CXV. *On Sir J. J. Thomson's Model of a Light-Quantum.*

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It is commonly accepted that the quantum theory is almost incompatible with Maxwell's equations ("Clerk Maxwell's Electromagnetic Theory," by H. A. Lorentz, the Rede Lecture, 1923, p. 34); however, Sir J. J. Thomson † constructed a most valuable model of a light-quantum which, as will be shown, is formally consistent with the system of Maxwell's equations.

By the way of examining the simplest case of a light-quantum, we shall explain the difference of the Faraday-Maxwell-J. J. Thomson's electrodynamics from the usual Maxwell-Hertz-Lorentz's interpretation of an electromagnetic field, the form of the differential equations for the free æther being the same in both theories.

The light-quantum in the model of Sir J. J. Thomson consists in a ring-shaped Faraday tube moving with the speed of light at right angles to its own plane. The ring is preceded and followed by ordinary waves of feeble intensity; their wave-length is equal to the circumference of the ring. The energy contained in such a ring will be of the form $h\nu$ (Planck's law) if we admit, with Sir J. J. Thomson, that the cross-section of the tube is proportional to the square of the radius of the ring.

These two fundamental laws, as will be shown, are formally in accordance with the system of Maxwell's equations under the condition that we accept Sir J. J. Thomson's interpretation of an electromagnetic field.

Let $\mathbf{E}_\alpha$ (fig. 1) be a closed ring-shaped Faraday tube referred to fixed axes. Let us suppose that at $t=0$ the plane of the ring coincides with the plane ZOY, and that $\mathbf{u}_x$ is the velocity of the ring. The Maxwell's equations for the free æther are of the form

$$\operatorname{div} \mathbf{E} = 0, \qquad (1)$$

$$\operatorname{curl} \mathbf{M} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \qquad (2)$$

$$\operatorname{div} \mathbf{M} = 0, \qquad (3)$$

$$\operatorname{curl} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{M}}{\partial t}, \qquad (4)$$

* Communicated by the Author.

† "A Suggestion as to the Structure of Light," Phil. Mag. [6] xlviii. p. 737 (1924); "The Structure of Light," Phil. Mag. 1. p. 1182 (1925).

where c is the velocity of light, $\mathbf{E}$ the electric and $\mathbf{M}$ the magnetic intensity. In the case of stationary motion the operation $\frac{1}{c}\frac{\partial}{\partial t}$ will be replaced by $-\beta_x\frac{\partial}{\partial x}$ if $\beta_x=\frac{\mathbf{u}_x}{c}$, and the equations (2) and (4) will be transformed into

$$\operatorname{curl} \overline{\mathbf{M}} = 0 \qquad (2')$$

and

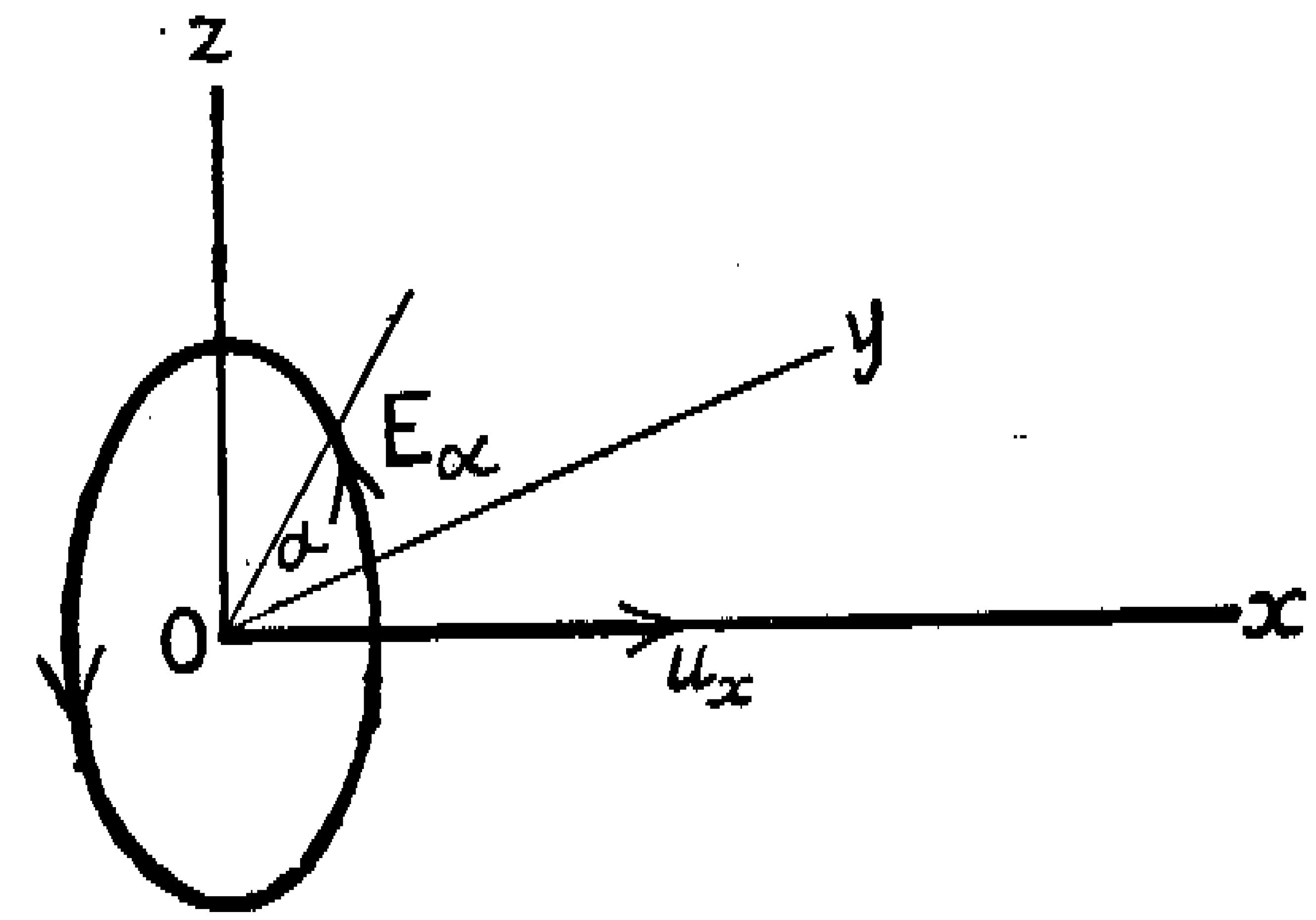
$$\operatorname{curl} \overline{\mathbf{E}} = 0, \qquad (4')$$

where $\overline{\mathbf{M}}$ has for components

$$\mathbf{M}_x,\ \mathbf{M}_y+\beta_x\mathbf{E}_z,\ \mathbf{M}_z-\beta_x\mathbf{E}_y$$

and $\overline{\mathbf{E}}$ ...... $\mathbf{E}_x,\ \mathbf{E}_y-\beta_x\mathbf{M}_z,\ \mathbf{E}_z+\beta_x\mathbf{M}_y$.

Fig. 1.

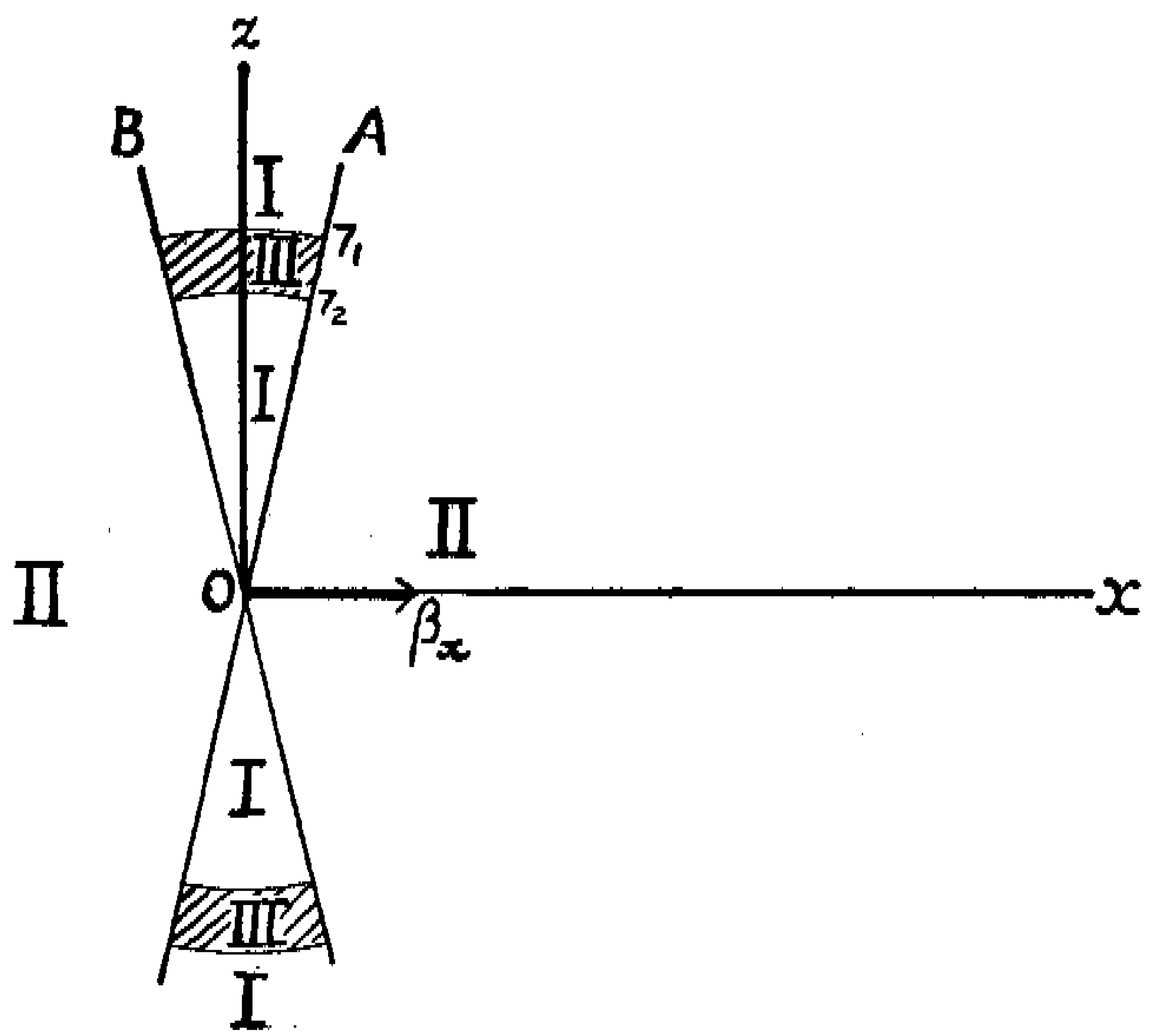


The simplest solutions of the equations (2′) and (4′) are the following: $\overline{\mathbf{M}}=0$ and $\overline{\mathbf{E}}=0$ for the whole space. Now $\overline{\mathbf{M}}=0$ gives $\mathbf{M}_x=0, \mathbf{M}_y=-\beta_x\mathbf{E}_z$, and $\mathbf{M}_z=\beta_x\mathbf{E}_y$. Since $\overline{\mathbf{E}}=0$, we have $\mathbf{E}_x=0$, $\mathbf{E}_y(1-\beta_x^2)=0$, and $\mathbf{E}_z(1-\beta_x^2)=0$. These equations can be satisfied by *two* different suppositions:

$$\mathbf{E}_x=0,\ \mathbf{E}_y=0,\ \mathbf{E}_z=0;\ \text{or}\ \mathbf{E}_x=0,\ \mathbf{E}_y\neq 0,\ \mathbf{E}_z\neq 0,\ 1-\beta_x^2=0.$$

Let us divide the whole space round the point O into *three* parts:—Part I, annular space between two cones (fig. 2), obtained by rotation of OA and OB round the x-axis, having a common vertex at O, the angle AOB being infinitesimal. Part II, the inside of the above-mentioned cones. Part III is taken out of I by the spheres of r_1 and r_2, with the centre at O. For parts I and II we take the first solution of the equations (2′) and (4′), *i. e.* $\mathbf{E}_x=0$, $\mathbf{E}_y=0$, $\mathbf{E}_z=0$; hence $\mathbf{M}_x=0$, $\mathbf{M}_y=0$, $\mathbf{M}_z=0$. For the region III we take the second solution: $\mathbf{E}_x=0$, $\mathbf{E}_y=-\mathbf{E}_\alpha \sin\alpha$, $\mathbf{E}_z=\mathbf{E}_\alpha \cos\alpha$, $\mathbf{E}_\alpha \neq 0$, $\beta_x=+1$; hence $\mathbf{M}_x=0$, $\mathbf{M}_y=-\mathbf{E}_\alpha \cos\alpha$, $\mathbf{M}_z=-\mathbf{E}_\alpha \sin\alpha$. Thus we obtain an electric field *only* in the region III. This field is moving with the velocity of light (first law of the light-quantum theory). If we transform the equation $\operatorname{div}\mathbf{E}=0$, using polar coordinates r, ϕ, α, we obtain $\frac{\partial \mathbf{E}_\alpha}{\partial \alpha}=0$. Thus $\mathbf{E}_\alpha$ is a function of r only; the cross-section of the Faraday tube is constant.

Fig. 2.



The equation $\operatorname{div}\mathbf{M}=0$, which is also to be solved for the region III, after transformation takes the form

$$\frac{\partial}{\partial r}(r^2\mathbf{M}_r)=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or

$$\frac{\partial}{\partial r}(r^2\mathbf{E}_\alpha)=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5')$$

Here, namely in this equation (5), lies the divergence between the Maxwell-Hertz-Lorentz electrodynamics and the theory of Faraday-Maxwell-J. J. Thomson. From the standpoint of the first theory the equation (5) is valid in the whole space occupied by the free æther, and consequently will be valid in regions I and III and at their boundaries. So that if we admit $\mathbf{E}_\alpha=0$ and $\mathbf{M}_r=0$ for the region I, the constant in the solution $\mathbf{E}_\alpha=\frac{\text{const.}}{r^2}$ of the equation (5′) for the region III must also vanish. So, according to the first theory, there can be no electromagnetic field in the region III, since there is no field in the regions I and II. We cannot admit the existence of a field in the region I, as it must have the form $\mathbf{E}_\alpha=\frac{\text{const.}}{r^2}$, which gives for the point O, $\mathbf{E}_\alpha=\infty$.

From the point of view of Sir J. J. Thomson's theory the structure of the field is *discontinuous*. The field consists in a system of Faraday tubes moving relatively to one another; the intensity of the magnetic field is determined by the intensity of the electric field and by the normal velocity β_n of the tube, from the equation $\mathbf{M}=\beta_n\mathbf{E}$. It follows (1) that the equation (2′) is satisfied identically; (2) that the intensity $\mathbf{M}$ has a definite value different from zero only where the Faraday tubes are in motion. The equation (3), $\operatorname{div}\mathbf{M}=0$, exists only where the Faraday tubes are in motion; in the rest of the free æther $\mathbf{M}$ does not exist. If we admit the theory of J. J. Thomson, the equation $\operatorname{div}\mathbf{M}=0$ must be satisfied in our case *only* in the region III up to its boundaries of II. At the boundary I–III we may have a discontinuity of $\mathbf{M}_r$ and consequently of $\mathbf{E}_\alpha$. Hence for the region III we may satisfy (5′) by the supposition $\mathbf{E}_\alpha r^2=$ finite constant.

Calculating the energy of the ring, we obtain

$$\frac{1}{8\pi}(\mathbf{E}^2+\mathbf{M}^2)\times\text{volume of the ring}=2\,.\,\tfrac{1}{2}\epsilon\,.\,\mathbf{E}_\alpha\,.\,2\pi r=\frac{2\pi^2\epsilon^2}{c\phi_0}\,.\,\nu,$$

where ϵ is the charge of the electron, $\frac{\nu}{c}=\frac{1}{2\pi r}$, and $\epsilon/2\phi_0$ the constant of integration. Thus we obtain the second law of the light-quantum theory (Law of Planck).

From the preceding considerations we may conclude that the Maxwell's equations actually lead to the solution given by Sir J. J. Thomson.

It seems to me also that the difference between the

Maxwell-Hertz-Lorentz electromagnetic theory and the theory of Faraday-Maxwell-J. J. Thomson might be *analytically* formulated as follows :—(1) For the free æther the differential equations are the same in both theories, being the Maxwell's system :

$$\operatorname{div} \mathbf{E} = 0, \quad \ldots \quad (1)$$

$$\operatorname{curl} \mathbf{M} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \quad \ldots \quad (2)$$

$$\operatorname{div} \mathbf{M} = 0, \quad \ldots \quad (3)$$

and

$$\operatorname{curl} \mathbf{E} = \frac{1}{c}\frac{\partial \mathbf{M}}{\partial t}. \quad \ldots \quad (4)$$

(2) The first theory postulated that all these equations are valid in the *whole* space filled by the free æther ; in the second theory the equation (3) occurs *only* where we have a moving electric field. Formulating this difference *geometrically*, we may say that the electric lines of force in the free æther are everywhere continuous, while the magnetic may be *discontinuous* : they must end on the boundaries dividing the moving Faraday tubes from those at rest. This circumstance gives the possibility of introducing *discontinuities* in electromagnetic field, and enormously increases the variability of the special instances, which satisfy the Maxwell systems of equations. The Maxwell equations, as it has been shown above, admit such forms of electromagnetic fields as have the same properties as the light-quanta.

It seems to me that this interpretation enables us to construct, on the basis of Maxwell's equations, investigating them for the case $\beta_n^2 = 1$, a *rational* quantum theory.

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1 Moscow University,
1 April, 1926.

CXVI. *The Resistance of High-Frequency Circuits. By* R. R. Ramsey, *Ph.D., Professor of Physics, Indiana University* *.

THE resistance of a high-frequency circuit is apparently easy to determine. However, the separation of the resistance of the coil from that of the condenser is a difficult matter.

It has been customary to consider the resistance of the condenser small enough to be neglected, so that the entire resistance of the circuit is ascribed to the coil.

The first attempt to measure the resistance of a condenser at high frequency was made a little more than a year ago by Weyl and Harris (Institute of Radio Engineers, Proc., vol. xiii. p. 109, Feb., 1925). In this work the resistance of a rectangular coil approximately eighteen feet by twenty-five feet was calculated, assuming the resistance of the rectangle at the given frequency to be the same as the high frequency resistance of a straight wire of the same diameter, whose length was the same as the perimeter of the coil. The resistance obtained in this manner varied with the capacity of the condenser from one ohm at ·0005 microfarad to about twenty ohms at ·00005 microfarad.

Callis (Phil. Mag. vol. i. p. 428, Feb. 1926) has made measurements in which he used coils made of No. 36 and No. 40 copper wire. The method used was to measure the resistance of a circuit with a coil and condenser at a given setting using the resistance variation method, then to measure the resistance of the circuit again after the first coil had been replaced by one made exactly like the first. Then the two were connected in series opposition and their position adjusted, so the combined inductance of the two was the same as the inductance of a single coil, and the combined resistance was measured again. From these results the resistance of the condenser could be eliminated and the resistance of the coils determined. Since the results gave the resistance of the coils to be the same as the D.C. resistance, it was assumed that the resistance of small wire wound in a coil was the same as the resistance of the wire when straight, which was very close to its D.C. resistance at a frequency of one million. From these same equations the resistance of the condenser was determined. It was found that the resistance of a

* Communicated by the Author.