terms of the *displacements* of the points of application of the forces acting on its surface, on the supposition that the passive forces are replaced by any active forces which can be in equilibrium with the applied forces, then, since

$$\delta U = \Sigma\{X\delta u + Y\delta v + Z\delta w\},$$

where u, v, w are the component displacements of the point of application of the force whose components are X, Y, Z, we have

$$\frac{dU}{du}=X, \quad \frac{dU}{dv}=Y, \quad \frac{dU}{dw}=Z,$$

equations which express a property of U analogous to the well-known property of the potential energy of a particle consequent on its vicinity to an attracting body, and to the properties of the accumulation of *vis viva* of a dynamical system.

LXI. *On the Elementary Relations between Electrical Measurements. By* Professor J. Clerk Maxwell and Fleeming Jenkin, *Esq.**

Part I.—*Introductory.*

1. *Objects of Treatise.*—The progressive extension of the electric telegraph has made a practical knowledge of electric and magnetic phenomena necessary to a large number of persons who are more or less occupied in the construction and working of the lines, and interesting to many others who are unwilling to be ignorant of the use of the network of wires which surrounds them. The discoveries of Volta and Galvani, of Œrsted, and of Faraday are familiar in the mouths of all who talk of science, while the results of those discoveries are the foundation of branches of industry conducted by many who have perhaps never heard of those illustrious names. Between the student's mere knowledge of the history of discovery and the workman's practical familiarity with particular operations which can only be communicated to others by direct imitation, we are in want of a set of rules, or rather principles, by which the laws remembered in their abstract form can be applied to estimate the forces required to effect any given practical result.

We may be called on to construct electrical apparatus for a particular purpose. In order to know how many cells are required for the battery, and of what size they should be, we require to know the strength of the current required, the electromotive force of the cells, and the resistance of the circuit. If

* Reprinted from the Reports of the British Association for 1863.

we know the results of previous scientific inquiry, and are acquainted with the method of adapting them to the case before us, we may discover the proper arrangement at once. If we are unable to make any estimate of what is required before constructing the apparatus, we may have to encounter numerous failures which might have been avoided if we had known how to make a proper use of existing data.

All exact knowledge is founded on the comparison of one quantity with another. In many experimental researches conducted by single individuals, the absolute values of those quantities are of no importance; but whenever many persons are to act together, it is necessary that they should have a common understanding of the measures to be employed. The object of the present treatise is to assist in attaining this common understanding as to electrical measurements.

2. *Derivation of Units from fundamental Standards.*—Every distinct kind of quantity requires a standard of its own, and these standards might be chosen quite independently of each other, and in many cases have been so chosen; but it is possible to deduce all standards of quantity from the fundamental standards adopted for length, time, and mass; and it is of great scientific and practical importance to deduce them from these standards in a systematic manner. Thus it is easy to understand what a square foot is when we know what a linear foot is, or to find the number of cubic feet in a room from its length, breadth, and height—because the foot, the square foot, and the cubic foot are parts of the same system of units. But the pint, gallon, &c., form another set of measures of volume, which has been formed without reference to the system based on length; and in order to reduce the one set of numbers to the other, we have to multiply by a troublesome fraction, difficult to remember, and therefore a fruitful source of error.

The varieties of weights and measures which formerly prevailed in this country, when different measures were adopted for different kinds of goods, may be taken as an example of the principle of unsystematized standards, while the modern French system, in which everything is derived from the elementary standards, exhibits the simplicity of the systematic arrangement.

In the opinion of the most practical and the most scientific men, a system in which every unit is derived from the primary units with decimal subdivisions is the best whenever it can be introduced. It is easily learnt; it renders calculations of all kinds simpler; it is more readily accepted by the world at large; and it bears the stamp of the authority, not of this or that legislator or man of science, but of nature.

The phenomena by which electricity is known to us are of a mechanical kind, and therefore they must be measured by mechanical units or standards. Our task is to explain how these units may be derived from the elementary ones; in other words, we shall endeavour to show how all electric phenomena may be measured in terms of time, mass, and space only, referring briefly in each case to a practical method of effecting the observation.

3. *Standard Mechanical Units.*—In this country the standard of length is one yard, but a foot is the unit popularly adopted. In France it is the ten millionth part of the distance from the pole to the equator, measured along the earth's surface, according to the calculations of Delambre; and this measure is called a metre, and is equal to 3·280899 feet, or 39·37079 inches.

The standard unit of time in all civilized countries is deduced from the time of rotation of the earth about its axis. The sidereal day, or the true period of rotation of the earth, can be ascertained with great exactness by the ordinary observations of astronomers; and the mean solar day can be deduced from this by our knowledge of the length of the year. The unit of time adopted in all physical researches is one second of mean solar time.

The standard unit of mass is in this country the avoirdupois pound, as we received it from our ancestors. The grain is one 7000th of a pound. In the French system it is the gramme, derived from the unit of length by the use of water at a standard temperature as a standard of density. One cubic centimetre of water is a gramme $=15{\cdot}43235$ grains $={\cdot}00220462$ lb.

A Table showing the relative value of the standard and derived units in the British and metrical systems is given in § 55.

The unit of force adopted in this treatise is that force which will produce a unit of velocity in a free unit mass, by acting on it during a unit of time. This unit of force is equal to the weight of the unit mass divided by g, where g is the accelerating force of gravity

$$\left.\begin{array}{l} =32{\cdot}088\,(1+0{\cdot}005133\sin^2\lambda) \text{ in Brit. units} \\ \text{or } =9{\cdot}78024\,(1+0{\cdot}005133\sin^2\lambda) \text{ in met. units} \end{array}\right\} \text{at the level of the sea,}$$

λ being the latitude of the place of observation. A unit of force still very generally adopted is the weight of the standard mass. The value of the new unit is $\frac{1}{g}$ times the old or gravitation unit.

The unit of work adopted in this treatise is the unit of force, defined as above, acting through the unit of space (*vide* § 55).

4. *Dimensions of Derived Units.*—Every measurement of which

we have to speak involves as factors measurements of time, space, and mass only; but these measurements enter sometimes at one power, and sometimes at another. In passing from one set of fundamental units to another, and for other purposes, it is useful to know at what power each of these fundamental measurements enters into the derived measure.

Thus the value of a force is directly proportional to a length and a mass, but inversely proportional to the square of a time. This is expressed by saying that the *dimensions* of a force are $\frac{LM}{T^2}$; in other words, if we wish to pass from the English to the French system of measurements, the French unit of force will be to the English as $\frac{3{\cdot}28\times 15{\cdot}43}{1}:1$, or as 50·6 to 1; because there are 3·28 feet in a metre, and 15·43 grains in a gramme. If the minute were chosen as the unit of time, the unit of force would, in either system, be $\frac{1}{3600}$ of that founded on the second as unit.

A Table of the dimensions of every unit adopted in the present treatise is given in § 55.

Part II.—*The Measurement of Magnetic Phenomena.*

5. *Magnets and Magnetic Poles.*—Certain natural bodies, as the iron ore called loadstone, the earth itself, and pieces of steel after being subjected to certain treatment, are found to possess the following properties, and are called magnets.

If one of these bodies be free to turn in any direction, the presence of another will cause it to set itself in a position which is conveniently described or defined by reference to certain imaginary lines occupying a fixed position in the two bodies, and called their magnetic axes. One object of our magnetic measurements will be to determine the force which one magnet exerts upon another. It is found by experiment that the greatest manifestation of force exerted by one long thin magnet on another occurs very near the ends of the two bars, and that the two ends of any one long thin magnet possess opposite qualities. This peculiarity has caused the name of "poles" to be given to the ends of long magnets; and this conception of a magnet, as having two poles capable of exerting opposite forces joined by a bar exerting no force, is so much the most familiar that we shall not hesitate to employ it, especially as many of the properties of magnets may be correctly expressed in this way; but it must be borne in mind, in speaking of poles, that they do not really exist as points or centres of force at the ends of the bar, except in the case of long, infinitely thin, uniformly magnetized rods.

If we mark the poles of any two magnets which possess similar qualities, we find that the two marked poles repel each other, that two unmarked poles also repel each other, but that a marked and an unmarked pole attract each other. The pole which is repelled from the northern regions of the earth is called a positive pole; the other end the negative pole. The negative pole is generally marked N by British instrument-makers, and is sometimes called the north pole of the magnet, whereas it is obviously similar to the earth's south pole.

The strength of a pole is necessarily defined as proportional to the force it is capable of exerting on any other pole. Hence the force f exerted between two poles of the strengths m and m_1 must be proportional to the product mm_1. The force, f, is also found to be inversely proportional to the square of the distance, D, separating the poles, and to depend on no other quantity; hence we have, unless an absurd and useless coefficient be introduced,

$$f=\frac{m\,m_1}{D^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

from which equation it follows that the unit pole will be that which at unit distance repels another similar pole with unit force; f will be an attraction or a repulsion according as the poles are of opposite or the same kinds. The dimensions of the unit magnetic pole are $\frac{L^{\frac{3}{2}}M^{\frac{1}{2}}}{T}$.

6. *Magnetic Field.*—It is clear that the presence of a magnet in some way modifies the surrounding space, since any other magnet brought into that space experiences a peculiar force. The neighbourhood of a magnet is, for convenience, called a magnetic field; and for the same reason the effect produced by a magnet is often spoken of as due to the magnetic field instead of to the magnet itself. This mode of expression is the more proper, inasmuch as the same or a similar condition of space may be produced by the passage of electrical currents in the neighbourhood, without the presence of a magnet. Since the peculiarity of the magnetic field consists in the presence of a certain force, we may numerically express the properties of the field by measuring the strength and direction of the force, or, as it may be worded, the intensity of the field and the direction of the lines of force.

This direction at any point is the direction in which the force tends to move a free pole; and the intensity, H, of the field is necessarily defined as proportional to the force, f, with which it

acts on a free pole; but this force, f, is also proportional to the strength, m, of the pole introduced into the field, and it depends on no other quantities; hence

$$f=mH, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and therefore the field of unit intensity will be that which acts with unit force on the unit pole.

The dimensions of H are $\frac{M^{\frac{1}{2}}}{L^{\frac{1}{2}}T}$.

The lines of force produced by a long thin bar magnet near its poles will radiate from the poles, and the intensity of the field will be equal to the quotient of the strength of the pole divided by the square of the distance from the pole; thus the unit field will be produced at the unit distance from the unit pole. In a uniform magnetic field the lines of force, as may be demonstrated, will be parallel; such a field can only be produced by special combinations of magnets, but a small field at a great distance from any one pole will be sensibly uniform. Thus, in any room unaffected by the neighbourhood of iron or magnets, the magnetic field due to the earth will be sensibly uniform; its direction will be that assumed by the dipping-needle.

7. *Magnetic Moment.*—In reality we can never have a single pole entirely free or disconnected from its opposite pole; and it is time to pass to the consideration of the effect produced on a material bar magnet in a magnetic field. In a uniform field two equal opposite and parallel forces act on its poles, and tend to set it with the line joining those poles in the direction of the force of the field. When the magnet is so placed that the line joining the poles is at right angles to the lines of force in the field, this tendency to turn or "couple," G, is proportional to the intensity of the field, H, the strength of the poles, m, and the distance between them, l; or

$$G=mlH. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

ml, or the product of the strength of the poles into the length between them, is called the magnetic moment of the magnet; and from equation (3) it follows, that, in a field of unit intensity, the couple actually experienced by any magnet in the above position measures its moment. The dimensions of the unit of magnetic moment are evidently $\frac{L^{\frac{5}{2}}M^{\frac{1}{2}}}{T}$.

8. *Intensity of Magnetization.*—The intensity of magnetization of a magnet may be measured by its magnetic moment divided by its volume.

The dimensions of the unit of magnetization are therefore $\frac{M^{\frac{1}{2}}}{L^{\frac{1}{2}}T}$, the same as in the case of intensity of field.

9. *Coefficient of Magnetic Induction.*—When certain bodies, such as soft iron, &c., are placed in the magnetic field, they become magnetized by "induction"; so that the intensity of magnetization is (except when great) nearly proportional to the intensity of the field.

In diamagnetic bodies, such as bismuth, the direction of magnetization is opposite to that of the field. In paramagnetic bodies, such as iron, nickel, &c., the direction of magnetization is the same as that of the field.

The coefficient of magnetic induction is the ratio of the intensity of magnetization to the intensity of the field, and is therefore a *numerical* quantity, positive for paramagnetic bodies, negative for diamagnetic bodies.

10. *Magnetic Potentials and Equipotential Surfaces.*—If we take a very long magnet, and, keeping one pole well out of the way, move the other pole from one point to another of the magnetic field, we shall find that the forces in the field do work on the pole, or that they act as a resistance to its motion, according as the motion is with or contrary to the force acting on the pole. If the pole moves at right angles to the force, no work is done.

The *magnetic potential* at any point in a magnetic field is measured by the work done by the magnetic forces on a unit pole during its motion from an infinite distance from the magnet producing the field to the point in question, supposing the unit pole to exercise no influence on the magnetic field in question. The idea of potential as a mathematical quantity having different values at different points of space, was brought into form by Laplace*. The name of potential, and the application to a great number of electric and magnetic investigations, were introduced by George Green, in his Essay on Electricity (Nottingham, 1828).

An equipotential surface in a magnetic field is a surface so drawn that the potential of all its points shall be equal. By drawing a series of equipotential surfaces corresponding to potentials 1, 2, 3 . . . . . . n, we may map out any magnetic field so as to indicate its properties.

The magnetic force at any point is perpendicular to the equipotential surface at that point, and its intensity is the reciprocal of the distance between one surface and the next at that

* *Mécanique Céleste*, liv. iii.

point. The dimensions of the unit of magnetic potential are $\frac{L^{\frac{1}{2}}M^{\frac{1}{2}}}{T}$.

11. *Lines of Magnetic Force.*—There is another way of exploring the magnetic field, and indicating the direction and magnitude of the force at any point. The conception and application of this method in all its completeness are due to Faraday*. The full importance of this method cannot be recognized till we come to electromagnetic phenomena (§§ 22, 23, & 24).

A line, whose direction at any point always coincides with that of the force acting on the pole of a magnet at that point, is called a line of magnetic force. By drawing a sufficient number of such lines, we may indicate the *direction* of the force in every part of the magnetic field; but by drawing them according to rule, we may indicate the intensity of the force at any point as well as its direction. It has been shown† that if, in any part of their course, the number of lines passing through unit of area is proportional to the intensity there, the same proportion between the number of lines in unit of area and the intensity will hold good in every part of the course of the lines.

All that we have to do, therefore, is to space out the lines in any part of their course, so that the number of lines which start from unit or area is *equal* to the number representing the intensity of the field there. The intensity at any other part of the field will then be measured by the number of lines which pass through unit of area there; each line indicates a constant and equal force.

12. *Relation between Lines of Force and Equipotential Surfaces.*—The lines of force are always perpendicular to the equipotential surfaces; and the number of lines passing through unit of area of an equipotential surface is the reciprocal of the distance between that equipotential surface and the next in order—a statement made above in slightly different language.

In a uniform field the lines of force are straight, parallel, and equidistant; and the equipotential surfaces are planes perpendicular to the lines of force, and equidistant from each other.

If one magnetic pole of strength m be alone in the field, its lines of force are straight lines, radiating from the pole equally in all directions; and their number is $4\pi m$. The equipotential surfaces are a series of spheres, whose centres are at the pole, and whose radii are m, $\frac{1}{2}m$, $\frac{1}{3}m$, $\frac{1}{4}m$, &c. In other magnetic arrangements these lines and surfaces are more complicated;

* Experimental Researches, vol. iii. art. 3122 *et passim*.

† *Vide* Maxwell on Faraday's Lines of Force, Cambridge Phil. Trans. 1857.

but in all cases the calculation is simple, and in many cases the lines and surfaces can be graphically constructed without any calculation.

Part III.—*Measurement of Electric Phenomena by their Electromagnetic Effects.*

13. *Preliminary.*—Before treating of electrical measurements, the exact meaning in which the words "quantity," "current," "electromotive force," and "resistance" are used will be explained. But, in giving these explanations, we shall assume the reader to be acquainted with the meaning of such expressions as conductor, insulator, voltaic battery, &c.

14. *Meaning of the words "Electric Quantity."*—When two light conducting bodies are connected with the same pole of a voltaic battery, while the other pole is connected with the earth, they may be observed to repel one another. The two poles produce equal and similar effects. When the two bodies are connected with opposite poles, they attract one another. Bodies, when in a condition to exert this peculiar force one on the other, are said to be electrified, or charged with electricity. These words are mere names given to a peculiar condition of matter. If a piece of glass and a piece of resin are rubbed together, the glass will be found to be in the same condition as an insulated body connected with the copper pole of the battery, and the resin in the same condition as the body connected with the zinc pole of the battery. The former is said to be positively, and the latter negatively electrified. The propriety of this antithesis will soon appear. The force with which one electrified body acts on another, even at a constant distance, varies with different circumstances. When the force between the two bodies at a constant distance, and separated by air, is observed to increase, it is said to be due to an increase in the quantity of electricity; and the quantity at any spot is defined as proportional to the force with which it acts, through air, on some other constant quantity at a distance. If two bodies, charged each with a given quantity of electricity, are incorporated, the single body thus composed will be charged with the sum of the two quantities. It is this fact which justifies the use of the word "quantity."

Thus the quality in virtue of which a body exerts the peculiar force described is called electricity, and its quantity is measured (*cæteris paribus*) by measuring force.

The quantity thus defined produced on two similar balls similarly circumstanced, but connected with opposite poles of a voltaic battery, is equal, but opposite; so that the sum of these two equal and opposite quantities is zero; hence the conception of positive and negative quantities.

In speaking of a quantity of electricity, we need not conceive it as a separate thing, or entity distinct from ponderable matter, any more than in speaking of sound we conceive it as having a distinct existence. Still it is convenient to speak of the intensity or velocity of sound, to avoid tedious circumlocution; and quite similarly we may speak of electricity, without for a moment imagining that any real electric fluid exists.

The laws according to which the force described varies, as the shape of the conductors, their combinations, and their distances are varied, have been established by Coulomb, Poisson, Green, W. Thomson, and others. These will be found accurately described, independently of all hypothesis, in papers by Professor W. Thomson, published in the Cambridge Mathematical Journal, vol. i. p. 75 (1846), and a series of papers in 1848 and 1849.

15. *Meaning of the words "Electric Current."*—When two balls charged by the opposite poles of a battery, with opposite and equal quantities of electricity, are joined by a conductor, they lose in a very short time their peculiar properties, and assume a neutral condition intermediate between the positive and negative states, exhibiting no electrical symptoms whatever, and hence described as unelectrified, or containing no electricity. But during the first moment of their junction, the conductor is found to possess certain new and peculiar properties: any one part of the conductor exerts a force upon any other part of the conductor; it exerts a force on any magnet in the neighbourhood; and if any part of the conductor be formed by one of those compound bodies called electrolytes, a certain portion of this body will be decomposed. These peculiar effects are said to be due to a current of electricity in the conductor. The positive quantity, or excess, is conceived as flowing into the deficiency caused by the negative quantity; so that the whole combination is reduced to the neutral condition. This neutral condition is similar to that of the earth where the experiment is tried. If the balls are continually recharged by the battery, and discharged or neutralized by the wire, a rapid succession of the so-called currents will be sent; and it is found that the force with which a magnet is deflected by this rapid succession of currents is proportional (*cæteris paribus*) to the quantity of electricity passed through the conductor or neutralized per second; it is also found that the amount of chemical action, measured by the weights of the bodies decomposed, is proportional to the same quantity. The currents just described are intermittent; but a wire or conductor, used simply to join the two poles of a battery, acquires permanently the same properties as when used to discharge the balls as above with great rapidity; and the greater the rapidity with which the balls are discharged, the more perfect the similarity of

the condition of the wire in the two cases. The wire in the latter case is therefore said to convey a permanent current of electricity, the magnitude or strength of which is defined as proportional to the quantity conveyed per second. This definition is expressed by the equation

$$C=\frac{Q}{t}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

where C is the current, Q the quantity, and t the time. A permanent current flowing through a wire may be measured by the force which it exerts on a magnet; the actual quantity it conveys may be obtained by comparing this force with the force exerted under otherwise similar conditions, when a known quantity is sent through the same wire by discharges. The strength of a permanent current is found at any one time to be equal in all parts of the conductor. Conductors conveying currents exert a peculiar force one upon another; and during their increase or decrease they produce currents in neighbouring conductors. Similar effects are produced as they approach or recede from neighbouring conductors. The laws according to which currents act upon magnets and upon one another will be found in the writings of Ampère and Weber.

16. *Meaning of the words "Electromotive Force."*—Hitherto we have spoken simply of statical effects; but it is found that a current of electricity, as above defined, cannot exist without effecting work or its equivalent. Thus it either heats the conductor, or raises a weight, or magnetizes soft iron, or effects chemical decomposition; in fine, in some shape it effects work, and this work bears a definite relation to the current. Work done presupposes a force in action. The immediate force producing a current, or, in other words, causing the transfer of a certain quantity of electricity, is called an electromotive force. This force is necessarily assumed as ultimately due to that part of a circuit where a "degradation" or consumption of energy takes place: thus we speak of the electromotive force of the voltaic or thermoelectric couple; but the term is used also independently of the source of power, to express the fact that, however caused, a certain force tending to do work by setting electricity in motion does, under certain circumstances, exist between two points of a conductor or between two separate bodies. But equal quantities of electricity transferred in a given time do not necessarily or usually produce equal amounts of work; and the electromotive force between two points, the proximate cause of the work, is defined as proportional to the amount of work done between those points when a given quantity of electricity is transferred from one point to another. Thus if, with equal currents in two distinct conductors, the work done in the one is

double that done in the second in the same time, the electromotive force in the first case is said to be double that in the second; but if the work done in two circuits is found strictly proportional to the two currents, the electromotive force acting on the two currents is said to be the same. Defined in this way, the electromotive force of a voltaic battery is found to be constant so long as the materials of which it is formed remain in a similar or constant condition. The above definitions, in mathematical language, give $W=ECt$, or

$$E=\frac{W}{Ct}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (5)$$

where E is the electromotive force, and W the work done. Thus the electromotive force producing a current in a conductor is equal to the ratio between the work done in the unit of time and the current effecting the work. This conception of the relations of work, electromotive force, current, and quantity will be aided by the following analogy:—A quantity of electricity may be compared to a quantity or given mass of water; currents of water in pipes in which equal quantities passed each spot in equal times would then correspond to equal currents of electricity; electromotive force would correspond to the head of water producing the current. Thus if, with two pipes conveying equal currents, the head forcing the water through the first was double that forcing it through the second, the work done by the water in flowing through the first pipe would necessarily be twice that done by the water in the second pipe; but if twice as much water passed through the first pipe as passed through the second, the work done by water in the first pipe would again be doubled. This corresponds exactly with the increase of work done by the electrical current when the electromotive force is doubled, and when the quantity is doubled.

Thus, to recapitulate, the quality of a battery or source of electricity, in virtue of which it tends to do work by the transfer of electricity from one point to another, is called its electromotive force, and this force is measured by measuring the work done during the transfer of a given quantity of electricity between those points. The relations between electromotive force and work were first fully explained in a paper by Prof. W. Thomson "On the application of the principle of Mechanical Effect to the Measurement of Electromotive Forces," published in the Philosophical Magazine for December 1851.

17. *Meaning of the words "Electric Resistance."*—It is found by experiment, that even when the electromotive force between two points remains constant, so that the work done by the transfer of a given quantity of electricity remains constant, nevertheless, by modifying the material and form of the conductor, this transfer

may be made to take place in very different times; or, in other words, currents of very different magnitudes are produced, and very different amounts of work are done in the unit of time. The quality of the conductor in virtue of which it prevents the performance of more than a certain amount of work in a given time by a given electromotive force is called its electrical resistance. The resistance of a conductor is therefore inversely proportional to the work done in it when a given electromotive force is maintained between its two ends; and hence, by equation (5), it is inversely proportional to the currents which will then be produced in the respective conductors. But it is found by experiment that the current produced in any case in any one conductor is simply proportional to the electromotive force between its ends; hence the ratio $\frac{E}{C}$ will be a constant quantity, to which the resistance as above defined must be proportional, and may with convenience be made equal; thus

$$R=\frac{E}{C}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

an equation expressing Ohm's law. In order to carry on the parallel with the pipes of water, the resistance overcome by the water must be of such nature that twice the quantity of water will flow through any one pipe when twice the head is applied. This would not be the result of a constant mechanical resistance, but of a resistance which increased in direct proportion to the speed of the current; thus the electrical resistance must not be looked on as analogous to a simple mechanical resistance, but rather to a coefficient by which the speed of the current must be multiplied to obtain the whole mechanical resistance. Thus if the electrical resistance of a conductor be called R, the work W is not equal to CRt, but $C \times CR \times t$, or

$$W=C^2Rt*, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where C may be looked on as analogous to a quantity moving at a certain speed, and CR as analogous to the mechanical resistance which it meets with in its progress, and which increases in direct proportion to the quantity conveyed in the unit of time.

18. *Measurement of Electric Currents by their Action on a Magnetic Needle.*—In 1820, Oersted discovered the action of an electric current upon a magnet at a distance; and one method of measurement may be based on this action. Let us suppose the current to be in the circumference of a vertical circle, so that in the upper part it runs from left to right. Then a magnet suspended in the centre of the circle will turn with the end which points to the north away from the observer. This may be taken as the simplest case, as every part of the circuit is at the same distance from the magnet, and tends to turn it the same way. The force is proportional to the moment of the magnet, to the strength of the current as defined by § 15, to its length, and inversely to the square of its distance from the magnet.

Let the moment of the magnet be ml, the strength of the current C, the radius of the circle k, the number of times the current passes round the circle n, the angle between the axis of the magnet and the plane of the circle θ, and the moment tending to turn the magnet G, then

$$G=mlC\,.\,2\pi nk\frac{1}{k^2}\cos\theta, \quad . \quad . \quad . \quad . \quad . \quad (8)$$

which will be unity if ml, C, k, and the length of the circuit be unity, and if $\theta=0°$.

The unit of current founded on this relation, and called the electromagnetic unit, is therefore that current of which the unit of length placed along the circumference of a circle of unit radius produces a unit of magnetic force at the centre.

The usual way of measuring C, the strength of a current, is by making it describe a circle about a magnet, the plane of the circle being vertical and magnetic north and south. Thus, if H be the intensity of the horizontal component of terrestrial magnetism, and G the moment of this on the magnet, $G=mlH\sin\theta$, whence the strength of the current

$$C=\frac{k^2}{2\pi n}H\tan\theta, \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where k is the radius of the circle, n the number of turns, H the intensity of the horizontal part of the earth's magnetic force as determined by the usual method, and θ the angle of deviation of the magnet suspended in the centre of the circle. As the strength of the current is proportional to the tangent of the angle θ, an instrument constructed on this plan is called a tangent galvanometer. The instrument called a sine galvanometer may also be used, provided the coil is circular. The equation is similar to that just given, substituting $\sin\theta$ for $\tan\theta$.

To find the dimensions of C, we must consider that what we observe is the force acting between a magnetic pole, m, and a current of given length, L, at a given distance, L_1, and that this force $=\frac{mCL}{L_1^2}$. Hence the dimensions of C, an electric current thus measured, are $\frac{L^{\frac{1}{2}}M^{\frac{1}{2}}}{T}$.

19. *Measurement of Electric Currents by their mutual action*

* By equation (5) we have $W=CEt$; but by equation (6) $R=\frac{E}{C}$; hence $W=C^2Rt$.—Q.E.D.

on one another.—Hitherto we have spoken of the measurement of currents as dependent on their action upon magnets; but this measurement in the same units can as simply be founded on their mutual action upon one another. Ampère has investigated the laws of mechanical action between conductors carrying currents. He has shown that the action of a small closed circuit at a distance is the same as that of a small magnet, provided the axis of the magnet be placed normal to the plane of the circuit, and the moment of the magnet be equal to the product of the current into the area of the circuit which it traverses.

Thus, let two small circuits having areas A and A_1 be placed at a great distance D from each other in such a way that their planes are at right angles to each other, and that the line D is in the intersection of the planes. Now let currents C and C_1 circulate in these conductors; a force will act between them tending to make their planes parallel, and the direction of the currents opposite. The moment of this couple will be

$$G=\frac{AC\times A_1C_1}{D^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Hence the unit electric current conducted round two circuits of unit area in vertical planes at right angles to each other, one circuit being at a great distance, D, vertically above the other, will cause a couple to act between the circuits of a magnitude $\frac{1}{D^3}$. The definition of the unit current (identical with the unit founded on the relations given in § 18) might be founded on this action quite independently of the idea of magnetism.

20. *Weber's Electro-Dynamometer*.—The measurement described in the last paragraph is only accurate when D is very great, and therefore the moment to be measured very small. Hence it is better to make the experimental measurements in another form. For this purpose, let a length (l) of wire be made into a circular coil of radius k; let a length (l_1) of wire be made into a coil of very much smaller radius, k_1. Let the second coil be hung in the centre of the first, the planes being vertical and at the angle θ. Then, if a current C traverses both coils, the moment of the force tending to bring them parallel will be

$$G=\tfrac{1}{2}C^2\frac{ll_1k_1}{k^2}\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

This force may be measured in mechanical units by the angle through which it turns the suspended coil, the forces called into play by the mechanical arrangements of suspension being known from the construction of the instrument. Weber used a bifilar suspension, by which the weight of the smaller coil was used to resist the moment produced by the action of the currents.

21. *Comparison of the Electromagnetic and Electrochemical action of Currents*.—Currents of electricity, when passed through certain compound substances, decompose them; and it is found that, with any given substance, the weight of the body decomposed in a given time is proportional to the strength of the current as already defined with reference to its electromagnetic effect. The voltameter is an apparatus of this kind, in which water is the substance decomposed. Special precautions have to be taken, in carrying this method of measurement into effect, to prevent variations in the resistance of the circuit, and consequently in the strength of the current. This subject is more fully treated in Part V. §§ 53, 54.

22. *Magnetic Field near a Current*.—Since a current exerts a force on the pole of a magnet in its neighbourhood, it may be said to produce a magnetic field (§ 6), and, by exploring this field with a magnet, we may draw lines of force and equipotential surfaces of the same nature as those already described for magnetic fields caused by the presence of magnets.

When the current is a straight line of indefinite length, like a telegraph-wire, a magnetic pole in its neighbourhood is urged by a force tending to turn it round the wire, so that this force is at any point perpendicular to the plane passing through this point and the axis of the current.

The equipotential surfaces are therefore a series of planes passing through the axis of the current, and inclined at equal angles to each other. The number of these planes is $4\pi C$, where C is the strength of the current.

The lines of magnetic force are circles, having their centres in the axis of the current, and their planes perpendicular to it. The intensity of the magnetic force at a distance, k, from the current is the reciprocal of the distance between two equipotential surfaces, which shows the forces to be $\frac{2C}{k}$.

The work done on a unit magnetic pole in going completely round the current is $4\pi C$, whatever the path which the pole describes.

23. *Mechanical Action of a Magnetic Field on a closed Conductor conveying a Current*.—When there is mechanical action between a conductor carrying a current and a magnet, the force acting on the conductor must be equal and opposite to that acting on the magnet. Every part of the conductor is therefore acted on by a force perpendicular to the plane passing through its own direction and the lines of magnetic force due to the magnet, and equal to the product of the length of the conductor

into the strength of the current, the intensity of the magnetic field, and the sine of the angle between the lines of force and the direction of the current. This may be more concisely expressed by saying that if a conductor carrying a current is moved in a magnetic field, the work done on the conductor by the electromagnetic forces is equal to the product of the strength of the current into the *number* of lines of force which it cuts during its motion.

Hence we arrive at the following general law, for determining the mechanical action on a closed conductor carrying a current and placed in a magnetic field:—

Draw the lines of magnetic force. Count the number which pass through the circuit of the conductor, then any motion which increases this number will be aided by the electromagnetic forces, so that the work done during the motion will be the product of the strength of the current and the number of additional lines of force.

For instance, let the lines of force be due to a single magnetic pole of strength m. These are $4\pi m$ in number, and are in this case straight lines radiating equally in all directions from the pole. Describe a sphere about the pole, and project the circuit on its surface by lines drawn to the pole. The surface of the area so described on the sphere will measure the solid angle subtended by the circuit at the pole. Let this solid angle $=\omega$, then the number of lines passing through the closed surface will be $m\omega$; and if C be the strength of the current, the amount of work done by bringing the magnet and circuit from an infinite distance to their present position will be $Cm\omega$. This shows that the magnetic potential of a closed circuit carrying a unit current with respect to a unit magnetic pole placed at any point is equal to the solid angle which the circuit subtends at that point.

By considering at what points the circuit subtends equal solid angles, we may form an idea of the surfaces of equal potential. They form a series of sheets, all intersecting each other in the circuit itself, which forms the boundary of every sheet. The number of sheets is $4\pi C$, where C is the strength of the current. The lines of magnetic force intersect these surfaces at right angles, and therefore form a system of rings, encircling every point of the circuit. When we have studied the general form of the lines of force, we can form some idea of the electromagnetic action of that current, after which the difficulties of numerical calculation arise entirely from the imperfection of our mathematical skill.

24. *General Law of the Mechanical Action between Electric Currents and other Electric Currents or Magnets.*—Draw the lines of magnetic force due to all the currents, magnets, &c., in the field, supposing the strength of each current or magnet to be reduced from its actual value to unity. Call the number of lines of force due to a circuit or magnet, which pass through another circuit, the potential coefficient between the one and the other. This number is to be reckoned positive when the lines of force pass through the circuit in the same direction as those due to a current in that circuit, and negative when they pass in the opposite direction.

If we now ascertain the change of the potential coefficient due to any displacement, this increment multiplied by the product of the strengths of the currents or magnets will be the amount of work done by the mutual action of these two bodies during the displacement. The determination of the actual value of the potential coefficient of two things, in various cases, is an important part of mathematics as applied to electricity. (See the mathematical discussion of the experiment, Appendix D. Brit. Assoc. Reports, 1863, p. 168.)

25. *Electromagnetic Measurement of Electric Quantity.*—A conducting body insulated at all points from the neighbouring conductors may in various ways be electrified, or made to hold a quantity of electricity. This quantity (§ 14) is perfectly definite in any given circumstances; it cannot be augmented or diminished so long as the conductor is insulated, and is called the charge of the conductor. Its magnitude depends on the dimensions and shape and position of the insulated and the neighbouring conductors, on the insulating material, and finally on the electromotive force between the insulated and the neighbouring conductors, at the moment when the charge was produced. The well-known Leyden jar is an arrangement by which a considerable charge can be obtained on a small conductor with moderate electromotive force between the inner and outer coatings, which constitute respectively the "insulated" and "neighbouring" conductors referred to in general. We need not enter into the general laws determining the charge, since our object is only to show how it may be measured when already existing; but it may be well to state that the quantity on the charged insulated conductor necessarily implies an equal and opposite quantity on the surrounding or neighbouring conductors.

We have already defined the magnitude of a current of electricity as simply proportional to the quantity of electricity conveyed in a given time, and we have shown a method of measuring consonant with this definition. The unit quantity will therefore be that conveyed by the unit current as above defined in the unit of time. Thus, if a unit current is allowed to flow for a unit of time in any wire connecting the two coatings of a

Leyden phial, the quantity which one coating loses, or which the other gains, is the electromagnetic unit quantity*. The measurement thus defined of the quantity in a given statical charge can be made by observing the swing of a galvanometer-needle produced by allowing the charge to pass through the coil of the galvanometer in a time extremely short compared with that occupied by an oscillation of the needle.

Let Q be the whole quantity of electricity in an instantaneous current, then

$$Q=2\frac{C_1 t}{\pi}\sin\tfrac{1}{2}i, \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where C_1 = the strength of a current giving a unit deflection (45° on a tangent or 90° on a sine galvanometer), t = half the period or time of a complete oscillation of the needle of the galvanometer under the influence of terrestrial magnetism alone, and i = the angle to which the needle is observed to swing from a position of rest, when the discharge takes place. C_1 is a constant which need only be determined once for each instrument, provided the horizontal force of the earth's magnetism remain unchanged. In the case of the tangent galvanometer, the formula for obtaining it has already been given. From equations (9) and (12) we have for a tangent galvanometer

$$Q=\frac{k}{\pi^2 n}Ht\sin\tfrac{1}{2}i, \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where, as before, k = the radius of the coil, and n = the number of turns made by the wire round the coil.

The quantity in a given charge which can be continually reproduced under fixed conditions may be measured by allowing a succession of discharges to pass at regular and very short intervals through a galvanometer, so as to produce a permanent deflection. The value of a current producing this deflection can be ascertained; and the quotient of this value by the number of discharges taking place in the "second" gives the value of each charge in electromagnetic measure.

To find the dimensions of Q, we simply observe that the unit of electricity is that which is transferred by the unit current in the unit of time. Multiplying the dimensions of C by T, we find the dimensions of Q are $L^{\frac{1}{2}}M^{\frac{1}{2}}$.

26. *Electric Capacity of a Conductor.*—It is found by experiment that, other circumstances remaining the same, the charge on an insulated conductor is simply proportional to the electromotive force between it and the surrounding conductors, or, in other words, to the difference of potentials (47). The charge

* Weber calls this quantity two units—a fact which must not be lost sight of in comparing his results with those of the Committee.

that would be produced by the unit electromotive force is said to measure the electric capacity of a conductor. Thus, generally, the capacity of a conductor $S=\frac{Q}{E}$, where Q is the whole quantity in the charge produced by the electromotive force E. When the electromotive force producing the charge is capable of maintaining a current, the capacity of the conductor may be obtained without a knowledge of the value either of Q or E, provided we have the means of measuring the resistance of a circuit in electromagnetic measure. For let R be the resistance of a circuit, in which the given electromotive force, E, will produce the unit deflection on a tangent galvanometer, then, from equations (6) and (12), we have

$$S=2\frac{t\sin\frac{1}{2}i}{\pi R_1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where t and i retain the same signification as in equation (13) (§ 25).

27. *Direct Measurement of Electromotive Force.*—The meaning of the words "electromotive force" has already been explained (§ 16). This force tends to do work by means of a current or transfer of electricity, and may therefore be said to produce and maintain the current. In any given combination in which electric currents flow, the immediate source of the power by which the work is done is said to produce the electromotive force. The sources of power producing electromotive force are various. Of these, chemical action in the voltaic battery, unequal distribution of temperature in circuits of different conductors, the friction of different substances, magnetoelectric induction, and simple electric induction are the most familiar. An electromotive force may exist between two points of a conductor, or between two points of an insulator, or between an insulator and a conductor, —in fine, between any points whatever. This electromotive force may be capable of maintaining a current for a long time, as in a voltaic battery, or may instantly cease after producing a current of no sensible duration, as when two points of the atmosphere at different potentials (§ 47) are joined by a conductor; but in every case in which a constant electromotive force, E, is maintained between any two points, however situated, the work spent or gained in transferring a quantity, Q, of electricity from one of those points to the other will be constant; nor will this work be affected by the manner or method of the transfer. If the electricity be slowly conveyed as a static charge on an insulated ball, the work will be spent or gained in accelerating or retarding the ball; if the electricity be conveyed rapidly through a conductor of small resistance, or more slowly through a con-

ductor of great resistance, the work may be spent in heating the conductor, or it may electrolyze a solution, or be thermoelectrically or mechanically used; but in all cases the change effected, measured as equivalent to work done, will be the same, and equal to EQ. Hence the electromotive force between two points is unity, if a unit of mechanical work is spent (or gained) in the transfer of a unit of electricity from one point to the other. This general definition is due to Professor W. Thomson.

The direct measurement of electromotive force would be given by the measure, in any given case, of the work done by the transfer of a given quantity of electricity. The ratio between the numbers measuring the work done, and the quantity transferred, would measure the electromotive force. This measurement has been made by Dr. Joule and Professor Thomson, by determining the heat developed in a wire by a given current measured as in (§ 18)*.

28. *Indirect Measurements of Electromotive Force.*—The direct method of measurement is in most cases inconvenient, and in many impossible; but the indirect methods are numerous and easily applied. The relation between the current, C, the resistance, R, and the electromotive force, E, expressed by Ohm's law (equation 6), will determine the electromotive force of a battery whenever R and C are known. A second indirect method depends on the measurement of the statical force with which two bodies attract one another when the given electromotive force is maintained between them. This method is fully treated in Part IV. (43). The phenomenon on which it is based admits of an easy comparison between various electromotive forces by electrometers. This method is applicable even to those cases in which the electromotive force to be measured is incapable of maintaining a current. The laws of chemical electrolysis and electromagnetic induction afford two other indirect methods of estimating electromotive force in special cases (54 and 31).

29. *Measurement of Electric Resistance.*—We have already stated that the resistance of a conductor is that property in virtue of which it limits the amount of work performed by a given electromotive force in a given time, and we have shown that it may be measured by the ratio $\frac{E}{C}$ of the electromotive force between two ends of a conductor to the current maintained by it. The unit resistance is therefore that in which the unit electromotive force produces the unit current, and therefore performs the unit of work, in the unit of time. If in any circuit

* Phil. Mag. S. 4. vol. ii. (1851), p. 551.

we can measure the current and electromotive force, or even the ratio of these magnitudes, we should, *ipso facto*, have measured the resistance of the circuit. The methods by which this ratio has been measured, founded on the laws of electromagnetic induction, are fully described in Appendix D (Brit. Assoc. Reports, p. 163). Other methods may be founded on the measurement of currents and electromotive forces described in 18, 19, 20, 27, and 28. Lastly, a method founded on the gradual loss of charge through very great resistances will be found in Part IV. (45). The equation (25) there given for electrostatic measure is applicable to electromagnetic measure when the capacity and difference of potentials are expressed in electromagnetic units.

30. *Electric Resistance in Electromagnetic Units is measured by an Absolute Velocity.*—The dimensions of R are found, by comparing those of E and C, to be $\frac{L}{T}$, or those of a simple velocity. This velocity, as was pointed out by Weber, is an absolute velocity in nature, quite independent of the magnitude of the fundamental units in which it is expressed. The following illustration, due to Professor Thomson, will show how a velocity may express a resistance, and also how that expression may be independent of the magnitude of the units of time and space.

Let a wire of any material be bent into an arc of $57\frac{1}{4}°$ with any radius, k. Let this arc be placed in the magnetic meridian of any magnetic field, with a magnet of any strength freely suspended in the centre of the arc. Let two vertical wires or rails, separated by a distance equal to k, be attached to the ends of the arc; and let a cross piece slide along these rails, inducing a current in the arc. Then it may be shown that the speed required to produce a deflection of 45° on the magnet will measure the resistance of the circuit, which is assumed to be constant. This speed will be the same whatever be the value of k, or the intensity of the magnetic field, or the moment of the magnet. In this form the experiment could not be easily carried out; but if a length, l, of wire be taken and rolled into a circular coil at the radius k, and the distance between the vertical rails be taken equal to $\frac{k^2}{l}$, then, if the resistance of the circuit be the same as in the previous case, the deflection of 45° will be produced by the same velocity in the cross piece, measuring that resistance; or, generally, if the distance between the rails be $p\frac{k^2}{l}$, then p times the velocity required to produce the unit deflection (45°) will measure the resistance. The truth of this proposition can easily be established when the laws of magneto-electric induction have been understood (31).

31. *Magneto-electric Induction.*—Let a conducting circuit be placed in a magnetic field. Let C be the intensity of any current in that circuit; E the magnitude of the electromotive force acting in the circuit. Let the circuit be so moved that the number of lines of magnetic force (11) passing through it is increased by N in the time t, then (23) the electromagnetic forces will contribute towards the motion an amount of work measured by CN. Now Q, the quantity of electricity which passes, is equal to Ct; so that the work done on the current is EQ or CEt. By the principle of conservation of energy, the work done by the electromagnetic forces must be at the expense of that done by the electromotive forces, or

$$CN+CEt=0;$$

or dividing by Ct, we find that

$$E=-\frac{N}{t}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

or, in other words, if the number of lines of force passing through a circuit be increased, an electromotive force in the negative direction will act in the circuit measured by the number of lines of force added per second.

If R be the resistance of the circuit, we have by Ohm's law (quation 6) $E=CR$; and therefore

$$N=-Et=-RCt=-RQ; \quad . \quad . \quad . \quad . \quad (16)$$

or, in other words, if the number of lines of magnetic force passing through the circuit is altered, a current will be produced in the circuit in the direction opposite to that of a current which would have produced lines of force in the direction of those added, and the quantity of electricity which passes, multiplied by the resistance of the circuit, measures the number of additional lines passing through the circuit.

The facts of magneto-electric induction were discovered by Faraday, and described by him in the First Series of his "Experimental Researches in Electricity," read to the Royal Society, November 24th, 1831.

He has shown* the relation between the induced current and the lines of force cut by the circuit, and he has also described the state of a conductor in a field of force as a state the change of which is a cause of currents. He calls it the electrotonic state; and, as we have just seen, the electrotonic state may be *measured* by the number of lines of force which pass through the circuit at any time.

* Experimental Researches, 3082, &c.

The measure of electromotive force used by W. Weber, and derived by him (independently of the principle of conservation of energy) from the motion of a conductor in a magnetic field, is the same as that at which we have arrived; for, from equation (15), we find that the unit electromotive force will be produced by motion in a magnetic field when one line of force is added (or subtracted) per unit of time; and this will occur when in a field of unit intensity a straight bar of unit length, forming part of a circuit otherwise at rest, is moved with unit velocity perpendicularly to the lines of force and to its own direction.

To W. Weber, whose numerical determinations of electrical magnitudes are the starting-point of exact science in electricity, we owe this, the first definition of the unit of electromotive force; but to Professor Helmholtz* and to Professor W. Thomson†, working independently of each other, we owe the proof of the necessary existence of magneto-electric induction, and the determination of electromotive force on strictly mechanical principles.

32. *On Material Standards for the Measurement of Electrical Magnitudes.*—The comparison between two different electrical magnitudes of the same nature, *e. g.* between two currents or between two resistances, is in all cases much simpler than the direct measurement of these magnitudes in terms of time, mass, and space, as described in the foregoing pages. Much labour is therefore saved by the use of standards of each magnitude; and the construction and diffusion of those standards form part of the duties of the Committee.

Electric currents are most simply compared by "electro-dynamometers" (20)—instruments which, unlike galvanometers, are practically independent of the intensity of the earth's magnetism. When an instrument of this kind has been constructed, with which the values of the currents corresponding to each deflection have been measured (19), (20), other instruments may easily be so compared with this standard that the relative value of the deflections produced by equal currents on the standard and the copies shall be known. Hence the absolute value of the current indicated by each deflection of each copy will be known in absolute measure. In other words, in order to obtain the electromagnetic measure of a current in the system described, each observer in possession of an electro-dynamometer which has been compared with the standard instrument will simply multiply by a constant number the deflection produced by the current on his instrument (or the tangent or sine of the deflection, according to the particular construction of the instrument).

* Paper read before the Physical Society of Berlin, 1847 (*vide* Taylor's Scientific Memoirs, part 2. Feb. 1853, p. 114).

† Reports of the British Association, 1848; Phil. Mag. Dec. 1851.

Electric quantities may be compared by the swing of the needle of a galvanometer of any kind. They may be measured by any one in possession of a standard electro-dynamometer, or resistance-coil, since the observer will then be in a position directly to determine C_1 in equation (12), or R_1 in equation (14).

Capacities may be compared by the methods described (26); and a Leyden jar or condenser (41) of unit capacity, and copies derived from it, may be prepared and distributed. The owner of such a condenser, if he can measure electromotive force, can determine the quantity in his condenser.

The material standard for *electromotive force* derived from electromagnetic phenomena would naturally be a conductor of known shape and dimensions, moving in a known magnetic field. Such a standard as this would be far too complex to be practically useful: fortunately a very simple and practical standard or gauge of electromotive force can be based on its statical effects, and will be described in treating of those effects (Part IV. 43). A practical standard for approximate measurements might be formed by a voltaic couple, the constituent parts of which were in a standard condition. It is probable that the Daniell's cell may form a practical standard of reference in this way, when its value in electromagnetic measure is known. This value lies between 9×10^7 and 11×10^7.

Resistances are compared by comparing currents produced in the several conductors by one and the same electromotive force. The unit resistance, determined as in Appendix D (Brit. Assoc. Reports, p. 163), will be represented by a material conductor; simple coils of insulated wire compared with this standard, and issued by the Committee, will allow any observer to measure any resistance in electromagnetic measure.

[To be continued.]

LXII. *On a Theorem relating to Five Points in a Plane.* *By* A. Cayley, *F.R.S.**

Two triangles, ABC, A′B′C′ which are such that the lines AA′, BB′, CC′ meet in a point, are said to be in perspective; and a triangle A′B′C′, the angles A′, B′, C′ of which lie in the sides BC, CA, AB respectively, is said to be inscribed in the triangle ABC; hence, if A′, B′, C′ are the intersections of the sides by the lines AO, BO, CO respectively (where O is any point whatever), the triangle A′B′C′ is said to be perspectively inscribed in the triangle ABC, viz. it is so inscribed by means of the point O.

We have the following theorem, relating to any triangle

* Communicated by the Author.

ABC, and two points O, O′. If in the triangle ABC, by means of the point O, we inscribe a triangle A′B′C′, and in the triangle A′B′C′, by means of the point O′, we inscribe a triangle $\alpha\beta\gamma$, then the triangles ABC, $\alpha\beta\gamma$ are in perspective, viz. the lines Aα, Bβ, Cγ will meet in a point.

This is very easily proved analytically; in fact, taking $x=0$, $y=0$, $z=0$ for the equations of the lines B′C′, C′A′, A′B′ respectively, and (X, Y, Z) for the coordinates of the point O, then the coordinates of (A, B, C) are found to be $(-X, Y, Z)$, $(X, -Y, Z)$, $(X, Y, -Z)$ respectively. Moreover, if (X′, Y′, Z′) are the coordinates of the point O′, then the coordinates of (α, β, γ) are found to be

$$(0, Y', Z'),\ (X', 0, Y),\ (X', Y', 0)$$

respectively. Hence the equations of the lines Aα, Bβ, Cγ are respectively

$$\begin{vmatrix} x, & y, & z \\ -X, & Y, & Z \\ 0, & Y', & Z' \end{vmatrix}=0,\quad \begin{vmatrix} x, & y, & z \\ X, & -Y, & Z \\ X', & 0, & Z' \end{vmatrix}=0,\quad \begin{vmatrix} x, & y, & z \\ X, & Y, & -Z \\ X', & Y', & 0 \end{vmatrix}=0;$$

that is,

$$\begin{aligned} &x(\ YZ'-Y'Z)+y(\qquad Z'X)+z(-XY'\qquad)=0,\\ &x(-YZ'\qquad)+y(\ ZX'-Z'X)+z(\qquad X'Y)=0,\\ &x(\qquad +Y'Z)+y(-ZX'\qquad)+z(\ XY'-X'Y)=0, \end{aligned}$$

which are obviously the equations of three lines which meet in a point.

But the theorem may be exhibited as a theorem relating to a quadrangle 1234 and a point O′; for writing 1, 2, 3, 4 in place of A, B, C, O, the triangle A′B′C′ is in fact the triangle formed by the three centres 41.23, 42.31, 43.12 of the quadrangle 1234, hence the triangle in question must be similarly related to each of the four triangles 423, 431, 412, 123; or, forming the diagram

	P	Q	R	S
41.23	4	3	2	1
42.31	3	4	1	2
43.12	2	1	4	3,

we have the following form of the theorem: viz. the lines

$\alpha4$, $\beta3$, $\gamma2$ meet in a point P,
$\alpha3$, $\beta4$, $\gamma1$ „ „ Q,
$\alpha2$, $\beta1$, $\gamma4$ „ „ R,
$\alpha1$, $\beta2$, $\gamma3$ „ „ S,

to the margin of the retina, as if they were related to the distribution of its blood-vessels and hence it was probable that the paralysis of the corresponding parts of the retina was produced by their pressure. This opinion might have long remained merely a reasonable explanation of hemiopsy, had not a phenomenon presented itself to me which places it beyond a doubt. When I had a rather severe attack, which never took place unless I had been reading for a long time the small print of the 'Times' newspaper, and which was never accompanied either with headache or gastric irritation, I went accidentally into a dark room, when I was surprised to observe that all the parts of the retina which were affected were slightly luminous, an effect invariably produced by pressure upon that membrane. If these views be correct, hemiopsy cannot be regarded as a case of amaurosis, or in any way connected, as has been supposed, with cerebral disturbance.

Dr. Wollaston endeavoured to explain the phenomena of hemiopsy, and the fact of single vision with two eyes, by what he calls the semidecussation of the optic nerves, a doctrine which Sir Isaac Newton had suggested, and employed to account for single vision*. A fibre of the right-hand side of the optic nerve is supposed to semidecussate or divide itself into two fibres, sending *one* to the right side of the right eye, and *another* to the right side of the left eye, while a fibre on the left-hand side of the optic nerve also semidecussates, sending *one* fibre to the left side of the left eye, and *another* to the left side of the right eye. Hence Sir Isaac Newton drew the conclusion, that an impression on each of the two half-fibres would convey a single sensation to the brain; and hence Dr. Wollaston concluded that hemiopsy in one eye must be accompanied with hemiopsy in the other.

Ingenious as these explanations are, the anatomical facts by which alone they could be supported have not been established. Dr. Alison†, who has adopted the opinion of Newton, and reasoned upon it, admits that the anatomical evidence is still defective; and the late Mr. Twining‡ has adduced nine cases of disease in the optic nerves and thalami, which stand in direct opposition to the hypothesis of semidecussation. Dr. Mackenzie, too, adopting the same view of the subject as Mr. Twining, distinctly asserts that "the great mass of facts in Pathology and Experimental Anatomy, touching this question, go to prove that injuries and diseases affecting one side of the brain, instead of *hemiopsia* in both eyes, produce amaurosis only in the opposite eye."

* Optics, p. 320.
† Edinburgh Transactions, vol. xiii. p. 479.
‡ Trans. Med. Soc. Calcutta, vol. ii. p. 151; or Edinburgh Journal o Science, July 1828, vol. ix. p. 143.

The two great facts of hemiopsy in both eyes, and of what is called single vision with two eyes, do not require the hypothesis of semidecussation to explain them. If hemiopsy is produced by the distended blood-vessels of the retina, these vessels must be similarly distributed in each eye, and similarly affected by any change in the system; and consequently must produce the same effect upon each retina, and upon the same part of it.

In explaining single vision with two eyes, we have no occasion to appeal to double fibres in the optic nerves, or to corresponding points on the retina. There is, in reality, no such thing as single vision, that is, a single image seen by both eyes. With two sound eyes every object is seen double, and it appears single only when, by the law of visible position, the one image is placed above the other. But even in this case the object is seen double, by means of two dissimilar images of it which are not coincident. By shutting the right eye we lose sight of a part on the right side of the double image, which is seen only by the right eye; and by shutting the left eye we lose sight of a part on the left side of the double image, which is seen only by the left eye. If one eye gives a better picture than the other, the duplicity of the apparently single image is more easily seen. By shutting the good eye the imperfect picture is seen, and by shutting the bad eye we insulate the perfect picture. It is difficult to understand how optical writers and physiologists should have so long demanded a single sensation for the production of a single picture from the two pictures imprinted on the two retinas. If we had the hundred eyes of Argus, the production of an apparently single picture would have been the necessary result of the Law of Visible Position.

LXX. *On the Elementary Relations between Electrical Measurements. By* Professor J. Clerk Maxwell and Fleeming Jenkin, *Esq.*

[Concluded from p. 460.]

Part IV.—*Measurement of Electric Phenomena by Statical Effects.*

33. *Electrostatic Measure of Electric Quantity.*—By the application of a sufficient electromotive force between two parts of a conductor which does not form a circuit, it is possible to communicate to either part a *charge* of electricity which may be maintained in both parts, if properly insulated (14). With the ordinary electromotive forces due to induction or chemical action, and the ordinary size of insulated conductors, the charge of

electricity in electromagnetic measure is exceedingly small; but when the capacity of the conductor is great, as in the case of long submarine cables, the charge may be considerable. By making use of the electromotive force produced by the friction of unlike substances, the charge or electrification even of small bodies may be made to produce visible effects. The electricity in a charge is not essentially in motion, as is the case with the electricity in a current. In other words, a charge may be permanently maintained without the performance of work. Electricity in this condition is therefore frequently spoken of as statical electricity; and its effects, to distinguish them from those produced by currents, may be called statical effects. The peculiar properties of electrically charged bodies are these:—

1. When one body is charged positively (14), some other body or bodies must be charged negatively to the same extent.

2. Two bodies repel one another when both are charged positively, or both negatively, and attract when oppositely charged.

3. These forces are inversely proportional to the square of the distance of the attracting or repelling charges of electricity.

4. If a body electrified in any given invariable manner be placed in the neighbourhood of any number of electrified bodies, it will experience a force which is the resultant of the forces that would be separately exerted upon it by the different bodies if they were placed in succession in the positions which they actually occupy, without any alteration in their electrical conditions.

From these propositions it follows that, at a given distance, the force, f, with which two small electrified bodies repel one another is proportional to the product of the charges, q and q_1, upon them; but when the distance varies, this force, f, is inversely proportional to the square of the distance, d, between them. Hence

$$f=\frac{qq_1}{d^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

When q and q_1 are of dissimilar signs, f becomes negative; *i. e.* there is an attraction, and not a repulsion. This equation is incompatible with the electromagnetic definitions given in Part III., and, if it be allowed to be fundamental, gives a new definition of the unit quantity of electricity, as that quantity which, if placed at unit distance from another equal quantity of the same kind, repels it with unit force.

34. *Electrostatic System of Units.*—This new measurement of quantity forms the foundation of a distinct system or series of units, which may be called the electrostatic units; and measurements in these units will in these pages be designated by the use of small letters: thus, as Q, C, &c., signified quantity, current, &c., in *electromagnetic* measure, so q, c, e, and r, &c., will represent the *electrostatic* measure of quantity, current, electromotive force, resistance, &c.

The relations between current and quantity, between work, current, and electromotive force, and between electromotive force, current, and resistance, remain unchanged by the change from the electromagnetic to the electrostatic system.

35. *Ratio between Electrostatic and Electromagnetic Measures of Quantity.*—Since the expression forming the second member of equation (17) represents a force the dimensions of which are $\frac{LM}{T^2}$, the dimensions of q are $\frac{L^{\frac{3}{2}}M^{\frac{1}{2}}}{T}$. The dimensions of the unit of electricity, Q, in the electromagnetic system are $L^{\frac{1}{2}}M^{\frac{1}{2}}$ (25). Hence, since in passing from the one system to the other we must employ the ratio $\frac{q}{Q}$, this ratio will be of the dimensions $\frac{L}{T}$; that is to say, the ratio $\frac{q}{Q}$ is a velocity. In the present treatise this velocity will be designated by the letter v.

The first estimate of the relation between quantity of electricity measured statically and the quantity transferred by a current in a given time was made by Faraday*. A careful experimental investigation by MM. Weber and Kohlrausch† not only confirms the conclusion that the two kinds of measurement are consistent, but shows that the velocity $v=\frac{q}{Q}$ is 310,740,000 metres per second—a velocity not differing from the estimated velocity of light more than the different determinations of the latter quantity differ from each other. v must always be a constant, real velocity in nature, and should be measured in terms of the system of fundamental units adopted in electrical measurements (3) and (55). A redetermination of v (46) will form part of the present Committee's business in 1863–64. It will be seen that, by definition, the quantity transmitted by an electromagnetic unit current in the unit time is equal to v electrostatic units of quantity.

36. *Electrostatic Measure of Currents.*—In any coherent system, a current is measured by the quantity of electricity which passes in the unit of time (15); if both current and quantity

* Experimental Researches, series iii. § 361, &c.

† *Abhandlungen der König. Sächsischen Ges.* vol. iii. (1857) p. 260; or Poggendorff's *Annalen*, vol. xcix. p. 10 (Aug. 1856).

are measured in electrostatic units, then

$$c=\frac{q}{t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The dimensions of c are therefore $\frac{L^{\frac{3}{2}}M^{\frac{1}{2}}}{T^2}$; and in order to reduce a current from electromagnetic to electrostatic measure, we must multiply C by v, or

$$c=v\text{C}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

37. *Electrostatic Measure of Electromotive Force.*—The statical measure of an electromotive force is the work which would be done by electrical forces during the passage of a unit of electricity from one point to another. The only difference between this definition and the electromagnetic definition (16 and 27) consists in the change of the unit of electricity from the electromagnetic to the electrostatic.

Hence, if q units of electricity are transferred from one place to another, the electromotive force between those places being e, the work done during the transfer will be qe; but we found (27) that if E and Q be the electromagnetic measures of the same quantities, the work done would be expressed by QE; hence

$$qe=\text{QE};$$

but (35)

$$q=v\text{Q},$$

therefore

$$e=\frac{\text{E}}{v}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Thus, to reduce electromotive force from electromagnetic to electrostatic measure, we must divide by v.

The dimensions of e are $\frac{L^{\frac{1}{2}}M^{\frac{1}{2}}}{T}$.

38. *Electrostatic Measure of Resistance.*—If an electromotive force, e, act on a conductor whose resistance in electrostatic measure is r, and produce a current, c, then by Ohm's law

$$r=\frac{e}{c}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Substituting for e and c their equivalents in electromagnetic measure (equations 19 and 20), we have

$$r=\frac{1}{v^2}\frac{\text{E}}{\text{C}}$$

but (eq. 7)

$$\text{R}=\frac{\text{E}}{\text{C}},$$

and therefore

$$r=\frac{1}{v^2}\text{R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

To reduce a resistance measured in electromagnetic units to its electrostatic value, we must divide by v^2.

The dimensions of r are $\frac{T}{L}$, or the reciprocal of a velocity.

39. *Electric Resistance in Electrostatic Units is measured by the Reciprocal of an Absolute Velocity.*—We have seen from the last paragraph that the dimensions of r establish this proposition; but the following independent definition, due to Professor W. Thomson, assists the mind in receiving this conception as a necessary natural truth. Conceive a sphere of radius k, charged with a given quantity of electricity Q. The potential of the sphere, when at a distance from all other bodies, will be $\frac{\text{Q}}{k}$ (40, 41, and 47). Let it now be discharged through a certain resistance, r. Then, if the sphere could collapse with such a velocity that its potential should remain constant (or, in other words, that the ratio of the quantity on the sphere to its radius should remain constant, during the discharge), the time occupied by its radius in shrinking the unit of length would measure the resistance of the discharging conductor in electrostatic measure, or the velocity with which its radius diminished would measure the conducting-power (50) of the discharging conductor. Thus the conducting-power of a few yards of silk in dry weather might be an inch per second, in damp weather a yard per second. The resistance of 1000 miles of pure copper wire, $\frac{1}{16}$ inch in diameter, would be about 0·00000141 of a second per metre, or its conducting-power one metre per 0·00000141 of a second, or 708980 metres per second.

40. *Electrostatic Measure of the Capacity of a Conductor.*—The electrostatic capacity of a conductor is equal to the quantity of electricity with which it can be charged by the unit electromotive force. This definition is identical with that given of capacity measured in electromagnetic units (26). Let s be the capacity of a conductor, q the electricity in it, and e the electromotive force charging it; then

$$q=se. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

From this equation we can see that the dimension of the quantity s is a length only. It will also be seen that

$$s=v^2\text{S}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where S is the electromagnetic measure of the capacity of the conductor with the electrostatic capacity s.

The capacity of a spherical conductor in an open space is, in electrostatic measure, equal to the radius of the sphere,—a fact demonstrable from the fundamental equation (17).

Experimentally to determine s, the capacity of the conductor in electrostatic measure, charge it with a quantity, q, of electricity, and measure in any unit its potential (47) or tension (49), e. Then bring it into electrical connexion with another conductor whose capacity, s_1, is known. Measure the potential, e_1, of s and s_1 after the charge is divided between them; then

$$q = se = (s + s_1)e_1,$$

and hence

$$s = \frac{e_1}{e - e_1} s_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

In this measurement we do not require to know e and e_1 in absolute measure, since the ratio of these two quantities only is required. We must, however, know the value of s_1; and hence we must begin either with a spherical conductor in a large open space, whose capacity is measured by its radius, or with some other form of absolute condenser alluded to in the following paragraph.

41. *Absolute Condenser. Practical Measurement of Quantity.* —As soon as the electromotive force of a source of electricity is known in electrostatic measure, the quantity which it will produce in the form of charge on simple forms is known by the laws of electrical distribution experimentally proved by Coulomb. Simple forms of this kind may be termed *absolute condensers.* A sphere in an open space is such a condenser, and the quantity it contains is se (equation 23). A more convenient form is a sphere of radius x, suspended in the centre of a hollow sphere, radius y, the latter being in communication with the earth. The capacity, s, of the internal sphere is then, by calculation,

$$s = \frac{xy}{y - x}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

By a series of condensers of increasing capacity we may measure the capacity of any condenser, however large. The comparison is made by the method described above (40). Thus, the practical method of measuring quantity in electrostatic measure is first to determine the capacity of the conductor containing the charge, and then to multiply that capacity by the electromotive force producing the charge (43).

42. *Practical Measurement of Currents:*—The electrostatic value of currents can be obtained from equation (21) when e and r are known, or from equation (19) when v and C are known, or by comparison with a succession of discharges of known quantities from an absolute condenser.

43. *Practical Measurement of Electromotive Force.*—The relations expressed by equations (17) and (23) show that in any given circumstances the force exerted between two bodies due to the effects of statical electricity will be proportional to the electromotive force or difference of potential (47) between them. This fact allows us to construct gauges of electromotive force, or instruments so arranged that a given electromotive force between two parts of the apparatus brings an index into a sighted position. In order that the gauge may serve to *measure* the electromotive force absolutely, it is necessary that two things should be known,—first, the distribution of the electricity over the two attracting or repelling masses (or, in other words, the capacity of each part); secondly, the absolute force exerted between them. For simple forms, the distribution, or capacity of each part, can be calculated from the fundamental principles (33); the force actually exerted can be weighed by a balance. By these means Professor W. Thomson* determined the electromotive force of a Daniell's cell to be 0·0021 in British electrostatic units, or 0·0002951 in metrical electrostatic units†. This proposition is equivalent to saying that two balls of a metre radius, at a distance d apart in a large open space, and in connexion with the opposite poles of a Daniell's cell, would attract one another with a force equal to $\frac{(0{\cdot}0002951)^2}{4\,d^2}$ absolute units, or $\frac{0{\cdot}000{,}000{,}00888}{d^2}$ gramme weight‡.

An apparatus by which such a measurement as the foregoing can be carried out is called an absolute electrometer. It will be observed that, although the definition of electromotive force is founded on the idea of work, its practical measurement is effected by observing a force, inasmuch as when this force exerted between two conductors of simple shape is known, the work which the passage of a unit of electricity between them would perform may be calculated by known laws.

44. *Comparison of Electromotive Forces by their Statical Effects.* —This comparison is simpler than the absolute measurement, inasmuch as it is not necessary, in comparing two forces, to know the absolute values of either. Instruments by which the comparison can be made are called electrometers. Their arrange-

* Paper read before the Royal Society, February 1860. *Vide* Proceedings of the Royal Society, vol. x. p. 319, and Phil. Mag. S. 4. vol. xx. (1860) p. 233.

† [Note added May 5, 1865.—In electromagnetic measure this would make the electromotive force of a Daniell's cell equal to about 91,700. Other observers have found a value of about 100,000 (metrical system, based on metre, gramme, and second).]

‡ This value was erroneously given in the original paper.

ment is of necessity such that the force exerted between two given parts of the instrument shall be proportional to the difference of potential between them. This force may be variable and measured by the torsion of a wire, as in Thomson's reflecting electrometer; or it may be constant, and the electromotive forces producing it may be compared by measuring the distance required in each case between the two electrified bodies to produce that constant force. The latter arrangement is adopted in Professor Thomson's portable electrometer, first exhibited at the present Meeting of the Association. The indications of a gauge or electrometer not in itself absolute may be reduced to absolute measurement by multiplication into a constant coefficient.

45. *Practical Measurement of Electric Resistance.*—The electrostatic resistance of a conductor of great resistance (such as gutta percha or india rubber) might be directly obtained in the following manner:—Let a body of known capacity, s (40), be charged to a given potential, P (47), and let it be gradually discharged through the conductor of great resistance, r. Let the time, t, be noted at the end of which the potential of the body has fallen to p. The rate of loss of electricity will then be $\frac{p}{sr}$. Hence $p=\mathrm{P}_\epsilon{}^{-\frac{t}{sr}}$ and $\frac{t}{sr}=\log_\epsilon\frac{\mathrm{P}}{p}$. Hence

$$r=\frac{t}{s\log_\epsilon\frac{\mathrm{P}}{p}}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

from which equation r can be deduced if s, t, and the ratio $\frac{\mathrm{P}}{p}$ be known; t can be directly observed; s can be measured (40); and the ratio $\frac{\mathrm{P}}{p}$ can be measured by an electrometer (44) in constant connexion with the charged body. This ratio can also be measured by the relative discharges through a galvanometer, first, immediately after the body has been charged to the potential P, and again when, after having been recharged to the potential P, it has, after a time t, fallen to potential p. (This latter plan has long been practically used by Messrs. Siemens, although the results have not been expressed in absolute measure.)

Unfortunately, in those bodies, such as gutta percha and india rubber, the resistance of which is sufficiently great to make t a measurable number, the phenomenon of absorption due to continued electrification* so complicates the experiment as to

* *Vide* British Association Report, 1859, Trans. of Sec. p. 248, and Report of the Committee of Board of Trade on Submarine Cables, pp. 136 & 464.

render it practically unavailable for any exact determination. The apparent effect of absorption is to cause r, the resistance of the material, to be a quantity variable with the time t, and the laws of the variation are very imperfectly known.

46. *Experimental Determination of the Ratio, v, between Electromagnetic and Electrostatic Measures of Quantity.*—In order to obtain the value of v, it is necessary and sufficient that we should obtain a common electrostatic and electromagnetic measure of some one quantity, current, resistance, electromotive force, or capacity. There are thus five known methods by which the value can be obtained.

(1) By a common measure of quantity. Let a condenser of known capacity, s, be prepared (40). Let it be charged to a given potential P (47). Then the quantity in the condenser will be sP in electrostatic measure. The charge can next be measured by discharge through a galvanometer (25) in electromagnetic measure. The ratio between the two numbers will give the value of v. The only difficulty in this method consists in the measurement of the potential P entailing the measurement of an absolute force between two electrified bodies. This method was proposed and adopted by Weber*.

(2) By a comparison of the measure of electromotive force. The electromotive force produced by a battery, in electrostatic measure, can be directly weighed (43). Its electromotive force, in electromagnetic measure, can be obtained from the current it produces in a given resistance (28). The ratio of the two numbers will give the value of v. This method has been carried out by Professor W. Thomson, who was not, however, at the time in possession of the means of determining accurately either the absolute resistance of his circuit or the absolute value of the current†.

(3) By a common measure of resistance. We know (29 and 45) how to measure resistances in electromagnetic and electrostatic measure. The ratio between these measures is equal to v^2. The measure of resistance in electrostatic measure is not as yet susceptible of great accuracy.

(4) By a comparison of currents. The electromagnetic value of a current produced by a continuous succession of discharges from a condenser of capacity s can be measured (18, 19). The electrostatic value of the current will be known if the potential to which the condenser is charged be known. The ratio of the two numbers is equal to v.

* Pogg. *Ann.* August 1856, vol. xcix. p. 10. *Abhandlungen der Kön. Sächsischen Gesellschaft*, vol. iii. (1857), p. 266.

† Paper read before the Royal Society, February 1860. *Vide* Proceedings of the Royal Society, vol. x. p. 319, and Phil. Mag. S. 4. vol. xx. p. 233.

(5) By a common measure of capacity. The two measurements can be effected by the methods given (26 and 40). The ratio between the two measurements will give v^2. This method would probably yield very accurate results.

PART V.—*Electrical Measurements derived from the five elementary Measurements; and Conclusion.*

47. *Electric Potential.*—The word "potential," as applied by G. Green to the condition of an electrified body and the space surrounding it, is now coming into extensive use, but is perhaps less generally understood than any other electrical term. Electric potential is defined by Professor W. Thomson as follows*.

"The potential, at any point in the neighbourhood of or within an electrified body, is the quantity of work that would be required to bring a unit of positive electricity from an infinite distance to that point, if the given distribution of electricity remained unaltered."

It will be observed that this definition is exactly analogous to that given of magnetic potential (10), with the substitution of the unit quantity of electricity for the unit magnetic pole. (Analogous definitions might be given of gravitation-potential, heat-potential; and every one of these potentials coexist at every point of space quite independently one of the other.) In another paper† Professor Thomson describes electric potential as follows:—"The amount of work required to move a unit of electricity, against electric repulsion, from any one position to any other position, is equal to the excess of the electric potential of the first position above the electric potential of the second position."

The two definitions given are virtually identical, since the potential at every point of infinity is zero, and it will be seen that the difference of potential defined in the second passage quoted is identical with what we have called the electromotive force between the two points (16 and 27).

When, instead of a difference of potentials, *the potential* simply of a point is spoken of, the difference of potential between the point and the earth is referred to, or, as we might say, the electromotive force between the point and the earth.

The potential at all points close to the surface and in the interior of any simple metallic body is constant; that is to say, no electromotive force can be produced in a single metallic body by mere electrical distribution; the potential *at* the body may therefore be called the potential *of* the body. The potential of a metallic body varies according to the distribution, dimensions, position, and electrification of all surrounding bodies. It also depends on the substance forming the dielectric.

* Paper read before the British Association, 1852. *Vide* Phil. Mag. 1853, p. 288.

† Paper read before the Royal Society, February 1860. *Vide* Proceedings of the Royal Society, vol. x. p. 334, and Phil. Mag. S. 4 vol. xx. p. 323.

In any given circumstances, the potential of the body will be simply proportional to the quantity of electricity with which it is charged; but if the circumstances are altered, the potential will vary although the total amount of the charge may remain constant.

In a closed circuit in which a current circulates, the potential of all parts of the circuit is different; the difference depends on the resistance of each part and on the electromotive force of the source of electricity, *i. e.* on the difference of potentials which it is capable of causing when its two electrodes are separated by an insulator or dielectric. The different parts of a conductor moving in a magnetic field are maintained at different potentials, inasmuch as we have shown that an electromotive force is produced in this case. The potential of a body moving in an electric field (*i. e.* in the neighbourhood of electrified bodies) is constantly changing, but at any given moment the potential of all the parts is equal. The use of the word "potential" has the following advantages. It enables us to be more concise than if we were continually obliged to use the circumlocution, "electromotive force between the point and the earth;" and it avoids the conception of a force capable of generating a current, which almost necessarily, although falsely, is attached to "electromotive force."

Equipotential surfaces and lines of force in an electric field may be conceived for statically electrified bodies; these surfaces and lines would be drawn on similar principles and possess analogous properties to those described in a magnetic field (10). It is hardly necessary to observe that the magnetic and the electric fields are totally distinct, and coexist without producing any mutual influence or interference.

The rate of variation of electric potential per unit of length along a line of force is at any point equal to the electrostatic force at that point, *i. e.* to the force which a unit of electricity placed there would experience. The unit difference of potential is identical with the unit electromotive force; and the electrometer spoken of as measuring electromotive force measures potentials or differences of potential.

48. *Density, Resultant Electric Force, Electric Pressure.*—The three following definitions are taken almost literally from a paper by Professor W. Thomson*. Our treatise would be in-

* Paper read before the Royal Society, February 1860. *Vide* Proceedings of the Royal Society, vol. x. p. 333 (1860), and Phil. Mag. S. 4. vol. xx. p. 322.

complete without reference to these terms, and Professor Thomson's definitions can hardly be improved.

"*Electric Density*.—This term was introduced by Coulomb to designate the quantity of electricity per unit of area in any part of the surface of a conductor. He showed how to measure it, though not in absolute measure, by his proof-plane.

"*Resultant Electric Force*.—The resultant force in air or other insulating fluid in the neighbourhood of an electrified body is the force which a unit of electricity concentrated at that point would experience if it exercised no influence on the electric distributions in the neighbourhood. The resultant force at any point in the air close to the surface of a conductor is perpendicular to the surface, and equal to $4\pi\rho$, if ρ designates the electric density of the surface in the neighbourhood.

"*Electric Pressure from the Surface of a Conductor balanced by Air*.—A thin metallic shell or liquid film, as for instance a soap-bubble, if electrified, experiences a real mechanical force in a direction perpendicular to the surface outwards, equal in amount per unit of area to $2\pi\rho^2$, ρ denoting, as before, the electric density at the part of the surface considered. In the case of a soap-bubble its effect will be to cause a slight enlargement of the bubble on electrification with either vitreous or resinous electricity, and a corresponding collapse on being perfectly discharged. In every case we may consider it as constituting a deduction from the amount of air-pressure which the body experiences when unelectrified. The amount of deduction being different at different parts according to the square of the electric density, its resultant action on the whole body disturbs its equilibrium, and constitutes in fact the resultant electric force experienced by the body."

49. *Tension*.—The use of this word has been intentionally avoided by us in this treatise, because the term has been somewhat loosely used by various writers, sometimes apparently expressing what we have called the density, and at others diminution of air-pressure. By the most accurate writers it has been used in the sense of a magnitude proportional to potential or difference of potentials, but without the conception of absolute measurement, or without reference to the idea of work essential in the conception of potential. We believe also that it has not been generally, if ever, applied to that condition of an insulating fluid in virtue of which each point has an electric potential, although no sensible quantity of electricity be present at the point. The expression "tension" might be used to designate what we have termed the potential of a body. The tension between two points would then be equivalent to the electromotive force between those points, or to their difference of potentials, and would be measured in the same unit.

50. *Conducting-Power, Specific Resistance, and Specific Conducting-Power.*

Conducting-Power, or Conductivity.—These expressions are employed to signify the reciprocal of the resistance of any conductor. Thus, if the resistance of a wire be expressed by the number 2, its conducting-power will be 0·5.

Specific Resistance referred to Unit of Mass.—The specific resistance of a material at a given temperature may be defined as the resistance of the unit mass formed into a conductor of unit length and of uniform section. Thus the specific resistance of a metal in the metrical system is the resistance of a wire of that metal, one metre long, and weighing one gramme.

The Specific Conducting-Power of a material is the reciprocal of its specific resistance.

Specific resistance, referred to unit of volume, is the resistance opposed by the unit cube of the material to the passage of electricity between two opposed faces. It may easily be deduced from the specific resistance referred to unit of mass, when the specific gravity of the material is known.

Specific conducting-power may also be referred to unit of volume. It is of course the reciprocal of the specific resistance referred to the same unit.

It is somewhat more convenient to refer to the unit of mass with long uniform conductors, such as metal wires, of which the size is frequently and easily measured by the weight per foot or metre; and it is, on the other hand, more convenient to refer to the unit of volume bodies, such as gutta percha, glass, &c., which do not generally occur as conducting-rods of uniform section, while theird imensions can always be measured with at least as much accuracy as their weights.

51. *Specific Inductive Capacity*.—Faraday* discovered that the capacity of a conductor does not depend simply on its dimensions or on its position relatively to other conductors, but is influenced in amount by the nature of the insulator or dielectric separating it from them. The laws of induction are assumed to be the same in all insulating materials, although the amount be different. The name "inductive capacity" is given to that quality of an insulator in virtue of which it affects the capacity of the conductor it surrounds, and this quality is measured by reference to air, which is assumed to possess the unit inductive capacity. The specific inductive capacity of a material is therefore equal to the quotient of the capacity of any conductor insulated by that material from the surrounding conductors, divided by the capacity of the same conductor in the same position separated

* Experimental Researches, series xi.

from them by air only. It is not improbable that this view of induction may be hereafter modified.

52. *Heat produced in a Conductor by a Current.*—The work done in driving a current, C, for a unit of time through a conductor whose resistance is R, by an electromotive force E, is $EC = RC^2$ (§ 17). This work is lost as electrical energy, and is transformed into heat. As Dr. Joule has ascertained the quantity of mechanical work equivalent to one unit of heat, we can calculate the quantity of heat produced in a conductor in a given time, if we know C and R in absolute measure. In the metrical series of units founded on the metre, gramme, and second, if we call the total heat Θ, taking as unit the quantity required to raise one gramme of water one degree Centigrade, we have

$$\Theta = \frac{RC^2 t}{4157}. \quad \quad (28)$$

In the British system, founded on feet, grains, and seconds, with a unit of heat equal to the quantity required to raise one grain of water one degree Fahrenheit, we must substitute the number 24861 for 4157 in the above equation.

53. *Electrochemical Equivalents.*—Dr. Faraday has shown* that when an electric current passes through certain substances and decomposes them, the quantity of each substance decomposed is proportional to the quantity of electricity which passes. Hence we may call that quantity of a substance which is decomposed by unit current in unit time the electrochemical equivalent of that substance.

This equivalent is a certain number of grammes of the substance. The equivalents of different substances are in the proportion of their combining-numbers; and if all chemical compounds were electrolytes, we should be able to construct experimentally a table of equivalents, in which the weight of each substance decomposed by a unit of electricity would be given. The electrochemical equivalent of water, in electromagnetic measure, is about 0·02 in British, 0·0092† in the metrical system. The electrochemical equivalents of all other electrolytes can be deduced from this measurement with the aid of their combining-numbers.

54. *Electromotive Force of Chemical Affinity.*—When two substances having a tendency to combine are brought together and enter into combination, they enter into a new state, in which the intrinsic energy of the system is generally less than it was before; that is, the substances are less able to effect chemical changes, or to produce heat or mechanical action, than before.

* Experimental Researches, series vii.

† ·009375 by Weber and Kohlrausch.

The energy thus lost appears during the combination as heat or electrical or mechanical action, and can in many cases be measured*.

The energy given out during the combination of two substances may, like all other forms of energy, be considered as the product of two factors†—the tendency to combine, and the amount of combination effected. Now the amount of combination may be measured by the number of electrochemical equivalents which enter into combination; so that the tendency to combine may also be ascertained by dividing the energy given out by the number of electrochemical equivalents which enter into combination.

If the whole energy appears in the form of electric currents, the energy of the current is measured by the product of the electromotive force and the quantity of electricity which passes. Now the quantity of electricity which passes is equal to the number of electrochemical equivalents which enter on either side into combination. Hence the total energy given out, divided by this number, will give the electromotive force of combination. Thus, if N electrochemical equivalents enter into combination under a chemical affinity I, and in doing so give out energy equal to W, either as heat or as electrical action, then

$$NI = W.$$

But if W be given out as electrical action, and causes a quantity of electricity Q to traverse a conductor under an electromotive force E, we shall have

$$W = EQ.$$

By the definition of electrochemical equivalents,

$$E = N,$$

therefore

$$I = E;$$

or the force of chemical affinity may in these cases be measured as electromotive force.

This method of ascertaining the electromotive force due to chemical combination, which gives us a clear insight into the meaning and the measurement of "chemical affinity," is due to Professor W. Thomson‡.

The field of investigation presented to us by these considera-

* Report of the British Association, 1850, p. 63; and Phil. Mag. S. 3. vol. xxxii. See papers by Professor Andrews, and Favre and Silbermann, "On the Heat given out in Chemical Action," *Comptes Rendus*, vols. xxxvi. and xxxvii.

† See Rankine "On the General Law of Transformation of Energy," Phil. Mag. 1853.

‡ "On the Mechanical Theory of Electrolysis," Phil. Mag. Dec. 1851.

tions is very wide. We have to measure the intrinsic energy of substances as dependent on volume, temperature, and state of combination. When this is done, the energy due to any combination will be found by subtracting the energy of the compound from that of the components before combination.

As the tendency to increase in volume is measured as pressure, and as the tendency to part with heat is measured by the temperature, so in chemical dynamics the tendency to combine will be properly measured by the electromotive force of combination.

55. *Tables of Dimensions and other Constants:—*

Fundamental Units.

Length = L. Time = T. Mass = M.

Derived Mechanical Units.

$$\text{Work} = W = \frac{L^2M}{T^2}. \quad \text{Force} = F = \frac{LM}{T^2}. \quad \text{Velocity} = V = \frac{L}{T}.$$

Derived Magnetical Units.

Strength of the pole of a magnet . $m = L^{\frac{3}{2}}\ T^{-1}\ M^{\frac{1}{2}}$

Moment of a magnet . . . . . . $ml = L^{\frac{5}{2}}\ T^{-1}\ M^{\frac{1}{2}}$

Intensity of magnetic field . . . $H = L^{-\frac{1}{2}}T^{-1}\ M^{\frac{1}{2}}$

Electromagnetic System of Units.

Quantity of electricity . . . . . . $Q = L^{\frac{1}{2}} \times M^{\frac{1}{2}}$

Strength of electric current . . . $C = L^{\frac{1}{2}}T^{-1}\ M^{\frac{1}{2}}$

Electromotive force . . . . $E = L^{\frac{3}{2}}T^{-2}\ M^{\frac{1}{2}}$

Resistance of conductor . . . . . . $R = L\ T^{-1}$

Electrostatic System of Units.

Quantity of electricity . . . . . . $q = L^{\frac{3}{2}}\ T^{-1}\ M^{\frac{1}{2}}$

Strength of electric currents . . . $c = L^{\frac{3}{2}}\ T^{-2}\ M^{\frac{1}{2}}$

Electromotive force . . . . $e = L^{\frac{1}{2}}\ T^{-1}\ M^{\frac{1}{2}}$

Resistance of conductor . . . . . . $r = L^{-1}T$

Let v be the ratio of the electrostatic to the electromagnetic unit of quantity (35 and 46); then $v = 310{,}740{,}000$ metres per second approximately, and we have

$$q = vQ \quad \Big| \quad c = vC \quad \Big| \quad e = \frac{1}{v}E \quad \Big| \quad r = \frac{1}{v^2}R \quad \Big| \quad s = v^2S$$

TABLE for the Conversion of British (foot-grain-second) System to Metrical (metre-gramme-second) System.

	Number of metrical units contained in a British unit.	Log.	Log.	Number of British units contained in a metrical unit.
1. For M	0·0647989	$\bar{2}$·8115678	1·1884321	15·43235
2. For L, $\frac{v}{l}$, R, $\frac{l}{r}$, and V.	0·3047945	$\bar{1}$·4840071	0·5159929	3·280899
3. For F (also for foot-grains and metre-grammes)	0·0197504	$\bar{2}$·2955749	1·7044250	50·6320
4. For W	0·0060198	$\bar{3}$·7795820	2·2204179	166·1185
5. For H and electro-chemical equivalents.	0·461085	$\bar{1}$·6637804	0·3362196	2·16880
6. For Q, C, and e.........	0·140536	$\bar{1}$·1477874	0·8522125	7·11561
7. For E, m, q, and c ...	0·0428346	$\bar{2}$·6317949	1·3682051	23·3456
8. For heat..................	0·0359994	$\bar{2}$·5562953	1·4437046	27·7782

British System.—Relations between Absolute and other Units.

One absolute unit of $\begin{cases}\text{force}\\ \text{work}\end{cases} = 0{\cdot}0310666$ $\begin{matrix}\text{weight of a grain}\\ \text{foot-grains}\end{matrix}$ in London.

In London $\begin{cases}\text{weight of a grain}\\ \text{one foot-grain}\end{cases} = 32{\cdot}1889$ absolute units of $\begin{matrix}\text{force}\\ \text{work.}\end{matrix}$

One absolute unit of $\begin{cases}\text{force}\\ \text{work}\end{cases} = \dfrac{1}{g}$ $\begin{matrix}\text{unit weight}\\ \text{unit weight} \times \text{unit length}\end{matrix}$ everywhere.

g in British system $= 32{\cdot}088\ (1 + 0{\cdot}005133 \sin^2 \lambda)$, where $\lambda =$ the latitude of the place at which the observation is made.

Heat.—The unit of heat is the quantity required to raise the temperature of one grain of water at its maximum density 1° Fahrenheit.

Absolute mechanical equivalent of unit of heat $= 24861 = 772$ foot-grains at *Manchester*.

Thermal equivalent of an absolute unit of work $= 0{\cdot}000040224$.

Thermal equivalent of a foot-grain at Manchester $= 0{\cdot}0012953$.

Electrochemical equivalent of water $= 0{\cdot}02$, nearly.

Metrical System.—Relation between Absolute and other Units.

One absolute unit of $\begin{cases}\text{force}\\ \text{work}\end{cases} = 0{\cdot}0809821$ $\begin{matrix}\text{weight of a gramme}\\ \text{metre-gramme}\end{matrix}$ at Paris.

At Paris $\begin{cases}\text{the weight of a gramme}\\ \text{or metre-gramme}\end{cases} = 9{\cdot}80868$ absolute units of $\begin{matrix}\text{force}\\ \text{work.}\end{matrix}$

One absolute unit of $\begin{cases}\text{force}\\ \text{work}\end{cases} = \dfrac{1}{g}$ $\begin{matrix}\text{unit weight}\\ \text{unit weight} \times \text{unit length}\end{matrix}$ everywhere.

g in metrical system $= 9{\cdot}78024(1 + 0{\cdot}005133 \sin^2 \lambda)$, where $\lambda =$ the latitude of the place where the experiment is made.

Heat.—The unit of heat is the quantity required to raise one gramme of water at its maximum density 1° Centigrade.

Absolute mechanical equivalent of the unit of heat $=4157{\cdot}25$ $=423{\cdot}542$ metre-grammes at *Manchester*.

Thermal equivalent of an absolute unit of work $=0{\cdot}00024054$.

Thermal equivalent of a metr-grm. at Manchester $=0{\cdot}00236154$.

Electrochemical equivalent of water $=0{\cdot}0092$, nearly.

56. *Note to the Table of Dimensions, by Professor Clerk Maxwell.*—All the measurements of which we have hitherto treated are supposed to be made in the same medium—ordinary air; but Faraday has shown that other media have different properties. Paramagnetic bodies, such as oxygen and salts of iron, when placed in media less paramagnetic than themselves, behave as paramagnetic bodies; but when placed in media more paramagnetic than themselves, they behave as diamagnetic bodies.

Hence magnetic phenomena are influenced by the nature of the medium in which the bodies are placed, and the system of units and of measurements which we adopt depends on the nature of the medium in which our experiments are made. If we made our experiments in highly condensed oxygen, magnets would attract each other less, and currents would attract each other more, than they do in common air; and the reverse would be the case if we worked in a sea of melted bismuth.

Now if we take into account the "coefficient of magnetic induction" of the medium in which we work, and instead of assuming that of common air to be unity, assume it proportional to the density of that part of the medium to which the magnetic action is due, we shall have the repulsion of two poles $=\frac{mm'}{\mu r^2}$, where mm' are the two poles, μ the density of the magnetic medium, and r the distance. Now a density is a mass, M_1, divided by L^3, the unit of volume. Hence the dimensions of m are $\sqrt{\frac{MM_1}{T^2}}$; or if we can measure the density of the magnetic medium in the same unit of mass as that employed for other purposes, the dimensions of m will be simply $\frac{M}{T}$; those of H will then be $\frac{L}{T}$, or a velocity.

If we suppose the density of the magnetic medium to be taken account of in the electromagnetic units, their dimensions become

Quantity of electricity .	$Q=L^2$, or equivalent to an area.
Strength of current . .	$C=\frac{L^2}{T}$
Electromotive force . .	$E=\frac{M}{T^2}$
Resistance of conductor	$R=\frac{M}{L^2T}$

The electromagnetic unit of quantity of electricity is equal to the electrostatic unit multiplied by a certain velocity, depending on the elasticity of the magnetic medium, and proportional or probably equal to the velocity of propagation of vibrations in it. Hence the dimensions of

Electrostatic quantity	$q=LT$
Electrostatic current	$c=L$
Electrostatic electromotive force .	$e=\frac{LM}{T^3}$
Resistance	$r=\frac{M}{T^3}$

As we have no knowledge of the density, elasticity, &c., of the magnetic medium, we assume it as having a standard state in common air; and supposing all measurements to be made in air, the original table of dimensions is sufficient for expressing measurements made according to one system in terms of any other system.

57. *Magnitude of Units and Nomenclature.*—In connexion with the system of measurement explained in this treatise, two points hitherto unmentioned deserve attention—first, the absolute magnitude of the units, and secondly the nomenclature.

The absolute magnitude is in most cases an inconvenient one, leading to the use either of exceedingly small or exceedingly large numbers. Thus the units of electromagnetic resistance and electromotive force and quantity, and of electrostatic currents, are inconveniently small; the unit of electrostatic resistance is inconveniently large. Decimal multiples and submultiples of these units will therefore probably have to be adopted in practice. The choice of these multiples and submultiples forms part of the business of the Committee.

The nomenclature hitherto adopted is extremely defective. In referring to each measurement, we have to say that the number expresses the value in electrostatic or electromagnetic absolute units: if a multiple is to be used, this multiple will also have to be named; and further, the standard units of length, mass, and time have to be referred to, inasmuch as some writers use the pound and some the grain, some the metre and some the millimetre, as fundamental units. This cumbrous diction, and the risk of error imported by it, would be avoided if each unit received a short distinctive name in the manner proposed by Sir Charles Bright and Mr. Latimer Clark, is a paper read before the British Association at Manchester, 1861.