is almost exactly the mean of the powers of the two liquids which compose it—account being taken of the proportion of each in the mixture, and of the alteration of density, in this case very small. On the contrary, when the proportion of acid or of water is more than two tenths, the rotatory power increases in a much less proportion than the density, proving that the combination diminishes the molecular magneto-rotatory power. It is true that in this case there is a strong chemical action, as is proved by the great disengagement of heat. I am disposed to believe that in all cases there is formed a compound of water and acid, of which the magneto-rotatory power is always less than the mean of the rotatory powers of the water and the acid in combination, even allowing for the contraction, as we have seen above. Only, when the proportions of the two substances mixed are very different, the compound, dissolved in a large quantity of the liquid which is in excess, cannot sensibly modify the mean rotatory power. It is not so when the proportions of the two liquids are nearly equal, the compound being in proportionally much larger quantity in the mixture. Magnetic rotatory polarization, therefore, might with advantage be used for the purpose of distinguishing simple physical solutions or weak combinations from definite chemical compounds.

4. Our fourth conclusion is that the phenomenon of magneto-rotatory polarization presents a means of penetrating into the intimate constitution of bodies, and may thus be really serviceable to science. For example, the still very imperfect investigation we have made shows already that the relations of ponderable particles to the ether in which they are immersed do not depend solely on the nature of the particles, but also on their mode of grouping in the combinations which they form; for simple mixture is not sufficient to modify these relations; combination is necessary in order that modification may take place. I think, therefore, that it will be especially by operating on solutions and isomeric bodies that we shall succeed in throwing some light both on the nature of the phenomenon of magnetic rotatory polarization and on the atomic constitution of bodies, by determining for this purpose the differences existing between the magneto-rotatory powers of isomers on the one hand, and on the other the differences presented by the rotatory powers of simple mixtures compared with those of true chemical combinations. I shall endeavour, if I can procure the necessary substances, to study these two points more completely than I have been able to study them in the present memoir.

L. On Hills and Dales.

By J. Clerk Maxwell, *LL.D., F.R.S.*

To the Editors of the Philosophical Magazine and Journal.

Gentlemen,

I FIND that in the greater part of the substance of the following paper I have been anticipated by Professor Cayley, in a memoir "On Contour and Slope Lines," published in the Philosophical Magazine in 1859 (S. 4. vol. xviii. p. 264). An exact knowledge of the first elements of physical geography, however, is so important, and loose notions on the subject are so prevalent, that I have no hesitation in sending you what you, I hope, will have no scruple in rejecting if you think it superfluous after what has been done by Professor Cayley.

I am, Gentlemen,

Your obedient Servant,

J. Clerk Maxwell.

Glenlair, Dalbeattie,
October 12, 1870.

1. On Contour-lines and Measurement of Heights.

The results of the survey of the surface of a country are most conveniently exhibited by means of a map on which are traced contour-lines, each contour-line representing the intersection of a level surface with the surface of the earth, and being distinguished by a numeral which indicates the level surface to which it belongs.

When the extent of country surveyed is small, the contour-lines are defined with sufficient accuracy by the number of feet above the mean level of the sea; but when the survey is so extensive that the variation of the force of gravity must be taken into account, we must adopt a new definition of the height of a place in order to be mathematically accurate. If we could determine the exact form of the surface of equilibrium of the sea, so as to know its position in the interior of a continent, we might draw a normal to this surface from the top of a mountain, and call this the height of the mountain. This would be perfectly definite in the case when the surface of equilibrium is everywhere convex; but the lines of equal height would not be level surfaces.

Level surfaces are surfaces of equilibrium, and they are not equidistant. The only thing which is constant is the amount of work required to rise from one to another. Hence the only consistent definition of a level surface is obtained by assuming a standard station, say, at the mean level of the sea at a particular place, and defining every other level surface by the work required to raise unit of mass from the standard station to that level surface. This work must, of course, be expressed in absolute measure, not in local foot-pounds.

At every step, therefore, in ascertaining the difference of level of two places, the surveyor should ascertain the force of gravity, and multiply the linear difference of level observed by the numerical value of the force of gravity.

The height of a place, according to this system, will be defined by a number which represents, not a lineal quantity, but the half square of the velocity which an unresisted body would acquire in sliding along any path from that place to the standard station. This is the only definition of the height of a place consistent with the condition that places of equal height should be on the same level. If by any means we can ascertain the mean value of gravity along the line of force drawn from the place to the standard level surface, then, if we divide the number already found by this mean value, we shall obtain the length of this line of force, which may be called the linear height of the place.

On the Forms of Contour-lines.

Let us begin with a level surface entirely within the solid part of the earth, and let us suppose it to ascend till it reaches the bottom of the deepest sea. At that point it will touch the surface of the earth; and if it continues to ascend, a contour-line will be formed surrounding this bottom (or Immit, as it is called by Professor Cayley) and enclosing a region of depression. As the level surface continues to ascend, it will reach the next deepest bottom of the sea; and as it ascends it will form another contour-line, surrounding this point, and enclosing another region of depression below the level surface. As the level surface rises these regions of depression will continually expand, and new ones will be formed corresponding to the different lowest points of the earth's surface.

At first there is but one region of depression, the whole of the rest of the earth's surface forming a region of elevation surrounding it. The number of regions of elevation and depression can be altered in two ways.

1st. Two regions of depression may expand till they meet and so run into one. If a contour-line be drawn through the point where they meet, it forms a closed curve having a double point at this place. This contour-line encloses two regions of depression. We shall call the point where these two regions meet a Bar.

It may happen that more than two regions run into each other at once. Such cases are singular, and we shall reserve them for separate consideration.

2ndly. A region of depression may thrust out arms, which may meet each other and thus cut off a region of elevation in the midst of the region of depression, which thus becomes a cyclic region, while a new region of elevation is introduced. The contour-line through the point of meeting cuts off two regions of elevation from one region of depression, and the point itself is called a Pass. There may be in singular cases passes between more than two regions of elevation.

3rdly. As the level surface rises, the regions of elevation contract and at last are reduced to points. These points are called Summits or Tops.

Relation between the Number of Summits and Passes.

At first the whole earth is a region of elevation. For every new region of elevation there is a Pass, and for every region of elevation reduced to a point there is a Summit. And at last the whole surface of the earth is a region of depression. Hence the number of Summits is one more than the number of Passes. If S is the number of Summits and P the number of passes,

$$S=P+1.$$

Relation between the Number of Bottoms and Bars.

For every new region of depression there is a Bottom, and for every diminution of the number of these regions there is a Bar. Hence the number of Bottoms is one more than the number of Bars. If I is the number of Bottoms or Immits and B the number of Bars, then

$$I=B+1.$$

From this it is plain that if, in the singular cases of passes and bars, we reckon a pass as single, double, or n-ple, according as two, three, or $n+1$ regions of elevation meet at that point, and a bar as single, double, or n-ple, as two, three, or $n+1$ regions of depression meet at that point, then the census may be taken as before, giving each singular point its proper number. If one region of depression meets another in several places at once, one of these must be taken as a bar and the rest as passes.

The whole of this theory applies to the case of the maxima and minima of a function of two variables which is everywhere finite, determinate, and continuous. The summits correspond to maxima and the bottoms to minima. If there are p maxima and q minima, there must be $p+q-2$ cases of stationary values which are neither maxima nor minima. If we regard these points in themselves, we cannot make any distinction among them; but if we consider the regions cut off by the curves of constant value of the function, we may call $p-1$ of them false maxima and $q-1$ of them false minima.

On Functions of Three Variables.

If we suppose the three variables to be the three coordinates of a point, and the regions where the function is greater or less than a given value to be called the positive and the negative regions, then, as the given value increases, for every negative region formed there will be a minimum, and the positive region will have an increase of its periphraxy. For every junction of two *different* negative regions there will be a false minimum, and the positive region will have a diminution of its periphraxy. Hence if there are q true minima there will be $q-1$ false minima.

There are different orders of these stationary points according to the number of regions which meet in them. The first order is when two negative regions meet surrounded by a positive region, the second order when three negative regions meet, and so on. Points of the second order count for two, those of the third for three, and so on, in this relation between the true minima and the false ones.

In like manner, when a negative region expands round a hollow part and at last surrounds it, thus cutting off a new positive region, the negative region acquires periphraxy, a new positive region is formed, and at the point of contact there is a false maximum.

When any positive region is reduced to a point and vanishes, the negative region loses periphraxy and there is a true maximum. Hence if there are p maxima there are $p-1$ false maxima.

But these are not the only forms of stationary points; for a negative region may thrust out arms which may meet in a stationary point. The negative and the positive region both become cyclic. Again, a cyclic region may close in so as to become acyclic, forming another kind of stationary point where the ring first fills up. If there are r points at which cyclosis is gained and r' points at which it is lost, then we know that

$$r=r';$$

but we cannot determine any relation between the number of these points and that of either the true or the false maxima and minima.

If the function of three variables is a potential function, the true maxima are points of stable equilibrium, the true minima points of equilibrium unstable in every direction, and at the other

stationary points the equilibrium is stable in some directions and unstable in others.

On Lines of Slope.

Lines drawn so as to be everywhere at right angles to the contour-lines are called lines of slope. At every point of such a line there is an upward and a downward direction. If we follow the upward direction we shall *in general* reach a summit, and if we follow the downward direction we shall *in general* reach a bottom. In particular cases, however, we may reach a pass or a bar.

On Hills and Dales.

Hence each point of the earth's surface has a line of slope, which begins at a certain summit and ends in a certain bottom. Districts whose lines of slope run to the same bottom are called Basins or Dales. Those whose lines of slope come from the same summit may be called, for want of a better name, Hills.

Hence the whole earth may be naturally divided into Basins or Dales, and also, by an independent division, into hills, each point of the surface belonging to a certain dale and also to a certain hill.

On Watersheds and Watercourses.

Dales are divided from each other by Watersheds, and Hills by Watercourses.

To draw these lines, begin at a pass or a bar. Here the ground is level, so that we cannot begin to draw a line of slope; but if we draw a very small closed curve round this point, it will have highest and lowest points, the number of maxima being equal to the number of minima, and each one more than the index number of the pass or bar. From each maximum point draw a line of slope upwards till it reaches a summit. This will be a line of watershed. From each minimum point draw a line of slope downwards till it reaches a bottom. This will be a line of Watercourse. Lines of Watershed are the only lines of slope which do not reach a bottom, and lines of Watercourse are the only lines of slope which do not reach a summit. All other lines of slope diverge from some summit and converge to some bottom, remaining throughout their course in the district belonging to that summit and that bottom, which is bounded by two watersheds and two watercourses.

In the pure theory of surfaces there is no method of determining a line of watershed or of watercourse, except by first finding a pass or a bar and drawing the line of slope from that point. In nature, water actually trickles down the lines of slope, which generally converge towards the mathematical watercourses, though they do not actually join them; but when the streams increase in quantity, they join and excavate courses for themselves; and these actually run into the main watercourse which bounds the district, and so cut out a river-bed, which, whether full or empty, forms a visible mark on the earth's surface. No such action takes place at a watershed, which therefore generally remains invisible.

There is another difficulty in the application of the mathematical theory, on account of the principal regions of depression being covered with water, so that very little is known about the positions of the singular points from which the lines of watershed must be drawn to the summits of hills near the coast. A complete division of the dry land into districts, therefore, requires some knowledge of the form of the bottom of the sea and of lakes.

On the Number of Natural Districts.

Let p_1 be the number of single passes, p_2 that of double passes, and so on. Let b_1, b_2, &c. be the numbers of single, double, &c. bars. Then the number of summits will be, by what we have proved,

$$S=1+p_1+2p_2+\&c.,$$

and the number of bottoms will be

$$I=1+b_1+2b_2+\&c.$$

The number of watersheds will be

$$W=2(b_1+p_1)+3(b_2+p_2)+\&c.$$

The number of watercourses will be the same.

Now, to find the number of faces, we have by Listing's rule

$$P-L+F-R=0,$$

where P is the number of points, L that of lines, F that of Faces, and R that of regions, there being in this case no instance of cyclosis or periphraxy. Here R=2, viz. the earth and the surrounding space; hence

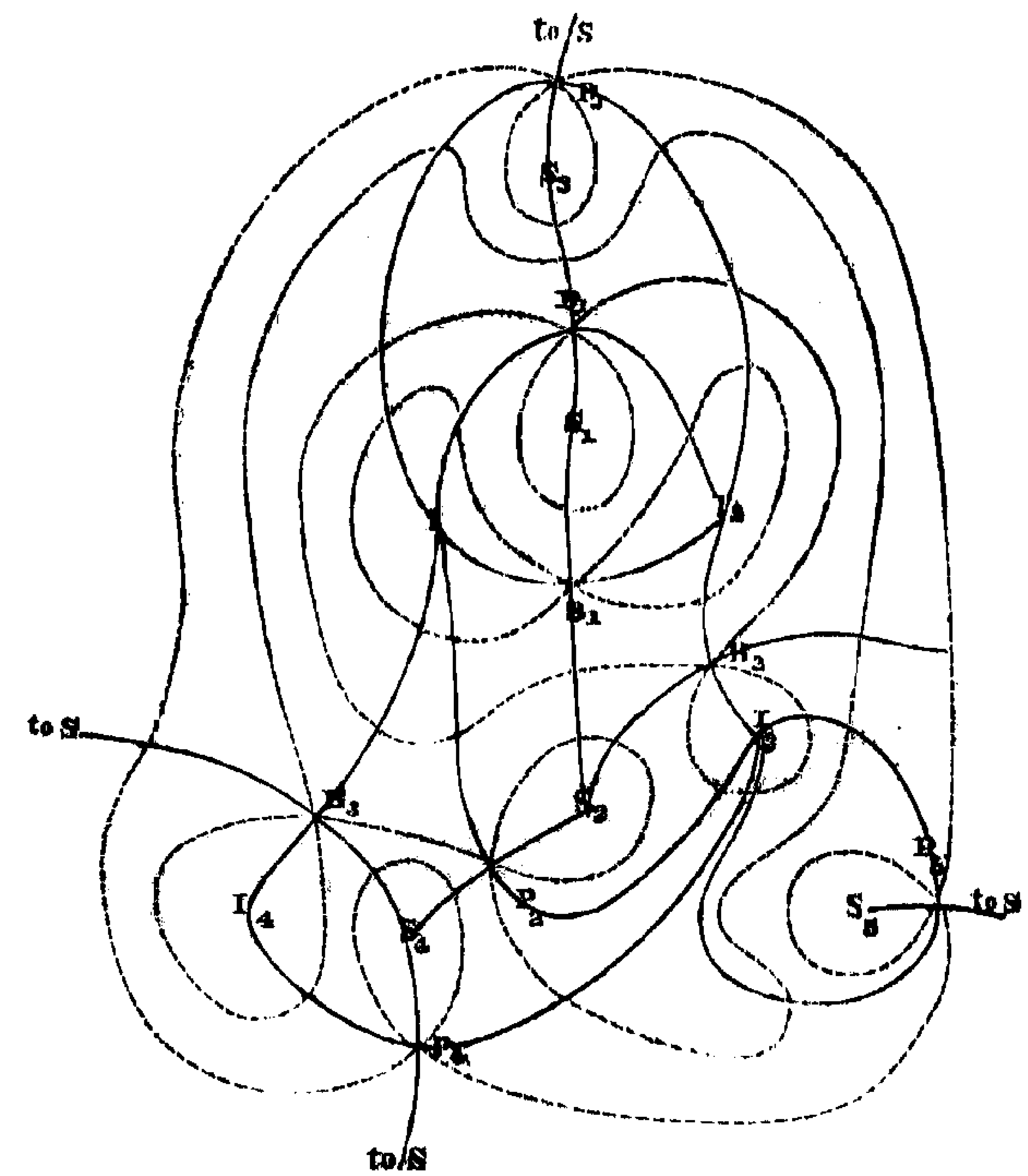
$$F=L-P+2.$$

If we put L equal to the number of watersheds, and P equal to that of summits, passes, and bars, then F is the number of Dales, which is evidently equal to the number of bottoms.

If we put L for the number of watercourses, and P for the number of passes, bars, and bottoms, then F is the number of Hills, which is evidently equal to the number of summits.

If we put L equal to the whole number of lines, and P equal to the whole number of points, we find that F, the number of natural districts named from a hill and a dale together, is equal to W, the number of watersheds or watercourses, or to the whole number of summits, bottoms, passes, and bars diminished by 2.

Chart of an Inland Basin.



I_1, I_2, I_3, I_4.	Lowest points, Bottoms or Immits.
S_1, S_2, S_4, S_5.	Highest points, Tops or Summits.
B_1, B_2, B_3.	Bars between regions of depression.
P_1, P_2, P_3, P_4, P_5.	Passes between regions of elevation.
I_1 B_1 I_2 &c.	Lines of Watercourse.
S_1 P_1 S_3 &c.	Lines of Watershed.
Dotted line.	Contour-lines.

LI. *On Solar Protuberances (being an extract from a Supplement to his second Paper). By* Professor J. C. F. Zöllner*.

To Dr. Francis, F.L.S. &c.

2 Thurlow Place, Lower Norwood,
London, S.E., November 14, 1870.

My dear Sir,

As the Government has determined to afford assistance to the expeditions that will proceed to Spain and Sicily to observe the solar eclipse in December next, I send to you without loss of time a translation of a part of a paper by Professor Zöllner,

* From. No. 1772 of the *Astronomische Nachrichten.*