February 8, 1866.

Lieut.-General SABINE, President, in the Chair.

The BAKERIAN LECTURE was delivered by JAMES CLERK MAXWELL, M.A., F.R.S., "On the Viscosity or Internal Friction of Air and other Gases." The following is an abstract.

All bodies which are capable of having their form indefinitely altered, and which resist the change of form with a force depending on the rate of deformation, may be called Viscous Bodies. Taking tar or treacle as an instance in which both the change of form and the resistance opposed to it are easily observed, we may pass in one direction through the series of soft solids up to the materials commonly supposed to be most unyielding, such as glass and steel, and in the other direction through the series of liquids of various degrees of mobility to the gases, of which oxygen is the most viscous, and hydrogen the least.

The viscosity of elastic solids has been investigated by M. F. Kohlrausch * and Professor W. Thomson †; that of gases by Professor Stokes ‡, M. O. E. Meyer §, and Mr. Graham ||.

The author has investigated the laws of viscosity in air by causing three horizontal glass disks, 10·56 inches diameter, to perform rotatory oscillations about a vertical axis by means of the elasticity of a steel suspension wire about 4 feet long. The period of a complete oscillation was 72 seconds, and the maximum velocity of the edge of the disks was about $\frac{1}{12}$ inch per second.

The three disks were placed at known intervals on the vertical axis, and four larger fixed disks were so adjusted above and below them and in the intervals between them, that strata of air of known thickness were intercepted between the surfaces of the moving disks and the fixed disks. During the oscillations of the moveable disks, the viscosity of the air in these six strata caused a gradual diminution of the amplitude of oscillation, which was measured by means of the reflexion of a circular scale in a mirror attached to the axis.

The whole apparatus was enclosed in an air-tight case, so that the air might be exhausted or exchanged for another gas, or heated by a current of steam round the receiver. The observed diminution in the arc of oscillation is in part due to the viscosity of the suspending wire. To eliminate the effect of the wire from that of the air, the arrangement of the disks was altered, and the three disks, placed in contact, were made to oscillate midway between two fixed glass disks, at distances sometimes of 1 inch, and sometimes of ·5 inch.

* Pogg. Ann. cxix. (1863). † Proceedings of the Royal Society, May 18, 1865.
‡ Cambridge Philosophical Transactions, 1850. § Pogg. Ann. cxiii. (1861).
|| Phil. Trans. 1846 & 1849.

From these experiments on two strata of air, combined with three sets of experiments on six strata of thicknesses ·683, ·425, and ·1847 inches respectively, the value of the coefficient of viscosity or internal friction was determined.

Let two infinite planes be separated by a stratum of air whose thickness is unity. Let one of these planes be fixed, while the other moves in its own plane with a uniform velocity unity; then, if the air in immediate contact with either plane has the same velocity as the plane, every unit of surface of either plane will experience a tangential force μ, where μ is the coefficient of viscosity of the air between the planes.

The force μ is understood to be measured by the velocity which it would communicate in unit of time to unit of mass.

If L, M, T be the units of length, mass, and time, then the dimensions of μ are $L^{-1}M\,T^{-1}$.

In the actual experiment, the motion of the surfaces is rotatory instead of rectilinear, oscillatory instead of uniform, and the surfaces are bounded instead of infinite. These considerations introduce certain complications into the theory, which are separately considered.

The conclusions which are drawn from the experiments agree, as far as they go, with those of Mr. Graham on the Transpiration of Gases*. They are as follows:—

1. The coefficient of viscosity is independent of the density, the temperature being constant. No deviation from this law is observed between the atmospheric density and that corresponding to a pressure of half an inch of mercury.

This remarkable result was shown by the author in 1860† to be a consequence of the Dynamical Theory of Gases. It agrees with the conclusions of Mr. Graham, deduced from experiments on the transpiration of gases through capillary tubes. The considerable thickness of the strata of air in the present experiments shows that the property of air, to be equally viscous at all densities, is quite independent of any molecular action between its particles and those of solid surfaces, such as those of the capillary tubes employed by Graham.

2. The coefficient of viscosity increases with the temperature, and is proportional to $1+\alpha\theta$, where θ is the temperature and α is the coefficient of expansion per degree for air.

This result cannot be considered so well established as the former, owing to the difficulty of maintaining a high temperature constant in so large an apparatus, and measuring it without interfering with the motion. Experiments, in which the temperature ranged from 50° to 185° F., agreed with the theory to within 0·8 per cent., so that it is exceedingly probable that this is the true relation to the temperature.

The experiments of Graham led him to this conclusion also.

3. The coefficient of viscosity of hydrogen is much less than that of

* Phil. Trans. 1846.

† Phil. Mag. Jan. 1860.

air. I have never succeeded in filling my apparatus with perfectly pure hydrogen, for air leaks into the vacuum during the admission of so large a quantity of hydrogen as is required to fill it. The ratio of the viscosity of my hydrogen to that of air was ·5156. That obtained by Graham was ·4855.

4. The ratio for carbonic acid was found to be ·859. Graham makes it ·807. It is probable that the comparative results of Graham are more exact than those of this paper, owing to the difficulty of introducing so large a volume of gas without letting in any air during the time of filling the receiver. I find also that a very small proportion of air causes a considerable increase in the viscosity of hydrogen. This result also agrees with those of Mr. Graham.

5. Forty experiments on dry air were investigated to determine whether any slipping takes place between the glass and the air in immediate contact with it.

The result was, that if there were any slipping, it is of exceedingly small amount; and that the evidence in favour of the indicated amount being real is very precarious.

The results of the hypothesis, that there is no slipping, agree decidedly better with the experiments.

6. The actual value of the coefficient of viscosity of dry air was determined, from forty experiments of five different kinds, to be

$$\mu = \cdot 0000149\ (461^\circ + \theta),$$

where the inch, the grain, and the second are the units, and the temperature is on Fahrenheit's scale.

At 62° this gives $\mu = \cdot 007802$.

Professor Stokes, from the experiments of Baily on pendulums, has found

$$\sqrt{\frac{\mu}{\rho}} = \cdot 116,$$

which, with the average temperature and density of air, would give

$$\mu = \cdot 00417,$$

a much smaller value than that here found.

If the value of μ is expressed in feet instead of inches, so as to be uniform with the British measures of magnetic and electric phenomena, as recorded at the observatories,

$$\mu = \cdot 000179\ (461 + \theta)$$
$$= \cdot 08826 \text{ at } 32^\circ.$$

In metre-gramme-second measure and Centigrade temperature,

$$\mu = \cdot 01878\ (1 + \cdot 00366\ \theta).$$

M. O. E. Meyer (Pogg. Ann. cxiii. (1861) p. 383) makes μ at 18° C. $= \cdot 000360$ in centimetres, cubic centimetres of water, and seconds as units, or in metrical units, $\mu = \cdot 0360$.

According to the experiments here described, μ at 18° C. $= \cdot 02$.

M. Meyer's value is therefore nearly twice as great as that of this paper, while that of Professor Stokes is only half as great.

In M. Meyer's experiments, which were with one disk at a time in an open space of air, the influence of the air near the edge of the disk is very considerable; but M. Meyer (Crelle, 59; Pogg. cxiii. 76) seems to have arrived at the conclusion that the additional effect of the air at the edge is proportional to the thickness of the disk. If the additional force near the edge is underestimated, the resulting value of the viscosity will be in excess.

7. Each of the forty experiments on dry air was calculated from the concluded values of the viscosity of the air and of the wire, and the result compared with the observed result. In this way the error of mean square of each observation was determined, and from this the "probable error" of μ was found to be ·036 per cent. of its value. These experiments, it must be remembered, were made with five different arrangements of the disks, at pressures ranging from 0·5 inch to 30 inches, and at temperatures from 51° to 74° F.; so that their agreement does not arise from a mere repetition of the same conditions, but from an agreement between the properties of air and the theory made use of in the calculations.

February 15, 1866.

Lieut.-General SABINE, President, in the Chair.

The following communication was read :—

"Further Observations on the Spectra of some of the Nebulæ, with a Mode of determining the Brightness of these Bodies." By WILLIAM HUGGINS, F.R.S. Received January 30, 1866.

(Abstract.)

In the first part of this paper the author continues his observations on the spectra of nebulæ and clusters. The results already presented by him to the Royal Society are confirmed by his new observations, namely, that with his apparatus clusters and nebulæ give either a continuous spectrum or a spectrum consisting of one, two, or three bright lines. The positions in the spectrum of these lines are the same as those of the bright lines of the nebulæ described in his former papers.

On account of the faintness of these objects the author was not able to ascertain whether the continuous spectra which some of the nebulæ give are interrupted by dark lines in a manner similar to the spectra of the sun and fixed stars. Some of these spectra appear irregularly bright in some parts of the spectrum.

The nebulæ which follow have a spectrum of one, two, or three bright lines; in addition to which, in the case of some of them, a faint con-