due to the enormous increase of the mortality among mere infants under one year of age; and this increase is due not only to deaths at one age, but to deaths from one class of diseases, viz., bowel complaints. If the deaths from bowel complaints be deducted from the deaths from all causes, there remains an excess of deaths in the cold months, and a deficiency in the warm months. In other words, the curve of mortality is regulated by the large number of deaths from diseases of the respiratory organs. The curve of mortality for London, if mere infants be excepted, has thus an inverse relation to the temperature, rising as the temperature falls, and falling as the temperature rises. On the other hand, in Victoria, Australia, where the summers are hotter and the winters milder, the curves of mortality and temperature are directly related to each other—mortality and temperature rising and falling together; the reason being that in Victoria deaths from bowel complaints are much greater, and those from diseases of the respiratory organs much less than in London.

The curves show that the maximum annual mortality from the different diseases groups around certain specific conditions of temperature and moisture combined. Thus, *cold and moist weather* is accompanied with a high death-rate from rheumatism, heart diseases, diphtheria, and measles; *cold weather*, with a high death-rate from bronchitis, pneumonia, &c.; *cold and dry weather*, with a high death-rate from brain diseases, whooping-cough, convulsions; *warm and dry weather*, with a high death-rate from suicide and small-pox; *hot weather*, with a high death-rate from bowel complaints; and *warm moist weather* with a high death-rate from scarlet and typhoid fevers. (See CLIMATE.) (A. B.)

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ATMOSPHERIC RAILWAY, a railway in which the pressure of air is used directly or indirectly to propel carriages, as a substitute for steam. It was devised at a time when the principles of propulsion were not so well understood as they are now, and when the dangers and inconveniences attendant on the use of locomotives were very much exaggerated. It had been long known that small objects could be propelled for great distances through tubes by air pressure, but a Mr Vallance, of Brighton, seems to have been the first to propose the application of this system to passenger traffic. He projected (about 1825) an atmospheric railway, consisting of a wooden tube about 6 feet 6 inches in diameter, with a carriage running inside it. A diaphragm fitting the tube, approximately air-tight, was attached to the carriage, and the air exhausted from the front of it by a stationary engine, so that the atmospheric pressure behind drove the carriage forward. Later inventors, commencing with Henry Pinkus (1835), for the most part kept the carriages altogether outside the tube, and connected them by a bar with a piston working inside it, this piston being moved by atmospheric pressure in the way just mentioned. The tube was generally provided with a slot upon its upper side, closed by a continuous valve or its equivalent, and arrangements were made by which this valve should be opened to allow the passage of the driving bar without permitting great leakage of air. About 1840, Messrs Clegg & Samuda made various experiments with an atmospheric tube constructed on this principle upon a portion of the West London Railway, near Wormwood Scrubs. The apparent success of these induced the Dublin and Kingstown Railway to adopt Clegg & Samuda's scheme upon an extension of their line from Kingstown to Dalkey, where it was in operation in 1844. Later on, the same system was adopted on a part of the South Devon line and in several other places, and during the years 1844–1846 the English and French patent records show a very large number of more or less practicable and ingenious schemes for the tubes, valves, and driving gear of atmospheric railways. The atmospheric system was nowhere permanently successful, but in all cases was eventually superseded by locomotives, the last atmospheric line being probably that at St Germains, which was worked until 1862. Apart from difficulties in connection with the working of the valve, the maintenance of the vacuum, &c., other great practical difficulties, which had not been indicated by the experiments, soon made themselves known in the working of the lines. Above all, it was found that stationary engines, whether hauling a rope or exhausting a tube, could never work a railway with anything like the economy or the convenience of locomotives, a point which is now regarded as settled by engineers, but which was not so thoroughly understood thirty years ago. Lately, the principle of the atmospheric railway has been applied on a very large scale in London and elsewhere, under the name of "PNEUMATIC DESPATCH" (*q.v.*), to the transmission of small parcels in connection with postal and telegraph work, for which purpose it has proved admirably adapted. (See paper by Prof. Sternberg of Carlsruhe in Hensinger von Waldegg's *Handbuch für specielle Eisenbahntechnik*, vol. i. pt. 2, cap. xvii.

# ATOM

ATOM (ἄτομος) is a body which cannot be cut in two. The atomic theory is a theory of the constitution of bodies, which asserts that they are made up of atoms. The opposite theory is that of the homogeneity and continuity of bodies, and asserts, at least in the case of bodies having no apparent organisation, such, for instance, as water, that as we can divide a drop of water into two parts which are each of them drops of water, so we have reason to believe that these smaller drops can be divided again, and the theory goes on to assert that there is nothing in the nature of things to hinder this process of division from being repeated over and over again, times without end. This is the doctrine of the infinite divisibility of bodies, and it is in direct contradiction with the theory of atoms.

The atomists assert that after a certain number of such divisions the parts would be no longer divisible, because each of them would be an atom. The advocates of the continuity of matter assert that the smallest conceivable body has parts, and that whatever has parts may be divided.

In ancient times Democritus was the founder of the atomic theory, while Anaxagoras propounded that of continuity, under the name of the doctrine of homœomeria (Ὁμοιομέρια), or of the similarity of the parts of a body to the whole. The arguments of the atomists, and their replies to the objections of Anaxagoras, are to be found in Lucretius.

In modern times the study of nature has brought to light many properties of bodies which appear to depend on the magnitude and motions of their ultimate constituents, and the question of the existence of atoms has once more become conspicuous among scientific inquiries.

We shall begin by stating the opposing doctrines of atoms and of continuity before giving an outline of the state of molecular science as it now exists. In the earliest times the most ancient philosophers whose speculations are known to us seem to have discussed the ideas of number and of continuous magnitude, of space and time, of matter and motion, with a native power of thought which has probably never been surpassed. Their actual knowledge, however, and their scientific experience were necessarily limited, because in their days the records of human thought were only beginning to accumulate. It is probable that the first exact notions of quantity were founded on the consideration of number. It is by the help of numbers that concrete quantities are practically measured and calculated. Now, number is discontinuous. We pass from one number to the next *per saltum*. The magnitudes, on the other hand, which we meet with in geometry, are essentially continuous. The attempt to apply numerical methods to the comparison of geometrical quantities led to the doctrine of incommensurables, and to that of the infinite divisibility of space. Meanwhile, the same considerations had not been applied to time, so that in the days of Zeno of Elea time was still regarded as made up of a finite number of "moments," while space was confessed to be divisible without limit. This was the state of opinion when the celebrated arguments against the possibility of motion, of which that of Achilles and the tortoise is a specimen, were propounded by Zeno, and such, apparently, continued to be the state of opinion till Aristotle pointed out that time is divisible without limit, in precisely the same sense that space is. And the slowness of the development of scientific ideas may be estimated from the fact that Bayle does not see any force in this statement of Aristotle, but continues to admire the paradox of Zeno. (Bayle's *Dictionary*, art. "Zeno"). Thus the direction of true scientific progress was for many ages towards the recognition of the infinite divisibility of space and time.

It was easy to attempt to apply similar arguments to matter. If matter is extended and fills space, the same mental operation by which we recognise the divisibility of space may be applied, in imagination at least, to the matter which occupies space. From this point of view the atomic doctrine might be regarded as a relic of the old numerical way of conceiving magnitude, and the opposite doctrine of the infinite divisibility of matter might appear for a time the most scientific. The atomists, on the other hand, asserted very strongly the distinction between matter and space. The atoms, they said, do not fill up the universe; there are void spaces between them. If it were not so, Lucretius tells us, there could be no motion, for the atom which gives way first must have some empty place to move into.

> "Quapropter locus est intactus, inane, vacansque
> Quod si non esset, nulla ratione moveri
> Res possent; namque, officium quod corporis exstat,
> Officere atque obstare, id in omni tempore adesset
> Omnibus: haud igitur quicquam procedere posset,
> Principium quoniam cedendi nulla daret res."
> —*De Rerum Natura*, i. 335.

The opposite school maintained then, as they have always done, that there is no vacuum—that every part of space is full of matter, that there is a universal plenum, and that all motion is like that of a fish in the water, which yields in front of the fish because the fish leaves room for it behind.

> "Cedere squamigeris latices nitentibus aiunt
> Et liquidas aperire vias, quia post loca pisces
> Linquant, quo possint cedentes confluere undæ."
> —i. 373.

In modern times Descartes held that, as it is of the essence of matter to be extended in length, breadth, and thickness, so it is of the essence of extension to be occupied by matter, for extension cannot be an extension of nothing.

> "Ac proinde si quæratur quid fiet, si Deus auferat omne corpus quod in aliquo vase continetur, et nullum aliud in ablati locum venire permittat? respondendum est, vasis latera sibi invicem hoc ipso fore contigua. Cum enim inter duo corpora nihil interjacet, necesse est ut se mutuo tangant, ac manifeste repugnat ut distent, sive ut inter ipsa sit distantia, et tamen ut ista distantia sit nihil; quia omnis distantia est modus extensionis, et ideo sine substantia extensa esse non potest."—*Principia*, ii. 18.

This identification of extension with substance runs through the whole of Descartes's works, and it forms one of the ultimate foundations of the system of Spinoza. Descartes, consistently with this doctrine, denied the existence of atoms as parts of matter, which by their own nature are indivisible. He seems to admit, however, that the Deity might make certain particles of matter indivisible in this sense, that no creature should be able to divide them. These particles, however, would be still divisible by their own nature, because the Deity cannot diminish his own power, and therefore must retain his power of dividing them. Leibnitz, on the other hand, regarded his monad as the ultimate element of everything.

There are thus two modes of thinking about the constitution of bodies, which have had their adherents both in ancient and in modern times. They correspond to the two methods of regarding quantity—the arithmetical and the geometrical. To the atomist the true method of estimating the quantity of matter in a body is to count the atoms in it. The void spaces between the atoms count for nothing. To those who identify matter with extension, the volume of space occupied by a body is the only measure of the quantity of matter in it.

Of the different forms of the atomic theory, that of Boscovich may be taken as an example of the purest monadism. According to Boscovich matter is made up of atoms. Each atom is an indivisible point, having position in space, capable of motion in a continuous path, and possessing a certain mass, whereby a certain amount of force is required to produce a given change of motion. Besides this the atom is endowed with potential force, that is to say, that any two atoms attract or repel each other with a force depending on their distance apart. The law of this force, for all distances greater than say the thousandth of an inch, is an attraction varying as the inverse square of the distance. For smaller distances the force is an attraction for one distance and a repulsion for another, according to some law not yet discovered. Boscovich himself, in order to obviate the possibility of two atoms ever being in the same place, asserts that the ultimate force is a repulsion which increases without limit as the distance diminishes without limit, so that two atoms can never coincide. But this seems an unwarrantable concession to the vulgar opinion that two bodies cannot co-exist in the same place. This opinion is deduced from our experience of the behaviour of bodies of sensible size, but we have no experimental evidence that two atoms may not sometimes coincide. For instance, if oxygen and hydrogen combine to form water, we have no experimental evidence that the molecule of oxygen is not in the very same place with the two molecules of hydrogen. Many persons cannot get rid of the opinion that all matter is extended in length, breadth, and depth. This is a prejudice of the same kind with the last, arising from our experience of bodies consisting of immense multitudes of atoms. The system of atoms, according to Boscovich, occupies a certain region of space in virtue of the forces acting between the component atoms of the system and any other atoms when brought near them. No other system of atoms can occupy the same region of space at the same time, because, before it could do so, the mutual action of the atoms would have caused a repulsion between the two systems insuperable by any force which we can command. Thus, a number of soldiers with firearms may occupy an extensive region to the exclusion of the enemy's armies, though the space filled by their bodies is but small. In this way Boscovich explained the apparent extension of bodies consisting of atoms, each of which is devoid of extension. According to Boscovich's theory, all action between bodies is action at a distance. There is no such thing in nature as actual contact between two bodies. When two bodies are said in ordinary language to be in contact, all that is meant is that they are so near together that the repulsion between the nearest pairs of atoms belonging to the two bodies is very great.

Thus, in Boscovich's theory, the atom has continuity of existence in time and space. At any instant of time it is at some point of space, and it is never in more than one place at a time. It passes from one place to another along a continuous path. It has a definite mass which cannot be increased or diminished. Atoms are endowed with the power of acting on one another by attraction or repulsion, the amount of the force depending on the distance between them. On the other hand, the atom itself has no parts or dimensions. In its geometrical aspect it is a mere geometrical point. It has no extension in space. It has not the so-called property of Impenetrability, for two atoms may exist in the same place. This we may regard as one extreme of the various opinions about the constitution of bodies.

The opposite extreme, that of Anaxagoras—the theory that bodies apparently homogeneous and continuous are so in reality—is, in its extreme form, a theory incapable of development. To explain the properties of any substance by this theory is impossible. We can only admit the observed properties of such substance as ultimate facts. There is a certain stage, however, of scientific progress in which a method corresponding to this theory is of service. In hydrostatics, for instance, we define a fluid by means of one of its known properties, and from this definition we make the system of deductions which constitutes the science of hydrostatics. In this way the science of hydrostatics may be built upon an experimental basis, without any consideration of the constitution of a fluid as to whether it is molecular or continuous. In like manner, after the French mathematicians had attempted, with more or less ingenuity, to construct a theory of elastic solids from the hypothesis that they consist of atoms in equilibrium under the action of their mutual forces, Stokes and others showed that all the results of this hypothesis, so far at least as they agreed with facts, might be deduced from the postulate that elastic bodies exist, and from the hypothesis that the smallest portions into which we can divide them are sensibly homogeneous. In this way the principle of continuity, which is the basis of the method of Fluxions and the whole of modern mathematics, may be applied to the analysis of problems connected with material bodies by assuming them, for the purpose of this analysis, to be homogeneous. All that is required to make the results applicable to the real case is that the smallest portions of the substance of which we take any notice shall be sensibly of the same kind. Thus, if a railway contractor has to make a tunnel through a hill of gravel, and if one cubic yard of the gravel is so like another cubic yard that for the purposes of the contract they may be taken as equivalent, then, in estimating the work required to remove the gravel from the tunnel, he may, without fear of error, make his calculations as if the gravel were a continuous substance. But if a worm has to make his way through the gravel, it makes the greatest possible difference to him whether he tries to push right against a piece of gravel, or directs his course through one of the intervals between the pieces; to him, therefore, the gravel is by no means a homogeneous and continuous substance.

In the same way, a theory that some particular substance, say water, is homogeneous and continuous may be a good working theory up to a certain point, but may fail when we come to deal with quantities so minute or so attenuated that their heterogeneity of structure comes into prominence. Whether this heterogeneity of structure is or is not consistent with homogeneity and continuity of substance is another question.

The extreme form of the doctrine of continuity is that stated by Descartes, who maintains that the whole universe is equally full of matter, and that this matter is all of one kind, having no essential property besides that of extension. All the properties which we perceive in matter he reduces to its parts being movable among one another, and so capable of all the varieties which we can perceive to follow from the motion of its parts (*Principia*, ii. 23). Descartes's own attempts to deduce the different qualities and actions of bodies in this way are not of much value. More than a century was required to invent methods of investigating the conditions of the motion of systems of bodies such as Descartes imagined. But the hydrodynamical discovery of Helmholtz that a vortex in a perfect liquid possesses certain permanent characteristics, has been applied by Sir W. Thomson to form a theory of vortex atoms in a homogeneous, incompressible, and frictionless liquid, to which we shall return at the proper time.

### Outline of Modern Molecular Science, and in particular of the Molecular Theory of Gases.

We begin by assuming that bodies are made up of parts, each of which is capable of motion, and that these parts act on each other in a manner consistent with the principle of the conservation of energy. In making these assumptions, we are justified by the facts that bodies may be divided into smaller parts, and that all bodies with which we are acquainted are conservative systems, which would not be the case unless their parts were also conservative systems.

We may also assume that these small parts are in motion. This is the most general assumption we can make, for it includes, as a particular case, the theory that the small parts are at rest. The phenomena of the diffusion of gases and liquids through each other show that there may be a motion of the small parts of a body which is not perceptible to us.

We make no assumption with respect to the nature of the small parts—whether they are all of one magnitude. We do not even assume them to have extension and figure. Each of them must be measured by its mass, and any two of them must, like visible bodies, have the power of acting on one another when they come near enough to do so. The properties of the body, or medium, are determined by the configuration and motion of its small parts.

The first step in the investigation is to determine the amount of motion which exists among the small parts, independent of the visible motion of the medium as a whole. For this purpose it is convenient to make use of a general theorem in dynamics due to Clausius.

When the motion of a material system is such that the time average of the quantity $\Sigma(mx^2)$ remains constant, the state of the system is said to be that of stationary motion. When the motion of a material system is such that the sum of the moments of inertia of the system, about three axes at right angles through its centre of mass, never varies by more than small quantities from a constant value, the system is said to be in a state of stationary motion.

The kinetic energy of a particle is half the product of its mass into the square of its velocity, and the kinetic energy of a system is the sum of the kinetic energy of all its parts.

When an attraction or repulsion exists between two points, half the product of this stress into the distance between the two points is called the *virial* of the stress, and is reckoned positive when the stress is an attraction, and negative when it is a repulsion. The virial of a system is the sum of the virials of the stresses which exist in it. If the system is subjected to the external stress of the pressure of the sides of a vessel in which it is contained, this stress will introduce an amount of virial $\frac{3}{2}p\mathrm{V}$, where $p$ is the pressure on unit of area and V is the volume of the vessel.

The theorem of Clausius may now be stated as follows:—In a material system in a state of stationary motion the time-average of the kinetic energy is equal to the time-average of the virial. In the case of a fluid enclosed in a vessel

$$\tfrac{1}{2}\Sigma(m\overline{v^2}) = \tfrac{3}{2}p\mathrm{V} + \tfrac{1}{2}\Sigma\Sigma(\mathrm{R}r),$$

where the first term denotes the kinetic energy, and is half the sum of the product of each mass into the mean square of its velocity. In the second term, $p$ is the pressure on unit of surface of the vessel, whose volume is V, and the third term expresses the virial due to the internal actions between the parts of the system. A double symbol of summation is used, because every pair of parts between which any action exists must be taken into account. We have next to show that in gases the principal part of the pressure arises from the motion of the small parts of the medium, and not from a repulsion between them.

In the first place, if the pressure of a gas arises from the repulsion of its parts, the law of repulsion must be inversely as the distance. For, consider a cube filled with the gas at pressure $p$, and let the cube expand till each side is $n$ times its former length. The pressure on unit of surface according to Boyle's law is now $\frac{p}{n^3}$, and since the area of a face of the cube is $n^2$ times what it was, the whole pressure on the face of the cube is $\frac{1}{n}$ of its original value. But since everything has been expanded symmetrically, the distance between corresponding parts of the air is now $n$ times what it was, and the force is $n$ times less than it was. Hence the force must vary inversely as the distance.

But Newton has shown (*Principia*, bk. i. prop. 93) that this law is inadmissible, as it makes the effect of the distant parts of the medium on a particle greater than that of the neighbouring parts. Indeed, we should arrive at the conclusion that the pressure depends not only on the density of the air but on the form and dimensions of the vessel which contains it, which we know not to be the case.

If, on the other hand, we suppose the pressure to arise entirely from the motion of the molecules of the gas, the interpretation of Boyle's law becomes very simple. For, in this case

$$p\mathrm{V} = \tfrac{1}{3}\Sigma(m\overline{v^2}).$$

The first term is the product of the pressure and the volume, which according to Boyle's law is constant for the same quantity of gas at the same temperature. The second term is two-thirds of the kinetic energy of the system, and we have every reason to believe that in gases when the temperature is constant the kinetic energy of unit of mass is also constant. If we admit that the kinetic energy of unit of mass is in a given gas proportional to the absolute temperature, this equation is the expression of the law of Charles as well as of that of Boyle, and may be written—

$$p\mathrm{V} = \mathrm{R}\theta,$$

where $\theta$ is the temperature reckoned from absolute zero, and R is a constant. The fact that this equation expresses with considerable accuracy the relation between the volume, pressure, and temperature of a gas when in an extremely rarified state, and that as the gas is more and more compressed the deviation from this equation becomes more apparent, shows that the pressure of a gas is due almost entirely to the motion of its molecules when the gas is rare, and that it is only when the density of the gas is considerably increased that the effect of direct action between the molecules becomes apparent.

The effect of the direct action of the molecules on each other depends on the number of pairs of molecules which at a given instant are near enough to act on one another. The number of such pairs is proportional to the square of the number of molecules in unit of volume, that is, to the square of the density of the gas. Hence, as long as the medium is so rare that the encounter between two molecules is not affected by the presence of others, the deviation from Boyle's law will be proportional to the square of the density. If the action between the molecules is on the whole repulsive, the pressure will be greater than that given by Boyle's law. If it is, on the whole, attractive, the pressure will be less than that given by Boyle's law. It appears, by the experiments of Regnault and others, that the pressure does deviate from Boyle's law when the density of the gas is increased. In the case of carbonic acid and other gases which are easily liquefied, this deviation is very great. In all cases, however, except that of hydrogen, the pressure is less than that given by Boyle's law, showing that the virial is on the whole due to *attractive* forces between the molecules.

Another kind of evidence as to the nature of the action between the molecules is furnished by an experiment made by Dr Joule. Of two vessels, one was exhausted and the other filled with a gas at a pressure of 20 atmospheres; and both were placed side by side in a vessel of water, which was constantly stirred. The temperature of the whole was observed. Then a communication was opened between the vessels, the compressed gas expanded to twice its volume, and the work of expansion, which at first produced a strong current in the gas, was soon converted into heat by the internal friction of the gas. When all was again at rest, and the temperature uniform, the temperature was again observed. In Dr Joule's original experiments the observed temperature was the same as before. In a series of experiments, conducted by Dr Joule and Sir W. Thomson on a different plan, by which the thermal effect of free expansion can be more accurately measured, a slight cooling effect was observed in all the gases examined except hydrogen. Since the temperature depends on the velocity of agitation of the molecules, it appears that when a gas expands without doing external work the velocity of agitation is not much affected, but that in most cases it is slightly diminished. Now, if the molecules during their mutual separation act on each other, their velocity will increase or diminish according as the force is repulsive or attractive. It appears, therefore, from the experiments on the free expansion of gases, that the force between the molecules is small but, on the whole, attractive.

Having thus justified the hypothesis that a gas consists of molecules in motion, which act on each other only when they come very close together during an encounter, but which, during the intervals between their encounters which constitute the greater part of their existence, are describing free paths, and are not acted on by any molecular force, we proceed to investigate the motion of such a system.

The mathematical investigation of the properties of such a system of molecules in motion is the foundation of molecular science. Clausius was the first to express the relation between the density of the gas, the length of the free paths of its molecules, and the distance at which they encounter each other. He assumed, however, at least in his earlier investigations, that the velocities of all the molecules are equal. The mode in which the velocities are distributed was first investigated by the present writer, who showed that in the moving system the velocities of the molecules range from zero to infinity, but that the number of molecules whose velocities lie within given limits can be expressed by a formula identical with that which expresses in the theory of errors the number of errors of observation lying within corresponding limits. The proof of this theorem has been carefully investigated by Boltzmann,[1] who has strengthened it where it appeared weak, and to whom the method of taking into account the action of external forces is entirely due.

The mean kinetic energy of a molecule, however, has a definite value, which is easily expressed in terms of the quantities which enter into the expression for the distribution of velocities. The most important result of this investigation is that when several kinds of molecules are in motion and acting on one another, the mean kinetic energy of a molecule is the same whatever be its mass, the molecules of greater mass having smaller mean velocities. Now, when gases are mixed their temperatures become equal. Hence we conclude that the physical condition which determines that the temperature of two gases shall be the same is that the mean kinetic energies of agitation of the individual molecules of the two gases are equal. This result is of great importance in the theory of heat, though we are not yet able to establish any similar result for bodies in the liquid or solid state.

In the next place, we know that in the case in which the whole pressure of the medium is due to the motion of its molecules, the pressure on unit of area is numerically equal to two-thirds of the kinetic energy in unit of volume. Hence, if equal volumes of two gases are at equal pressures the kinetic energy is the same in each. If they are also at equal temperatures the mean kinetic energy of each molecule is the same in each. If, therefore, equal volumes of two gases are at equal temperatures and pressures, the number of molecules in each is the same, and therefore, the masses of the two kinds of molecules are in the same ratio as the densities of the gases to which they belong.

This statement has been believed by chemists since the time of Gay-Lussac, who first established that the weights of the chemical equivalents of different substances are proportional to the densities of these substances when in the form of gas. The definition of the word molecule, however, as employed in the statement of Gay-Lussac's law is by no means identical with the definition of the same word as in the kinetic theory of gases. The chemists ascertain by experiment the ratios of the masses of the different substances in a compound. From these they deduce the chemical equivalents of the different substances, that of a particular substance, say hydrogen, being taken as unity. The only evidence made use of is that furnished by chemical combinations. It is also assumed, in order to account for the facts of combination, that the reason why substances combine in definite ratios is that the molecules of the substances are in the ratio of their chemical equivalents, and that what we call combination is an action which takes place by a union of a molecule of one substance to a molecule of the other.

This kind of reasoning, when presented in a proper form and sustained by proper evidence, has a high degree of cogency. But it is purely chemical reasoning; it is not dynamical reasoning. It is founded on chemical experience, not on the laws of motion.

Our definition of a molecule is purely dynamical. A molecule is that minute portion of a substance which moves about as a whole, so that its parts, if it has any, do not part company during the motion of agitation of the gas. The result of the kinetic theory, therefore, is to give us information about the relative masses of molecules considered as moving bodies. The consistency of this information with the deductions of chemists from the phenomena of combination, greatly strengthens the evidence in favour of the actual existence and motion of gaseous molecules.

Another confirmation of the theory of molecules is derived from the experiments of Dulong and Petit on the specific heat of gases, from which they deduced the law which bears their name, and which asserts that the specific heats of equal weights of gases are inversely as their combining weights, or, in other words, that the capacities for heat of the chemical equivalents of different gases are equal. We have seen that the temperature is determined by the kinetic energy of agitation of each molecule. The molecule has also a certain amount of energy of internal motion, whether of rotation or of vibration, but the hypothesis of Clausius, that the mean value of the internal energy always bears a proportion fixed for each gas to the energy of agitation, seems highly probable and consistent with experiment. The whole kinetic energy is therefore equal to the energy of agitation multiplied by a certain factor. Thus the energy communicated to a gas by heating it is divided in a certain proportion between the energy of agitation and that of the internal motion of each molecule. For a given rise of temperature the energy of agitation, say of a million molecules, is increased by the same amount whatever be the gas. The heat spent in raising the temperature is measured by the increase of the whole kinetic energy. The thermal capacities, therefore, of equal numbers of molecules of different gases are in the ratio of the factors by which the energy of agitation must be multiplied to obtain the whole energy. As this factor appears to be nearly the same for all gases of the same degree of atomicity, Dulong and Petit's law is true for such gases.

Another result of this investigation is of considerable importance in relation to certain theories,[2] which assume the existence of æthers or rare media consisting of molecules very much smaller than those of ordinary gases. According to our result, such a medium would be neither more nor less than a gas. Supposing its molecules so small that they can penetrate between the molecules of solid substances such as glass, a so-called vacuum would be full of this rare gas at the observed temperature, and at the pressure, whatever it may be, of the ætherial medium in space. The specific heat, therefore, of the medium in the so-called vacuum will be equal to that of the same volume of any other gas at the same temperature and pressure. Now, the purpose for which this molecular æther is assumed in these theories is to act on bodies by its pressure, and for this purpose the pressure is generally assumed to be very great. Hence, according to these theories, we should find the specific heat of a so-called vacuum very considerable compared with that of a quantity of air filling the same space.

We have now made a certain definite amount of progress towards a complete molecular theory of gases. We know the mean velocity of the molecules of each gas in metres per second, and we know the relative masses of the molecules of different gases. We also know that the molecules of one and the same gas are all equal in mass. For if they

[1] *Sitzungsberichte der K. K. Akad., Wien*, 8th Oct. 1868.

[2] See Gustav Hansemann, *Die Atome und ihre Bewegungen*. 1871. (H. G. Mayer.)

are not, the method of dialysis, as employed by Graham, would enable us to separate the molecules of smaller mass from those of greater, as they would stream through porous substances with greater velocity. We should thus be able to separate a gas, say hydrogen, into two portions, having different densities and other physical properties, different combining weights, and probably different chemical properties of other kinds. As no chemist has yet obtained specimens of hydrogen differing in this way from other specimens, we conclude that all the molecules of hydrogen are of sensibly the same mass, and not merely that their mean mass is a statistical constant of great stability.

But as yet we have not considered the phenomena which enable us to form an estimate of the actual mass and dimensions of a molecule. It is to Clausius that we owe the first definite conception of the free path of a molecule and of the mean distance travelled by a molecule between successive encounters. He showed that the number of encounters of a molecule in a given time is proportional to the velocity, to the number of molecules in unit of volume, and to the square of the distance between the centres of two molecules when they act on one another so as to have an encounter. From this it appears that if we call this distance of the centres the diameter of a molecule, and the volume of a sphere having this diameter the volume of a molecule, and the sum of the volumes of all the molecules the molecular volume of the gas, then the diameter of a molecule is a certain multiple of the quantity obtained by diminishing the free path in the ratio of the molecular volume of the gas to the whole volume of the gas. The numerical value of this multiple differs slightly, according to the hypothesis we assume about the law of distribution of velocities. It also depends on the definition of an encounter. When the molecules are regarded as elastic spheres we know what is meant by an encounter, but if they act on each other at a distance by attractive or repulsive forces of finite magnitude, the distance of their centres varies during an encounter, and is not a definite quantity. Nevertheless, the above statement of Clausius enables us, if we know the length of the mean path and the molecular volume of a gas, to form a tolerably near estimate of the diameter of the sphere of the intense action of a molecule, and thence of the number of molecules in unit of volume and the actual mass of each molecule. To complete the investigation we have, therefore, to determine the mean path and the molecular volume. The first numerical estimate of the mean path of a gaseous molecule was made by the present writer from data derived from the internal friction of air. There are three phenomena which depend on the length of the free path of the molecules of a gas. It is evident that the greater the free path the more rapidly will the molecules travel from one part of the medium to another, because their direction will not be so often altered by encounters with other molecules. If the molecules in different parts of the medium are of different kinds, their progress from one part of the medium to another can be easily traced by analysing portions of the medium taken from different places. The rate of diffusion thus found furnishes one method of estimating the length of the free path of a molecule. This kind of diffusion goes on not only between the molecules of different gases, but among the molecules of the same gas, only in the latter case the results of the diffusion cannot be traced by analysis. But the diffusing molecules carry with them in their free paths the momentum and the energy which they happen at a given instant to have. The diffusion of momentum tends to equalise the apparent motion of different parts of the medium, and constitutes the phenomenon called the internal friction or viscosity of gases. The diffusion of energy tends to equalise the temperature of different parts of the medium, and constitutes the phenomenon of the conduction of heat in gases.

These three phenomena—the diffusion of matter, of motion, and of heat in gases—have been experimentally investigated,—the diffusion of matter by Graham and Loschmidt, the diffusion of motion by Oscar Meyer and Clerk Maxwell, and that of heat by Stefan.

These three kinds of experiments give results which in the present imperfect state of the theory and the extreme difficulty of the experiments, especially those on the conduction of heat, may be regarded as tolerably consistent with each other. At the pressure of our atmosphere, and at the temperature of melting ice, the mean path of a molecule of hydrogen is about the 10,000th of a millimetre, or about the fifth part of a wave-length of green light. The mean path of the molecules of other gases is shorter than that of hydrogen.

The determination of the molecular volume of a gas is subject as yet to considerable uncertainty. The most obvious method is that of compressing the gas till it assumes the liquid form. It seems probable, from the great resistance of liquids to compression, that their molecules are at about the same distance from each other as that at which two molecules of the same substance in the gaseous form act on each other during an encounter. If this is the case, the molecular volume of a gas is somewhat less than the volume of the liquid into which it would be condensed by pressure, or, in other words, the density of the molecules is somewhat greater than that of the liquid.

Now, we know the relative weights of different molecules with great accuracy, and, from a knowledge of the mean path, we can calculate their relative diameters approximately. From these we can deduce the relative densities of different kinds of molecules. The relative densities so calculated have been compared by Lorenz Meyer with the observed densities of the liquids into which the gases may be condensed, and he finds a remarkable correspondence between them. There is considerable doubt, however, as to the relation between the molecules of a liquid and those of its vapour, so that till a larger number of comparisons have been made, we must not place too much reliance on the calculated densities of molecules. Another, and perhaps a more refined, method is that adopted by M. Van der Waals, who deduces the molecular volume from the deviations of the pressure from Boyle's law as the gas is compressed.

The first numerical estimate of the diameter of a molecule was that made by Loschmidt in 1865 from the mean path and the molecular volume. Independently of him and of each other, Mr Stoney, in 1868, and Sir W. Thomson, in 1870, published results of a similar kind—those of Thomson being deduced not only in this way, but from considerations derived from the thickness of soap bubbles, and from the electric action between zinc and copper.

The diameter and the mass of a molecule, as estimated by these methods, are, of course, very small, but by no means infinitely so. About two millions of molecules of hydrogen in a row would occupy a millimetre, and about two hundred million million million of them would weigh a milligramme. These numbers must be considered as exceedingly rough guesses; they must be corrected by more extensive and accurate experiments as science advances; but the main result, which appears to be well established, is that the determination of the mass of a molecule is a legitimate object of scientific research, and that this mass is by no means immeasurably small.

Loschmidt illustrates these molecular measurements by a comparison with the smallest magnitudes visible by means of a microscope. Nobert, he tells us, can draw 4000 lines in the breadth of a millimetre. The intervals between these lines can be observed with a good microscope. A cube, whose side is the 4000th of a millimetre, may be taken as the *minimum visibile* for observers of the present day. Such a cube would contain from 60 to 100 million molecules of oxygen or of nitrogen; but since the molecules of organised substances contain on an average about 50 of the more elementary atoms, we may assume that the smallest organised particle visible under the microscope contains about two million molecules of organic matter. At least half of every living organism consists of water, so that the smallest living being visible under the microscope does not contain more than about a million organic molecules. Some exceedingly simple organism may be supposed built up of not more than a million similar molecules. It is impossible, however, to conceive so small a number sufficient to form a being furnished with a whole system of specialised organs.

Thus molecular science sets us face to face with physiological theories. It forbids the physiologist from imagining that structural details of infinitely small dimensions can furnish an explanation of the infinite variety which exists in the properties and functions of the most minute organisms.

A microscopic germ is, we know, capable of development into a highly organised animal. Another germ, equally microscopic, becomes, when developed, an animal of a totally different kind. Do all the differences, infinite in number, which distinguish the one animal from the other, arise each from some difference in the structure of the respective germs? Even if we admit this as possible, we shall be called upon by the advocates of Pangenesis to admit still greater marvels. For the microscopic germ, according to this theory, is no mere individual, but a representative body, containing members collected from every rank of the long-drawn ramification of the ancestral tree, the number of these members being amply sufficient not only to furnish the hereditary characteristics of every organ of the body and every habit of the animal from birth to death, but also to afford a stock of latent gemmules to be passed on in an inactive state from germ to germ, till at last the ancestral peculiarity which it represents is revived in some remote descendant.

Some of the exponents of this theory of heredity have attempted to elude the difficulty of placing a whole world of wonders within a body so small and so devoid of visible structure as a germ, by using the phrase structureless germs.[1] Now, one material system can differ from another only in the configuration and motion which it has at a given instant. To explain differences of function and development of a germ without assuming differences of structure is, therefore, to admit that the properties of a germ are not those of a purely material system.

The evidence as to the nature and motion of molecules, with which we have hitherto been occupied, has been derived from experiments upon gaseous media, the smallest sensible portion of which contains millions of millions of molecules. The constancy and uniformity of the properties of the gaseous medium is the direct result of the inconceivable irregularity of the motion of agitation of its molecules. Any cause which could introduce regularity into the motion of agitation, and marshal the molecules into order and method in their evolutions, might check or even reverse that tendency to diffusion of matter, motion, and energy, which is one of the most invariable phenomena of nature, and to which Thomson has given the name of the dissipation of energy.

Thus, when a sound-wave is passing through a mass of air, this motion is of a certain definite type, and if left to itself the whole motion is passed on to other masses of air, and the sound-wave passes on, leaving the air behind it at rest. Heat, on the other hand, never passes out of a hot body except to enter a colder body, so that the energy of sound-waves, or any other form of energy which is propagated so as to pass wholly out of one portion of the medium and into another, cannot be called heat.

We have now to turn our attention to a class of molecular motions, which are as remarkable for their regularity as the motion of agitation is for its irregularity.

It has been found, by means of the spectroscope, that the light emitted by incandescent substances is different according to their state of condensation. When they are in an extremely rarefied condition the spectrum of their light consists of a set of sharply-defined bright lines. As the substance approaches a denser condition the spectrum tends to become continuous, either by the lines becoming broader and less defined, or by new lines and bands appearing between them, till the spectrum at length loses all its characteristics and becomes identical with that of solid bodies when raised to the same temperature.

Hence the vibrating systems, which are the source of the emitted light, must be vibrating in a different manner in these two cases. When the spectrum consists of a number of bright lines, the motion of the system must be compounded of a corresponding number of types of harmonic vibration.

In order that a bright line may be sharply defined, the vibratory motion which produces it must be kept up in a perfectly regular manner for some hundreds or thousands of vibrations. If the motion of each of the vibrating bodies is kept up only during a small number of vibrations, then, however regular may be the vibrations of each body while it lasts, the resultant disturbance of the luminiferous medium, when analysed by the prism, will be found to contain, besides the part due to the regular vibrations, other motions, depending on the starting and stopping of each particular vibrating body, which will become manifest as a diffused luminosity scattered over the whole length of the spectrum. A spectrum of bright lines, therefore, indicates that the vibrating bodies when set in motion are allowed to vibrate in accordance with the conditions of their internal structure for some time before they are again interfered with by external forces.

It appears, therefore, from spectroscopic evidence that each molecule of a rarefied gas is, during the greater part of its existence, at such a distance from all other molecules that it executes its vibrations in an undisturbed and regular manner. This is the same conclusion to which we were led by considerations of another kind at p. 39.

We may therefore regard the bright lines in the spectrum of a gas as the result of the vibrations executed by the molecules while describing their free paths. When two molecules separate from one another after an encounter, each of them is in a state of vibration, arising from the unequal action on different parts of the same molecule during the encounter. Hence, though the centre of mass of the molecule describing its free path moves with uniform velocity, the parts of the molecule have a vibratory motion with respect to the centre of mass of the whole molecule, and it is the disturbance of the luminiferous medium communicated to it by the vibrating molecules which constitutes the emitted light.

We may compare the vibrating molecule to a bell. When struck, the bell is set in motion. This motion is compounded of harmonic vibrations of many different periods, each of which acts on the air, producing notes of as many different pitches. As the bell communicates its motion to the air, these vibrations necessarily decay, some

[1] See F. Galton, "On Blood Relationship," *Proc. Roy. Soc.*, June 13, 1872.

of them faster than others, so that the sound contains fewer and fewer notes, till at last it is reduced to the fundamental note of the bell.[1] If we suppose that there are a great many bells precisely similar to each other, and that they are struck, first one and then another, in a perfectly irregular manner, yet so that, on an average, as many bells are struck in one second of time as in another, and also in such a way that, on an average, any one bell is not again struck till it has ceased to vibrate, then the audible result will appear a continuous sound, composed of the sound emitted by bells in all states of vibration, from the clang of the actual stroke to the final hum of the dying fundamental tone.

But now let the number of bells be reduced while the same number of strokes are given in a second. Each bell will now be struck before it has ceased to vibrate, so that in the resulting sound there will be less of the fundamental tone and more of the original clang, till at last, when the peal is reduced to one bell, on which innumerable hammers are continually plying their strokes all out of time, the sound will become a mere noise, in which no musical note can be distinguished.

In the case of a gas we have an immense number of molecules, each of which is set in vibration when it encounters another molecule, and continues to vibrate as it describes its free path. The molecule is a material system, the parts of which are connected in some definite way, and from the fact that the bright lines of the emitted light have always the same wave-lengths, we learn that the vibrations corresponding to these lines are always executed in the same periodic time, and therefore the force tending to restore any part of the molecule to its position of equilibrium in the molecule must be proportional to its displacement relative to that position.

From the mathematical theory of the motion of such a system, it appears that the whole motion may be analysed into the following parts, which may be considered each independently of the others:—In the first place, the centre of mass of the system moves with uniform velocity in a straight line. This velocity may have any value. In the second place, there may be a motion of rotation, the angular momentum of the system about its centre of mass remaining during the free path constant in magnitude and direction. This angular momentum may have any value whatever, and its axis may have any direction. In the third place, the remainder of the motion is made up of a number of component motions, each of which is an harmonic vibration of a given type. In each type of vibration the periodic time of vibration is determined by the nature of the system, and is invariable for the same system. The relative amount of motion in different parts of the system is also determinate for each type, but the absolute amount of motion and the phase of the vibration of each type are determined by the particular circumstances of the last encounter, and may vary in any manner from one encounter to another.

The values of the periodic times of the different types of vibration are given by the roots of a certain equation, the form of which depends on the nature of the connections of the system. In certain exceptionally simple cases, as, for instance, in that of a uniform string stretched between two fixed points, the roots of the equation are connected by simple arithmetical relations, and if the internal structure of a molecule had an analogous kind of simplicity, we might expect to find in the spectrum of the molecule a series of bright lines, whose wave-lengths are in simple arithmetical ratios.

But if we suppose the molecule to be constituted according to some different type, as, for instance, if it is an elastic sphere, or if it consists of a finite number of atoms kept in their places by attractive and repulsive forces, the roots of the equation will not be connected with each other by any simple relations, but each may be made to vary independently of the others by a suitable change of the connections of the system. Hence, we have no right to expect any definite numerical relations among the wave-lengths of the bright lines of a gas.

The bright lines of the spectrum of an incandescent gas are therefore due to the harmonic vibrations of the molecules of the gas during their free paths. The only effect of the motion of the centre of mass of the molecule is to alter the time of vibration of the light as received by a stationary observer. When the molecule is coming towards the observer, each successive impulse will have a shorter distance to travel before it reaches his eye, and therefore the impulses will appear to succeed each other more rapidly than if the molecule were at rest, and the contrary will be the case if the molecule is receding from the observer. The bright line corresponding to the vibration will therefore be shifted in the spectrum towards the blue end when the molecule is approaching, and towards the red end when it is receding from the observer. By observations of the displacement of certain lines in the spectrum, Dr Huggins and others have measured the rate of approach or of recession of certain stars with respect to the earth, and Mr Lockyer has determined the rate of motion of tornadoes in the sun. But Lord Rayleigh has pointed out that according to the dynamical theory of gases the molecules are moving hither and thither with so great velocity that, however narrow and sharply-defined any bright line due to a single molecule may be, the displacement of the line towards the blue by the approaching molecules, and towards the red by the receding molecules, will produce a certain amount of widening and blurring of the line in the spectrum, so that there is a limit to the sharpness of definition of the lines of a gas. The widening of the lines due to this cause will be in proportion to the velocity of agitation of the molecules. It will be greatest for the molecules of smallest mass, as those of hydrogen, and it will increase with the temperature. Hence the measurement of the breadth of the hydrogen lines, such as C or F in the spectrum of the solar prominences, may furnish evidence that the temperature of the sun cannot exceed a certain value.

### On the Theory of Vortex Atoms.

The equations which form the foundations of the mathematical theory of fluid motion were fully laid down by Lagrange and the great mathematicians of the end of last century, but the number of solutions of cases of fluid motion which had been actually worked out remained very small, and almost all of these belonged to a particular type of fluid motion, which has been since named the irrotational type. It had been shown, indeed, by Lagrange, that a perfect fluid, if its motion is at any time irrotational, will continue in all time coming to move in an irrotational manner, so that, by assuming that the fluid was at one time at rest, the calculation of its subsequent motion may be very much simplified.

It was reserved for Helmholtz to point out the very remarkable properties of rotational motion in a homogeneous incompressible fluid devoid of all viscosity. We must first define the physical properties of such a fluid. In the first place, it is a material substance. Its motion is

[1] Part of the energy of motion is, in the case of the bell, dissipated in the substance of the bell in virtue of the viscosity of the metal, and assumes the form of heat, but it is not necessary, for the purpose of illustration, to take this cause of the decay of vibrations into account.

continuous in space and time, and if we follow any portion of it as it moves, the mass of that portion remains invariable. These properties it shares with all material substances. In the next place, it is incompressible. The form of a given portion of the fluid may change, but its volume remains invariable; in other words, the density of the fluid remains the same during its motion. Besides this, the fluid is homogeneous, or the density of all parts of the fluid is the same. It is also continuous, so that the mass of the fluid contained within any closed surface is always *exactly* proportional to the volume contained within that surface. This is equivalent to asserting that the fluid is not made up of molecules; for, if it were, the mass would vary in a discontinuous manner as the volume increases continuously, because first one and then another molecule would be included within the closed surface. Lastly, it is a perfect fluid, or, in other words, the stress between one portion and a contiguous portion is always normal to the surface which separates these portions, and this whether the fluid is at rest or in motion.

We have seen that in a molecular fluid the interdiffusion of the molecules causes an interdiffusion of motion of different parts of the fluid, so that the action between contiguous parts is no longer normal but in a direction tending to diminish their relative motion. Hence the perfect fluid cannot be molecular.

All that is necessary in order to form a correct mathematical theory of a material system is that its properties shall be clearly defined and shall be consistent with each other. This is essential; but whether a substance having such properties actually exists is a question which comes to be considered only when we propose to make some practical application of the results of the mathematical theory. The properties of our perfect liquid are clearly defined and consistent with each other, and from the mathematical theory we can deduce remarkable results, some of which may be illustrated in a rough way by means of fluids which are by no means perfect in the sense of not being viscous, such, for instance, as air and water.

The motion of a fluid is said to be irrotational when it is such that if a spherical portion of the fluid were suddenly solidified, the solid sphere so formed would not be rotating about any axis. When the motion of the fluid is rotational the axis and angular velocity of the rotation of any small part of the fluid are those of a *small* spherical portion suddenly solidified.

The mathematical expression of these definitions is as follows:—Let $u$, $v$, $w$ be the components of the velocity of the fluid at the point $(x, y, z)$, and let

$$\alpha = \frac{dv}{dz} - \frac{dw}{dy}\ ,\ \beta = \frac{dw}{dx} - \frac{du}{dz}\ ,\ \gamma = \frac{du}{dy} - \frac{dv}{dx} \qquad (1),$$

then $\alpha$, $\beta$, $\gamma$ are the components of the velocity of rotation of the fluid at the point $(x, y, z)$. The axis of rotation is in the direction of the resultant of $\alpha$, $\beta$, and $\gamma$, and the velocity of rotation, $\omega$, is measured by this resultant.

A line drawn in the fluid, so that at every point of the line

$$\frac{1}{\alpha}\frac{dx}{ds} = \frac{1}{\beta}\frac{dy}{ds} = \frac{1}{\gamma}\frac{dz}{ds} = \frac{1}{\omega} \quad . \quad . \quad (2),$$

where $s$ is the length of the line up to the point $x$, $y$, $z$, is called a vortex line. Its direction coincides at every point with that of the axis of rotation of the fluid.

We may now prove the theorem of Helmholtz, that the points of the fluid which at any instant lie in the same vortex line continue to lie in the same vortex line during the whole motion of the fluid.

The equations of motion of a fluid are of the form

$$\rho\frac{\partial u}{\partial t} + \frac{dp}{dx} + \rho\frac{dV}{dx} = 0 \quad . \quad . \quad (3),$$

when $\rho$ is the density, which in the case of our homogeneous incompressible fluid we may assume to be unity, the operator $\frac{\partial}{\partial t}$ represents the rate of variation of the symbol to which it is prefixed at a point which is carried forward with the fluid, so that

$$\frac{\partial u}{\partial t} = \frac{du}{dt} + u\frac{du}{dx} + v\frac{du}{dy} + w\frac{du}{dz} \quad . \quad . \quad (4),$$

$p$ is the pressure, and V is the potential of external forces. There are two other equations of similar form in $y$ and $z$. Differentiating the equation in $y$ with respect to $z$, and that in $z$ with respect to $y$, and subtracting the second from the first, we find

$$\frac{d}{dz}\frac{\partial v}{\partial t} - \frac{d}{dy}\frac{\partial w}{\partial t} = 0 \quad . \quad . \quad . \quad (5).$$

Performing the differentiations and remembering equations (1) and also the condition of incompressibility,

$$\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz} = 0 \quad . \quad . \quad . \quad (6),$$

we find

$$\frac{\partial\alpha}{\partial t} = \alpha\frac{du}{dx} + \beta\frac{du}{dy} + \gamma\frac{du}{dz} \quad . \quad . \quad (7).$$

Now, let us suppose a vortex line drawn in the fluid so as always to begin at the same particle of the fluid. The components of the velocity of this point are $u$, $v$, $w$. Let us find those of a point on the moving vortex line at a distance $ds$ from this point where

$$ds = \omega d\sigma \quad . \quad . \quad . \quad (8).$$

The co-ordinates of this point are

$$x + \alpha d\sigma\ ,\ y + \beta d\sigma\ ,\ z + \gamma d\sigma \quad . \quad . \quad (9),$$

and the components of its velocity are

$$u + \frac{\partial\alpha}{\partial t}d\sigma\ ,\ v + \frac{\partial\beta}{\partial t}d\sigma\ ,\ w + \frac{\partial\gamma}{\partial t}d\sigma \quad . \quad (10).$$

Consider the first of these components. In virtue of equation (7) we may write it

$$u + \frac{du}{dx}\alpha d\sigma + \frac{du}{dy}\beta d\sigma + \frac{du}{dz}\gamma d\sigma \quad . \quad . \quad (11),$$

or

$$u + \frac{du}{dx}\frac{dx}{ds}d\sigma + \frac{du}{dy}\frac{dy}{ds}d\sigma + \frac{du}{dz}\frac{dz}{ds}d\sigma \quad . \quad (12),$$

or

$$u + \frac{du}{ds}d\sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13).$$

But this represents the value of the component $u$ of the velocity of the fluid itself at the same point, and the same thing may be proved of the other components.

Hence the velocity of the second point on the vortex line is identical with that of the fluid at that point. In other words, the vortex line swims along with the fluid, and is always formed of the same row of fluid particles. The vortex line is therefore no mere mathematical symbol, but has a physical existence continuous in time and space.

By differentiating equations (1) with respect to $x$, $y$, and $z$ respectively, and adding the results, we obtain the equation—

$$\frac{d\alpha}{dx} + \frac{d\beta}{dy} + \frac{d\gamma}{dz} = 0 \quad . \quad . \quad . \quad (14).$$

This is an equation of the same form with (6), which expresses the condition of flow of a fluid of invariable density. Hence, if we imagine a fluid, quite independent of the original fluid, whose components of velocity are $\alpha$, $\beta$, $\gamma$, this imaginary fluid will flow without altering its density.

Now, consider a closed curve in space, and let vortex lines be drawn in both directions from every point of this curve. These vortex lines will form a tubular surface, which is called a vortex tube or a vortex filament. Since the imaginary fluid flows along the vortex lines without change of density, the quantity which in unit of time flows through any section of the same vortex tube must be the same. Hence, at any section of a vortex tube the product of the area of the section into the mean velocity of rotation is the same. This quantity is called the *strength* of the vortex tube.

A vortex tube cannot begin or end within the fluid; for, if it did, the imaginary fluid, whose velocity components are $\alpha$, $\beta$, $\gamma$, would be generated from nothing at the beginning of the tube, and reduced to nothing at the end of it. Hence, if the tube has a beginning and an end, they must lie on the surface of the fluid mass. If the fluid is infinite the vortex tube must be infinite, or else it must return into itself.

We have thus arrived at the following remarkable theorems relating to a finite vortex tube in an infinite fluid:—(1.) It returns into itself, forming a closed ring. We may therefore describe it as a vortex *ring*. (2.) It always consists of the same portion of the fluid. Hence its volume is invariable. (3.) Its strength remains always the same. Hence the velocity of rotation at any section varies inversely as the area of that section, and that of any segment varies directly as the length of that segment. (4.) No part of the fluid which is not originally in a state of rotational motion can ever enter into that state, and no part of the fluid whose motion is rotational can ever cease to move rotationally. (5.) No vortex tube can ever pass through any other vortex tube, or through any of its own convolutions. Hence, if two vortex tubes are linked together, they can never be separated, and if a single vortex tube is knotted on itself, it can never become untied. (6.) The motion at any instant of every part of the fluid, including the vortex rings themselves, may be accurately represented by conceiving an electric current to occupy the place of each vortex ring, the strength of the current being proportional to that of the ring. The magnetic force at any point of space will then represent in direction and magnitude the velocity of the fluid at the corresponding point of the fluid.

These properties of vortex rings suggested to Sir William Thomson[1] the possibility of founding on them a new form of the atomic theory. The conditions which must be satisfied by an atom are—permanence in magnitude, capability of internal motion or vibration, and a sufficient amount of possible characteristics to account for the difference between atoms of different kinds.

The small hard body imagined by Lucretius, and adopted by Newton, was invented for the express purpose of accounting for the permanence of the properties of bodies. But it fails to account for the vibrations of a molecule as revealed by the spectroscope. We may indeed suppose the atom elastic, but this is to endow it with the very property for the explanation of which, as exhibited in aggregate bodies, the atomic constitution was originally assumed. The massive centres of force imagined by Boscovich may have more to recommend them to the mathematician, who has no scruple in supposing them to be invested with the power of attracting and repelling according to any law of the distance which it may please him to assign. Such centres of force are no doubt in their own nature indivisible, but then they are also, singly, incapable of vibration. To obtain vibrations we must imagine molecules consisting of many such centres, but, in so doing, the possibility of these centres being separated altogether is again introduced. Besides, it is in questionable scientific taste, after using atoms so freely to get rid of forces acting at sensible distances, to make the whole function of the atoms an action at insensible distances.

On the other hand, the vortex ring of Helmholtz, imagined as the true form of the atom by Thomson, satisfies more of the conditions than any atom hitherto imagined. In the first place, it is quantitatively permanent, as regards its volume and its strength,—two independent quantities. It is also qualitatively permanent as regards its degree of implication, whether "knottedness" on itself or "linkedness" with other vortex rings. At the same time, it is capable of infinite changes of form, and may execute vibrations of different periods, as we know that molecules do. And the number of essentially different implications of vortex rings may be very great without supposing the degree of implication of any of them very high.

But the greatest recommendation of this theory, from a philosophical point of view, is that its success in explaining phenomena does not depend on the ingenuity with which its contrivers "save appearances," by introducing first one hypothetical force and then another. When the vortex atom is once set in motion, all its properties are absolutely fixed and determined by the laws of motion of the primitive fluid, which are fully expressed in the fundamental equations. The disciple of Lucretius may cut and carve his solid atoms in the hope of getting them to combine into worlds; the follower of Boscovich may imagine new laws of force to meet the requirements of each new phenomenon; but he who dares to plant his feet in the path opened up by Helmholtz and Thomson has no such resources. His primitive fluid has no other properties than inertia, invariable density, and perfect mobility, and the method by which the motion of this fluid is to be traced is pure mathematical analysis. The difficulties of this method are enormous, but the glory of surmounting them would be unique.

There seems to be little doubt that an encounter between two vortex atoms would be in its general character similar to those which we have already described. Indeed, the encounter between two smoke rings in air gives a very lively illustration of the elasticity of vortex rings.

But one of the first, if not the very first desideratum in a complete theory of matter is to explain—first, mass, and second, gravitation. To explain mass may seem an absurd achievement. We generally suppose that it is of the essence of matter to be the receptacle of momentum and energy, and even Thomson, in his definition of his primitive fluid, attributes to it the possession of mass. But according to Thomson, though the primitive fluid is the only true matter, yet that which we call matter is not the primitive fluid itself, but a mode of motion of that primitive fluid. It is the mode of motion which constitutes the vortex rings, and which furnishes us with examples of that permanence and continuity of existence which we are accustomed to attribute to matter itself. The primitive fluid, the only true matter, entirely eludes our perceptions when it is not endued with the mode of motion which converts certain portions of it into vortex rings, and thus renders it molecular.

In Thomson's theory, therefore, the mass of bodies requires explanation. We have to explain the inertia of what is only a mode of motion, and inertia is a property of matter, not of modes of motion. It is true that a vortex ring at any given instant has a definite momentum and a definite energy, but to show that bodies built up of vortex rings would have such momentum and energy as we know them to have is, in the present state of the theory, a very difficult task.

It may seem hard to say of an infant theory that it is bound to explain gravitation. Since the time of Newton,

[1] "On Vortex Atoms," *Proc. Roy. Soc. Edin.*, 18th February 1867.

the doctrine of gravitation has been admitted and expounded, till it has gradually acquired the character rather of an ultimate fact than of a fact to be explained.

It seems doubtful whether Lucretius considers gravitation to be an essential property of matter, as he seems to assert in the very remarkable lines—

"Nam si tantundem est in lanæ glomere, quantum
Corporis in plumbo est, tantundem pendere par est:
Corporis officium est quoniam premere omnia deorsum."
—*De Rerum Natura*, i. 361.

If this is the true opinion of Lucretius, and if the downward flight of the atoms arises, in his view, from their own gravity, it seems very doubtful whether he attributed the weight of sensible bodies to the impact of the atoms. The latter opinion is that of Le Sage, of Geneva, propounded in his *Lucrèce Newtonien*, and in his *Traité de Physique Mécanique*, published, along with a second treatise of his own, by Pierre Prevost, of Geneva, in 1818.[1] The theory of Le Sage is that the gravitation of bodies towards each other is caused by the impact of streams of atoms flying in all directions through space. These atoms he calls ultramundane corpuscules, because he conceives them to come in all directions from regions far beyond that part of the system of the world which is in any way known to us. He supposes each of them to be so small that a collision with another ultramundane corpuscule is an event of very rare occurrence. It is by striking against the molecules of gross matter that they discharge their function of drawing bodies towards each other. A body placed by itself in free space and exposed to the impacts of these corpuscules would be bandied about by them in all directions, but because, on the whole, it receives as many blows on one side as on another, it cannot thereby acquire any sensible velocity. But if there are two bodies in space, each of them will screen the other from a certain proportion of the corpuscular bombardment, so that a smaller number of corpuscules will strike either body on that side which is next the other body, while the number of corpuscules which strike it in other directions remains the same.

Each body will therefore be urged towards the other by the effect of the excess of the impacts it receives on the side furthest from the other. If we take account of the impacts of those corpuscules only which come directly from infinite space, and leave out of consideration those which have already struck mundane bodies, it is easy to calculate the result on the two bodies, supposing their dimensions small compared with the distance between them.

The force of attraction would vary directly as the product of the areas of the sections of the bodies taken normal to the distance and inversely as the square of the distance between them.

Now, the attraction of gravitation varies as the product of the *masses* of the bodies between which it acts, and inversely as the square of the distance between them. If, then, we can imagine a constitution of bodies such that the effective areas of the bodies are proportional to their masses, we shall make the two laws coincide. Here, then, seems to be a path leading towards an explanation of the law of gravitation, which, if it can be shown to be in other respects consistent with facts, may turn out to be a royal road into the very arcana of science.

Le Sage himself shows that, in order to make the effective area of a body, in virtue of which it acts as a screen to the streams of ultramundane corpuscules, proportional to the mass of the body, whether the body be large or small, we must admit that the size of the solid atoms of the body is exceedingly small compared with the distances between them, so that a very small proportion of the corpuscules are stopped even by the densest and largest bodies. We may picture to ourselves the streams of corpuscules coming in every direction, like light from a uniformly illuminated sky. We may imagine a material body to consist of a congeries of atoms at considerable distances from each other, and we may represent this by a swarm of insects flying in the air. To an observer at a distance this swarm will be visible as a slight darkening of the sky in a certain quarter. This darkening will represent the action of the material body in stopping the flight of the corpuscules. Now, if the proportion of light stopped by the swarm is very small, two such swarms will stop nearly the same amount of light, whether they are in a line with the eye or not, but if one of them stops an appreciable proportion of light, there will not be so much left to be stopped by the other, and the effect of two swarms in a line with the eye will be less than the sum of the two effects separately.

Now, we know that the effect of the attraction of the sun and earth on the moon is not appreciably different when the moon is eclipsed than on other occasions when full moon occurs without an eclipse. This shows that the number of the corpuscules which are stopped by bodies of the size and mass of the earth, and even the sun, is very small compared with the number which pass straight through the earth or the sun without striking a single molecule. To the streams of corpuscules the earth and the sun are mere systems of atoms scattered in space, which present far more openings than obstacles to their rectilinear flight.

Such is the ingenious doctrine of Le Sage, by which he endeavours to explain universal gravitation. Let us try to form some estimate of this continual bombardment of ultramundane corpuscules which is being kept up on all sides of us.

We have seen that the sun stops but a very small fraction of the corpuscules which enter it. The earth, being a smaller body, stops a still smaller proportion of them. The proportion of those which are stopped by a small body, say a 1 lb shot, must be smaller still in an enormous degree, because its thickness is exceedingly small compared with that of the earth.

Now, the weight of the ball, or its tendency towards the earth, is produced, according to this theory, by the excess of the impacts of the corpuscules which come from above over the impacts of those which come from below, and have passed through the earth. Either of these quantities is an exceedingly small fraction of the momentum of the whole number of corpuscules which pass through the ball in a second, and their difference is a small fraction of either, and yet it is equivalent to the weight of a pound. The velocity of the corpuscules must be enormously greater than that of any of the heavenly bodies, otherwise, as may easily be shown, they would act as a resisting medium opposing the motion of the planets. Now, the energy of a moving system is half the product of its momentum into its velocity. Hence the energy of the corpuscules, which by their impacts on the ball during one second urge it towards the earth, must be a number of foot-pounds equal to the number of feet over which a corpuscule travels in a second, that is to say, not less than thousands of millions. But this is only a small fraction of the energy of all the impacts which the atoms of the ball receive from the innumerable streams of corpuscules which fall upon it in all directions.

Hence the rate at which the energy of the corpuscules is spent in order to maintain the gravitating property of a single pound, is at least millions of millions of foot-pounds per second.

What becomes of this enormous quantity of energy? If the corpuscules, after striking the atoms, fly off with a

[1] See also *Constitution de la Matière, &c.*, par le P. Leray, Paris, 1869.

velocity equal to that which they had before, they will carry their energy away with them into the ultramundane regions. But if this be the case, then the corpuscules rebounding from the body in any given direction will be both in number and in velocity exactly equivalent to those which are prevented from proceeding in that direction by being deflected by the body, and it may be shown that this will be the case whatever be the shape of the body, and however many bodies may be present in the field. Thus, the rebounding corpuscules exactly make up for those which are deflected by the body, and there will be no excess of the impacts on any other body in one direction or another.

The explanation of gravitation, therefore, falls to the ground if the corpuscules are like perfectly elastic spheres, and rebound with a velocity of separation equal to that of approach. If, on the other hand, they rebound with a smaller velocity, the effect of attraction between the bodies will no doubt be produced, but then we have to find what becomes of the energy which the molecules have brought with them but have not carried away.

If any appreciable fraction of this energy is communicated to the body in the form of heat, the amount of heat so generated would in a few seconds raise it, and in like manner the whole material universe, to a white heat.

It has been suggested by Sir W. Thomson that the corpuscules may be so constructed as to carry off their energy with them, provided that part of their kinetic energy is transformed, during impact, from energy of translation to energy of rotation or vibration. For this purpose the corpuscules must be material systems, not mere points. Thomson suggests that they are vortex atoms, which are set into a state of vibration at impact, and go off with a smaller velocity of translation, but in a state of violent vibration. He has also suggested the possibility of the vortex corpuscule regaining its swiftness and losing part of its vibratory agitation by communion with its kindred corpuscules in infinite space.

We have devoted more space to this theory than it seems to deserve, because it is ingenious, and because it is the only theory of the cause of gravitation which has been so far developed as to be capable of being attacked and defended. It does not appear to us that it can account for the temperature of bodies remaining moderate while their atoms are exposed to the bombardment. The temperature of bodies must tend to approach that at which the average kinetic energy of a molecule of the body would be equal to the average kinetic energy of an ultramundane corpuscule.

Now, suppose a plane surface to exist which stops *all* the corpuscules. The pressure on this plane will be $p = NMu^2$ where M is the mass of a corpuscule, N the number in unit of volume, and $u$ its velocity normal to the plane. Now, we know that the very greatest pressure existing in the universe must be much less than the pressure $p$, which would be exerted against a body which stops all the corpuscules. We are also tolerably certain that N, the number of corpuscules which are at any one time within unit of volume, is small compared with the value of N for the molecules of ordinary bodies. Hence, $Mu^2$ must be enormous compared with the corresponding quantity for ordinary bodies, and it follows that the impact of the corpuscules would raise all bodies to an enormous temperature. We may also observe that according to this theory the habitable universe, which we are accustomed to regard as the scene of a magnificent illustration of the conservation of energy as the fundamental principle of all nature, is in reality maintained in working order only by an enormous expenditure of external power, which would be nothing less than ruinous if the supply were drawn from anywhere else than from the infinitude of space, and which, if the contrivances of the most eminent mathematicians should be found in any respect defective, might at any moment tear the whole universe atom from atom.

We must now leave these speculations about the nature of molecules and the cause of gravitation, and contemplate the material universe as made up of molecules. Every molecule, so far as we know, belongs to one of a definite number of species. The list of chemical elements may be taken as representing the known species which have been examined in the laboratories. Several of these have been discovered by means of the spectroscope, and more may yet remain to be discovered in the same way. The spectroscope has also been applied to analyse the light of the sun, the brighter stars, and some of the nebulæ and comets, and has shown that the character of the light emitted by these bodies is similar in some cases to that emitted by terrestrial molecules, and in others to light from which the molecules have absorbed certain rays. In this way a considerable number of coincidences have been traced between the systems of lines belonging to particular terrestrial substances and corresponding lines in the spectra of the heavenly bodies.

The value of the evidence furnished by such coincidences may be estimated by considering the degree of accuracy with which one such coincidence may be observed. The interval between the two lines which form Fraunhofer's line D is about the five hundredth part of the interval between B and G on Kirchhoff's scale. A discordance between the positions of two lines amounting to the tenth part of this interval, that is to say, the five thousandth part of the length of the bright part of the spectrum, would be very perceptible in a spectroscope of moderate power. We may define the power of the spectroscope to be the number of times which the smallest measurable interval is contained in the length of the visible spectrum. Let us denote this by $p$. In the case we have supposed $p$ will be about 5000.

If the spectrum of the sun contains $n$ lines of a certain degree of intensity, the probability that any one line of the spectrum of a gas will coincide with one of these $n$ lines is

$$1 - \left(1 - \frac{1}{p}\right) = \frac{n}{p}\left(1 - \frac{n-1}{2}\frac{1}{p} + \&c.\right),$$

and when $p$ is large compared with $n$, this becomes nearly $\frac{n}{p}$. If there are $r$ lines in the spectrum of the gas, the probability that each and every one shall coincide with a line in the solar spectrum is approximately $\frac{n^r}{p^r}$. Hence, in the case of a gas whose spectrum contains several lines, we have to compare the results of two hypotheses. If a large amount of the gas exists in the sun, we have the strongest reason for expecting to find all the $r$ lines in the solar spectrum. If it does not exist, the probability that $r$ lines out of the $n$ observed lines shall coincide with the lines of the gas is exceedingly small. If, then, we find all the $r$ lines in their proper places in the solar spectrum, we have very strong grounds for believing that the gas exists in the sun. The probability that the gas exists in the sun is greatly strengthened if the character of the lines as to relative intensity and breadth is found to correspond in the two spectra.

The absence of one or more lines of the gas in the solar spectrum tends of course to weaken the probability, but the amount to be deducted from the probability must depend on what we know of the variation in the relative intensity of the lines when the temperature and the pressure of the gas are made to vary.

Coincidences observed, in the case of several terrestrial substances, with several systems of lines in the spectra of the heavenly bodies, tend to increase the evidence for the doctrine that terrestrial substances exist in the heavenly bodies, while the discovery of particular lines in a celestial spectrum which do not coincide with any line in a terrestrial spectrum does not much weaken the general argument, but rather indicates either that a substance exists in the heavenly body not yet detected by chemists on earth, or that the temperature of the heavenly body is such that some substance, undecomposable by our methods, is there split up into components unknown to us in their separate state.

We are thus led to believe that in widely-separated parts of the visible universe molecules exist of various kinds, the molecules of each kind having their various periods of vibration either identical, or so nearly identical that our spectroscopes cannot distinguish them. We might argue from this that these molecules are alike in all other respects, as, for instance, in mass. But it is sufficient for our present purpose to observe that the same kind of molecule, say that of hydrogen, has the same set of periods of vibration, whether we procure the hydrogen from water, from coal, or from meteoric iron, and that light, having the same set of periods of vibration, comes to us from the sun, from Sirius, and from Arcturus.

The same kind of reasoning which led us to believe that hydrogen exists in the sun and stars, also leads us to believe that the molecules of hydrogen in all these bodies had a common origin. For a material system capable of vibration may have for its periods of vibration any set of values whatever. The probability, therefore, that two material systems, quite independent of each other, shall have, to the degree of accuracy of modern spectroscopic measurements, the same set of periods of vibration, is so very small that we are forced to believe that the two systems are not independent of each other. When, instead of two such systems, we have innumerable multitudes all having the same set of periods, the argument is immensely strengthened.

Admitting, then, that there is a real relation between any two molecules of hydrogen, let us consider what this relation may be.

We may conceive of a mutual action between one body and another tending to assimilate them. Two clocks, for instance, will keep time with each other if connected by a wooden rod, though they have different rates if they were disconnected. But even if the properties of a molecule were as capable of modification as those of a clock, there is no physical connection of a sufficient kind between Sirius and Arcturus.

There are also methods by which a large number of bodies differing from each other may be sorted into sets, so that those in each set more or less resemble each other. In the manufacture of small shot this is done by making the shot roll down an inclined plane. The largest specimens acquire the greatest velocities, and are projected farther than the smaller ones. In this way the various pellets, which differ both in size and in roundness, are sorted into different kinds, those belonging to each kind being nearly of the same size, and those which are not tolerably spherical being rejected altogether.

If the molecules were originally as various as these leaden pellets, and were afterwards sorted into kinds, we should have to account for the disappearance of all the molecules which did not fall under one of the very limited number of kinds known to us; and to get rid of a number of indestructible bodies, exceeding by far the number of the molecules of all the recognised kinds, would be one of the severest labours ever proposed to a cosmogonist.

It is well known that living beings may be grouped into a certain number of species, defined with more or less precision, and that it is difficult or impossible to find a series of individuals forming the links of a continuous chain between one species and another. In the case of living beings, however, the generation of individuals is always going on, each individual differing more or less from its parents. Each individual during its whole life is undergoing modification, and it either survives and propagates its species, or dies early, accordingly as it is more or less adapted to the circumstances of its environment. Hence, it has been found possible to frame a theory of the distribution of organisms into species by means of generation, variation, and discriminative destruction. But a theory of evolution of this kind cannot be applied to the case of molecules, for the individual molecules neither are born nor die, they have neither parents nor offspring, and so far from being modified by their environment, we find that two molecules of the same kind, say of hydrogen, have the same properties, though one has been compounded with carbon and buried in the earth as coal for untold ages, while the other has been "occluded" in the iron of a meteorite, and after unknown wanderings in the heavens has at last fallen into the hands of some terrestrial chemist.

The process by which the molecules become distributed into distinct species is not one of which we know any instances going on at present, or of which we have as yet been able to form any mental representation. If we suppose that the molecules known to us are built up each of some moderate number of atoms, these atoms being all of them exactly alike, then we may attribute the limited number of molecular species to the limited number of ways in which the primitive atoms may be combined so as to form a permanent system.

But though this hypothesis gets rid of the difficulty of accounting for the independent origin of different species of molecules, it merely transfers the difficulty from the known molecules to the primitive atoms. How did the atoms come to be all alike in those properties which are in themselves capable of assuming any value?

If we adopt the theory of Boscovich, and assert that the primitive atom is a mere centre of force, having a certain definite mass, we may get over the difficulty about the equality of the mass of all atoms by laying it down as a doctrine which cannot be disproved by experiment, that mass is not a quantity capable of continuous increase or diminution, but that it is in its own nature discontinuous, like number, the atom being the unit, and all masses being multiples of that unit. We have no evidence that it is possible for the ratio of two masses to be an incommensurable quantity, for the incommensurable quantities in geometry are supposed to be traced out in a continuous medium. If matter is atomic, and therefore discontinuous, it is unfitted for the construction of perfect geometrical models, but in other respects it may fulfil its functions.

But even if we adopt a theory which makes the equality of the mass of different atoms a result depending on the nature of mass rather than on any quantitative adjustment, the correspondence of the periods of vibration of actual molecules is a fact of a different order.

We know that radiations exist having periods of vibration of every value between those corresponding to the limits of the visible spectrum, and probably far beyond these limits on both sides. The most powerful spectroscope can detect no gap or discontinuity in the spectrum of the light emitted by incandescent lime.

The period of vibration of a luminous particle is therefore a quantity which in itself is capable of assuming any one of a series of values, which, if not mathematically continuous, is such that consecutive observed values differ from each other by less than the ten thousandth part of either. There is, therefore, nothing in the nature of time itself to prevent the period of vibration of a molecule from assuming any one of many thousand different observable values. That which determines the period of any particular kind of vibration is the relation which subsists between the corresponding type of displacement and the force of restitution thereby called into play, a relation involving constants of space and time as well as of mass.

It is the equality of these space- and time-constants for all molecules of the same kind which we have next to consider. We have seen that the very different circumstances in which different molecules of the same kind have been placed have not, even in the course of many ages, produced any appreciable difference in the values of these constants. If, then, the various processes of nature to which these molecules have been subjected since the world began have not been able in all that time to produce any appreciable difference between the constants of one molecule and those of another, we are forced to conclude that it is not to the operation of any of these processes that the uniformity of the constants is due.

The formation of the molecule is therefore an event not belonging to that order of nature under which we live. It is an operation of a kind which is not, so far as we are aware, going on on earth or in the sun or the stars, either now or since these bodies began to be formed. It must be referred to the epoch, not of the formation of the earth or of the solar system, but of the establishment of the existing order of nature, and till not only these worlds and systems, but the very order of nature itself is dissolved, we have no reason to expect the occurrence of any operation of a similar kind.

In the present state of science, therefore, we have strong reasons for believing that in a molecule, or if not in a molecule, in one of its component atoms, we have something which has existed either from eternity or at least from times anterior to the existing order of nature. But besides this atom, there are immense numbers of other atoms of the same kind, and the constants of each of these atoms are incapable of adjustment by any process now in action. Each is physically independent of all the others.

Whether or not the conception of a multitude of beings existing from all eternity is in itself self-contradictory, the conception becomes palpably absurd when we attribute a relation of quantitative equality to all these beings. We are then forced to look beyond them to some common cause or common origin to explain why this singular relation of equality exists, rather than any one of the infinite number of possible relations of inequality.

Science is incompetent to reason upon the creation of matter itself out of nothing. We have reached the utmost limit of our thinking faculties when we have admitted that, because matter cannot be eternal and self-existent, it must have been created. It is only when we contemplate not matter in itself, but the form in which it actually exists, that our mind finds something on which it can lay hold.

That matter, as such, should have certain fundamental properties, that it should have a continuous existence in space and time, that all action should be between two portions of matter, and so on, are truths which may, for aught we know, be of the kind which metaphysicians call necessary. We may use our knowledge of such truths for purposes of deduction, but we have no data for speculating on their origin.

But the equality of the constants of the molecules is a fact of a very different order. It arises from a particular distribution of matter, a *collocation*, to use the expression of Dr Chalmers, of things which we have no difficulty in imagining to have been arranged otherwise. But many of the ordinary instances of collocation are adjustments of constants, which are not only arbitrary in their own nature, but in which variations actually occur; and when it is pointed out that these adjustments are beneficial to living beings, and are therefore instances of benevolent design, it is replied that those variations which are not conducive to the growth and multiplication of living beings tend to their destruction, and to the removal thereby of the evidence of any adjustment not beneficial.

The constitution of an atom, however, is such as to render it, so far as we can judge, independent of all the dangers arising from the struggle for existence. Plausible reasons may, no doubt, be assigned for believing that if the constants had varied from atom to atom through any sensible range, the bodies formed by aggregates of such atoms would not have been so well fitted for the construction of the world as the bodies which actually exist. But as we have no experience of bodies formed of such variable atoms this must remain a bare conjecture.

Atoms have been compared by Sir J. Herschel to manufactured articles, on account of their uniformity. The uniformity of manufactured articles may be traced to very different motives on the part of the manufacturer. In certain cases it is found to be less expensive as regards trouble, as well as cost, to make a great many objects exactly alike than to adapt each to its special requirements. Thus, shoes for soldiers are made in large numbers without any designed adaptation to the feet of particular men. In another class of cases the uniformity is intentional, and is designed to make the manufactured article more valuable. Thus, Whitworth's bolts are made in a certain number of sizes, so that if one bolt is lost, another may be got at once, and accurately fitted to its place. The identity of the arrangement of the words in the different copies of a document or book is a matter of great practical importance, and it is more perfectly secured by the process of printing than by that of manuscript copying.

In a third class not a part only but the whole of the value of the object arises from its exact conformity to a given standard. Weights and measures belong to this class, and the existence of many well-adjusted material standards of weight and measure in any country furnishes evidence of the existence of a system of law regulating the transactions of the inhabitants, and enjoining in all professed measures a conformity to the national standard.

There are thus three kinds of usefulness in manufactured articles—cheapness, serviceableness, and quantitative accuracy. Which of these was present to the mind of Sir J. Herschel we cannot now positively affirm, but it was at least as likely to have been the last as the first, though it seems more probable that he meant to assert that a number of exactly similar things cannot be each of them eternal and self-existent, and must therefore have been made, and that he used the phrase "manufactured article" to suggest the idea of their being made in great numbers.

(J. C. M.)

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ATOOI, one of the larger Sandwich Islands, in the North Pacific Ocean. Towards the N.E. and N.W. the country is rugged and broken, but to the southward it is more level. The hills rise from the sea with a gentle acclivity, and, at a little distance back, are covered with wood; the central peaks attain an elevation of 7000 feet. The chief ports are Waimea and Hanalei. The island was one of the stations chosen for the observation of the transit of Venus in 1874. It is nearly 40 miles in length, and contains about 10,000 inhabitants. Long. 159° 40′ W., lat. 21° 57′ N.

ATRATO, a river of Colombia, South America, which,