

the body must experience during Alpine ascensions. Professor Plantamour, of Geneva,\* from observations made at Geneva and the Great St. Bernard, concludes that there is no marked difference between the hygrometric states at various altitudes. According, however, to Dr. Lombard, who has considerable knowledge and experience of climate, the air appears to be, as a rule, much drier above 1,500 metres than below that altitude.

III. "On Stresses in Rarefied Gases arising from Inequalities of Temperature." By J. CLERK MAXWELL, F.R.S., Professor of Experimental Physics in the University of Cambridge. Received March 19, 1878.

(Abstract.)

1. In this paper I have followed the method given in my paper "On the Dynamical Theory of Gases" (Phil. Trans., 1867, p. 49). I have shown that when inequalities of temperature exist in a gas, the pressure at a given point is not the same in all directions, and that the difference between the maximum and the minimum pressure at a point may be of considerable magnitude when the density of the gas is small enough, and when the inequalities of temperature are produced by small solid bodies at a higher or lower temperature than the vessel containing the gas.

2. The nature of this stress may be thus defined: let the distance from the given point, measured in a given direction, be denoted by  $h$ , and the absolute temperature by  $\theta$ ; then the space-variation of the temperature for a point moving along this line will be denoted by  $\frac{d\theta}{dh}$ , and the space-variation of this quantity along the same line by  $\frac{d^2\theta}{dh^2}$ . There is in general a particular direction of the line  $h$ , for which  $\frac{d^2\theta}{dh^2}$  is a maximum, another for which it is a minimum, and a third for which it is a maximum-minimum. These three directions are at right angles to each other, and are the axes of principal stress at the given point; and the part of the stress arising from inequalities of temperature is in each of these principal axes a pressure equal to—

$$3 \frac{\mu^2}{\rho\theta} \frac{d^2\theta}{dh^2}$$

where  $\mu$  is the coefficient of viscosity,  $\rho$  the density, and  $\theta$  the absolute temperature.

3. Now, for dry air at 15° C.,  $\mu=1.9 \times 10^{-4}$  in centimetre-gramme-

\* Lombard. "Climat des Montagnes," 1873.

second measure, and  $\frac{3\mu^2}{\rho\theta} = \frac{1}{p} 0.315$ , where  $p$  is the pressure, the unit of pressure being one dyne per square centimetre, or nearly one-millionth part of an atmosphere.

If a sphere of one centimetre in diameter is  $T$  degrees centigrade hotter than the air at a distance from it, then, when the flow of heat has become steady, the temperature at a distance of  $r$  centimetres will be

$$\theta = T_0 + \frac{T}{2r}, \text{ and } \frac{d^2\theta}{dr^2} = \frac{T}{r^3}.$$

Hence, at a distance of one centimetre from the centre of the sphere, the pressure in the direction of the radius arising from inequality of temperature will be

$$\frac{T}{p} 0.315 \text{ dynes per square centimetre.}$$

4. In Mr. Crookes' experiments the pressure,  $p$ , was often so small that this stress would be capable, if it existed alone, of producing rapid motion in small masses.

Indeed, if we were to consider only the normal part of the stress exerted on solid bodies immersed in the gas, most of the phenomena observed by Mr. Crookes could be readily explained.

5. Let us take the case of two small bodies symmetrical with respect to the axis joining their centres of figure. If both bodies are warmer than the air at a distance from them, then in any section perpendicular to the axis joining their centres, the point where it cuts this line will have the highest temperature, and there will be a flow of heat outwards from this axis in all directions.

Hence  $\frac{d^2\theta}{dh^2}$  will be positive for the axis, and it will be a line of maximum pressure, so that the bodies will repel each other.

If both bodies are colder than the air at a distance, everything will be reversed; the axis will be a line of minimum pressure, and the bodies will attract each other.

If one body is hotter, and the other colder, than the air at a distance, the effect will be smaller; and it will depend on the relative sizes of the bodies, and on their exact temperatures, whether the action is attractive or repulsive.

6. If the bodies are two parallel disks, very near to each other, the central parts will produce very little effect, because between the disks the temperature varies uniformly and  $\frac{d^2\theta}{dh^2} = 0$ . Only near the edges will there be any stress arising from inequality of temperature in the gas.

7. If the bodies are encircled by a ring having its axis in the line joining the bodies, then the repulsion between the two bodies, when

they are warmer than the air in general, may be converted into attraction by heating the ring, so as to produce a flow of heat inwards towards the axis.

8. If a body in the form of a cup or bowl is warmer than the air, the distribution of temperature in the surrounding gas is similar to the distribution of electric potential near a body of the same form, which has been investigated by Sir W. Thomson.\* Near the convex surface the value of  $\frac{d^2\theta}{dh^2}$  is nearly the same as if the body had been a complete sphere, namely  $2T\frac{1}{a^2}$ , where T is the excess of temperature, and  $a$  is the radius of the sphere. Near the concave surface the variation of temperature is exceedingly small. Hence the normal pressure on the convex surface will be greater than on the concave surface, as Mr. Crookes has shown by the motion of his radiometers.

Since the expressions for the stress are linear as regards the temperature, everything will be reversed when the cup is colder than the surrounding air.

9. In a spherical vessel, if the two polar regions are made hotter than the equatorial zone, the pressure in the direction of the axis will be greater than that parallel to the equatorial plane, and the reverse will be the case if the polar regions are made colder than the equatorial zone.

10. All such explanations of the observed phenomena must be subjected to careful criticism. They have been obtained by considering the normal stresses alone, to the exclusion of the tangential stresses; and it is much easier to give an elementary exposition of the former than of the latter.

If, however, we go on to calculate the forces acting on any portion of the gas in virtue of the stresses on its surface, we find that when the flow of heat is steady, these forces are in equilibrium. Mr. Crookes tells us that there is no molar current, or wind, in his radiometer vessels. It may not be easy to prove this by experiment, but it is satisfactory to find that the system of stresses here described as arising from inequalities of temperature will not, when the flow of heat is steady, generate currents.

11. Consider, then, the case in which there are no currents of gas, but a steady flow of heat, the condition of which is

$$\frac{d^2\theta}{dx^2} + \frac{d^2\theta}{dy^2} + \frac{d^2\theta}{dz^2} (= -\nabla^2\theta) = 0.$$

(In the absence of external forces, such as gravity, and if the gas in contact with solid bodies does not slide over them, this is always a solution of the equations, and it is the only permanent solution.) In

\* Reprint of Papers on Electrostatics, p. 178.

this case the equations of motion show that every particle of the gas is in equilibrium under the stresses acting on it.

Hence any finite portion of the gas is also in equilibrium; also, since the stresses are linear functions of the temperature, if we superpose one system of temperatures on another, we also superpose the corresponding systems of forces. Now the system of temperatures due to a solid sphere of uniform temperature, immersed in the gas, cannot of itself give rise to any force tending to move the sphere in one direction rather than in another. Let the sphere be placed within the finite portion of gas which, as we have said, is already in equilibrium. The equilibrium will not be disturbed. We may introduce any number of spheres at different temperatures into the portion of gas, and when the flow of heat has become steady the whole system will be in equilibrium.

12. How, then, are we to account for the observed fact that forces act between solid bodies immersed in rarefied gases, and this, apparently, as long as inequalities of temperature are maintained?

I think we must look for an explanation in the fact discovered in the case of liquids by Helmholtz and Piotrowski,\* and for gases by Kundt and Warburg,† that the fluid in contact with the surface of a solid must slide over it with a finite velocity in order to produce a finite tangential stress.

The theoretical treatment of the boundary conditions between a gas and a solid is difficult, and it becomes more difficult if we consider that the gas close to the surface is probably in an unknown state of condensation. We shall, therefore, accept the results obtained by Kundt and Warburg on their experimental evidence.

They have found that the velocity of sliding of the gas over the surface due to a given tangential stress varies inversely as the pressure

The coefficient of sliding for air on glass was found to be  $\lambda = \frac{10}{p}$

centimetres, where  $p$  is the pressure in millionths of an atmosphere. Hence at ordinary pressures  $\lambda$  is insensible, but in the vessels exhausted by Mr. Crookes it may be considerable.

Hence if close to the surface of a solid there is a tangential stress,  $S$ , acting on a surface parallel to that of the body, in a direction,  $h$ , parallel to that surface, there will also be a sliding of the gas in contact with the solid over its surface in the direction  $h$ , with a finite velocity  $= S \frac{\lambda}{\mu}$ .

13. I have not attempted to enter on the calculation of the effect of this sliding motion, but it is easy to see that if we begin with the case in which there is no sliding, the effect of permission being given to the gas to slide must be in the first place to diminish the action of

\* Wiener Sitzb., xl (1860), p. 607.

† Pogg. Ann., clv (1875), p. 337.

all tangential stresses on the surface without affecting the normal stresses; and, in the second place to set up currents sweeping over the surfaces of solid bodies, thus completely destroying the simplicity of our first solution of the problem.

14. When external forces, such as gravity, act on the gas, and when the thermal phenomena produce differences of density in different parts of the vessel, then the well-known convection currents are set up. These also interfere with the simplicity of the problem and introduce very complicated effects. All that we know is that the rarer the gas and the smaller the vessel, the less is the velocity of the convection currents; so that in Mr. Crookes' experiments they play a very small part.

#### IV. "Note on the Existence of Carbon in the Coronal Atmosphere of the Sun." By J. N. LOCKYER, F.R.S. Received March 20, 1878.

It is now four years since I obtained evidence, which seemed to me conclusive, as to the existence of carbon in the sun's atmosphere. There were two points, however, which remained to be settled before the matter could be considered to be placed beyond all doubt.

The first was to establish that the fluted bands generally present in the spectrum of the arc, as photographed, which bands vary very considerably in strength according to the volatility of the metal under experiment, were really bands of carbon—a point denied by Ångström and Thalèn.

This point is settled by the photographs submitted to the Society with this communication. In these the carbon bands remain the same, though one spectrum is that of carbon in air, the other of carbon in chlorine dried with great care, and the proof that it cannot be the spectrum of a combination of carbon with oxygen lies in the fact that in the chlorine it is more brilliant than in the oxygen. Now, assuming the chlorine to have been but imperfectly dried, this would not have happened if a compound of oxygen had been in question.

The next point was to obtain evidence that there was absolutely no shift in the carbon bands, which sometimes happens when the part of the arc photographed is not perfectly in the prolongation of the axis of the collimator.

A photograph has been obtained which supplies such evidence. There are metallic lines close to the carbon bands, which are prolongations of Fraunhofer lines, while the lines which I have already mapped at W. L., 39·27 and 39·295, in the spectrum of iron, are also absolute prolongations. Therefore there is no shift in the carbon flutings. Now the individual lines in the brightest portion of the