Theory of Secondary Electron Emission by High-Speed Ions

E. J. STERNGLASS

Westinghouse Research Laboratories, Pittsburgh, Pennsylvania

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The physical mechanism of secondary electron emission under the impact of high-speed heavy particles is analyzed. The treatment is based on the formation of secondaries according to the Bohr-Bethe theory of ionization, the diffusion of the slow secondaries to the surface, and their subsequent escape in the vacuum. The yield is found to be proportional to the rate of energy loss of the incident particles, and it is shown to be essentially the same for all metals, independent of their work function, conductivity, and other bulk properties. The observed energy distribution of the secondaries, the effect of adsorbed layers and the dependence of the yield on temperature, particle charge, and velocity are found to be explained in terms of this mechanism. The application to the general problem of the escape of electrons from metals and to the study of electron capture and loss by ions passing through solids is discussed.

I. INTRODUCTION

THEORETICAL descriptions of secondary electron emission from metals under fast-ion bombardment available so far have not been in good agreement with experimental observations.[1] The most complete theory published is that of Kapitza.[2] His approach is based on the simplifying hypothesis that the secondary electrons derive their energy from local heating produced by the incident ions. As Kapitza himself pointed out, this implies the doubtful assumption that thermodynamic equilibrium considerations may be employed in treating the very rapid and highly localized energy exchange of fast particles with matter. Since the resulting theory leads to a dependence on the thermal constants and thermionic work function of the material, in diagreement with subsequent experimental measurements, it seemed desirable to try an approach that is not based on the assumption of statistical equilibrium.

II. OUTLINE OF PHYSICAL MECHANISM

The process of secondary electron emission will be regarded as composed of two essentially independent parts, namely the formation of secondaries and their subsequent escape. Energy lost by the primary particles

[1] See the review of the subject in H. S. W. Massey and E. H. S. Burhop, *Electronic and Ionic Impact Phenomena* (Oxford University Press, New York, 1952), Chap. 9, Sec. 2.

[2] P. Kapitza, Phil. Mag. 45, 989 (1923).

as a result of excitation and ionization processes will be assumed to result in the formation of internal secondary electrons. The secondaries formed will be considered to lose their energy in various types of collision processes so that only a small fraction of all those formed are able to reach the surface with sufficient energy to escape from the solid. The number of such secondaries per incident particle, or the yield Δ, is the experimentally observed quantity whose magnitude is to be calculated as a function of primary particle energy and target characteristics.

In order to allow the use of simple expressions for the rate of energy loss of the incident ions, free from the complicating effects of electron capture and loss, the present treatment will be primarily concerned with high-velocity ions. The theory will therefore first be developed for the case of protons in the Mev energy range, for which experimental measurements have recently become available. Once the principal features of the theory for the case of high-energy protons have been tested, it becomes possible to extend it to lower energies and other types of ions.

By first developing the theory for the case of high-velocity ions, another important simplification can be made. This results from the fact that the secondary electrons formed can only escape from a depth small compared to the penetration depth of these ions. Whereas secondary electrons in metals are known to

FIG. 1. Formation of secondary electrons (SE)
and δ rays by heavy ion.

come from a region between 10^{-7} to 10^{-6} cm thick,[3] ions with energies of the order of Mev penetrate to depths[4] of the order of 10^{-3} cm. As a result, to a very good approximation, one may consider the secondary electrons which escape to have originated in a region in which the incident ions possess nearly their original energy. The amount of energy lost per unit path length which is available for secondary electron formation may accordingly be taken as constant throughout this region.

The energy lost by fast ions is given up in two types of collision processes.[5] In the first process, frequently referred to as distant collisions, the ion produces only a small perturbation in the atoms of the stopping material so that correspondingly small amounts of energy are transferred in each collision. The second type of collision involves a close approach between the incident ion and an atomic electron in an essentially free collision, resulting in large amounts of energy transferred to individual atomic electrons. The former type accounts for the slow secondaries formed in a direct primary process, referred to hereafter simply as secondaries, whereas the latter gives rise to the relatively rare energetic knock-on electrons (δ rays), which, in turn, produce secondaries in higher order collisions. (See the schematic representation in Fig. 1.) Although the number of δ rays is very small, the total energy going into their formation approximately equals that going into the direct production of slow secondaries at high ion velocities.[5] As a result, the contribution of the energetic collisions must be considered in calculating the yield. As shown in Fig. 1, δ rays are emitted preferentially in forward directions and possess effective ranges that exceed the distances secondaries can travel in metals. As a result, the number of secondaries formed per unit layer resulting from the formation of δ rays is less near the surface than in the interior of the solid (see Fig. 1). Accordingly, when calculating the number of secondaries formed per unit layer of the material near the surface, all of the energy lost per unit path length in distant collisions, but only a fraction of the energy going into close collisions, must be considered.

For very high ion velocities, corresponding to those of protons in the Mev energy region, the range of the δ rays is much larger than the depth from which secondaries can escape in metals,[6] as indicated by the shading in Fig. 1. Under these conditions, which are the ones of principal interest here, the contribution of the fast electrons to the secondary electron yield from the entrance surface becomes very small. As a result, it is not necessary to know the contribution of the close collisions with great accuracy in order to calculate the yield for high-velocity ions.

III. FORMATION OF SECONDARY ELECTRONS

According to the above considerations, the number of low-energy secondary electrons originating in unit layer dx at depth x may be calculated in two parts: that produced by direct interaction $n_{se}^{(1)}$ and that produced by the fast δ rays, $n_{se}^{(2)}$. In terms of the mean energy loss per secondary formed, $\bar{E}_0$, and the mean energy loss per unit distance going directly into slow secondary production $\langle dE_i/dx\rangle_{Av}^{(1)}$, one has

$$n_{se}^{(1)}(v_i,x) = \frac{1}{\bar{E}_0}\left\langle\frac{dE_i}{dx}\right\rangle_{Av}^{(1)}, \qquad (1a)$$

where v_i is the incident ion velocity.

In order to obtain $n_{se}^{(2)}$, it is necessary to sum over the contributions made by the δ rays formed all along the track of the ion. As shown in the appendix, carrying out this summation in the case where the rate of energy loss is essentially constant over the region of interest leads to the expression

$$n_{se}^{(2)}(x,v_i) = f(v_i,x)\frac{1}{\bar{E}_0}\left\langle\frac{dE_i}{dx}\right\rangle_{Av}^{(2)}. \qquad (1b)$$

Here, $\langle dE_i/dx\rangle_{Av}^{(2)}$ is the energy loss per unit path length going into the formation of δ rays, and $f(v_i,x)$ is a factor that represents the fraction of $\langle dE_i/dx\rangle_{Av}^{(2)}$ available for the formation of secondaries in higher order processes at depth x.

The theories of Bohr[5] and Bethe[7] show that at high ion velocities one-half of the total energy loss goes into each of the two kinds of collision processes, i.e.,

$$\langle dE_i/dx\rangle_{Av}^{(1)} = \langle dE_i/dx\rangle_{Av}^{(2)} = \tfrac{1}{2}\langle dE_i/dx\rangle_{Av},$$

where $\langle dE_i/dx\rangle_{Av}$ is the total energy loss per unit path length. This is the case when $v_p > 2z_iv_0$, where z_i is the effective charge of the incident particle and v_0 is the Bohr hydrogen orbital velocity. In the case of protons ($z_i=1$), this condition holds for all energies above about 100 kev, so that for the energy range of primary interest

[3] H. Bruining, *Physics and Applications of Secondary Electron Emission* (McGraw-Hill Book Company, Inc., New York, 1954), p. 100.

[4] H. A. Bethe and J. Ashkin in *Experimental Nuclear Physics*, edited by E. Segrè (John Wiley and Sons, Inc., New York, 1953), Vol. 1, Sec. 1A.

[5] N. Bohr, Kgl. Danske Videnskab. Selskab., Math.-fys. Medd. 18, No. 8 (1948), Chap. 3.

[6] For a 2-Mev proton, the fastest knock-on electron has an energy of $4(1/1835)\times2$ Mev$=4360$ ev. This corresponds to a range of about 0.25×10^{-4} cm in Al [see R. O. Lane and D. J. Zaffarano, Phys. Rev. 94, 960 (1954)].

[7] H. A. Bethe, Ann. Physik 5, 325 (1930).

at present, Eq. (1) becomes

$$n_{se}(v_i,x) = \left(\frac{1}{\bar{E}_0}\right)\frac{1}{2}\left\langle\frac{dE_i}{dx}\right\rangle_{Av}[1+f(v_i,x)]. \quad (2)$$

For the total energy loss per unit path length, the theory in the approximation used by Bohr[5] gives

$$\left\langle\frac{dE_i}{dx}\right\rangle_{Av} = \frac{2\pi N\epsilon^4 z_i{}^2}{E_{eq}}\sum_{n,l} Z_{n,l}\ln\left(\frac{4E_{eq}}{I_{n,l}}\right), \quad (3)$$

where $E_{eq}=\frac{1}{2}m_0v_i{}^2=(m_0/M)E_i$. Here N is the number of atoms per unit volume, ϵ is the electronic charge, z_i, the charge of the incident particle, and M, E_i and v_i, its mass, energy, and velocity, respectively. The quantity m_0 is the electronic mass, $Z_{n,l}$, the number of electrons in the n,l shell, and $I_{n,l}$, the binding energy of these electrons. The summation is to be taken over all shells for which the logarithm remains positive. Equation (3) differs from the more exact form given by Bethe[7] only in that $I_{n,l}$ is used to approximate the excitation potential and $Z_{n,l}$, the oscillator strength of each shell.[5]

For sufficiently high primary velocities, the summation extends over all shells, in which case Eq. (3) may be written as

$$\left\langle\frac{dE_i}{dx}\right\rangle_{Av} = 2\pi N\epsilon^4 z_i{}^2\left[\frac{Z}{E_{eq}}\ln\left(\frac{4E_{eq}}{\bar{I}}\right)\right], \quad (4)$$

where $\bar{I}$ is the mean excitation potential for the atom defined by the relation

$$Z\ln\bar{I} = \sum_{n,l} Z_{n,l}\ln I_{n,l}. \quad (5)$$

This quantity may be evaluated experimentally from stopping-power measurements in the absence of reliable theoretical values.[8] For lower particle velocities, only the outer shells participate, and it is necessary to sum over these shells alone. For this case, Bohr[9] derived an approximate expression based on the Thomas-Fermi model of the atom

$$\left\langle\frac{dE_i}{dx}\right\rangle_{Av} = 2\pi N\epsilon^4 z_i{}^2\left[\frac{4Z^{\frac{1}{3}}}{I_0^{\frac{1}{3}}E_{eq}^{\frac{1}{2}}}\right], \quad (6)$$

where I_0 is the Rydberg energy.

As to the mean energy expended per ion formed, $\bar{E}_0$, it may be expected to have essentially the same magnitude as observed in gases. This is due to the fact that, as Eq. (3) shows, the incident particles interact predominantly with the heavily populated bound shells of an atom whose binding energies are affected relatively little by changes in the state of aggregation. Detailed theoretical estimates for $\bar{E}_0$ have so far been obtained

for only a few elements[7,10] so that it is necessary to fall back on experimental data. The qualitative considerations of Fano indicate that $\bar{E}_0$ should not vary greatly from element to element, despite large differences in ionization potentials and excitation probabilities, in agreement with the observed facts for many different atomic and molecular gases.[11] Since for most of the heavier gases, $\bar{E}_0$ ranges between 20 and 30 ev, a value of 25 ev will be adopted as a reasonable estimate for solids. This is also the average value found empirically in the analysis of secondary electron emission from metals under electron bombardment.[12]

IV. ESCAPE OF SECONDARY ELECTRONS

It will now be necessary to consider the process whereby the internal secondary electrons lose their energy and are prevented from escaping.

There are two types of energy-loss processes that can occur, namely inelastic and elastic collisions. The inelastic collisions are those processes in which other electrons are raised to various excited levels resulting in large energy transfers per collision. By contrast, the elastic collisions do not lead to electronic excitation and involve only small amounts of energy transfer to the thermal vibrational modes of the lattice. Since, in metals, any amount of energy can be transferred to a valence electron in a single collision, it takes on the average only a few collisions to reduce the energy of a secondary below the minimum value necessary to overcome the surface potential barrier. Thus, it is to be expected that the dominant fraction of all collisions determining the depth from which secondaries can escape from metals are of the inelastic type. It is important to note, furthermore, that the energy of many secondaries is sufficient to excite not only the outermost valence and conduction electrons, but also some of the more firmly bound electrons in the filled levels next to the valence shell.

The physical situation in the case of highly excited secondary electrons moving through a solid therefore appears to resemble much more closely the diffusion of electrons in a gas than the motion of conduction electrons through a periodic lattice. In the treatment that follows, it will therefore be assumed that the solid may be considered as a collection of atoms whose electron scattering cross sections are essentially similar to those of a gas.[13]

It remains to arrive at an expression for the probability $P(x)$ that a secondary electron formed at a depth x below the surface will be able to reach the surface and escape.

[8] Reference 4, Sec. 1A, Sec. 4.
[9] Reference 5, p. 102.

[10] U. Fano, Phys. Rev. **70**, 44 (1946).
[11] Reference 4, Sec. 1B, Sec. 2.
[12] E. J. Sternglass, Phys. Rev. **95**, 609(A) (1954).
[13] This assumption is supported by recent experimental evidence on the similarity of the characteristic energy losses of electrons in metal vapors and foils [L. B. Leder and L. Marton, Bull. Am. Phys. Soc. Ser. II, **2**, 68 (1957); L. B. Leder, Phys. Rev. **107**, 1569 (1957)].

For purposes of the present treatment, it will be assumed that diffusion theory[14] may be used to approximate the motion of the secondaries. This does not appear unreasonable in view of the evidence for preferred energy losses of a few electron volts in various metals,[15] which suggests that a typical secondary of 2–15 ev can undergo anywhere between 2–5 collisions before losing its ability to escape. Thus, the actual absorption process probably lies somewhere between a simple "go–no-go" type of behavior and ordinary diffusion accompanied by absorption. In both cases, one is led essentially to an exponential form for $P(x)$, with a characteristic length L_s of the order of the distance between inelastic collisions, i.e.,

$$P(x) = TA \exp(-x/L_s), \qquad (7)$$

where T and A are constants to be discussed below.

That L_s is of the order of the mean free path for inelastic collisions λ_{sc} follows from the small number of such collisions, $\bar{n}_c$, required before the secondary loses its ability to escape. For the case of isotropic scattering, which is known to be a good approximation for the scattering of slow electrons in gases,[16] diffusion theory[14] gives for L_s the expression

$$L_s = (\tfrac{1}{3}\lambda_{sa}\lambda_{sc})^{\frac{1}{2}}. \qquad (8)$$

Here, λ_{sa} is the mean free path for absorption, which in the present case may be set equal to $\bar{n}_c\lambda_{sc}$. According to the above discussion, $\bar{n}_c$ may be expected to have a value of the order 2–5, giving

$$L_s = (\tfrac{1}{3}\bar{n}_c)^{\frac{1}{2}}\lambda_{sc}, \qquad (9)$$

so that $L_s \simeq \lambda_{sc}$.

As to the value of λ_{sc}, on the basis of the present assumptions it is determined by the total collision cross sections of slow electrons moving through a collection of gas atoms, since almost every collision can lead to an energy loss when the allowed levels are broadened into overlapping bands. These cross sections are known to be of the order of the geometric areas of the atoms, both from experimental measurements and theoretical considerations.[17] This will also be assumed to hold for secondary electron scattering in a solid. More precisely, the cross section for secondary electron scattering σ_{sc} will be taken as proportional to the geometric area σ_g of the outermost filled shells as determined by the covalent radii, or

$$\sigma_{sc} = \alpha\sigma_g. \qquad (10)$$

Here, α is a constant, which, on the basis of the assumed similarity to the case of gases, should vary only slowly with the electron velocity.[18] It follows that $\lambda_{sc} = (N\sigma_{sc})^{-1}$ $= (\alpha N\sigma_g)^{-1}$. In view of Eq. (9), this allows one to write for L_s

$$L_s = (\alpha'N\sigma_g)^{-1}, \qquad (11)$$

where $\alpha' = (3/\bar{n}_c)^{\frac{1}{2}}\alpha$. If the assumption that the solid may be treated as a collection of free atoms is valid, α' should have closely the same value for all metals. Thus, once it has been determined from electron absorption measurements in one metal, it should fit all others, independent of their crystal structure, Fermi energy or number of free electrons per atom.

The constant A in Eq. (8) is determined by the distribution of the initial velocities of the secondaries and by the ratio $\lambda_{sa}/\lambda_{sc} = \bar{n}_c$. For a symmetrical distribution of initial directions about a plane parallel to the surface and $\bar{n}_c$ as estimated above, A has a value slightly larger than $\tfrac{1}{2}$, or approximately 0.6.[19] The great majority of slow electrons are emitted at nearly right angles to the track of the ion as a result of momentum and energy conservation considerations so that the assumption of a symmetrical distribution of initial directions seems to be adequate for the present calculation.[20]

The constant T in Eq. (7) is the surface transmission coefficient, representing the probability that an electron arriving at the surface from the interior will be able to escape. Since the secondaries possess energies well above the zero energy level represented by an electron at rest in the vacuum, reflection of the electrons due to the wave-mechanical change in refractive index is negligible compared to the purely electrostatic reflection of electrons approaching the surface potential step at different angles.

The magnitude of T may be obtained by examining the nature of the surface potential barrier appropriate for a secondary electron. The secondaries are formed in the interior of the metal where the mean electrostatic potential has a magnitude differing from that in the vacuum outside. This potential difference results from a surface dipole layer which has its origin in the asymmetric electron charge cloud of the surface atoms projecting outward beyond the positive charges of the ion core.[21] As shown by Wigner and Bardeen,[22] this surface dipole layer may be regarded as part of the total work required to remove an electron from a metal, the remainder consisting of the energy difference between the highest filled level and the mean electrostatic potential in the interior. The latter part of the

[14] Marshak, Brooks, and Hurwitz, Nucleonics 5, No. 5, 10 (1949); 5, No. 6, 43 (1949); 5, No. 7, 53 (1949); 5, No. 8, 59 (1949).
[15] Marton, Leder, and Mendlowitz in *Advances in Electronics and Electron Physics* (Academic Press, Inc., New York, 1955), Vol. VII, p. 183.
[16] Reference 1, Chap. II, Sec. 7.3.
[17] Reference 1, Chap. II.
[18] Reference 5, Sec. 1.5.
[19] From calculations by Coltman, Ebbinghausen, and Altar, J. Appl. Phys. 18, 530 (1947), for the analogous case of photons escaping from an absorbing and scattering medium, one obtains values of A between 0.61 and 0.63 for the estimated range of $\bar{n}_c$ values.
[20] The angle ϕ made by the velocity vector of a secondary of energy E_s relative to the direction of an incident ion of mass $M \gg m_0$ may be shown to be given by the relation $\cos^2\phi = E_s/4E_{eq}$ (reference 7).
[21] For a detailed discussion of the origin of the dipole layer and the velocity dependence of the work function, see the review article of C. Herring and M. H. Nichols, Revs. Modern Phys. 21, 185 (1949), Chap. II.
[22] E. Wigner and J. Bardeen, Phys. Rev. 48, 84 (1935).

work of removal is a volume property of the metal independent of surface condition. It may be regarded as supplied by the incident ion in the course of the formation process in the interior of the solid when energy is transferred to one of the atomic electrons. Since the secondary electron moves toward the surface in a time short compared to the relaxation time of the conduction electrons in the solid (10^{-13}—10^{-14} sec), it is reasonable to expect that polarization effects that give rise to the usual exchange, correlation, and image forces do not have time to come into full effect. In the absence of a detailed theory applicable to such rapidly moving electrons, it appears reasonable to assume that the only work that remains to be done when the electron arrives at the surface is that required to overcome the surface dipole potential Φ_D as shown schematically in Fig. 2. The magnitude of Φ_D has been calculated explicitly for the case of sodium by Bardeen,[23] who found a value of 0.4 ev for this quantity. Estimates based on observed work function variations between different crystal faces of various metals as well as qualitative theoretical considerations indicate that for most metals, the surface potential barrier should be small compared with the total work function.[22]

In the case of an isotropic distribution of electron velocities approaching a uniform surface potential barrier, it is readily shown that $\mathcal{T}$ is given by

$$\mathcal{T}=1-[\Phi_D/(\bar{E}_s+\Phi_D)]^{\frac{1}{2}}, \qquad (12)$$

where $\bar{E}_s$ is the energy of the secondary as measured relative to the zero vacuum level outside the metal. Substituting for $\bar{E}_s$ typical average energies of secondary electrons (6–8 ev) and for Φ_D values estimated by Bardeen for monovalent metals ($\Phi_D \simeq 0.1$–0.5 ev) gives values for $\mathcal{T}$ between 0.8 and 0.9, making $\mathcal{T}A \simeq 0.5$. It is therefore reasonable to expect that the escape probability $P(x)$ will have the value

$$P(x) \simeq 0.5 \exp(-x/L_s) \qquad (13)$$

for all smooth[24] metal surfaces, essentially independent of differences in total work function, conductivity or crystal structure.
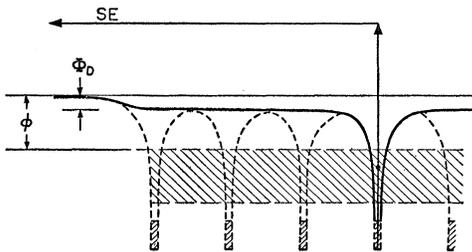
FIG. 2. Schematic diagram to illustrate the formation of a fast SE in the interior of a metal and its escape across the surface potential barrier.

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[23] J. Bardeen, Phys. Rev. **49**, 653 (1936).
[24] Abnormally rough surfaces may have values of $\mathcal{T}$ appreciably less than 0.9, as indicated by the observed reduction of yield in such cases (see Bruining, reference 3, p. 42).

V. FORMULATION OF YIELD EQUATION

A number of secondary electrons escaping from the surface per incident ion, Δ, may now be obtained as follows. The yield from a thin layer of width dx, located at a depth x, is given by

$$d\Delta = n_{se}(v_i, x)\, P(x) dx. \qquad (14)$$

Using Eq. (7) for $P(x)$, Eq. (2) for $n_{se}(v_i, x)$, and summing over all layers gives

$$\Delta = \int_0^\infty \frac{1}{2}\frac{1}{\bar{E}_0} \left\langle \frac{dE_i}{dx} \right\rangle_{Av} [1+f(v_i, x)]\, \mathcal{T}A\, \exp(-x/L_s) dx. \qquad (15)$$

As shown in the Appendix, $f(v_i, x)$ for low atomic numbers has the form

$$f(v_i, x) = 1 - \exp[-x/L_\delta(v_i)], \qquad (16)$$

where $L_\delta(v_i)$ is the effective penetration distance of the δ rays. Substituting Eq. (16) in Eq. (15) and remembering that $\langle dE_i/dx \rangle_{Av}$ may be taken as constant over a surface region of width large compared with L_s, the integration leads to the expression

$$\Delta = \frac{1}{2}\frac{1}{\bar{E}_0} \left\langle \frac{dE_i}{dx} \right\rangle_{Av} \mathcal{T}AL_s[1+F(v_i)], \qquad (17)$$

in which

$$F(v_i) = (1+L_\delta/L_s)^{-1}. \qquad (18a)$$

For the heavier elements, using the more accurate expression for $f(v_i, x)$ derived in the Appendix, one obtains

$$F(v_i) = \left[\left(1+\frac{L_\delta}{L_s}\right)^{-1} + \frac{L_\delta'}{L_\delta}\right]\left(1+\frac{L_\delta'}{L_\delta}\right)^{-1}. \qquad (18b)$$

As shown in the Appendix [Eq. (XVIII)],

$$L_\delta(v_i)/L_s \simeq E_{eq}/100, \qquad (19)$$

where E_{eq} is expressed in electron volts. At proton energies greater than 2 Mev, $E_{eq} > 1000$ ev and $F(v_i) < 0.3$ for all elements so that the dominant term is the yield due to secondaries resulting from primary energy loss in "distant" collisions.

When $F(v_i)$ may be neglected, the complete expression for Δ becomes, using Eq. (3) for $\langle dE_i/dx \rangle_{Av}$ and Eq. (11) for L_s,

$$\Delta = \frac{\mathcal{T}A\pi\epsilon^4 z_i^2}{\bar{E}_0 \alpha' \sigma_g} \sum_{n,l} \frac{Z_{n,l}}{E_{eq}} \ln\left(\frac{4E_{eq}}{I_{n,l}}\right), \qquad (20)$$

In order to obtain a rough test of the theory and its over-all prediction as to the variation of Δ with the atomic number and other material constants, Bohr's approximation for $\langle dE_i/dx \rangle_{Av}$ [Eq. (6)] is most useful. In this case, Eq. (20) becomes

$$\Delta = \frac{\mathcal{T}A\pi\epsilon^4 z_i^2}{\bar{E}_0 \alpha' \sigma_g}\left(\frac{4Z^{\frac{1}{3}}}{I_0^{\frac{1}{3}}E_{eq}^{\frac{1}{2}}}\right). \qquad (21)$$
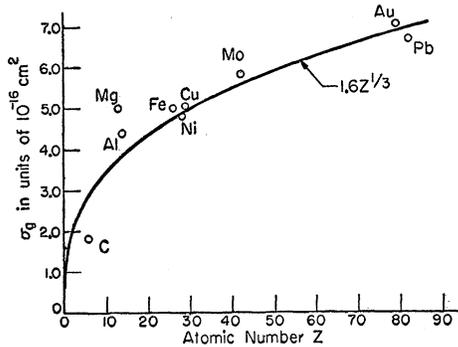
FIG. 3. Geometric areas of atoms σ_g as a function of atomic number Z.

VI. COMPARISON OF YIELD EQUATION WITH EXPERIMENT

The expression for the yield is most conveniently tested by using a simple empirical relation for σ_g that represents its over-all dependence on atomic number in the same way that Eq. (6) provides an estimate for the variation of the energy loss with Z. As shown in Fig. 3, the geometric areas calculated from the covalent radii for typical elements can be fitted quite closely by the empirical equation $\sigma_g = 1.6Z^{\frac{1}{3}} \times 10^{-16}$ cm^2. Inserting this in Eq. (21) and substituting numerical values for ϵ and I_0 leads to a first approximation for the yield:

$$\Delta = 440 \left(\frac{TA}{\bar{E}_0 \alpha'} \right) z_i^2 E_{eq}^{-\frac{1}{2}}, \qquad (22)$$

where $\bar{E}_0$ and E_{eq} are expressed in electron volts, and the explicit Z dependence has dropped out.

The most striking prediction of Eq. (22) is that the yield for a given ion charge and velocity should be essentially the same in all metals. This follows from the discussion of Secs. III and IV, according to which T, A, $\bar{E}_0$ and α' cannot be expected to vary systematically with Z, nor should these quantities depend on the work function, Fermi energy, crystal structure, or density of free electrons. This is precisely the surprising result that has been obtained by Aarset[25] and many earlier investigators working with heavier ions and at lower energies.[1] The physical explanation lies in the proportionality between the cross sections for secondary formation and absorption. This, in turn, follows directly from the assumption that both the formation and escape processes can be treated essentially as if the atoms were free.

Not only does Eq. (22) account qualitatively for the absence of any strong material dependence of Δ, but it also leads to the right order of magnitude for the yield and its variation with ion velocity. Substituting $z_i = 1$, $TA = 0.5$, $\bar{E}_0 = 25$ ev, and $\alpha' = 0.23$ obtained from Goldschmidt and Dember's experimental value for

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[25] Aarset, Cloud, and Trump, J. Appl. Phys. 25, 1365 (1954).

$L_s = 10.8 \times 10^{-8}$ cm in the case of Pt,[26] Eq. (22) reduces to

$$\Delta = 38 V_{eq}^{-\frac{1}{2}} \qquad (23)$$

for the case of protons. This equation is plotted as Curve 1 in Fig. 4, together with the data of Hill, Buechner, Clark, and Fisk,[27] and of Aarset, Cloud, and Trump.[25] Despite the cursory nature of the approximations involved in the derivation of Bohr's expression for the energy loss, and the uncertainty in the precise values of $\bar{E}_0$ and α', the agreement with the data is surprisingly good. It is seen that the approximate expression overestimates the yield somewhat, but that it fits the trend of the data rather well at intermediate energies, the observed yield decreasing somewhat more rapidly than according to $E^{-\frac{1}{2}}$ at higher energies. The deviation at high and low energies is not surprising, since the inverse half-power law of Bohr's expression [Eq. (6)] is a good approximation to the actual logarithmic form of Eq. (4) only over a limited range. To fit a power law to the logarithmic expression at higher energies, the exponent would have to be increased to 0.7, in agreement with the trend of the experimental data above one Mev.

It is therefore to be expected that the use of the logarithmic form of the expression for the energy loss will give a better fit both at the low energies, where the logarithmic term results in the occurrence of a maximum in the yield, and at the high energies, where the yield drops more rapidly than predicted by the half-power approximation. This is confirmed by Curve 2 in Fig. 4, using the values of $\langle dE_i/dx \rangle_{Av}$ as calculated for aluminum according to Eq. (4).[28]

Examination of this plot reveals that the high-energy data of Aarset et al.[1] are very closely represented by Curve 2 both as to absolute value and dependence on energy. As expected, at low energies the theoretical
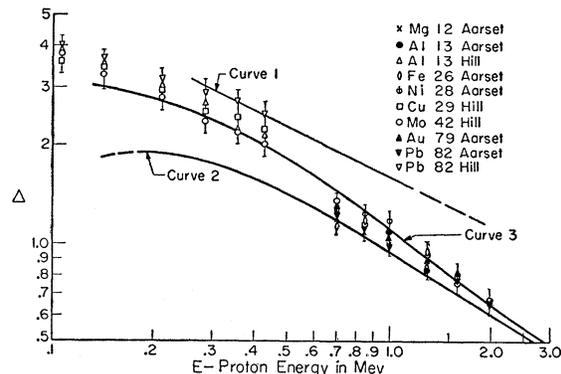
FIG. 4. Secondary electron yield Δ as a function of proton energy. Curve 1, Eq. (21); curve 2, Eq. (17) [using Eq. (4)] with $F(v_i) = 0$; curve 3, Eq. (17) [using Eq. (4)] with $F(v_i)$ given by Eq. (18a).

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[26] H. Goldschmidt and H. Dember, Z. tech. Phys. 7, 137 (1926).
[27] Hill, Buechner, Clark, and Fisk, Phys. Rev. 55, 463 (1939).
[28] For Al, I has been determined accurately by stopping-power measurements as equal to 150 ev (see reference 4, p. 203). All other constants are the same as those used for Curve 1.

curve shows a maximum in the region where the data points level off. However, since Curve 2 is calculated for $F(v_i) = 0$, it drops increasingly below the experimental points at low energies. When E_i has been reduced to 100 kev, the maximum energy of the knock-on electrons (equal to $4E_{eq}$) becomes 218 ev so that most of these electrons will have effective ranges of the order of L_s, causing $F(v_i)$ to approach unity. The experimental values in this energy range should therefore be just about twice what the high-energy theory predicts, and this is, in fact, seen to be the case. By using the approximate relation for $F(v_i)$ [Eq. (18a)], the theoretical curve for Al (Curve 3) is seen to follow the experimental values within the estimated accuracy of the experimental data. The tendency for the data points representing Pb to lie above those for the lighter elements reflects the effect of backscattered δ rays for which the more accurate expression for $F(v_i)$ must be used.

Although Curves 2 and 3 were calculated for the case of aluminum, the data for the heavier elements also appear to be surprisingly well reproduced, despite the fact that one might expect differences in the mean excitation potential $\bar{I}$ to shift the location of the maximum in the yield curve in this region. The reason for the absence of such a shift lies in the nature of the expression for $\langle dE_i/dx \rangle_{Av}$. As pointed out in the discussion of Eqs. (4) and (5) above, only those atomic shells for which the logarithmic term remains positive can make a contribution to the energy loss. It follows that the mean excitation potential at low energies must be determined by averaging only over the outer shells of heavy atoms. Since the outer-shell binding energies of all elements are of the same order of magnitude, the *effective* mean excitation potentials of heavy elements at low energies will be of the same order as for a light element such as aluminum. As a result, the maximum in $\langle dE_i/dx \rangle_{Av}$ occurs at roughly the same energy for all substances.

In view of the close agreement between the theoretically predicted and experimentally observed values of the yield, it becomes possible to test the internal consistency of the physical model on which the present theory is based. Thus, the value of $A \simeq 0.60$ in Eq. (8) hinges on the condition that $\bar{n}_c \lesssim 5$. This represents an upper limit to $\bar{n}_c$, which, in turn, provides a rough estimate for the maximum value of α for a given size of α' by virtue of the relation $\alpha = \alpha'(\bar{n}_c/3)^{\frac{1}{2}}$. Taking $\bar{n}_c = 5$ and $\alpha' = 0.23$ as obtained above gives $\alpha = 0.3$, approximately the same for all elements. Physically, this means that the collision cross section for secondaries is comparable in size to the "core" of filled shells, confirming the assumption that the scattering of secondaries in metals is primarily determined by atomic properties, in close similarity to the scattering of slow electrons in a gas.

It is also possible to test the conclusion that the surface transmission coefficient T must be of the order of 0.8–0.9, independent of work function or Fermi energy. Thus, if it had been assumed that T is determined by the simple picture of reflection at a uniform potential barrier having the full height of the work function φ plus Fermi energy E_F,[29] then T would be given by

$$T = 1 - [(\varphi + E_F)/\bar{E}_s']^{\frac{1}{2}}, \qquad (24)$$

where $\bar{E}_s'$ is the energy of the secondary electron measured relative to the bottom of the conduction band. Using 6–8 ev for the energy of the slow electrons outside the metal together with typical values of φ and E_F, one obtains 0.15–0.20. In order to reach agreement with the magnitude of the observed yield without changing the shape of the yield curve, one only has the quantities A, $\bar{E}_0$, and α' available for adjustment. The maximum value that A could possibly have is unity, so that it could account at most for a factor of 1.6, but not for the required factor of 5–6. The quantity α' is limited by photoelectric[25] and secondary emission[30] experiments to magnitudes differing by no more than a factor of two from the value used to that only $\bar{E}_0$ remains. To obtain agreement, $\bar{E}_0$ would have to be reduced to a value of the order of 5 ev. This is clearly much too small since it is even less than the observed values of the average energy of secondary electrons, in addition to which at least an amount of energy equal to the work function has to be supplied by the primary particle. The quantity $\bar{E}_0$ must necessarily be larger than the sum of these two quantities, not only because many electrons come from levels below the top of the valence band, but also because a large amount of energy is required to provide for the unavoidable "wastage" in excitation to bound states. On the other hand, a value of $\bar{E}_0$ of the order of 20–30 ev as observed for gases is consistent with the whole physical picture underlying the present approach, in which the formation process is taken to be essentially the same as in a free atom.

The conclusion, therefore, seems to be inescapable that T cannot be determined by the magnitude of the total work function but only by the component due to the surface dipole layer, whose magnitude is small compared to the average energy of the secondaries.[31]

[29] See, for instance, D. E. Wooldridge, Phys. Rev. **56**, 562 (1939); P. A. Wolff, Phys. Rev. **95**, 56 (1954); and E. M. Baroody, Phys. Rev. **78**, 780 (1950).

[30] E. J. Sternglass and M. M. Wachtel, Phys. Rev. **99**, 646(A) (1955).

[31] This conclusion finds added support in recent theoretical calculations of Auger ejection of electrons by slow ions impinging on tungsten [H. D. Hagstrum, Phys. Rev. **96**, 336 (1954)]. It is found that if T is calculated according to Eq. (23), using an isotropic distribution of initial directions and a barrier of height $\varphi + E_F$, the calculated yields fall below the observed values by factors of two to five. This discrepancy may be removed if, as suggested in the present paper, T is calculated according to Eq. (12) using $\Phi_D = 1.5$ ev, which is of the order of observed differences in work functions between different crystal faces of tungsten. A more detailed discussion of this problem will be presented elsewhere.

VII. ENERGY DISTRIBUTION OF SECONDARIES

It remains to examine briefly the theoretical predictions for other aspects of the phenomenon. As to the energy distribution of the secondaries, because the formation and absorption process is closely similar in all metals, it should have essentially the same form in all cases, independent of work function or density of conduction electrons. Furthermore, since according to the underlying physical picture discussed in Sec. IV, σ_{sc} is not a very rapid function of secondary energy, one is dealing with a process that tends to preserve substantially the original energy distribution as determined by the formation process. The fact that the value of L_s that fits the yield data under proton bombardment is of the same order of magnitude as for the somewhat slower photoelectrons measured by Goldschmidt and Dember[26] ($\sim$0.3–1.3 ev) suggests that the cross section for inelastic collisions is essentially constant between $\sim$0.3–10 ev. Accordingly, it is to be expected that the form of the energy distribution curves, at least above 10 ev, is very closely equal to the theoretical shape predicted for free atoms.[7] This is confirmed by comparison with data for gases,[32] where the energy distribution is almost completely unaffected by the escape process because of the small probability that a secondary will collide with another atom before being collected.

The similarity of the energy distribution for all metals has, in fact, been noted by many investigators.[1,27,33] Comparison with data obtained by Ishino[32] on air and H_2 in the analogous case of emission under electron bombardment shows the same shape and absolute value of the spectral distribution for metals and gases below about 10–15 ev.[34] Above this energy, the number of secondary electrons emitted by metals drops sharply below that in gases, indicating a pronounced increase in the absorption cross section for secondaries of higher energies as interactions with the more heavily populated filled shells become important.

VIII. DEPENDENCE ON ANGLE OF INCIDENCE

Still another aspect of secondary emission under ion bombardment that must be explained by the present theory is the dependence of Δ on angle of incidence of the ions. Since the heavy particles possess ranges much greater than L_s, it is evident that changing this angle simply increases the length of the ion's track within the "escape zone." Accordingly, $\Delta(\theta)$ must be given in terms of the yield at normal incidence Δ_0 by the relation

$$\Delta(\theta) = \Delta_0 \sec\theta. \qquad (25)$$

This is in fact the form of $\Delta(\theta)$ that has been found to hold empirically.[1,35]

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[32] M. Ishino, Phil. Mag. **32**, 202 (1916).

[33] L. H. Linford, Phys. Rev. **47**, 279 (1935).

[34] E. J. Sternglass, thesis, Cornell University, September 1953 (unpublished); Phys. Rev. **93**, 929(A) (1954).

[35] J. S. Allen, Phys. Rev. **55**, 336 (1939).

IX. TEMPERATURE DEPENDENCE

The present theory also sheds some light on another puzzling feature of the secondary electron emission phenomenon, namely its dependence on temperature. Already early investigations with low-energy ions (1–2 kev) by Oliphant[36] and Moon[37] appeared to show that the yield from Ni under He bombardment decreased with increasing temperature. A similar decrease was reported by Linford[33] using 0.7–2.35-Mev Hg ions on targets of Mo and W heated to an orange temperature, the yield dropping to 50–75% of its value for cold targets and returning nearly to its full value upon cooling. Allen,[35] using protons between 72 and 212 kev on C, Cu, Ni, and Pt, also found a reversible decrease of about 50% when measuring the yield at a "bright red heat." The following tentative explanation of these effects suggests itself on the basis of the present treatment.

The temperature cannot reasonably be expected to influence the very energetic formation process, so that the most likely effect on the yield appears to be through a change in the escape probability. According to the foregoing discussion, the yield should not be strongly affected by changes in the surface dipole barrier. Since N does not appear in the expression for the yield, a change in density due to thermal expansion cannot by itself account for a temperature effect, so that there remains only a possible temperature dependence of L_s. Now an increase in temperature results in increased vibrations of the atoms about their equilibrium positions which should reduce L_s in analogy to the case of electrical conductivity, thereby decreasing the yield according to Eq. (15).

A rough quantitative test of this hypothesis may therefore be made by assuming that L_s varies with the absolute temperature T in a similar manner to the electrical conductivity, i.e.,

$$\frac{L_s(T_1)}{L_s(T_2)} = \frac{1+\beta T_2}{1+\beta T_1} = \frac{\Delta(T_1)}{\Delta(T_2)}. \qquad (26)$$

Substituting room temperature for T_1 and 1000° K for T_2 as the temperature corresponding to "bright red heat," and using the observed ratio of the yields $\Delta(T_2)/\Delta(T_1) = 0.5$, one obtains $\beta \simeq 2.5 \times 10^{-3}$. This is of the same order of magnitude as the temperature coefficient of resistivity for most metals, and small enough to be consistent with the dominant importance of inelastic collision processes.

The present theory therefore suggests that the observed temperature effect in the case of secondary emission under positive-ion bombardment is a real effect, attributable to a decrease in secondary mean free path, and that it is not a spurious effect due to changes in surface conditions.[38]

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[36] M. L. Oliphant, Proc. Roy. Soc. (London) **A127**, 373 (1930).

[37] P. B. Moon, Proc. Cambridge Phil. Soc. **27**, 570 (1931).

[38] This conclusion is corroborated by recent observations of a similar decrease in yield with increasing temperature in the case

X. EFFECTS OF GAS ADSORPTION AND OXIDE FORMATION ON THE YIELD

Two different cases must be distinguished in discussing surface effects in metals, namely the presence of very thin and very thick layers of gas or oxide. In the former case, the large majority of secondaries are formed in the bulk of the metal, so that the principal effect of the adsorbed gases will be to alter the surface transmission coefficient T. According to the discussion of this quantity in Sec. IV, this could come about through a change in the surface dipole moment potentials Φ_D of Eq. (12). In view of the relatively large energy of the secondaries, Eq. (12) indicates that any such effects will be small. Thus, increasing Φ_D from 0.5 ev to 1.0 ev, or by 100%, causes the value of T to decrease by only 12 %. This explains why such surprisingly good agreement exists between the yields measured for high proton velocities by different authors, despite the presence of thin layers of adsorbed gases unavoidable under the vacuum conditions employed.

In contrast, when heavy layers of gases or thick oxides are present or when ion velocities are low, a large number of secondaries will be formed in the surface layer. The dominant effects will then be due to the combined effect of a change in the number of secondaries formed and their ability to diffuse to the surface. The number formed per unit distance may be larger or smaller than for the pure metal, depending on the atomic stopping power of the adsorbed gas or oxide relative to that of the metallic base. Thus, a particularly strong effect is to be expected in the case of low-Z metals, such as Be and Li, when these metals are oxidized, the stopping power of oxygen exceeding that of these elements. As to the diffusion length for secondaries L_s, it is known experimentally to be orders of magnitude larger in insulators than in metals.[30,39] The explanation is to be found in the inability of very low-energy electrons in insulators to make inelastic collisions with the bound electrons.[40] According to Eq. (17), the yield from insulators should therefore be much larger than from metals of comparable stopping power. Such an increase in yield has in fact been observed in all cases where thick oxides were allowed to form[32,34] or where heavy layers of adsorbed gases were known to exist prior to outgassing.[1,26] For freshly formed alkali metal[33] and beryllium[35] layers, the yield was found to increase with time towards a limiting value many times the original yield. On the other hand, for the less reactive metals where all but a few atomic layers can be removed easily, it was found that the yield decreases

FIG. 5. Dependence of secondary electron yield on ion mass and charge, illustrating the scaling property according to ion velocity. Curves for heavier ions calculated from theoretical curve for protons (curve 3, Fig. 4). Data for protons (∇) and helium ions ($\bigcirc$) taken from Hill et al. (reference 27) and Aarset et al. (reference 25).

towards that of a clean metal surface with time of bombardment or outgassing.[1,27]

XI. MASS AND CHARGE DEPENDENCE OF YIELD

As an inspection of Eq. (20) for the yield shows, the mass of the ion enters only by determining E_{eq} or the velocity of the ion for a given energy. The yield, just as $\langle dE_i/dx \rangle_{Av}$, is therefore only a function of the velocity and the charge of the particle, allowing one to reduce all yield curves to a single universal curve by a simple scaling process.[41]

This particular consequence of the present theory clarifies at once a troublesome discrepancy in the observed effect of ion mass on yield. A number of investigators had found that at low ion energies ($\lesssim 50$ kev) the yield varies oppositely to the ionic mass,[42-44] whereas at high energies, the reverse has been observed.[27] This is, however, exactly what the scaling property implies for the two regions below and above the peak in the yield curves. This may be seen from Fig. 5, where the theoretical yield curves have been drawn for protons, deuterons, and α particles incident on aluminum, making use of the scaling property to derive the curves for the heavier ions from the proton curve (Curve 3, Fig. 4). The experimental points for protons and α particles are those obtained by Hill[27] and Aarset.[25] The data for α particles were obtained using singly charged helium ions. The fact that the maximum yield for these ions attains almost the value expected for a doubly charged α particle must be interpreted as indicating that the ion is stripped of its remaining electron within a distance from the surface comparable with L_s. It also illustrates that measurements of the

of electron emission under electron bombardment for C, Pt, and Ta [E. J. Sternglass, Phys. Rev. **90**, 380(A) (1953)]; and for Ge [J. B. Johnson and K. G. McKay, Phys. Rev. **93**, 668 (1954)]; as well as for the yield of photoelectrons ejected from metals by ultraviolet quanta [Walker, Wainfan, and Weissler, Phys. Rev. **98**, 562(A) (1955) and H. E. Hinteregger, Phys. Rev. **96**, 538 (1954)].

[39] Knoll, Hachenberg, and Randmer, Z. Physik **122**, 137 (1944).
[40] H. A. Bethe, Phys. Rev. **59**, 940(A) (1941).

[41] That a scaling process according to the ionic velocity and not according to ion momentum or energy is required to bring the yields for isotopic ions into coincidence has been shown experimentally by W. Ploch [Z. Physik **130**, 174 (1951)].
[42] M. Healea, Phys. Rev. **55**, 984 (1939); M. Healea and C. Houtermans, Phys. Rev. **58**, 608 (1940).
[43] Higatsberger, Demorest, and Nier, J. Appl. Phys. **25**, 883 (1954).
[44] Barnett, Evans, and Stier, Rev. Sci. Instr. **25**, 1112 (1954).

secondary electron yield on the entrance and exit sides of thin foils may prove useful in studying the phenomenon of electron capture and loss by moving ions in solids.

XII. SUMMARY AND CONCLUSIONS

The principal features of the mechanism of secondary electron emission under fast-ion bombardment arrived at may be summarized as follows:

1. The yield is directly proportional to the number of low-energy electrons formed by the incident particle in a thin layer near the surface of a solid. The ionization process is essentially the same as in a gas, and for a given material it is determined only by the velocity and effective charge of the ion.

2. The width of the zone from which slow secondaries can escape is determined by the diffusion length of these electrons in the material. For metals, this is only of the order of a few atomic layers, due to the possibility of large energy losses per collision in inelastic processes. In insulators, where slow electrons can make only elastic collisions, the diffusion length may be orders of magnitude larger.

3. The cross section for scattering and absorption of secondary electrons is approximately proportional to the cross section for secondary formation by the incident particle. It shows no dependence on the number of conduction electrons or other bulk properties of the metal.

4. The majority of all secondaries arriving at the surface are able to escape. The work function appears to play a role in the escape process only insofar as it involves the part due to the dipole layer formed by the asymmetric electron cloud of the surface atoms. The potential step to which this asymmetry gives rise is small compared to the energy of the secondaries so that differences in work function have a negligible effect on the yield. Owing to the proportionality between the inelastic cross sections for the incident particles and the secondaries, the yield is essentially the same for all metals at a given ion velocity and charge.

It is of interest to consider briefly the most important implications of these results. Point 1 indicates that the yield of secondary electrons may be used directly to study the ionization or stopping process of all types of fast charged particles in matter. The only condition is that they pass through the surface zone without appreciable scattering or energy loss.[45] The "escape zone" near the surface in effect acts as an extremely thin, shallow "ionization chamber" from which a fixed fraction of all ejected electrons is collected. Thus, knowing the mass and velocity of a particle, its effective charge may be determined from the secondary yield. Similarly, either its mass or velocity can be obtained from the measured yield if the other two quantities are known. This would appear to have direct application to such problems as the study of electron capture and loss processes for heavy ions in solids where the effective instantaneous charge of the ion is desired.

Point 2 indicates that secondary emission under ion bombardment may be used to study the motion of slow electrons in different materials. Knowing the stopping power of a material from the Bohr-Bethe theory allows a determination of the diffusion length for the secondaries directly from the yield. Thus, secondary electron emission measurements for protons in the Mev energy range provide a new tool for studying the diffusion of slow electrons in metals and insulators.

Point 3 indicates that as far as the scattering of electrons in highly excited states of solids is concerned, the approximation of simple band or free-electron theories are not applicable. Instead, the evidence indicates that, as suggested by Slater,[46] the interaction of electrons capable of making inelastic collisions with the atoms of a solid is determined primarily by the properties of the free atoms, modified by the proximity of nearest neighbors.

Point 4 explains the absence of strong work-function effects in the escape of energetic electrons from solids, be it secondary electrons produced by fast ions,[1,27] slow ions,[47] electrons,[48,49] or hard ultraviolet radiation.[50] It emphasizes the fact that the simplified model of a uniform potential barrier at the surface of a solid equal to the full work function cannot be used to calculate the surface transmission coefficient in those cases where energetic electrons are formed in the interior of a solid and subsequently escape from the surface. These considerations indicate that secondary electron emission may become a useful tool not only in investigating the stopping of fast particles, but also for gaining information on the behavior of slow electrons in excited states of solids.

APPENDIX

The contribution of the fast δ rays to the yield of secondaries may be arrived at as follows.

Referring to Fig. 6, the average number of slow secondaries formed in a layer dx at depth x due to fast δ rays originating at a depth $x=z$ is given by

$$dn_{se}^{(2)} = \frac{1}{\bar{E}_0}\left\langle\frac{dE_i}{dz}\right\rangle_{Av}^{(2)} g(x-z, v_i)dz, \qquad (I)$$

where $g(x-z, v_i)$ represents the spatial distribution of

[45] This condition limits the present treatment to particles that are heavy compared to the rest mass of the electron, or in the case of fast electrons, to thin foils for which backscattering may be neglected. The general case of electrons incident on solids, where scattering cannot be neglected, has been treated in reference 54 and will form the subject of a separate communication.

[46] J. C. Slater, Trans. Faraday Soc. 34, 828 (1938).
[47] H. Paetow and W. Walcher, Z. Physik 110, 69 (1938).
[48] H. Bruining, reference 3, Sec. 5.3, p. 72.
[49] E. J. Sternglass, Phys. Rev. 80, 925 (1950).
[50] H. E. Hinteregger, Phys. Rev. 96, 538 (1954); H. E. Hinteregger and K. Watanabe, J. Opt. Soc. Am. 43, 604 (1953).
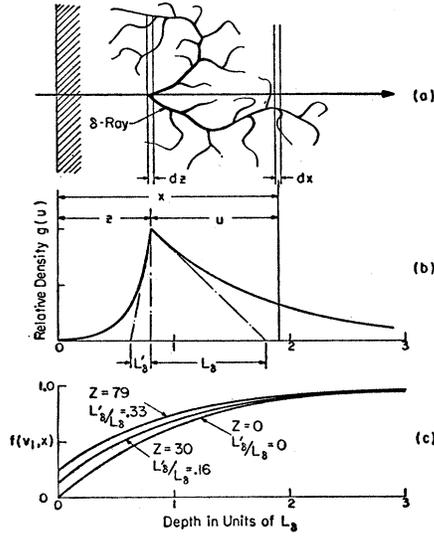
FIG. 6. Contribution of δ rays to the yield of low-energy secondaries. (a) Typical paths of δ rays originating in layer dz. (b) Density distribution of secondaries formed by δ rays from layer dz. (c) Integrated contribution of all δ rays for different ratios L_δ'/L_δ.

the energy carried away by the δ rays originating in dz. The total number of such "tertiary" electrons formed at depth x is therefore given by the integral

$$n_{se}^{(2)}(x,v_i) = \int_0^\infty \frac{1}{\bar{E}_0}\left\langle\frac{d\bar{E}_i}{dz}\right\rangle_{Av}^{(2)} g(x-z, v_i)dz. \quad \text{(II)}$$

For the present case, where the rate of energy loss $\langle dE_i/dz\rangle_{Av}$ may be taken as constant over the region of interest, Eq. (II) may be written in the form

$$n_{se}^{(2)}(x,v_i) = f(v_i,x)\frac{1}{\bar{E}_0}\left\langle\frac{dE_i}{dx}\right\rangle_{Av}^{(2)}, \quad \text{(III)}$$

where the factor

$$f(v_i,x) = \int_0^\infty g(x-z, v_i)dz \quad \text{(IV)}$$

is the quantity introduced in the text [Eq. (1b)].

The exact form of the function $g(x-z, v_i)$ cannot be readily obtained. However, for present purposes, only the principal features of $g(x-z, v_i)$ need to be known in order to arrive at a good estimate for the magnitude of $f(v_i,x)$ and the form of its dependence on the ion velocity v_i. This can be done by considering the δ rays as diffusing from a plane source located at depth z, following the general approach used by Bethe, Rose, and Smith[51] and Fano.[52]

Considering first the limiting case of very low atomic number materials where back diffusion is small, the δ rays will spend nearly all their energy in the forward direction from their point of origin. To a good approxi-

mation, one will not be too far wrong by taking for $g(x-z)=g(u)$ the solution of the diffusion equation for a plane source:

$$g(u) = C \exp[-u/L_\delta(v_i)] \quad \text{for } u > 0, \quad \text{(V)}$$

$$g(u) = 0 \quad \text{for } u < 0,$$

where C is normalizing constant. Here $L_\delta(v_i)$ is the so-called diffusion length, a function of the δ-ray velocity and therefore of v_i, which measures the depth of the region over which the δ rays diffuse while losing their energy. Substitution of Eq. (V) in the expression for $f(v_i,x)$ leads to the simple expression

$$f(v_i,x) = 1 - \exp(-x/L_\delta), \quad \text{(VI)}$$

applicable to materials of low atomic number Z.

For materials of higher Z, where an appreciable amount of the energy carried away by the δ rays is carried backwards as a result of back-diffusion, $g(u)$ may be taken as

$$g(u) = C' \exp[-u/L_\delta(v_i)] \quad \text{for } u > 0, \quad \text{(VII)}$$

$$g(u) = C' \exp[u/L_\delta'(v_i)] \quad \text{for } u < 0. \quad \text{(VIII)}$$

Here, diffusion in the backward direction is taken into account by using the solution of the diffusion equation with a different diffusion length L_δ' [see Fig. 6(b)]. Since the δ rays that find their way back to the plane from where they originated have lost most of their initial energy, their velocity will be small, making $L_\delta' < L_\delta$. To a first approximation, an estimate of an upper limit to the ratio of L_δ' to L_δ may be obtained from experimental data on the backscattering of electrons from solids. The ratio of the areas under the backward and forward portions of $g(u)$ is roughly equal to that of the energy carried away by the backscattered electrons to that remaining in the material for electrons falling on the surface of a solid. Experimental values for this ratio in the kilovolt region may be calculated from measured values[53] of the fraction of backscattered electrons η and their mean fractional energy $\bar{k}$ by means of the expression

$$L_\delta'/L_\delta = \bar{k}\eta/(1-\bar{k}\eta). \quad \text{(IX)}$$

Experimental values for this ratio range typically from 0.05 for Al to 0.25 for Au, indicating that the simple form for $f(v_i,x)$ [Eq. (IV)] is adequate for elements with $Z \lesssim 30$. For Z larger than this, use of the more accurate form of $g(u)$ in Eq. (IV) leads to

$$f(v_i,x) = (1+L_\delta'/L_\delta)^{-1}\left\{[1-\exp(-x/L_\delta)]+\frac{L_\delta'}{L_\delta}\right\}, \quad \text{(X)}$$

which reduces to Eq. (VI) for $(L_\delta'/L_\delta)\to 0$. Curves of $f(x,v_i)$ for a few values of L_δ'/L_δ are shown in Fig. 6(c).

It remains to find an expression for the diffusion length L_δ. This may be done by regarding the motion

[51] Bethe, Rose, and Smith, Proc. Am. Phil. Soc. 78, 573 (1938).
[52] U. Fano, Phys. Rev. 58, 544 (1940).
[53] E. J. Sternglass, Phys. Rev. 95, 345 (1954).

of the δ rays as a diffusion process[54] in the material, following the treatment suggested by Bethe, Rose, and Smith, and Fano. The diffusion length of the δ rays is defined by the relation

$$L_\delta = (\tfrac{1}{3}\lambda_{\delta a}\lambda_{\delta t})^{\frac{1}{2}}. \tag{XI}$$

Here, $\lambda_{\delta t}$ is the transport mean free path given in terms of the collision mean free path $\lambda_{\delta c}$ and the mean cosine of the angle of scattering per collision $\langle\cos\theta_\delta\rangle_{Av}$,

$$\lambda_{\delta t} = \lambda_{\delta c}[1 - \langle\cos\theta_\delta\rangle_{Av}]^{-1}. \tag{XII}$$

The absorption mean free path $\lambda_{\delta a}$ is equal to the actual range R_δ of the δ rays measured along their track length s according to the relation

$$\lambda_{\delta a} = R_\delta = \int_{E_\delta = E_{\delta 0}}^{E_\delta = 0} \frac{dE_\delta}{(dE_\delta/ds)}. \tag{XIII}$$

In Eq. (XIII), E_δ is the energy of the δ-rays at any point of their track and $E_{\delta 0}$, their initial energy upon ejection by the ion. The quantities $\lambda_{\delta c}$ and $\langle\cos\theta_\delta\rangle_{Av}$ are also functions of E_δ, but since they vary slowly with E_δ, to a first approximation they may be calculated for $E_\delta = E_{\delta 0}$.

For the rate of energy loss of the δ rays, use may be made of Bohr's expression [Eq. (6)]. Because of the close similarity in the stopping process for light and heavy particles, this relation requires only a small modification of the constant multiplying $Z^{\frac{1}{3}}$ when used for electrons instead of heavy ions, corresponding to a factor 2 instead of 4 in the logarithmic term of Eq. (4). Inserting Eq. (6) in Eq. (XIII) and carrying out the indicated operation gives

$$\lambda_{\delta a} = \frac{I_0^{\frac{1}{2}}}{2\pi N e^4 \times 4C_1 Z^{\frac{1}{3}}}\left(\frac{2}{3}\right)E_{\delta 0}^{\frac{3}{2}}. \tag{XIV}$$

The quantity $C_1 = 1/\sqrt{2}$ has been introduced to modify Eq. (5) for electrons. Similarly, Eq. (6) for the rate of energy loss may be used to obtain an estimate for $\lambda_{\delta c}(E_{\delta 0})$. The relation between $\lambda_{\delta c}$ and dE_δ/ds is

$$\lambda_{\delta c} = \frac{\bar{Q}}{\langle dE_\delta/ds\rangle_{Av}}, \tag{XV}$$

[54] This calculation is based on the author's thesis, Cornell University, February 1951 (unpublished).

where $\bar{Q}$ is defined as the mean energy loss per collision process in the energy range of interest. Inserting Eq. (XIV) and (XV) in Eq. (XI), making use of Eq. (XII) and Eq. (11), one obtains for L_δ/L_s

$$\frac{L_\delta}{L_s} = \frac{I_0^{\frac{1}{2}}\alpha'\times 1.6\times 10^{-16}}{8\pi e^4}\left(\frac{2}{3C_1}\right)\left(\frac{\bar{Q}}{1 - \langle\cos\theta\rangle_{Av}}\right)^{\frac{1}{2}}E_{\delta 0}. \tag{XVI}$$

The quantity $\bar{Q}$ has been shown by Bethe[7] to be of the same order of magnitude for all except the lightest elements and only slowly varying with electron energy. For nitrogen, the theoretical estimate in the case of kev electrons is 81 ev. Since $\bar{Q}$ enters only as the square root in Eq. (XVI), this value will be adopted as sufficiently representative for all elements. For the mean angle of scattering in an inelastic collision, experimental data[13] available for gases in the energy region of 100–500 ev indicate that $\bar{\theta}$ is of the order of 40°–60° for energy losses near 80 ev, leading to an estimate for $\{\bar{Q}[1 - \langle\cos\theta\rangle_{Av}]^{-1}\}^{\frac{1}{2}}$ between 12 and 18. Adopting a value of 15 for this quantity, using $C_1 = 1/\sqrt{2}$ and $\alpha' = 0.23$ as determined in Sec. VI, Eq. (XVI) reduces to

$$L_\delta/L_s \simeq E_{\delta 0}/380, \tag{XVII}$$

where $E_{\delta 0}$ is expressed in electron volts.

The maximum value of L_δ/L_s is that corresponding to the highest energy that can be transferred to an electron by a heavy ion, i.e., $E_{\delta 0} = 4E_{eq}$ where $E_{eq} = (\tfrac{1}{2}m_0 v_i^2)$. Adopting this value for $E_{\delta 0}$ tends to make L_δ somewhat too large to represent the average δ ray's penetration, but this is compensated by the fact that the penetration depth of electrons ejected preferentially in the forward direction is actually somewhat greater than L_δ since complete diffusion is reached only after a distance $\lambda_{\delta t}$ has been traversed. Inserting $E_{\delta 0} = 4E_{eq}$ into Eq. (XVII), the final estimate for L_δ/L_s becomes, in round numbers,

$$L_\delta(v_i)/L_s \simeq E_{eq}/100. \tag{XVIII}$$

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