in the respective positions to that which would enter it if the plate were in the immediate proximity of the opening, *i. e.* if r_1, r_2, and r_∞ are the distances in the respective cases, I calculate $\frac{K_1}{r_1^2} \times \frac{r_\infty^2}{K_\infty}$, and $\frac{K_2}{r_2^2} \times \frac{r_\infty^2}{K_\infty}$. For the two expressions we obtain the values 0·0827 and 0·00602 respectively. The difference between the two expressions is 0·077. If I assume that about equal scattered radiation is propagated backwards, this difference, which comes out as a difference in the apparent absorption, will amount to ca. 3·9 per cent. of the absorption caused by scattering. According to the figures given above it only amounts to $\frac{0·094}{0·094+0·142}$ of the total absorption. The difference will then be 1·5 per cent. of the total absorption. This is in fair agreement with the Aurén experiments.

In addition to the above calculation I wish to mention that, for small values of z, K is more readily denoted by developing in the formula (4), the integrand of a power series and integrating term by term. So the following expression is arrived at:

$$K=\sum_{s=0}^{\infty}\frac{z^{s+1}}{s+1}(-1)^s\sum_{\mu=0}^{s}\frac{s-\mu+1}{(\mu+1)\,!}\left(1+\frac{(s-\mu+2)(s-\mu+3)}{6}\right)k^{\mu+1}.$$

Nobel Institution of Physical Chemistry,
Stockholm, June 1918.

XIX. *Fizeau's Experiment and the Æther.* *By* Dr. R. A. Houstoun, *Lecturer on Physical Optics in the University of Glasgow**.

§ 1. IN a celebrated experiment performed in 1859 Fizeau showed that the velocity of light in a tube containing running water could be explained on the assumption that the æther in the tube was dragged with the water with a velocity $v(1-1/\mu^2)$, where v is the velocity of the water. The experiment was repeated with greater accuracy by Michelson and Morley in 1886, and Fizeau's result verified. According to the theory of H. A. Lorentz published in 1895, however, the velocity of the æther-drift in the tube is given by

$$v\left(1-\frac{1}{\mu^2}-\frac{\lambda}{\mu}\frac{d\mu}{d\lambda}\right),$$

* Communicated by the Author.

and not by $v(1-1/\mu^2)$, the numerical value of the new term being much smaller than the numerical value of either expression for the drift. The experiment was repeated with extreme care and accuracy by Zeeman in 1915, and not only was the existence of the new term verified, but also its variation with the wave-length.

It has hitherto been assumed, that the Lorentz expression for the æther-drift can be derived only on the basis of the theory of electrons, or from the equations of the theory of Relativity. I desire here to call attention to the fact, that it follows much more simply on the basis of the older elastic theories of light.

§ 2. Let us suppose that

$$\rho\frac{\partial^2 y}{\partial t^2}=E\frac{\partial^2 y}{\partial x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

represents the propagation of a light-wave in the æther, y denoting the displacement of the æther, ρ its density and E its elasticity; ρ is constant throughout all space. Let us suppose that inside matter there are particles attached to points in the æther, the mass of the distribution per unit volume being σ. These particles may be regarded as constituting a kind of "virtual" æther. They execute vibrations about the points to which they are attached. If η denotes their average displacement, their motion is given by

$$\sigma\frac{d^2\eta}{dt^2}+k(\eta-y)=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The force attaching them to the æther reacts on the latter, so that instead of (1) we obtain

$$\rho\frac{\partial^2 y}{\partial t^2}=E\frac{\partial^2 y}{\partial x^2}+k(\eta-y). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Add (2) and (3), and we obtain

$$\rho\frac{\partial^2 y}{\partial t^2}+\sigma\frac{d^2\eta}{dt^2}=E\frac{\partial^2 y}{\partial x^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Assume now that y and η vary as $\cos nt$. Then (2) gives

$$\eta(k-\sigma n^2)=ky,$$

and (4) becomes

$$\left(\rho+\frac{k\sigma}{k-\sigma n^2}\right)\frac{\partial^2 y}{\partial t^2}=E\frac{\partial^2 y}{\partial x^2}.$$

The index of refraction is given by

$$\mu^2=\frac{c^2}{\mathrm{E}}\left(\rho+\frac{k\sigma}{k-\sigma n^2}\right)=1+\frac{k\sigma c^2}{\mathrm{E}(k-\sigma n^2)}, \quad . \quad . \quad (5)$$

where c is the velocity of light *in vacuo*.

Now $n=2\pi c/\lambda$, where λ is the wave-length *in vacuo*. It will be found, by differentiating (5) with respect to λ and making the necessary substitutions, that

$$\mu^2-1-\mu\lambda\frac{d\mu}{d\lambda}=\frac{k^2\sigma c^2}{\mathrm{E}(k-\sigma n^2)^2}. \quad . \quad . \quad . \quad . \quad (6)$$

Suppose now that the refracting medium is moving through the æther with velocity v, in the direction in which the light-wave is travelling. Our co-ordinate axes are fixed in the æther. We assume that the light-wave is given by

$$y=\cos n(t-x/\mathrm{V}).$$

Since there is no damping, η is in phase with y, and is proportional to the same cosine. Equations (2) and (3) still hold, but in equation (2) now

$$\frac{d}{dt}=\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}.$$

Previously the $\partial/\partial x$ term was zero. This is the only difference between moving and stationary media. For one definite particle of the "virtual" æther

$$\frac{\partial}{\partial x}=-\frac{1}{\mathrm{V}}\frac{\partial}{\partial t}. \quad \text{Hence } \frac{d}{dt}=\left(1-\frac{v}{\mathrm{V}}\right)\frac{\partial}{\partial t}.$$

Substitute now in (2) and (4), and we obtain

$$\left[\rho+\frac{k\sigma(1-v/\mathrm{V})^2}{k-\sigma(1-v/\mathrm{V})^2n^2}\right]\frac{\partial^2 y}{\partial t^2}=\mathrm{E}\frac{\partial^2 y}{\partial x^2}.$$

Hence

$$\frac{1}{\mathrm{V}^2}=\frac{1}{\mathrm{E}}\left[\rho+\frac{k\sigma(1-v/\mathrm{V})^2}{k-\sigma(1-v/\mathrm{V})^2n^2}\right]. \quad . \quad . \quad . \quad (7)$$

This is absolutely true, no matter what the size of v/V is. Assume that v/V is small, and the equation reduces to

$$\frac{1}{\mathrm{V}^2}=\frac{1}{\mathrm{E}}\left[\rho+\frac{k\sigma}{k-\sigma n^2}-\frac{2v}{\mathrm{V}}\frac{k^2\sigma}{(k-\sigma n^2)^2}\right]$$
$$=\frac{\mu^2}{c^2}\left[1-\frac{2v}{\mathrm{V}}\left(1-\frac{1}{\mu}-\frac{\lambda}{\mu}\frac{d\mu}{d\lambda}\right)\right].$$

Invert and take the root of both sides, remembering that v/V is small. Then

$$\mathrm{V}=\frac{c}{\mu}\left[1+\frac{v}{\mathrm{V}}\left(1-\frac{1}{\mu^2}-\frac{\lambda}{\mu}\frac{d\mu}{d\lambda}\right)\right]$$
$$=\frac{c}{\mu}+v\left(1-\frac{1}{\mu^2}-\frac{\lambda}{\mu}\frac{d\mu}{d\lambda}\right),$$

on making the permissible assumption that $\mathrm{V}=c/\mu$ in the coefficient of v.

Thus the formula is proved. The proof can easily be extended to take in the case of several "virtual" æthers each with its own free period. If the medium is not moving in the direction of the light-wave, v is to be understood as the component of its velocity in that direction.

§ 3. Any theory which is to explain the phenomena of light must involve two vectors vibrating at right angles to one another in the same way as the electric and magnetic intensities do in the electromagnetic theory, *i. e.* it must have equations formally the same as the equations of the electromagnetic theory, and it must also have boundary conditions formally the same as the boundary conditions of the electromagnetic theory. The two vectors may, for example, represent displacement and rotation of an æther. There are various theories of this type, *e. g.* the elastic theory of W. Voigt. The above discussion shows, that if such a system of equations is coupled with a theory of dispersion of the type given by Maxwell or Sellmeier, it will apparently do all that the original theory of Lorentz does for moving media, and more simply than the latter, for it is simpler to suppose particles reacting on an elastic æther than to connect electrons with the light-wave by means of the displacement current. It is a case of phenomena being represented well from two different standpoints, which shows that each has only one aspect of the truth.

Now Lorentz's original theory explained astronomical aberration and the null effect of the earth's motion through space on all optical experiments to the order v/c, but did not explain the null effect of the earth's motion through space on the Michelson-Morley experiment in the order $(v/c)^2$. It is exactly the same with this theory. To explain the Michelson-Morley experiment on the Lorentz theory it is necessary to make the FitzGerald-Lorentz assumption, that the earth contracts in the direction of its motion. But if we assume an elastic æther we have another loophole of escape; we may suppose that the æther is moving with the earth in space, that it is participating in the earth's motion of translation, but not in the earth's motion of rotation. This assumption is not permissible to the Lorentz theory, for according to the latter the æther is merely empty space.

§ 4. Let us shortly examine the consequences involved in the hypothesis, that the æther has the same velocity of translation as the earth.

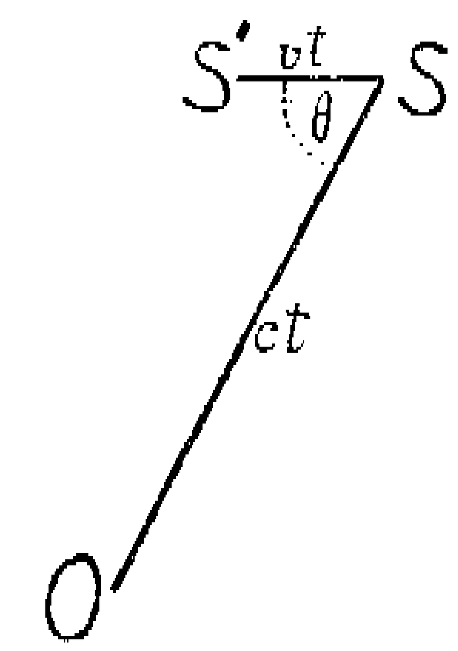
Seventy years ago this hypothesis would have been objected to on *a priori* grounds. The belief then was, that the æther was created first and then myriads of hard unchangeable material particles; these came together under the influence of certain "forces" of attraction, cohesion, heat, &c., and formed the planets. The object of science was to discover and interpret the Divine Plan, and, of course, it was unreasonable to suppose that the æther had any closer connexion with any one planet than any other.

Now the standpoint is rapidly changing; "laws of nature" have truth only with reference to our present state of knowledge, and to the degree of skill with which we have succeeded in giving expression to it. The æther is reached only by our intellectual processes; it is a conception to satisfy our phenomena, Hence we have a right to attach it to our own planet, if necessary, and the hypothesis must be judged solely on its merits.

If the æther participates in the earth's motion of translation, the only motion which the Michelson-Morley experiment might reveal, is that due to the earth's diurnal rotation. This amounts at the equator to ·291 miles per second, and hence gives a value of $v/c=1{\cdot}56\times10^{-6}$ instead of the 10^{-4} given by the orbital velocity. The effect to be observed would consequently diminish to $\frac{24}{100,000}$ of its value, and hence be 100 times too small for the sensitiveness of the apparatus. So the Michelson-Morley experiment is accounted for.

In addition to the annual astronomical aberration with the maximum value of $20''{\cdot}47$ there is a diurnal aberration ranging from zero to $0''{\cdot}31$. The peculiarity of this hypothesis is, that it explains them in different ways. The diurnal aberration is explained in the same way as the annual aberration is explained on the Lorentz theory. To explain the annual aberration it is necessary to proceed as is shown in the fig. O is the observer on the earth supposed at rest in the æther, S is the star and SO the path of a ray to the observer. When the ray arrives at O, the star has moved through the æther to S′, the earth and æther being regarded as fixed. The star is thus displaced behind its true position S′ by the angle $SS'\sin\theta/OS=v\sin\theta/c$, where v is

the relative velocity of star and observer. The case is exactly the same as locating an aeroplane by pointing a resonator in the direction in which the noise of the engine is loudest; the aeroplane will appear displaced behind the true position. If



θ is the apparent latitude and θ' the true latitude, this explanation makes $v/c=\sin(\theta'-\theta)/\sin\theta'$, whereas the usual explanation makes $v/c=\sin(\theta'-\theta)/\sin\theta$. But the difference is too small to observe.

Thus the hypothesis accounts for aberration, and seems to give a means of escaping from the FitzGerald-Lorentz assumption and its consequence, the Principle of Relativity, at the expense, of course, of a strictly electromagnetic explanation of matter.

XX. *On General Relativity.*

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

THE appearance of Dr. Silberstein's recent article * on "General Relativity without the Equivalence Hypothesis" encourages me to restate my own views on the subject. I am perhaps entitled to do this as my work on the subject of General Relativity was published before that of Einstein and Kottler, and appears to have been overlooked by recent writers. In 1909 I proposed a scheme of electromagnetic equations † which are covariant for all transformations of co-ordinates which are biuniform in the domain we are interested in. These equations were similar to Maxwell's equations, except that the familiar relations $B=\mu H$, $D=kE$ of Maxwell's theory were replaced by more

* Phil. Mag. July 1918.

† Proc. Lond. Math. Soc. ser. 2, vol. viii. p. 223 (1910).