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An Approach to Measure Both Emittance and Energy-Spread in the Extraction Region of CEBAF-ER

C.D. Tennant

Abstract

In a previous tech note (*JLAB-TN-03-004*), a two-stage process for obtaining emittance and energy-spread measurements in the extraction region was presented. However it was assumed that an energy-spread measurement could be made on the basis that the dispersion contribution to the beam size dominated, thus $\sigma \approx \eta\delta$. It turns out that the betatron and dispersion contributions are comparable. A new approach to measuring emittance and energy-spread using a quad scan and two harps is presented below.

Background

One can write the beam size as

$$\sigma^2 = \beta\varepsilon + (\eta\delta)^2$$

where β is the betatron function, ε the emittance, η the dispersion and δ the energy spread. It was shown in a previous note (*JLAB-TN-03-004*) that in a region of zero dispersion, that is $\beta\varepsilon >> (\eta\delta)^2$, one can use a harp and quad scan to obtain the beam emittance. Likewise, one can obtain the energy spread using the same method in a dispersive region, that is in a region where $(\eta\delta)^2 >> \beta\varepsilon$. In regards to CEBAF-ER, the previous note assumed that in the dumpline, the beam size would be dominated by the contribution due to dispersion and hence the energy spread would be a straightforward measurement. However, it was pointed out that using previous data from the Front End Test (FET) the contributions to the beam size from the betatron and the dispersion are comparable [1]. Using the table of values below, one finds that $\beta\varepsilon \approx (\eta\delta)^2 \approx 10^{-9}$.

Table 1: Approximate beam parameter values from FET at 45 MeV

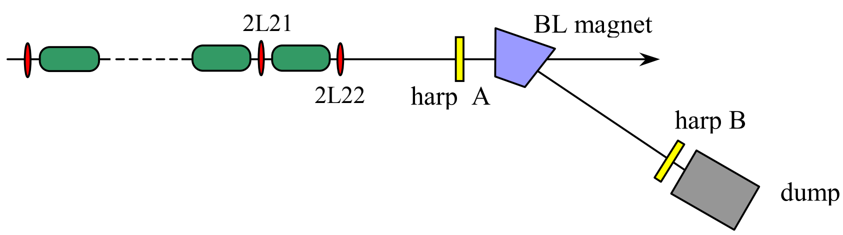
β	≈ 10 m
ε	≈ 2 nm-rad
η	≈ 1 m
δ	$\approx 10^{-4}$

Because of the betatron contribution to the beam size in the dumpline, a new approach to measuring the energy spread must be taken and is presented below [2].

Solution to Measuring Energy-Spread and Emittance

Consider the extraction region shown in Figure 2. After the quad at 2L22, the 45 MeV, energy-recovered beam enters a drift space until the bending magnet at which point it is bent into the dumpline. The harps are located prior to the bending magnet in a non-dispersive region and prior to the dump in a dispersive region.

Figure 2: Extraction Region



For convenience, denote a measurement at the first harp as A and a measurement at the second as B. In addition, let β, α, and γ be the betatron functions at the quad, let $\tilde{\beta}$ be the betatron function at A and $\hat{\beta}$ the betatron function at B. Following the general theory of emittance measurements [3], we can write

$$\begin{aligned}\sigma^2_{(A),1} &= \tilde{\beta}\varepsilon = M_{11}^2(\beta\varepsilon) - 2M_{11}M_{12}(\alpha\varepsilon) + M_{12}^2(\gamma\varepsilon)\\ \sigma^2_{(B),1} &= \hat{\beta}\varepsilon = N_{11}^2(\beta\varepsilon) - 2N_{11}N_{12}(\alpha\varepsilon) + N_{12}^2(\gamma\varepsilon) + (\eta\delta)^2\end{aligned} \tag{1}$$

where M_{ij} are elements of the transfer matrix that propagates beam from the quad to the first harp (quad-drift) and N_{ij} are the elements of the transfer matrix that propagates beam from the quad to the second harp (quad-drift-dipole-drift). By changing the quad strength, we alter the transfer matrix elements and can write

$$\begin{aligned}\sigma^2_{(A),2} &= \tilde{\beta}\varepsilon = \overline{M}_{11}^2(\beta\varepsilon) - 2\overline{M}_{11}\overline{M}_{12}(\alpha\varepsilon) + \overline{M}_{12}^2(\gamma\varepsilon)\\ \sigma^2_{(B),2} &= \hat{\beta}\varepsilon = \overline{N}_{11}^2(\beta\varepsilon) - 2\overline{N}_{11}\overline{N}_{12}(\alpha\varepsilon) + \overline{N}_{12}^2(\gamma\varepsilon) + (\eta\delta)^2\end{aligned} \tag{2}$$

The $\overline{M}_{ij}$ and $\overline{N}_{ij}$ elements are written with a bar to emphasize that *only* the matrix elements have changed as a result of changing the quad strength. Equations (1) and (2) constitute a system of four equations and four unknowns which can be solved for ε and δ.

Conclusion

With the procedure outlined above, we now have a multi-stage approach of obtaining both emittance and energy-spread measurements in the extraction region. The stages are as follows:

(1) Using a quad scan at 2L21, we can measure the emittance in the horizontal plane using the harp before the bending magnet. The resolution of the measurement is limited to our knowledge of the transfer matrix of the last cryomodule in the south linac.

(2) Using a quad scan at 2L22, we can measure the emittance in the vertical plane. As this measurement depends only on the transfer matrix of a quad and a drift, the resolution can be expected to be quite good.

(3) By adjusting the optics to produce a round spot size at the harp, a quad scan of 2L22 will allow measurement of the emittance in the horizontal and vertical planes simultaneously. The only potential drawback to this approach is the large beta functions that are produced at the end of the south linac.

(4) By once again scanning the quad at 2L22, and using two harp measurements, it was shown that the emittance and energy-spread can be measured. Without tinkering with the optics, the emittance in only the vertical plane can be measured with any degree of accuracy.

This multi-stage solution is attractive as it offers many cross-checks to insure we are making consistent measurements.

References

[1] D. Douglas, private communication

[2] The proceeding approach to obtaining the energy-spread and emittance measurement was worked out in collaboration with Y. Chao and D. Douglas

[3] Tennant, C.D., “An Overview of Emittance Measurements in CEBAF-ER”, JLAB-TN-03-004