illness, we, with heartfelt thankfulness, rejoice with YOUR MAJESTY and all YOUR MAJESTY'S subjects throughout the Empire in His ROYAL HIGHNESS'S happy recovery. That he may long be spared to YOUR MAJESTY, to his Family, and to the People of this Country, is the earnest wish and prayer of YOUR MAJESTY'S loyal and devoted subjects, the President, Council, and Fellows of the Royal Society of London."

On the motion of Mr. C. B. Vignoles, seconded by Dr. Webster, it was resolved that the Fellows do most cordially concur in the Address now read from the Chair.

The Address was then signed by the President on behalf of the Council and Fellows.

The following communications were read :—

I. "On the Induction of Electric Currents in an Infinite Plane Sheet of uniform conductivity." By Prof. J. CLERK MAXWELL, F.R.S. Received January 10, 1872.

1. When, on account of the motion or the change of strength of any magnet or electromagnet, a change takes place in the magnetic field, electromotive forces are called into play, and, if the material in which they act is a conductor, electric currents are produced. This is the phenomenon of the induction of electric currents, discovered by Faraday.

I propose to investigate the case in which the conducting substance is in the form of a thin stratum or sheet, bounded by parallel planes, and of indefinite extent. A system of magnets or electromagnets is supposed to exist on the positive side of this sheet, and to vary in any way by changing its position or its intensity. We have to determine the nature of the currents induced in the sheet, and their magnetic effect at any point, and, in particular, their reaction on the electromagnetic system which gave rise to them. The induced currents are due, partly to the direct action of the external system, and partly to their mutual inductive action; so that the problem appears, at first sight, somewhat difficult.

2. The result of the investigation, however, may be presented in a remarkably simple form, by the aid of the principle of images which was first applied to problems in electricity and hydrokinetics by Sir W. Thomson. The essential part of this principle is, that we conceive the state of things on the positive side of a certain closed or infinite surface (which is really caused by actions having their seat on that surface) to be due to an imaginary system on the negative side of the surface, which, if it existed, and if the action of the surface were abolished, would give rise to the actual state of things in the space on the positive side of the surface.

The state of things on the positive side of the surface is expressed by a mathematical function, which is different in form from that which expresses the state of things on the negative side, but which is identical with that which would be due to the existence, on the negative side, of a certain system which is called the Image.

The image, therefore, is what we should arrive at by *producing*, as it were, the mathematical function as far as it will go; just as, in optics, the virtual image is found by producing the rays, in straight lines, backwards from the place where their direction has been altered by reflexion or refraction.

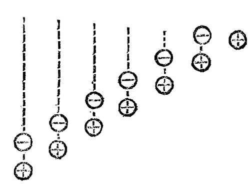
3. The position of the image of a point in a plane surface is found by drawing a perpendicular from the point to the surface and producing it to an equal distance on the other side of the surface. If the image is of the same sign as the point, as it is in hydrokinetics when the surface is a rigid plane, it is called a positive image. If it is of the opposite sign, as in statical electricity, when the surface is a conductor, it is called a negative image. The image of a conducting circuit is reckoned positive when the electric current flows in the corresponding directions through corresponding parts of the object and the image. The image is reckoned negative when the direction of the current is reversed.

In the case of the plane conducting sheet, the imaginary system on the negative side of the sheet is not the simple image, positive or negative, of the real magnet or electromagnet on the positive side, but consists of a moving train of images, the nature of which we now proceed to define.

4. Let the electric resistance of a rectangular portion of the sheet whose length is a, and whose breadth is $2\pi a$, be R.

R is to be measured on the electromagnetic system, and is therefore a velocity, the value of which is independent of the magnitude of the line a. (If ρ denotes the specific resistance of the material of the sheet for a unit cube, and if c is the thickness of the sheet, then $R=\frac{\rho}{2\pi c}$; and if σ denotes the specific resistance of the sheet for a unit (or any other) square, $R=\frac{\sigma}{2\pi}$.)

5. Let us begin by dividing the time into a number of equal intervals, each equal to δt. The smaller we take these intervals the more accurate will be the definition of the train of images which we shall now describe.



6. At a given time t, let a positive image of the magnet or electromagnet be formed on the negative side of the sheet.

As soon as it is formed, let this image begin to move away from the sheet in the direction of the normal, with the velocity R, its form and intensity remaining constantly the same as that which the magnet had at the time t.

After an interval δt, that is to say, at the time $t+\delta t$, let a negative image, equal in magnitude and opposite in sign to this positive image, be formed in the original position of the positive image, and let it then begin to move along the normal, after the positive image, with the velocity R. The interval of time between the arrival of these images at any point will be δt, and the distance between corresponding points will be $R\delta t$.

7. Leaving this pair of images to pursue their endless journey, let us attend to the real magnet, or electromagnet, as it is at the time $t+\delta t$. At this instant let a new positive image be formed of the magnet in its new position, and let this image also travel in the direction of the normal with the velocity R, and be followed after an interval of time δt by a corresponding negative image. Let these operations be repeated at equal intervals of time, each of these intervals being equal to δt.

8. Thus at any given instant there will be a train or trail of images, beginning with a single positive image, and followed by an endless succession of pairs of images. This trail, when once formed, continues unchangeable in form and intensity, and moves as a whole away from the conducting sheet with the constant velocity R.

9. If we now suppose the interval of time δt to be diminished without limit, and the train to be extended without limit in the negative direction, so as to include all the images which have been formed in all past time, the magnetic effect of this imaginary train at any point on the positive side of the conducting sheet will be identical with that of the electric currents which actually exist in the sheet.

Before proceeding to prove this statement, let us take notice of the form which it assumes in certain cases.

10. Let us suppose the real system to be an electromagnet, and that its intensity, originally zero, suddenly becomes I, and then remains constant. At this instant a positive image is formed, which begins to travel along the normal with velocity R. After an interval δt another positive image is formed; but at the same instant a second negative image is formed at the same place, which exactly neutralizes its effect. Hence the result is, that a single positive image travels by itself along the normal with velocity R. The magnetic effect of this image on the positive side of the sheet is equivalent to that of the currents of induction actually existing in the sheet, and the diminution of this effect, as the image moves away from the sheet, accurately represents the effect of the currents of induction, which gradually decay on account of the resistance of the sheet. After a sufficient time, the image is so distant that its effects are no longer sensible on the positive side of the sheet. If the current of the electromagnet be now broken, there will be no more images; but the last negative image of the train will be left unneutralized, and will move away from the sheet with velocity R. The currents in the sheet will therefore be of the same magnitude as those which followed the excitement of the electromagnet, but in the opposite direction.

11. It appears from this that, when the electromagnet is increasing in intensity, it will be acted on by a repulsive force from the sheet, and when its intensity is diminishing, it will be attracted towards the sheet.

It also appears that if any system of currents is produced in the sheet and then left to itself, the effect of the decay of the currents, as observed at a point on the positive side of the sheet, will be the same as if the sheet, with its currents remaining constant, had been carried away in the negative direction with velocity R.

12. If a magnetic pole of strength m is brought from an infinite distance along a normal to the sheet with a uniform velocity v towards the sheet, it will be repelled with a force

$$\frac{m^2}{4z^2}\frac{v}{\mathrm{R}+v},$$

where z is the distance from the sheet at the given instant.

This formula will not apply to the case of the pole moving away from the sheet, because in that case we must take account of the currents which are excited when the pole begins to move, which it does when near the sheet.

13. If the magnetic pole moves in a straight line parallel to the sheet, with uniform velocity v, it will be acted on by a force in the opposite direction to its motion, and equal to

$$\frac{m^2}{4z^2}\,v\,\frac{\sqrt{\mathrm{R}^2+v^2}+\mathrm{R}-v}{(\sqrt{\mathrm{R}^2+v^2}+\mathrm{R})^2}.$$

Besides this retarding force, it is acted on by a force repelling it from the sheet, equal to

$$\frac{m^2}{4z^2}\frac{v^2}{\mathrm{R}^2+v^2+\mathrm{R}\sqrt{\mathrm{R}^2+v^2}}.$$

14. If the pole moves uniformly in a circle, the trail is in the form of a helix, and the calculation of its effect is more difficult; it is easy, however, to see that, besides the retarding force and the repelling force, there is also a force towards the centre of the circle.

15. It is shown, in my treatise on Electricity and Magnetism (vol. ii. art. 600), that the currents in any system are the same, whether the conducting system or the inducing system be in motion, provided the relative motion is the same. Hence the results already given are directly applicable to the case of Arago's rotating disk, provided the induced currents are not sensibly affected by the limitation arising from the edge of the disk. These will introduce other sets of images, which we shall not now investigate.

16. The greater the resistance of the sheet, whether from its thinness or from the low conducting-power of its material, the greater is the velocity R. Hence in most actual cases R is very great compared with v, the velocity of the external system, and the trail of images is nearly normal to

the sheet, and the induced currents differ little from those which arise from the direct action of the external system (see § 1).

17. If the conductivity of the sheet were infinite, or its resistance zero, R would be zero. The images, once formed, would remain stationary, and all except the last formed positive image would be neutralized. Hence the trail would be reduced to a single positive image, and the sheet would exert a repulsive force $\frac{m^2}{4z^2}$ on the pole, whether the pole be in motion or at rest.

I need not say that this case does not occur in nature as we know it. Something of the kind is supposed to exist in the interior of molecules in Weber's Theory of Diamagnetism.

Mathematical Investigation.

18. Let the conducting sheet coincide with the plane of xy, and let its thickness be so small that we may neglect the variation of magnetic force at different points of the same normal within its substance, and that, for the same reason, the only currents which can produce sensible effects are those which are parallel to the surface of the sheet.

Current-function.

19. We shall define the currents in the sheet by means of the current-function ϕ. This function expresses the quantity of electricity which, in unit of time, crosses from right to left a curve drawn from a point at infinity to the point P.

This quantity will be the same for any two curves drawn from this point to P, provided no electricity enters or leaves the sheet at any point between these curves. Hence ϕ is a single-valued function of the position of the point P.

The quantity which crosses the element ds of any curve from right to left is

$$\frac{d\phi}{ds}ds.$$

By drawing ds first perpendicular to the axis of x, and then perpendicular to the axis of y, we obtain for the components of the electric current in the directions of x and of y respectively

$$u=\frac{d\phi}{dy}, \qquad v=-\frac{d\phi}{dx}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The curves for which ϕ is constant are called current lines.

20. The annular portion of the sheet included between the current lines ϕ and $\phi+\delta\phi$ is a conducting circuit round which an electric current of strength $\delta\phi$ is flowing in the positive direction, that is, from x towards y. Such a circuit is equivalent in its magnetic effects to a magnetic shell of strength $\delta\phi$, having the circuit for its edge*.

* W. Thomson, "Mathematical Theory of Magnetism," Phil. Trans. 1850.

The whole system of electric currents in the sheet will therefore be equivalent to a complex magnetic shell, consisting of all the simple shells, defined as above, into which it can be divided. The strength of the equivalent complex shell at any point will be ϕ.

We may suppose this shell to consist of two parallel plane sheets of imaginary magnetic matter at a very small distance c, the surface-density being $\frac{\phi}{c}$ on the positive sheet, and $-\frac{\phi}{c}$ on the negative sheet.

21. To find the magnetic potential due to this complex plane shell at any point not in its substance, let us begin by finding P, the potential at the point (ξ, η, ζ) due to a plane sheet of imaginary magnetic matter whose surface-density is ϕ, and which coincides with the plane of xy. The potential due to the positive sheet whose surface-density is $\frac{\phi}{c}$, and which is at a distance $\frac{1}{2}c$ on the positive side of the plane of xy, is

$$\frac{1}{c}\left(\mathrm{P}-\tfrac{1}{2}c\frac{d\mathrm{P}}{d\zeta}+\&\mathrm{c.}\right).$$

That due to the negative sheet, at a distance $\frac{1}{2}c$ on the negative side of the plane of xy, is

$$-\frac{1}{c}\left(\mathrm{P}+\tfrac{1}{2}c\frac{d\mathrm{P}}{d\zeta}+\&\mathrm{c.}\right).$$

Hence the magnetic potential of the shell is

$$\mathrm{V}=-\frac{d\mathrm{P}}{d\zeta}. \qquad (2)$$

This, therefore, is the value of the magnetic potential of the current-sheet at any given point on the positive side of it. Within the sheet there is no magnetic potential, and at any point $(\xi, \eta, -\zeta)$ on the negative side of the sheet the potential is equal and of opposite sign to that at the point (ξ, η, ζ) on the positive side.

22. At the positive surface the magnetic potential is

$$\mathrm{V}=-\frac{d\mathrm{P}}{d\zeta}=2\pi\phi. \qquad (3)$$

At the negative surface

$$\frac{d\mathrm{P}}{d\xi}=2\pi\phi. \qquad (4)$$

The normal component of magnetic force at the positive surface is

$$\gamma=-\frac{d\mathrm{V}}{d\zeta}=\frac{d^2\mathrm{P}}{d\zeta^2}. \qquad (5)$$

In the case of the magnetic shell, the magnetic force is discontinuous at the surface; but in the case of the current-sheet this expression gives the value of γ within the sheet itself, as well as in the space outside.

23. Let F, G, H be the components of the electromagnetic momentum at any point in the sheet, due to external electromagnetic action as well as to

that of the currents in the sheet, then the electromotive force in the directions of x is

$$-\frac{dF}{dt}-\frac{d\psi}{dx},$$

where ψ is the electric potential*; and by Ohm's law this is equal to σu, where σ is the specific resistance of the sheet.

Hence

$$\left.\begin{aligned}\sigma u&=-\frac{dF}{dt}-\frac{d\psi}{dx}.\\ \text{Similarly,}\quad \sigma v&=-\frac{dG}{dt}-\frac{d\psi}{dy}\end{aligned}\right\}\quad .\ .\ .\ .\ .\ .\ .\ .\ (6)$$

Let the external system be such that its magnetic potential is represented by $-\frac{dP_0}{dz}$, then the actual magnetic potential will be

$$V=-\frac{d}{dz}(P_0+P),\quad .\ .\ .\ .\ .\ .\ .\ .\ (7)$$

and

$$F=\frac{d}{dy}(P_0+P),\quad G=-\frac{d}{dx}(P_0+P),\quad H=0.\ .\ .\ .\ (8)$$

Hence equations (6) become, by introducing the stream-function ϕ from (1),

$$\left.\begin{aligned}\sigma\frac{d\phi}{dy}&=-\frac{d^2}{dtdy}(P_0+P)-\frac{d\psi}{dx},\\ -\sigma\frac{d\phi}{dx}&=\frac{d^2}{dtdx}(P_0+P)-\frac{d\psi}{dy}.\end{aligned}\right\}\quad .\ .\ .\ .\ .\ .\ (9)$$

A solution of these equations is

$$\sigma\phi=-\frac{d}{dt}(P_0+P),\quad \psi=\text{constant}.\ .\ .\ .\ .\ (10)$$

Substituting the value of ϕ in terms of P, as given in equation (4),

$$\frac{\sigma}{2\pi}\frac{dP}{dz}=\frac{d}{dt}(P_0+P).\ .\ .\ .\ .\ .\ .\ .\ .\ (11)$$

The quantity $\frac{\sigma}{2\pi}$ is evidently a velocity; let us therefore for conciseness call it R, then

$$\frac{dP}{dz}+\frac{dP}{dt}+\frac{dP_0}{dt}=0.\quad .\ .\ .\ .\ .\ .\ .\ .\ (12)$$

24. Let P_0' be the value of P_0 at the time $t-\tau$, and at a point on the negative side of the sheet, whose coordinates are x, y, $(z-R\tau)$, and let

$$Q=\int_0^\infty P_0'd\tau.\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .\ (13)$$

At the upper limit when τ is infinite P_0' vanishes. Hence at the lower limit, when $\tau=0$ and $P_0'=P_0$, we must have

$$P_0=\frac{dQ}{dt}+R\frac{dQ}{dz};\quad .\ .\ .\ .\ .\ .\ .\ .\ (14)$$

* "Dynamical Theory of the Electromagnetic Field," Phil. Trans. 1865, p. 483.

but by equation (12)

$$\frac{dP_0}{dt} = -\frac{dP}{dt} - R\frac{dP}{dz}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Hence the equation will be satisfied if we make

$$P = -\frac{dQ}{dt} = -\frac{d}{dt}\int_0^\infty P_0' d\tau. \quad . \quad . \quad . \quad . \quad (16)$$

25. This, then, is a solution of the problem. Any other solution must differ from this by a system of closed currents, depending on the initial state of the sheet, not due to any external cause, and which therefore must decay rapidly. Hence, since we assume an eternity of past time, this is the only solution of the problem.

This solution expresses P, a function due to the action of the induced current, in terms of P_0', and through this of P_0, a function of the same kind due to the external magnetic system. By differentiating P and P_0 with respect to z, we obtain the magnetic potential, and by differentiating them with respect to t, we obtain, by equation (10), the current-function. Hence the relation between P and P_0, as expressed by equation (16), is similar to the relation between the external system and its trail of images as expressed in the description of these images in the first part of this paper (§§ 6, 7, 8, 9), which is simply an explanation of the meaning of equation (16) combined with the definition of P_0' in § 24.

Note to the preceding Paper.

At the time when this paper was written, I was not able to refer to two papers by Prof. Felici, in Tortolini's 'Annali di Scienze' for 1853 and 1854, in which he discusses the induction of currents in solid homogeneous conductors and in a plane conducting sheet, and to two papers by E. Jochmann in Crelle's Journal for 1864, and one in Poggendorff's 'Annalen' for 1864, on the currents induced in a rotating conductor by a magnet.

Neither of these writers have attempted to take into account the inductive action of the currents on each other, though both have recognized the existence of such an action, and given equations expressing it. M. Felici considers the case of a magnetic pole placed almost in contact with a rotating disk. E. Jochmann solves the case in which the pole is at a finite distance from the plane of the disk. He has also drawn the forms of the current-lines and of the equipotential lines, in the case of a single pole, and in the case of two poles of opposite name at equal distances from the axis of the disk, but on opposite sides of it, and has pointed out why the current-lines are not, as Matteucci at first supposed, perpendicular to the equipotential lines, which he traced experimentally.

I am not aware that the principle of images, as described in the paper presented to the Royal Society, has been previously applied to the phenomena of induced currents, or that the problem of the induction of currents in an infinite plane sheet has been solved, taking into account the mutual induction of these currents, so as to make the solution applicable to a sheet of any degree of conductivity.

The statement in equation (10), that the motion of a magnetic system does not produce differences of potential in the infinite sheet, may appear somewhat strange, since we know that currents may be collected by electrodes touching the sheet at different points. These currents, however, depend entirely on the inductive action on the part of the circuit not included in the sheet; for if the whole circuit lies in the plane of the sheet, but is so arranged as not to interfere with the uniform conductivity of the sheet, there will be no difference of potential in any part of the circuit. This is pointed out by Felici, who shows that when the currents are induced by the instantaneous magnetization of a magnet, these currents are not accompanied with differences of potential in different parts of the sheet.

When the sheet is itself in motion, it appears, from art. 600 of my treatise 'On Electricity and Magnetism,' that the electric potential of any point, as measured by means of the electrodes of a fixed circuit, is

$$\psi = -\left(F\frac{\partial x}{\partial t} + G\frac{\partial y}{\partial t} + H\frac{\partial z}{\partial t}\right),$$

where $\frac{\partial x}{\partial t}$, $\frac{\partial y}{\partial t}$, $\frac{\partial z}{\partial t}$ are the components of the velocity of the part of the sheet to which the electrode is applied.

In the case of a sheet revolving with velocity ω about the axis of z, this becomes

$$\psi = \omega\left(x\frac{dP}{dx} + y\frac{dP}{dy}\right).$$

Note 2.—The velocity R for a copper plate of best quality 1 millimetre in thickness is about 25 metres per second. Hence it is only for *very* small velocities of the apparatus that we can obtain any approximation to the true result by neglecting the mutual induction of the currents.—Feb. 13.

II. "On some Derivatives of Uramidobenzoic Acid." By P. Griess, F.R.S. Received January 15, 1872.

This acid, of which I gave a short description some time ago*, has the composition $C_8H_8N_2O_3$. I obtained it in the first instance from the basic compound $C_{10}H_{12}N_2O_3$, which is one of the products of the action of cyanogen on an alcoholic solution of amidobenzoic acid. Its

* Zeitsch. f. Chem. 1868, p. 389.