XXVI. *On a Method of making a Direct Comparison of Electrostatic with Electromagnetic Force; with a Note on the Electromagnetic Theory of Light.* By J. Clerk Maxwell, *F.R.SS. L. & E.*

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There are two distinct and independent methods of measuring electrical quantities with reference to received standards of length, time, and mass.

The electrostatic method is founded on the attractions and repulsions between electrified bodies separated by a fluid dielectric medium, such as air; and the electrical units are determined so that the repulsion between two small electrified bodies at a considerable distance may be represented numerically by the product of the quantities of electricity, divided by the square of the distance.

The electromagnetic method is founded on the attractions and repulsions observed between conductors carrying electric currents, and separated by air; and the electrical units are determined so that if two equal straight conductors are placed parallel to each other, and at a very small distance compared with their length, the attraction between them may be represented numerically by the product of the currents multiplied by the sum of the lengths of the conductors, and divided by the distance between them.

These two methods lead to two different units by which the quantity of electricity is to be measured. The ratio of the two units is an important physical quantity, which we propose to measure. Let us consider the relation of these units to those of space, time, and force (that of force being a function of space, time, and mass).

In the electrostatic system we have a force equal to the product of two quantities of electricity divided by the square of the distance. The unit of electricity will therefore vary directly as the unit of length, and as the square root of the unit of force.

In the electromagnetic system we have a force equal to the product of two currents multiplied by the ratio of two lines. The unit of current in this system therefore varies as the square root of the unit of force; and the unit of electrical quantity, which is that which is transmitted by the unit current in unit of time, varies as the unit of time and as the square root of the unit of force.

The ratio of the electromagnetic unit to the electrostatic unit is therefore that of a certain distance to a certain time, or, in other words, this ratio is a *velocity*; and this velocity will be of the same absolute magnitude, whatever standards of length, time, and mass we adopt.

The electromagnetic value of the resistance of a conductor is also a quantity of the nature of a velocity, and therefore we may express the ratio of the two electrical units in terms of the resistance of a known standard coil; and this expression will be independent of the magnitude of our standards of length, time, and mass.

In the experiments here described no absolute measurements were made, either of length, time, or mass, the ratios only of these quantities being involved; and the velocity determined is expressed in terms of the British Association Unit of Resistance, so that whatever corrections may be discovered to be applicable to the absolute value of that unit must be also applied to the velocity here determined.

A resistance-coil whose resistance is equal to about 28·8 B. A. units would represent the velocity derived from the present experiments in a manner independent of all particular standards of measure.

The importance of the determination of this ratio in all cases in which electrostatic and electromagnetic actions are combined is obvious. Such cases occur in the ordinary working of all submarine telegraph-cables, in induction-coils, and in many other artificial arrangements. But a knowledge of this ratio is, I think, of still greater scientific importance when we consider that the velocity of propagation of electromagnetic disturbances through a dielectric medium depends on this ratio, and, according to my calculations*, is expressed by the very same number.

The first numerical determination of this quantity is that of WEBER and KOHLRAUSCH†, who measured the capacity of a condenser electrostatically by comparison with the capacity of a sphere of known radius, and electromagnetically by passing the discharge from the condenser through a galvanometer.

The Electrical Committee of the British Association have turned their attention to the means of obtaining an accurate measurement of this velocity, and for this purpose have devised new forms of condensers and contact-breakers; and Sir WILLIAM THOMSON has obtained numerical values of continually increasing accuracy by the constant improvement of his own methods.

A velocity which is so great compared with our ordinary units of space and time is probably most easily measured by steps, and by the use of several different instruments; but as it seemed probable that the time occupied in the construction and improvement of these instruments would be considerable, I determined to employ a more direct method of comparing electrostatic with electromagnetic effects.

I should not, however, have been able to do this, had not Mr. GASSIOT, with his usual liberality, placed at my disposal his magnificent battery of 2600 cells charged with corrosive sublimate, with the use of his laboratory to work in.

To Mr. WILLOUGHBY SMITH I am indebted for the use of his resistance-coils, giving a resistance of more than a million B. A. units, and to Messrs. FORDE and FLEEMING JENKIN for the use of a galvanometer and resistance-coils, a bridge and a key for double contacts.

Mr. C. HOCKIN, who has greatly assisted me with suggestions since I first devised the

* "A Dynamical Theory of the Electromagnetic Field," Philosophical Transactions, 1865.

† POGG. Ann. Aug. 1856, Bd. xcix. p. 10.

experiment, undertook the whole work of the comparison of the currents by means of the galvanometer and shunts. He has also tested the resistances, and in fact done everything except the actual observation of equilibrium, which I undertook myself.

The electrical balance itself was made for me by Mr. BECKER.

The electrostatic force observed was that between two parallel disks, of which one, 6 inches diameter, was insulated and maintained at a high potential, while the other, 4 inches diameter, was at the same potential as the case of the instrument.

In order to insure a known quantity of electricity on the surface of this disk, it was surrounded by the "guard-ring" introduced by Sir W. THOMSON, so that the surface of the disk when in position and that of the guard-ring were in one plane, at the same potential, and separated by a very narrow space. In this way the electrical action on the small disk was equal to that due to a uniform distribution over its front surface, while no electrical action could exist at its sides or back, as these were at the same potential with the surrounding surfaces.

The large disk was mounted on a slide worked by a micrometer-screw. The small disk was suspended on one arm of a torsion-balance so that in its position of equilibrium its surface and that of the guard-ring were in one plane.

If E is the difference of potential between the two disks in electromagnetic measure, the attraction between them is

$$\frac{\mathrm{E}^2}{8v^2}\frac{a^2}{b^2}, \quad \ldots \ldots \ldots \quad (1)$$

where a is the radius of the small disk, b its distance from the large one, and v is the velocity representing the ratio of the electromagnetic to the electrostatic unit of electricity.

The electromagnetic force observed was the repulsion between two circular coils, of which one was attached to the back of the suspended disk, and the other was placed behind the large disk, being separated from it by a plate of glass and a layer of HOOPER's compound. A current was made to pass through these coils in opposite directions, so as to produce a repulsion

$$=2\pi nn'\frac{2\mathrm{A}}{\mathrm{B}}y^2, \quad \ldots \ldots \ldots \quad (2)$$

where n and n' are the number of windings of each coil, y is the current, and

$$\frac{2\mathrm{A}}{\mathrm{B}}=\{\mathrm{E}_c\tan^2\gamma-2(\mathrm{F}_c-\mathrm{E}_c)\}\frac{b'\sin\gamma}{2\sqrt{a_1a_2}}, \quad \ldots \ldots \quad (3)$$

where

$$c=\sin\gamma=\frac{2\sqrt{a_1a_2}}{\sqrt{(a_1+a_2)^2+b'^2}}; \quad \ldots \ldots \ldots \quad (4)$$

a_1 and a_2 are the mean radii of the coils, and b' the mean distance of their planes, and E_c and F_c are the complete elliptic functions for modulus $c=\sin\gamma$.

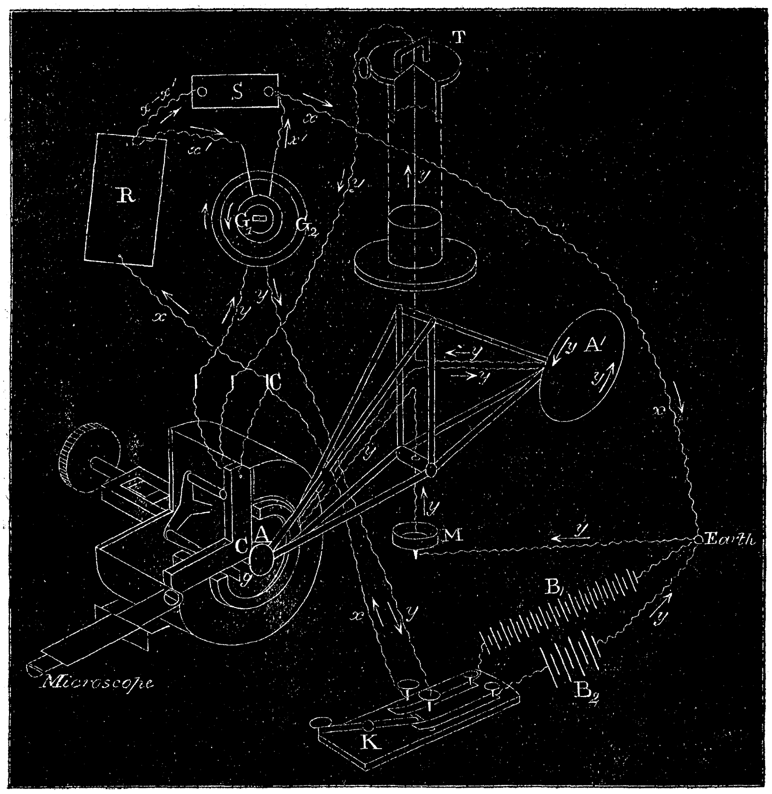
When b' is small compared with a', $\frac{2\mathrm{A}}{\mathrm{B}}$ becomes very nearly $\frac{2a'}{b'}$.

If we take into account the fact that the section of each coil is of sensible area, this formula would require correction; but in these coils the depth was made equal to the breadth of the section, whence it follows, by the differential equation of the potential of two coils, given at p. 508 of my paper on the Electromagnetic Field,

$$\frac{d^2M}{da^2}+\frac{d^2M}{db^2}+\frac{1}{a}\frac{dM}{da}=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

that the correction is a factor of the form $\left(1-\frac{1}{12}\frac{\alpha'^2}{a'^2}\right)$, where α' is the depth of the coil —a correction which is in this case about $1-\cdot000926$.

The suspended coil, besides the repulsion due to the fixed coil, experiences a couple due to the action of terrestrial magnetism. To balance this couple, a coil exactly similar was attached to the other arm of the torsion-balance, and the current in the second coil was made to flow in the opposite direction to that in the first. When the current was made to flow through both coils, no effect of terrestrial magnetism could be observed.



A. Suspended disk and coil.
A′. Counterpoise disk and coil.
C. Fixed disk and coil.
B_1. Great battery. B_2. Small battery.
G_1. Primary coil of galvanometer. G_2. Secondary coil.
R. Great resistance. S. Shunt.
K. Double key. *g*. Graduated glass scale.
C. Electrode of fixed disk.
x. Current through R.
x′. Current through G_1. $x-x'$. Current through S.
y. Current through the three coils and G_2.
M. Mercury cup. T. Torsion head and tangent screw.

One quarter of the micrometer-box, disks, and coils is cut away to show the interior. The case of the instrument is not shown. The galvanometer and shunts were 10 feet from the Electric Balance.

The torsion-balance consisted of a light brass frame, to which the suspended coils and disks were attached so that the centre of each coil was about eight inches from the vertical axis of suspension. This frame was suspended by a copper wire (No. 20), the upper end of which was attached to the centre of a torsion head, graduated, and provided with a tangent screw for small angular adjustments. The torsion head was supported by a hollow pillar, the base of which was clamped to the lid of the instrument so as to admit of small adjustments in every direction.

The fixed disk and coil were mounted on a slide worked by a micrometer-screw, and were protected by a cylindrical brass box, the front of which, forming the guard-ring, 7 inches in diameter, had a circular aperture 4·26 inches diameter, within which the suspended disk, 4·13 inches diameter, was free to move, leaving an interval of ·065 of an inch between the disk and the aperture. A glass scale with divisions of $\frac{1}{100}$ of an inch was attached to the suspended disk on the side which was not electrified, and this was viewed by a microscope attached to the side of the instrument and provided with cross wires at the focus.

The disk worked by the micrometer was carefully adjusted by the maker, so as to be parallel to the inner surface of the guard-ring, or front face of the micrometer-box. This front face of the micrometer-box, when in position in the instrument, was made vertical by means of three adjusting screws. The suspended disk was then pressed against the fixed disk by means of a slight spring, and the fixed disk was gradually moved forward by the micrometer-screw, while at the same time the graduated scale was observed through the microscope. In this way the graduations on the scale were compared with the readings of the micrometer. This was continued till the large disk came into contact with the guard-ring at one point, when the regularity of the motion was interrupted. A very small motion was then sufficient to bring the whole circumference of the disk into contact with the guard-ring, when the motion ceased altogether. This motion was not much more than one-thousandth of an inch.

This disk was then brought to the position of first contact, and the microscope was adjusted so that a known division of the glass scale was bisected by the cross wires. A small piece of silvered glass was fastened to the outside of the guard-ring, and another to the back of the suspended disk; and these were adjusted so as to be in one plane, and to give a continuous image of reflected objects when the disks were in contact and the surface of the suspended disk was therefore in the plane of the surface of the guard-ring. The fixed disk was then screwed back, and the torsion-balance was adjusted so that the suspended disk when in equilibrium was in precisely the same position as before. This was tested by observing the coincidence of the zero division of the glass scale with the cross wires of the microscope, and by examining the reflections from the two pieces of silvered glass. The torsion-balance could be moved bodily in any horizontal direction by adjusting the base of the pillar; it could be raised or lowered by a winch, and it could be turned about any horizonal axis by sliding weights, and round the vertical axis by a tangent screw of the torsion head. In this way the position of equilibrium of the suspended disk could be made to coincide with the plane of the guard-ring to the thousandth of an inch; and the adjustment when made continued very good from day to day, soft copper wire, stretched straight, not having the tendency to untwist gradually which I have observed in steel wire. The weight of the torsion piece was about 1 lb. 3 oz., and the time of a double oscillation about fourteen seconds. The oscillations of the suspended disk, when near its sighted position, were found to subside very rapidly, the energy of the motion being expended in pumping the air through the narrow aperture between the guard-plate and the suspended disk.

The electrical arrangements were as follows:—

One electrode of Mr. GASSIOT'S great battery was connected with a key. When the key was pressed connexion was made to the fixed disk, and thence, through Mr. WILLOUGHBY SMITH'S resistance-coils, to a point where the current was divided between the principal coil of the galvanometer and a shunt, S, consisting of Mr. JENKIN'S resistance-coils. These partial currents reunited at a point where they were put in connexion with the other electrode of the battery, with the case of the instrument, and with the earth.

Another battery was employed to send a current through the coils. One electrode of this battery was connected with a second contact piece of the key, so that, when the key was pressed, the current went first through the secondary coil of the galvanometer, consisting of thirty windings of thick wire, then through the fixed coil, then to the suspension wire, and so through the two suspended coils to the brass frame of the torsion-balance and the suspended disk. A stout copper wire, well amalgamated, hanging from the centre of the torsion-balance into a cup of mercury, made metallic communication to the case, to earth, and to the other electrode of the battery.

When these arrangements had been made, the observer at the microscope, when the suspended disk was stationary at zero, made simultaneous contact with both batteries by means of the key. If the disk was attracted, the great battery was the more powerful, and the micrometer was worked so as to increase the distance of the disk. If the disk was repelled, the fixed disk had to be moved nearer to the suspended disk, till a distance was found at which, when the scale was at rest and at zero, no effect was produced by the simultaneous action of the batteries. With the forces actually employed the equilibrium of the scale at zero was unstable; so that when the adjustment was nearly perfect the force was always directed from zero, and contacts had to be made as the scale was approaching zero, in such a way as to bring it to rest, if possible, at zero.

In the meantime the other observer at the galvanometer was taking advantage of these contacts to alter the shunt S, till the effects of the two currents on the galvanometer-needle balanced each other.

When a satisfactory case of equilibrium had been observed simultaneously at the galvanometer and at the torsion-balance, the micrometer-reading and the resistance of the shunt were set down as the results of the experiment.

The chief difficulties experienced arose from the want of constancy in the batteries, the ratio of the currents varying very rapidly after first making contact. I think that by increasing considerably the resistance of the great battery-circuit, the current could be made more uniform.

When a sufficient number of experiments on equilibrium had been made, a current was made to pass through the secondary coil of the galvanometer, and was then divided between a shunt of 31 units B. A. and the primary coil of the galvanometer with a resistance S′ added. S′ was then varied till the needle was in equilibrium. In this way the magnetic effects of the two coils were compared.

The resistance of the galvanometer and of all the coils were tested by Mr. HOCKIN, who also made all the observations with the galvanometer and its adjusting shunts.

To determine v from these experiments, we have first, since the attraction is equal to the repulsion,

$$\frac{E^2}{8v^2}\frac{a^2}{b^2}=2\pi nn'\frac{2A}{B}y^2. \qquad (6)$$

If x is the current of the great battery passing through the great resistance R, and if x' of this passes through the galvanometer whose resistance is G, and $x-x'$ through the shunt S to earth, then

$$E=Rx+Gx' \qquad (7)$$

and

$$Gx'=S(x-x'). \qquad (8)$$

Also if g_1 is the magnetic effect of the principal coil of the galvanometer, and g_2 that of the secondary coil, then when the needle is in equilibrium

$$g_1x'=g_2y. \qquad (9)$$

In the comparison of the coils of the galvanometer, if x_1 and y_1 are the currents through each, we have

$$g_1x_1=g_2y_1. \qquad (10)$$

But y_1 is divided into two parts, of which x_1 passes through the galvanometer G and the shunt S′, and the other, y_1-x', passes through the shunt of 31 Ohms. Hence

$$x_1(G+S')=(y_1-x_1)31. \qquad (11)$$

From these equations we obtain as the value of v,

$$v=\frac{1}{4\sqrt{nn'\pi}}\frac{a}{b}\sqrt{\frac{B}{2A}}\left(\frac{RG}{S}+R+G\right)\frac{31}{G+S'+31}, \qquad (12)$$

an equation containing only known quantities on the right-hand side. Of these, n and n' are the numbers of windings on the two coils, a is the mean of the radii of the suspended disk and the aperture, b is the distance between the fixed disk and the suspended disk, $\frac{2A}{B}$ is found from a_1 and a_2, the mean radii of the coils, and b' their mean distance by equation (3).

R is the great resistance, G that of the galvanometer, S that of the shunt in the principal experiment, and S′ that of the additional resistance in the comparison of galvanometer-coils.

In this expression the only quantities which must be determined in absolute measure are the resistances. The other quantities which must be measured are the ratios of the radius of the disk to its distance from the fixed disk, and the ratio of the radius of the coils to the distance between them. These ratios and the number of windings in the coils are of course abstract numbers.

In the experiments,

$$n=144 \qquad n'=121$$
$$a=2\cdot0977 \text{ inches.} \qquad a'=1\cdot934 \text{ inch.}$$

To determine a', the circumference of every layer of the coils was measured with watch-spring, the thickness of which was ·008 inch.

One turn of the micrometer-screw was found by Mr. HOCKIN to be equal to ·0202 inch. If m is the micrometer-reading in terms of the screw,

$$b=m-12\cdot70, \qquad b'=m+26\cdot31.$$

In terms of the micrometer measure we have for a and a',

$$a=103\cdot85 \text{ turns}, \qquad a'=95\cdot75 \text{ turns.}$$

The resistances were determined by Mr. HOCKIN as follows:

R =1 102 000 Ohms.
G= 46 220 „

The experiments were made for two days, using a small battery charged with bichromate of potash. The current due to this battery was found to diminish so rapidly that a set of GROVE'S cells was used on the third day, which was found to be more constant than the great battery. A proper combination of the two batteries would perhaps produce a current which would diminish according to the same law as that of the great battery. Another difficulty arose from the fact that when the connexions were made, but before the key was pressed, if the micrometer was touched by the hand the disk was attracted. This I have not been able satisfactorily to account for, except by leakage of electricity from the great battery through the floor. When the micrometer was not touched, the disk remained at its proper zero. In certain experiments I kept my hand always on the micrometer in order to be able to adjust it more accurately. These experiments gave a value of v much too small, on account of the additional attraction. When I discovered the attraction, I took care to make the observations without touching the micrometer, and took advantage of the attraction to check the oscillations of the disk. The experiments in which these precautions were taken agree together as well as I could expect, and lead me to think that, with the experience I have acquired, still better results might be obtained by the same method. It must be borne in mind that none of the results were calculated till after the conclusion of all the experiments, and that the rejected experiments were condemned on account of errors observed while they were being made.

Any leakage arising from want of insulation of the fixed disk would introduce no error, as the difference of potentials between the two disks is measured by the current in the galvanometer, through a known resistance, independently of any leakage.

All that is essential to accuracy is that the position of equilibrium before making contact should be at true zero, the same as when there is no electrical action, and that this equilibrium should not be disturbed when simultaneous contact is made with both batteries.

Experiments on May 8. S′=1710 Ohms.

Number of experiment.	Great battery-cells.	Small battery-cells.	Distance of disks by micrometer.	Resistance of shunt S.	Value of v in Ohms.
1	1000	6	12·41	6870	28·591
2	1000	6	12·36	6940	28·430
8*	1800	8	16·99	5074	28·886
9	1800	8	17·02	5110	28·686
10	1800	8	19·91	4430	28·910
11	1800	7	20·07	4410	28·850
12	2600	9	25·08	3700	28·762
13	2600	9	25·12	3690	28·795
14	2600	9	25·29	3680	28·735
15	2600	9	25·18	3690	28·752
16	2600	9	25·19	3695	28·709
17	1800	7	19·69	4435	29·474

Mean value of v=28·798 Ohms, or B. A. units,
or 288 000 000 metres per second,
or 179 000 statute miles per second.

The "probable error" is about one-sixth per cent.

Experiments 3, 4, 5, 6, 7 were rejected on account of the micrometer being touched during the observation of equilibrium. These experiments gave an average value of v =27·39.

The value of v derived from these experiments is considerably smaller than that which was obtained by MM. Weber and Kohlrausch, which was 31·074 Ohms, or 310 740 000 metres per second.

Their method involved the determination of the electrostatic capacity of a condenser, the electrostatic determination of its potential when charged, and the electromagnetic determination of the quantity of electricity discharged through a galvanometer.

The capacity of the condenser was measured by dividing its charge repeatedly with a sphere of known radius. Now, since all condensers made with solid dielectrics exhibit the phenomena of "electric absorption," this method would give too large a value for the capacity, as the condenser would become recharged to a certain extent after each discharge, so that the repeated division of the charge would have too small an effect on the potential. The capacity being overestimated, the number of electrostatic units in the discharge would be overestimated, and the value of v would be too great.

In pointing out this as a probable source of error in the experiments of MM. Weber and Kohlrausch, I mean to indicate that I have such confidence in the ability and fidelity

* In experiment 8 Mr. Hockin and I changed places.

with which their investigation was conducted, that I am obliged to attribute the difference of their result from mine to a phenomenon the nature of which is now much better understood than when their experiments were made.

On the other hand, the result of present experiments depends on the accuracy of the experiments of the Committee of the British Association on Electric Resistance. The B. A. unit is about 8·8 per cent. larger than that determined by WEBER in 1862, and about 1·2 per cent. less than that derived by Dr. JOULE from his experiments on the dynamical equivalent of heat by comparing the heating effects of direct mechanical agitation with those of electric currents.

I believe that Sir WILLIAM THOMSON'S experiments, not yet published, give a value of v not very different from mine. His method, I believe, also depends on the value of the B. A. unit.

The lowest estimate of the velocity of light, that of the late M. FOUCAULT, is

298 000 000 metres per second.

Note on the Electromagnetic Theory of Light.

In a paper on the Electromagnetic Field* some years ago, I laid before the Royal Society the reasons which led me to believe that light is an electromagnetic phenomenon, the laws of which can be deduced from those of electricity and magnetism, on the theory that all these phenomena are affections of one and the same medium. Two papers appeared in POGGENDORFF'S 'Annalen' for 1867 bearing on the same subject. The first, by the late eminent mathematician BERNHARD RIEMANN, was presented in 1858 to the Royal Society of Göttingen, but was withdrawn before publication, and remained unknown till last year. RIEMANN shows that if for LAPLACE'S equation we substitute

$$\frac{d^2V}{dt^2}-\alpha^2\Delta^2V+\alpha^2 4\pi\varrho=0, \quad \ldots \ldots \quad (13)$$

V being the electrostatic potential, and α a velocity, the results will agree with known phenomena in all parts of electrical science. This equation is equivalent to a statement that the potential V is propagated through space with a certain velocity. The author, however, seems to avoid making explicit mention of any medium through which the propagation takes place, but he shows that this velocity is nearly, if not absolutely, equal to the known velocity of light.

The second paper, by M. LORENZ, shows that, on WEBER'S theory, periodic electric disturbances would be propagated with a velocity equal to that of light. The propagation of attraction through space forms part of this hypothesis also, though the medium is not explicitly recognized.

From the assumptions of both these papers we may draw the conclusions, first, that action and reaction are not always equal and opposite, and second, that apparatus may be constructed to generate any amount of work from its own resources.

* Philosophical Transactions, 1865, p. 459.

For let two oppositely electrified bodies A and B travel along the line joining them with equal velocities in the direction AB, then if either the potential or the attraction of the bodies at a given time is that due to their position at some former time (as these authors suppose), B, the foremost body, will attract A forwards more than A attracts B backwards.

Now let A and B be kept asunder by a rigid rod.

The combined system, if set in motion in the direction AB, will pull in that direction with a force which may either continually augment the velocity, or may be used as an inexhaustible source of energy.

I think that these remarkable deductions from the latest developments of WEBER and NEUMANN'S theory can only be avoided by recognizing the action of a medium in electrical phenomena.

The statement of the electromagnetic theory of light in my former paper was connected with several other electromagnetic investigations, and was therefore not easily understood when taken by itself. I propose, therefore, to state it in what I think the simplest form, deducing it from admitted facts, and showing the connexion between the experiments already described and those which determine the velocity of light.

The connexion of electromagnetic phenomena may be stated in the following manner.

Theorem A.—If a closed curve be drawn embracing an electric current, then the integral of the magnetic intensity taken round the closed curve is equal to the current multiplied by 4π.

The integral of the magnetic intensity may be otherwise defined as the work done on a unit magnetic pole carried completely round the closed curve.

This well-known theorem gives us the means of discovering the position and magnitude of electric currents, when we can ascertain the distribution of magnetic force in the field. It follows directly from the discovery of ŒRSTED.

Theorem B.—If a conducting circuit embraces a number of lines of magnetic force, and if, from any cause whatever, the number of these lines is diminished, an electromotive force will act round the circuit, the total amount of which will be equal to the decrement of the number of lines of magnetic force in unit of time.

The number of lines of magnetic force may be otherwise defined as the integral of the magnetic intensity resolved perpendicular to a surface, multiplied by the element of surface, and by the coefficient of magnetic induction, the integration being extended over any surface bounded by the conducting circuit.

This theorem is due to FARADAY, as the discoverer both of the facts and of this mode of expressing them, which I think the simplest and most comprehensive.

Theorem C.—When a dielectric is acted on by electromotive force it experiences what we may call electric polarization. If the direction of the electromotive force is called positive, and if we suppose the dielectric bounded by two conductors, A on the negative, and B on the positive side, then the surface of the conductor A is positively electrified, and that of B negatively. If we admit that the energy of the system so electrified resides in the polarized dielectric, we must also admit that within the dielectric there is a displacement of electricity in the direction of the electromotive force, the amount of this displacement being proportional to the electromotive force at each point, and depending also on the nature of the dielectric.

The energy stored up in any portion of the dielectric is half the product of the electromotive force and the electric displacement, multiplied by the volume of that portion.

It may also be shown that at every point of the dielectric there is a mechanical tension along the lines of electric force, combined with an equal pressure in all directions at right angles to these lines, the amount of this tension on unit of area being equal to the amount of energy in unit of volume.

I think that these statements are an accurate rendering of the ideas of FARADAY, as developed in various parts of his 'Experimental Researches.'

Theorem D.—When the electric displacement increases or diminishes, the effect is equivalent to that of an electric current in the positive or negative direction.

Thus, if the two conductors in the last case are now joined by a wire, there will be a current in the wire from A to B.

At the same time, since the electric displacement in the dielectric is diminishing, there will be an action electromagnetically equivalent to that of an electric current from B to A through the dielectric.

According to this view, the current produced in discharging a condenser is a complete circuit, and might be traced within the dielectric itself by a galvanometer properly constructed. I am not aware that this has been done, so that this part of the theory, though apparently a natural consequence of the former, has not been verified by direct experiment. The experiment would certainly be a very delicate and difficult one.

Let us now apply these four principles to the electromagnetic theory of light, considered as a disturbance propagated in plane waves.

Let the direction of propagation be taken as the axis of z, and let all the quantities be functions of z and of t the time; that is, let every portion of any plane perpendicular to z be in the same condition at the same instant.

Let us also suppose that the magnetic force is in the direction of the axis of y, and let β be the magnetic intensity in that direction at any point.

Let the closed curve of Theorem A consist of a parallelogram in the plane $y\,z$, two of whose sides are b along the axis of y, and z along the axis of z. The integral of the magnetic intensity taken round this parallelogram is $b(\beta_0-\beta)$, where β_0 is the value of β at the origin.

Now let p be the quantity of electric current in the direction of x per unit of area taken at any point, then the whole current through the parallelogram will be

$$\int_0^z bp\,dz,$$

and we have by (A),

$$b(\beta_0-\beta)=4\pi\int_0^z bp\,dz.$$

If we divide by b and differentiate with respect to z, we find

$$\frac{d\beta}{dz}=-4\pi p. \qquad (14)$$

Let us next consider a parallelogram in the plane of $x\,z$, two of whose sides are a along the axis of x, and z along the axis of z.

If P is the electromotive force per unit of length in the direction of x, then the total electromotive force round this parallelogram is $a(\mathrm{P}-\mathrm{P}_0)$.

If μ is the coefficient of magnetic induction, then the number of lines of force embraced by this parallelogram will be

$$\int_0^z a\mu\beta dz,$$

and since by (B) the total electromotive force is equal to the rate of diminution of the number of lines in unit of time,

$$a(\mathrm{P}-\mathrm{P}_0)=-\frac{d}{dt}\int_0^z a\mu\beta dz.$$

Dividing by a and differentiating with respect to z, we find

$$\frac{d\mathrm{P}}{dz}=-\mu\frac{d\beta}{dt}. \qquad (15)$$

Let the nature of the dielectric be such that an electric displacement f is produced by an electromotive force P,

$$\mathrm{P}=kf, \qquad (16)$$

where k is a quantity depending on the particular dielectric, which may be called its "electric elasticity."

Finally, let the current p, already considered, be supposed entirely due to the variation of f, the electric displacement, then

$$p=\frac{df}{dt}. \qquad (17)$$

We have now four equations, (14), (15), (16), (17), between the four quantities β, p, P, and f. If we eliminate p, P, and f, we find

$$\frac{d^2\beta}{dt^2}=\frac{k}{4\pi\mu}\frac{d^2\beta}{dz^2}. \qquad (18)$$

If we put

$$\frac{k}{4\pi\mu}=\mathrm{V}^2, \qquad (19)$$

the well-known solution of this equation is

$$\beta=\phi_1(z-\mathrm{V}t)+\phi_2(z+\mathrm{V}t), \qquad (20)$$

showing that the disturbance is propagated with the velocity V.

The other quantities p, P, and f can be deduced from β.

Thus, if

$$\left.\begin{aligned}\beta&=c\cos\frac{2\pi}{\lambda}(z-\mathrm{V}t),\\ p&=\frac{c}{2\lambda}\sin\frac{2\pi}{\lambda}(z-\mathrm{V}t),\\ \mathrm{P}&=c\mu\mathrm{V}\cos\frac{2\pi}{\lambda}(z-\mathrm{V}t),\\ f&=\frac{c}{4\pi\mathrm{V}}\cos\frac{2\pi}{\lambda}(z-\mathrm{V}t).\end{aligned}\right\}\quad\ldots\ldots\ldots\quad(21)$$

I have in the next place to show that the velocity V is the same quantity as that found from the experiments on electricity.

For this purpose let us consider a stratum of air of thickness b bounded by two parallel plane conducting surfaces of indefinite extent, the difference of whose potentials is E.

The electromotive force per unit of length between the surfaces is $\mathrm{P}=\frac{1}{b}\mathrm{E}$.

The electric displacement is $f=\frac{1}{k}\mathrm{P}$.

The energy in unit of volume and the tension along the lines of force per unit of area is $\frac{1}{2}\mathrm{P}f$.

The attraction X on an area πa^2 of either surface is

$$\left.\begin{aligned}\mathrm{X}&=\tfrac{1}{2}\pi a^2\mathrm{P}f\\ &=\tfrac{1}{2}\frac{\pi}{k}\frac{a^2}{b^2}\mathrm{E}^2\end{aligned}\right\}\quad\ldots\ldots\ldots\quad(22)$$

If this area is separated by a small interval from the rest of the plane surface, as in the experiment, and if this interval is small compared with the radius of the disk, the lines of force belonging to the disk will be separated from those belonging to the rest of the surface by a surface of revolution, the section of which, at any sensible distance from the surface, will be a circle whose radius is a mean between those of the disk and the aperture. This radius must be taken for a in the equation (22)*.

Let us next consider the magnetic force near a long straight conductor carrying a current y. The magnetic force will be in the direction of a tangent to a circle whose axis is the current; and the intensity will be uniform round this circle. If the radius is b, and the magnetic intensity β, the integral round the circle will be $2\pi b\beta=4\pi y$ by (A).

* [Note added Dec. 28, 1868.—I have since found that if a_1 is the radius of the disk, and a_2 that of the aperture of the guard-ring, and b the distance from the large fixed disk, then we must substitute for $\frac{a^2}{b^2}$ the more approximate expression $\frac{a_1^2}{b^2}+\frac{a_2^2-a_1^2}{2b(b+\alpha)}$, where α is a quantity which cannot exceed $\frac{\log_e 2}{\pi}(a_2-a_1)$.—J. C. M.]

Hence

$$\beta=2\frac{y}{b}. \qquad (23)$$

Let a wire carrying a current y' be placed parallel to the first at a distance b, and let us consider a portion of this wire of length l. This portion will be urged across the lines of magnetic force, and the electromagnetic force Y will be equal to the product of the length of the portion, multiplied by the current and by the number of lines which it crosses per unit of distance through which it moves, or, in symbols,

$$\left.\begin{aligned}Y&=ly'\mu\beta\\&=2\mu\frac{l}{b}yy'\end{aligned}\right\}. \qquad (24)$$

If the two wires instead of being straight are circular, of radius a', and if b' the distance between them is very small compared with the radius, the attraction will be the same as if they were straight, and will be

$$Y=2\mu\frac{2\pi a'}{b'}yy'. \qquad (25)$$

When b' is not very small compared with a', we must use the equation (3) to calculate the value of $\frac{2A}{B}$ by elliptic integrals.

Making $X=Y$ and comparing with equation (6), we find

$$v^2=\frac{\mu k}{4\pi}; \qquad (26)$$

but, by (19),

$$V^2=\frac{k}{4\pi\mu}.$$

Hence

$$v=\mu V, \qquad (27)$$

where v is the electromagnetic ratio and V is the velocity of light.

But since all the experiments are made in air, for which μ is *assumed* equal to unity, as the standard medium with which all others are compared, we have finally

$$v=V, \qquad (28)$$

or the number of electrostatic units in one electromagnetic unit of electricity is numerically equal to the velocity of light.