"THE MINOR" WATCH PARADOX: THE SOLUTION OF THE PROBLEM

I.A.Solomeshch

Abstract

"The minor" watch paradox (as well as "the major" one) consists in the diversion of readings of moving watches in the moment of the return, while evaluating them in two different ways.

Attention is called to the fact that this paradox (as well as "the major" one) arose because while transition from one way of evaluation to another, the involuntary substitution of one physical process with another was left unnoticed. The difference between these paradoxes is only in the processes participating in the substitution.

1 Introduction

In the first article [1, 1905] on the special theory of relativity (SR) Einstein got a strange, in his opinion, conclusion.

Let the watch of the inertial reference system (IRS) and the other identical watch, synchronized with it, be placed in some point of this IRS. If this other watch is moved along the straight way with a constant speed into some other point of this system and just then returned back , with the same speed and along the same way, then the readings of the moving watch will be less than the readings of the system watch, static all the time in the initial point. [1] .

The strangeness of this statement is, of course, in the fact that it contradicted the opinion in classical physics that the time read by both watches should coincide in the moment of the return of the moving watch.

Einstein's evaluation of this unusual finding increased with time ([1] , [2]) reaching its climax in his most detailed exposition of the SR, left unpublished during his lifetime [3].

Despite this fact, Einstein and the other great theoreticians of SR ([2], [4, ch.30], [5, ch.6, §5], [6, Lecture 10]) took this statement only as an unusual

[1]The wording is given here for the one-dimensional spatial event, satisfactory for the problems discussed in the work.

one, surprising (i.e. paradoxical in the broader sense), but nevertheless the unquestionable conclusion of the SR, that is not proved experimentally only because sufficiently distant and fast motion of the watches (either a device or a biological watch) had not yet been exercised. The attitude to the watch paradox had not changed with the appearance of reasonings ([7] , [8]) based on the SR in favor of equality of watches' readings while their meeting after the return of the moving watch.

The destiny of the other watch paradox was quite different. Between 1905 and 1911 a statement (based rather naively) on the SR was put forward that while returning the readings of the moving watch would be major. Together with Einstein's statement it formed a logical paradox that could ruin the foundations of the SR. If it really turned out that the paradox takes place, then the inner contradictions of the SR would follow.

In his work of 1911 [9] Langevin refuted (also rather naively) the second statement. Later there came a more profound proof of the second statement, but for its refuting, the same Langevin's reasons and the new variants of the proof of the first statement were put forward. It was completely left out that this logical paradox could be solved only while demonstrating the falsehood of the proof of the second statement. Until recently this problem had not yet been finally solved, see details in the work [10].

It is because of the importance laid on these paradoxes that in the work [10] the first of these paradoxes was called "the minor", and the second - "the major" watch (or twins) paradox.

Although the characteristics "the minor", "the major" are rather historical because if we consider the statement about the coincidence of watches' readings after the return of the moving watch more seriously, then it together with Einstein's statement will also form a logical paradox threatening to ruin the foundations of the SR no less than "the major" paradox.

The given work is fulfilled according to the following scheme. The generally accepted wording of the task about the readings of the moving watch on its return is given; its three possible solutions (answers) whose comparison leads to the paradoxes are put forward; additional suppositions used consciously or unconsciously to get these answers are analyzed.

As a result, it turns out that there is no "minor" watch paradox that it, as well as "the major" one, arose out of the fact that each time, unconsciously, different tasks taken as one were being solved, and the natural diversion of results was taken as the paradox.

2 The initial setting and solutions of the problem leading to the watch paradox

Let us turn to the more precise formulae.

2.1 *Task 1* [2]

S - is the IRS of the special theory of relativity, x - is the spatial axis of this system, t - is the time of the system S.

Let:

1. In the beginning of the coordinate system 0 *of the axis x there be the watch C of the system S fixed in* 0 *and the watch C', identical to the watch C and synchronized with it;*

2. At the moment $t = 0$ the watch C' begin to move with constant speed $v > 0$ in the positive direction ("to the right") along the axis x (event $\boldsymbol{B}$);

3. At the moment when the watch C' reaches the watch of the system S reading the time $t_{\boldsymbol{R}} > 0$ (i.e. at the moment $t = t_{\boldsymbol{R}}$ by the watch of the system S), the watch C' instantly changes its speed from v to $-v$ (event $\boldsymbol{R}$) and moves with this speed until it meets the watch C (event $\boldsymbol{E}$) [3].

It is necessary to determine the readings of the watches C and C' at the moment of their meeting (i.e. at the moment of event $\boldsymbol{E}$).

2.2 Let us denote the readings of the watches C and C' at the moment when at the point of their position there was some point event $\boldsymbol{A}$, by $t_{\boldsymbol{A}}(C)$ and $t_{\boldsymbol{A}}(C')$ accordingly. So, according to the terms 1 and 2 of the task

$$t_{\boldsymbol{B}}(C) = t_{\boldsymbol{B}}(C') = 0.$$

As to the readings of the watch C' at the moment of meeting, at different times there were three conclusions based on the SR given:

Conclusion 1 (Dingle [7], 1956; Essen [8], 1957) [4]

$$t_{\boldsymbol{E}}(C') = t_{\boldsymbol{E}}(C). \tag{2.1}$$

Conclusion 2 (Einstein [1], 1905)

$$t_{\boldsymbol{E}}(C') = 2t_{\boldsymbol{R}}/\gamma, \tag{2.2}$$

where

$$\gamma = (1 - v^2/c^2)^{-1/2}, \tag{2.3}$$

[2] The sense of the wording completely corresponds to the one adopted in special literature ([1], [5], [6], [11]).

[3] The justification of using the suppositions about the instant speed change is discussed in [10, 2.2].

[4] In fact, apparently, this conclusion appeared much earlier.

c - the light speed in the vacuum.

Conclusion 3 (appeared between 1905 and 1911)

$$t_{\boldsymbol{E}}(C') = 2t_{\boldsymbol{R}}\gamma. \tag{2.4}$$

As to the readings of the watch C at the moment of meeting, right from the definition of speed it follows

Conclusion 4

$$t_{\boldsymbol{E}}(C) = 2t_{\boldsymbol{R}}. \tag{2.5}$$

And this conclusion does not depend on the fact in the framework of which theory the problem is studied, and whether C' is the watch or simply a material point.

Combining Conclusion 4 in succession with Conclusions 1 - 3, we come to three different solutions of Task 1, arrived at different times by different scientists.

Solution 1

$$t_{\boldsymbol{E}}(C) = t_{\boldsymbol{E}}(C') = 2t_{\boldsymbol{R}}; \tag{2.6}$$

Solution 2

$$t_{\boldsymbol{E}}(C) = 2t_{\boldsymbol{R}}, \;\; t_{\boldsymbol{E}}(C') = 2t_{\boldsymbol{R}}/\gamma; \tag{2.7}$$

Solution 3

$$t_{\boldsymbol{E}}(C) = 2t_{\boldsymbol{R}}, \;\; t_{\boldsymbol{E}}(C') = 2t_{\boldsymbol{R}}\gamma. \tag{2.8}$$

2.3 Each of the solutions is logically consistent by itself, although Solutions 2 and 3 are surprising and intuitively unconceivable (at least, for their time), i.e. paradoxical. But every two of them taken together, make up a logical paradox:

Solutions 2 and 3 - "the major" watch paradox,

Solutions 1 and 2 - "the minor" watch paradox,

Solutions 1 and 3 - the analogue of "the minor" watch paradox.

In this situation there is nothing left as to demonstrate the falsehood of proof of two solutions at least, or to accept the inner inconsistency of the SR. While each solution is the claim that two conclusions be true: the unquestionably true Conclusion 4 and one of the Conclusions 1 - 3, then we should focus on the latter , because the irreproachable proof of just two of them would mean the contradictions in the SR.

3 Adjusting the watch in the SR

Further we shall measure the time in seconds, the distance - in meters, and consider the light signal as spreading in the vacuum.

Adjusting the watch is in the regulation of the watch rate and determining the readings at the moment of a specially set event, taking place at the point of the watch position (i.e. the determining of the initial readings). And the regulation of the watch rate is not necessarily exercised through changing the time of the process period used by the watch. One can change the number of periods determining the second.

In fact, the watch rate regulation means changing the durability of the second by this watch comparing with the initial durability, before the regulation. In [1] Einstein put forward the statements about the possibility of coordination (synchronization) of the watches in the IRS in the SR.

Let E and F be some points in the IRS, and let the light signal sent from E at the moment t_0 (by the watch in E) be reflected in F at the moment t_1 (by the watch in F) and return to point E at the moment t_2 (by the watch E).

Definition *It is said that a watch in point F is the watch synchronized with the watch in point E (at the moment* t_0*), if*

$$t_2 - t_1 = t_1 - t_0.$$

Then it was postulated –

The watches of the IRS can be adjusted so that the following requirements hold true:

Requirement 1 *As to the arbitrary chosen watch of the system at any moment, any other watch of the system is synchronic with it;*

Requirement 2 *For any points E and F and any* t_0

$$\frac{2r}{t_2 - t_0} = c,$$

where c is a constant universal for all IRS, r is the distance between E and F.

While fulfilling these requirements the watches in the IRS are called synchronized.

In [12] Einstein introduced a compact, but less constructive, equivalent definition of the watches' synchronization in the IRS in the SR in the form of

The principle of constant light speed in the vacuum:

"The watches can be synchronized so that the speed of every light ray in the vacuum, measured with the help of these watches is everywhere equal to the universal constant c."

4 *Solving "the minor" watch paradox*

To present a complete picture of the watch paradoxes' solutions, this section runs about the two other paradoxes except "the minor" one.

4.1 It is clear that in any IRS Requirement 2 uniquely determines the watch rate in it.

Let us analyze the IRS of the SR S and S^* moving relatively to each other with the speed that is not 0. Let the watches C_1 and C_2 be static as to S and let each of them be synchronic to the watch of the system S, situated in the same point. Let us move the watch C_1 into some point E of the system S^* and include it into this system, that is at the moment when it was placed at the point E let's adjust it so that the watch rate and the readings of it and those of the watch of the system S^* at the point E coincide. Here, possibly, the reading of the watch C_1 at the moment of its inclusion into the system S^* should be forcefully changed in a leap, i.e. the continuity of its readings will be broken.

Although if it is necessary to keep the continuity of readings of the watch C_1, it may be included into the reference system S^{**}, static to S^* and having the same coordinate axes but differing from S^* only in the fact that the readings of the watch S^{**} are shifted as to the readings of the watch placed in the same point of the system S^* by the constant, equal to the number, opposite the readings leap of C_1. S^{**} is also the IRS of SR, because the shift of the watch readings in the system S^* by the constant does not break either the law of inertia or the principle of constant light speed holding true for the system S^*. If it is necessary to include also the watch C_2 into the system S^{**}, then its readings at the moment of inclusion must become equal to the readings of the system S^{**} watch, placed at the point of inclusion, i.e. equal to the set quantity. That is why, generally speaking, there is no guarantee that the continuity of the watch readings C_2 will keep.

Let's return to task 1.

4.2 *About Conclusion 2*

All the variants of the proof of Conclusion 2 are clearly or unclearly based on the supposition that with the movement to the right the watch C' keeping the continuity of readings is included into one IRS, and with the movement to the left – to the other one.

So, in Einstein's proof [1], to calculate changes of the watch C' readings moving to the right as well as at its return, he uses the lemma of the slowing of the moving watch [10, 2.1.2]. But this lemma [5] was arrived at by Einstein [1]

[5] Einstein did not call this statement a lemma. This name is introduced in [10] for the convenience of references.

in the supposition that the moving watch is included into the IRS. So, with the distancing from the watch C the watch C' is in fact supposed to be included into one IRS (S' in [10]), and with the return - into the other (S'' in [10]). The continuity of readings of C' in both inclusions was taken for granted. The detailed calculations taking into account these additional suppositions are given in [10, 3.1.2].

So, Conclusion 2 does not refer to Task 1, but to Task 2 arrived at with the substitution of the expression "It is necessary" in the wording of Task 1 into "Considering that at the moment of the motion start (event $\boldsymbol{B}$) the watch C' keeping the continuity of readings is included into IRS S', moving with the same speed as C' to the right as to the axis x, and at the moment of turn (event $\boldsymbol{R}$) - also keeping the continuity of readings in IRS S'', moving with the same speed as C' to the left, it is necessary".

Task 2 coincides with Task 1 from [10, 2.2], if we consider that while the detailed solution of the latter one [10, 3.1] keeping the continuity of readings is taken into account [6].

Although, the diversion of wordings of Tasks 1 and 2 in the present work does not yet mean their non-equivalence. This problem will be analyzed further.

4.3 *About Conclusion 3*

There is only one proof of this conclusion. It is analyzed in details in [10, 3.2]. This proof is also based on the suppositions, not issuing from the wording of Task 1. They include the supposition that *while moving to the right the watch C' is included preserving the continuity of readings into one IRS (S' in [10, 3.2]), and with the moving to the left – into the other one S''', where keeping the continuity of readings the other watch C'_1 is included – the watch placed in the other than C' point of the axis x' of the system S' [10, 3.2].*

So, Conclusion 3 does not refer to Task 1, but to Task 3, arrived at with inclusion into the terms of Task 1 all the additional suppositions used while proving Conclusion 3. It is not necessary for our purposes to give a detailed wording of Task 3.

In [10, 4.2] it is demonstrated that in the supposition in italics (now the terms for Task 3) the readings of the watch C' while inclusion in the system S''' should be increased in a leap by some quantity (see [10], formula (4.6)) [7].

In the initial Task 1 the watch C' does not undergo the artificial adjust-

[6] The work [10] is fulfilled according to the scheme, analogous to the one accepted in the present work, and that is why the wordings of the tasks don't contain as requirements all the suppositions, introduced while solving them as taken for granted

[7] It is because of this leap in Task 3 that the reading of the watch C' in its return is greater than in Task 2.

ment, its readings are continuous. Hence, Task 3 and Task 1 are different in essence.

So, Conclusion 3 cannot refer to the solution of Task 1 and it should be ruled out of the analysis.

But then "the major" and the analogue of "the minor" watch paradoxes, in which Conclusion 3 participates, fall out. Only "the minor" paradox is left.

4.4 *About Conclusion 1*

The proof of Conclusion 1 is based on general ideas and can be summarized as follows [8].

In the terms of Task 1 on the way to the right the watch C' is static as to some IRS, but is not included into it; on the way to the left – the same, but as to the other IRS. But according to the principle of relativity all the IRS of the SR are quite equal, that is why the watch C' all the time (the start, the turn and the slowing - are instant) is in the same conditions as the watch C, constantly static as to the IRS S. Hence, there are no reasons why the watches C' and C should read different time at the moment of their meeting [9].

So, Conclusion 1 really refers to Task 1. Its proof is veritable, although the degree of its provability (at least, for the author of this work) is not clear.

Although in these conditions it is possible to prove that "the minor" paradox is not really a paradox.

In fact, "the minor" paradox arises in comparing Conclusions 2 and 1, referring accordingly to Tasks 2 and 1. But Task 2 is different from Task 1 only in the following:

in it at the moment of motion start the watch C' in the coincided point of the system S' is included in S' keeping the continuity of readings (i.e. only with the regulation, if necessary, the watch rate, uniquely determined for the watch of the system S') and analogous at the moment of turn – in S'';

and in Task 1 the watch C' is placed in the same points, the same time, with the same readings, but without regulation the watch rate.

So, the equivalence of these tasks is equal in strength to the following hypothesis.

The hypothesis about self-regulation of the watch

If the watch static at the point E of the system S and having the watch rate of this system is at some moment instantly moved in point E' of the system S', coinciding at this moment with E, and rests in it, then it turns

[8] The detailed variant is given.

[9] Such like reasons for the case of smooth start, turn and slowing are given in the last passage of [10].

out that the watch rate of it and the watch of the system S' coincide.

But this hypothesis has not yet presently been proved either experimentally or theoretically. That is why, at least until the hypothesis is not proved, there are no reasons to consider Task 2 equivalent to the initial Task 1. Hence, the differing results of these tasks' solution cannot be considered contradicting each other, i.e. there is no "minor" watch paradox as well yet.

5 Conclusion

5.1 So, there were three conclusions as to Task 1 (s.2.1), the efforts to solve which lead to the appearance of watch paradoxes – the readings of the moving watch at the moment of return are equal, less, greater as to the readings of the watch, constantly static.

The analysis of the proofs of these conclusions read the following (s.4):

a) Conclusion 3 refers to Task 3 from whose terms it follows that the requirement, which in terms of Task 1 must keep, is broken. I.e. Conclusion 3 does not relate to Task 1 and falls out that destroys the paradoxes – "the major" one and the one analogous to "the minor" [10].

b) Conclusion 2 refers to Task 2 and is unquestionably true in the terms of this task. Although the equivalence of Task 2 to the basic Task 1 is the same as the claim that the hypothesis about the self-regulation of the watch is true. Hence, "the minor" paradox , comparing Conclusions 2 and 1, as referring to Task 1, also does not exist, at least, till the proof of the hypothesis' correctness (that will probably not happen).

c) Conclusion 1 refers precisely to Task 1, but is not substantially proved [11]. Its truth for the watch paradoxes after everything mentioned in point b) is not important.

5.2 Now as we have already approached the problem, let's say some words about the slowing of aging in space travels.

One can speak as before about the model case with the instant start and slowing, discussed in the previous sections. The thing is that one can make the readings of the spacecraft watch C' (and also of the watch C in the starting place) at the return moment in this and a more real case of the smooth speed change, due to the enlargening the section of the uniform motion, coinciding, with arbitrarily little relative error.

Conclusion 2 (Einstein) that at the end of the travel the readings of the spacecraft watch will be γ times less than the readings of the watch in the starting place is got in the supposition that its watch rate was regulated (for

[10]The fact that "the major" paradox is not a paradox was demonstrated in [10].

[11]According to the opinion of the author of this work.

the inclusion in the inertial reference systems of the spacecraft on the ways to and back) twice – in the beginning of the way and at the moment of turn. On the contrary, in the problem of slowing the aging (as in Task 1) a free-moving watch is studied, i.e. simply as a passenger set into the spacecraft and moving together with it (as well as cosmonauts) without any artificial regulation with the aim of giving it the watch rate, providing, while calculations using it, the numerical quantity of double-sided light speed c, required by the principle of constant light speed (s.3, requirement 2). That is why the supposition that the watch rate of it and that of the spacecraft watch, included into the IRS of the spacecraft are the same (in other words, that the hypothesis about self-regulation of the watch is true) is doubtful, at least, – not founded.

In fact from the SR standpoint, the thing is probably as follows. Conclusion 1 is nevertheless true, and by return the watch that was in free flight and the watch resting in the starting place, will show the same time. In the case of the real flight the possible difference of their readings will be conditioned by accelerations, changing of the gravitation forces, possibly by other influences that the watch - passenger would undergo, but not with the sections of straight uniform motion.

Unfortunately, obviously the dream of arbitrarily far space flights in which cosmonauts will almost not age, due to the by-light speed of the straight uniform motion practically during the whole flight time, is illusory.

Only the spacecraft watch included in the IRS of the spacecraft, i.e. reading the Lorenz's time of the SR on board the spacecraft, will return with the less readings.

I gratefully thank Lyudmila Timoshenko Ph.D., Department of Foreign Languages at the Bashkir State University, for translation of the text of the manuscript.

REFERENCES

1. A. Einstein, Zur Elertrodynamik der bewegter Körper. Ann. Phys.,17, 891 - 921, 1905.
2. A. Einstein, Eine Dialog über Einwände gegen die Relativtätstheorie. Naturwiss., 6, 697 - 702, 1918.
3. A. Einstein, Manuscript on the Special Theory of Relativity. The Collected Papers of Albert Einstein, v.4, Princeton, 1996.
4. D. Bom , The special theory of relativity. New York – Amsterdam.1965.
5. M. Born, Einstein's theory of relativity. New York, 1962.
6. L.I. Mandelshtam, Lectures on optic, theory of relativity and quantum

mechanics. Nauka, Moscow 1972 (in Russian).
7. H. Dingle, Relativity and Space Travel. Nature, v.177, No. 4513, pp. 782-784, 1956.
8. L. Essen, The Clock Paradox of Relativity. Nature, v.180, No. 4594, pp. 1061-1062, 1957.
9. P. Langevin, L' évolution de léspace et du temps. Scientia, 10, 31, 1911.
10. I.A. Solomeshch, The Watch Paradox: Solution of the Problem, arXiv:0710.0197v1, 2007.
11. V.A. Fok, Theory of space, time and gravity. GITTL, Moscow, 1955 (in Russian).
12. A. Einstein, Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen. Jahrb. d. Radioaktivität u. Elektronik, 4, 411-462, 1907.