Volume 11, Number 4, (July, 1902), pp. 372-391. G. F. Stout stated, in 1902, “Psychologists generally hold the same type of theory for the two cases of space and time cognition, and the indications of individual views given under Extension (q. v.) hold largely also for time.” *Dictionary of Philosophy and Psychology*, Volume 2, Macmillan, New York, London, (1902), p. 705.

2419. W. Smith, “The Metaphysics of Time”, *The Philosophical Review*, Volume 11, Number 4, (July, 1902), pp. 372-391. Smith references F. A. Lange, *Logische Studien, ein Beitrag zur Neubegründung der formalen Logik und der Erkenntnisstheorie*, J. Baedeker, Iserlohn, (1877), p. 139.

2420. F. Ueberweg, English translation by G. S. Morris, “with additions” by N. Porter, *History of Philosophy from Thales to the Present Time*, Scribner, Armstrong & Co, New York, (1876), pp. 51-60.

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2422. Plato, English translation by B. Jowett, *Timæus* and *Parmenides*, both found in *The Dialogues of Plato*, Multiple Editions.

2423. Aristotle, *Physics*, *Metaphysics*, *On the Heavens* (*De cælo*), *On Generation and Corruption*, each in multiple editions, all reprinted in *Great Books of the Western World*.

2424. *Cf. The Gospel According to Mary Magdalene*, Chapter 4, Verses 22, 23, 30; Chapter 8, Verse 17.

2425. Philo Judæus, "On the Eternity of the World (*De Æternitate Mundi*)", *The Works of Philo*, Hendrickson Publishers, Inc., USA, (1993), pp. 707-724.
2426. Ocellus Lucanus, Translated by T. Taylor, *Ocellus Lucanus. On the nature of the universe. Taurus, the Platonic philosoher, On the eternity of the world. Julius Firmicus Maternus Of the thema mundi; in which the positions of the stars at the commencement of the several mundane periods is given. Select theorems on the perpetuity of time, by Proelus*, Thomas Taylor, London, (1831); Republished by the Philosophical Research Society, (1976/1999).
2427. Augustine, *Confessions*, Book 11, Multiple Editions.
2428. Ocellus Lucanus, Translated by T. Taylor, *Ocellus Lucanus. On the nature of the universe. Taurus, the Platonic philosoher, On the eternity of the world. Julius Firmicus Maternus Of the thema mundi; in which the positions of the stars at the commencement of the several mundane periods is given. Select theorems on the perpetuity of time, by Proelus*, Thomas Taylor, London, (1831); Republished by the Philosophical Research Society, (1976/1999).
2429. Ocellus Lucanus, Translated by T. Taylor, *Ocellus Lucanus. On the nature of the universe. Taurus, the Platonic philosoher, On the eternity of the world. Julius Firmicus Maternus Of the thema mundi; in which the positions of the stars at the commencement of the several mundane periods is given. Select theorems on the perpetuity of time, by Proelus*, Thomas Taylor, London, (1831); Republished by the Philosophical Research Society, (1976/1999).
2430. *Zohar*, I, 47*a*, "The answer is that God being omniscient knows all things and to him the past and the future are as the present; the past with its countless generations of men and the future enfolding everything that shall be in the course of ages to come, and this is the meaning involved in the above words, for evverything created and made by God cannot but be good."—N. De Manhar, *Zohar: Bereshith—Genesis: An Expository Translation from Hebrew*, Third Revised Edition, Wizards Bookshelf, San Diego, (1995), p. 204.
2431. G. Bruno, *De la causa, principio, et vno*, John Charleswood, London, (1584); English translation, *Cause, Principle, and Unity*, Multiple Editions; German translation, *Von der Ursache, dem Princip und dem Einen*, Multiple Editions; **and** *De l'Infinito Universo e Mondi*, John Charleswood, London, (1584); English translation, *Giordano Bruno, His Life and Thought. With Annotated Translation of his Work, On the Infinite Universe and Worlds*, Schuman, New York, (1950); German translation, *Zwiegespräche vom Unendlichen: All und den Welten*, E. Diedrich, Jena, (1892). Collected Works in German, *Gesammelte Werke*, E. Diedrich, Leipzig, (1904-1909).
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2433. J. Locke, *Essay Concerning Human Understanding*, Chapter 15, Section 12.
2434. I. Newton, *Principia*, Book I, Definition VIII, Scholium; **and** Book III, General Scholium.
2435. S. Clarke, *A Demonstration of the Being and Attributes of God And Other Writings*, Edited by E. Vialati, Cambridge University Press, (1998), pp. 19-20. *Cf.* Thomas Reid, *Essays on the Intellectual Powers of Man*, Essay III, Of Memory, CHAPTER III, OF DURATION, (1785); in *The Works of Thomas Reid, D.D. F.R.S. Edinburgh. Late Professor of Moral Philosophy in the University of Glasgow. With an Account of His Life and Writings*,

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2437. G. Berkeley, *Principles of Human Knowledge*, *De Motu*, *Three Dialogues between Hylas and Philonous*, *An Essay Towards a New Theory of Vision*; each in multiple editions, all reprinted in: D. M. Armstrong, Editor, *Berkeley's Philosophical Writings*, Collier Books, New York, (1965).

2438. D. Hartley, "Of the Being and Attributes of God, and of Natural Religion", *Observations on Man, His Frame, His Duty, and His Expectations in Two Parts*, Volume 2, Chapter 1, Printed by S. Richardson for James Leake and Wm. Frederick, booksellers in Bath and sold by Charles Hitch and Stephen Austen, booksellers in London, London, (1749), pp. 5-70.

2439. R. J. Boscovich, *A Theory of Natural Philosophy*, M.I.T. Press, Cambridge, Massachusetts, London, (1966).

2440. J. L. Lagrange (ca. 1797), "Application de Théorie des Fonctions a la Méchanique", *Œuvres de Lagrange*, Volume 9, Part 3, Chapter 1, Gauthier-Villars, Paris, (1881), pp. 337-344, at 337. *See also:* J. L. Lagrange, *Mecanique Analytic*, Chez la Veuve Desaint, Paris, (1788); **and** *Théorie des Fonctions Analytiques*, Volume 4, Third Part, Chapter 1, Section 1, Nouvelle édition, Courcier, Paris, (1813); fourth edition, Gauthier-Villars, Paris, (1881). ***See also:*** G. S. Klügel, "Abmessung", *Mathematisches Wörterbuch*, Volume 1, E. B. Schwickert, Leipzig, (1803), pp. 3-7, at 7.

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