IV. *On the Theory of Compound Colours, and the Relations of the Colours of the Spectrum.*

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§ I. *Introduction.*

According to Newton's analysis of light*, every colour in nature is produced by the mixture, in various proportions, of the different kinds of light into which white light is divided by refraction. By means of a prism we may analyse any coloured light, and determine the proportions in which the different homogeneous rays enter into it; and by means of a lens we may recombine these rays, and reproduce the original coloured light.

Newton has also shown† how to combine the different rays of the spectrum so as to form a single beam of light, and how to alter the proportions of the different colours so as to exhibit the result of combining them in any arbitrary manner.

The number of different kinds of homogeneous light being infinite, and the proportion in which each may be combined being also variable indefinitely, the results of such combinations could not be appreciated by the eye, unless the chromatic effect of every mixture, however complicated, could be expressed in some simpler form. Colours, as seen by the human eye of the normal type, can all be reduced to a few classes, and expressed by a few well-known names; and even those colours which have different names have obvious relations among themselves. Every colour, except purple, is similar to some colour of the spectrum‡, although less intense; and all purples may be compounded of blue and red, and diluted with white to any required tint. Brown colours, which at first sight seem different, are merely red, orange or yellow of feeble intensity, more or less diluted with white.

It appears therefore that the result of any mixture of colours, however complicated, may be defined by its relation to a certain small number of well-known colours. Having selected our standard colours, and determined the relations of a given colour to these, we have defined that colour completely as to its appearance. Any colour which has the same relation to the standard colours, will be identical in appearance, though its optical constitution, as revealed by the prism, may be very different.

* Optics, book i. part 2. prop. 7.

† Lectiones Opticæ, part 2. § 1. p. 100 to 105; and Optics, book i. part 2. prop. 11.

‡ Optics, book i. part 2. prop. 4.

We may express this by saying that two compound colours may be *chromatically* identical, but *optically* different. The *optical* properties of light are those which have reference to its origin and propagation through media, till it falls on the sensitive organ of vision; the *chromatical* properties of light are those which have reference to its power of exciting certain sensations of *colour*, perceived through the organ of vision.

The investigation of the chromatic relations of the rays of the spectrum must therefore be founded upon observations of the apparent identity of compound colours, as seen by an eye either of the normal or of some abnormal type; and the results to which the investigation leads must be regarded as partaking of a physiological, as well as of a physical character, and as indicating certain laws of sensation, depending on the constitution of the organ of vision, which may be different in different individuals. We have to determine the laws of the composition of colours in general, to reduce the number of standard colours to the smallest possible, to discover, if we can, what they are, and to ascertain the relation which the homogeneous light of different parts of the spectrum bears to the standard colours.

§ II. *History of the Theory of Compound Colours.*

The foundation of the theory of the composition of colours was laid by NEWTON*. He first shows that, by the mixture of homogeneal light, colours may be produced which are "like to the colours of homogeneal light as to the appearance of colour, but not as to the immutability of colour and constitution of light." Red and yellow give an orange colour, which is chromatically similar to the orange of the spectrum, but optically different, because it is resolved into its component colours by a prism, while the orange of the spectrum remains unchanged. When the colours to be mixed lie at a distance from one another in the spectrum, the resultant appears paler than that intermediate colour of the spectrum which it most resembles; and when several are mixed, the resultant may appear white. NEWTON† is always careful, however, not to call any mixture white, unless it agrees with common white light in its optical as well as its chromatical properties, and is a mixture of *all* the homogeneal colours. The theory of compound colours is first presented in a mathematical form in prop. 6, "*In a mixture of primary colours, the quantity and quality of each being given, to know the colour of the compound.*" He divides the circumference of a circle into seven parts, proportional to the seven musical intervals, in accordance with his opinion about the proportions of the colours in the spectrum. At the centre of gravity of each of these arcs he places a little circle, whose area is proportional to the number of rays of the corresponding colour which enter into the given mixture. The position of the centre of gravity of all these circles indicates the nature of the resultant colour. A radius drawn through it points out that colour of the spectrum which it most resembles, and the distance from the centre determines the fulness of its colour.

With respect to this construction, NEWTON says, "This rule I conceive accurate enough

* Optics, book i. part 2. props. 4, 5, 6.

† 7th and 8th Letters to Oldenburg.

for practice, though not mathematically accurate." He gives no reasons for the different parts of his rule, but we shall find that his method of finding the centre of gravity of the component colours is completely confirmed by my observations, and that it involves mathematically the theory of three elements of colour; but that the disposition of the colours on the circumference of a circle was only a provisional arrangement, and that the true relations of the colours of the spectrum can only be determined by direct observation.

YOUNG* appears to have originated the theory, that the three elements of colour are determined as much by the constitution of the sense of sight as by anything external to us. He conceives that three different sensations may be excited by light, but that the proportion in which each of the three is excited depends on the nature of the light. He conjectures that these primary sensations correspond to red, green, and violet. A blue ray, for example, though homogeneous in itself, he conceives capable of exciting both the green and the violet sensation, and therefore he would call blue a compound colour, though the colour of a simple kind of light. The *quality* of any colour depends, according to this theory, on the *ratios* of the intensities of the three sensations which it excites, and its *brightness* depends on the *sum* of these three intensities.

Sir DAVID BREWSTER, in his paper entitled "On a New Analysis of Solar Light, indicating three Primary Colours, forming Coincident Spectra of equal length†," regards the actual colours of the spectrum as arising from the intermixture, in various proportions, of three primary kinds of light, red, yellow, and blue, each of which is variable in intensity, but uniform in colour, from one end of the spectrum to the other; so that every colour in the spectrum is really compound, and might be shown to be so if we had the means of separating its elements.

Sir DAVID BREWSTER, in his researches, employed coloured media, which, according to him, absorb the three elements of a single prismatic colour in different degrees, and change their proportions, so as to alter the colour of the light, without altering its refrangibility.

In this paper I shall not enter into the very important questions affecting the physical theory of light, which can only be settled by a careful inquiry into the phenomena of absorption. The physiological facts, that we have a threefold sensation of colour, and that the three elements of this sensation are affected in different proportions by light of different refrangibilities, are equally true, whether we adopt the physical theory that there are three kinds of light corresponding to these three colour-sensations, or whether we regard light of definite refrangibility as an undulation of known length, and therefore variable only in intensity, but capable of producing different chemical actions on different substances, of being absorbed in different degrees by different media, and of exciting in different degrees the three different colour-sensations of the human eye.

* YOUNG's Lectures on Natural Philosophy, KELLAND's Edition, p. 345, or Quarto, 1807, vol. i. p. 441 see also YOUNG in Philosophical Transactions, 1801, or Works in Quarto, vol. ii. p. 617.

† Transactions of the Royal Society of Edinburgh, vol. xii. p. 123.

Sir DAVID BREWSTER has given a diagram of three curves, in which the base-line represents the length of the spectrum, and the ordinates of the curves represent, by estimation, the intensities of the three kinds of light at each point of the spectrum. I have employed a diagram of the same kind to express the results arrived at in this paper, the ordinates being made to represent the intensities of each of the three elements of colour, as calculated from the experiments.

The most complete series of experiments on the mixture of the colours of the spectrum, is that of Professor HELMHOLTZ* of Königsberg. By using two slits at right angles to one another, he formed two pure spectra, the fixed lines of which were seen crossing one another when viewed in the ordinary way by means of a telescope. The colours of these spectra were thus combined in every possible way, and the effect of the combination of any two could be seen separately by drawing the eye back from the eyepiece of the telescope, when the compound colour was seen by itself at the eye-hole. The proportion of the components was altered by turning the combined slits round in their own plane.

One result of these experiments was, that a colour, chromatically identical with white, could be formed by combining yellow with indigo. M. HELMHOLTZ was not then able to produce white with any other pair of simple colours, and considered that three simple colours were required in general to produce white, one from each of the three portions into which the spectrum is divided by the yellow and indigo.

Professor GRASSMANN† showed that NEWTON'S theory of compound colours implies that there are an infinite number of pairs of complementary colours in the spectrum, and pointed out the means of finding them. He also showed how colours may be represented by lines, and combined by the method of the parallelogram.

In a second memoir‡, M. HELMHOLTZ describes his method of ascertaining these pairs of complementary colours. He formed a pure spectrum by means of a slit, a prism, and a lens; and in this spectrum he placed an apparatus having two parallel slits which were capable of adjustment both in position and breadth, so as to let through any two portions of the spectrum, in any proportions. Behind this slit, these rays were united in an image of the prism, which was received on paper. By arranging the slits, the colour of this image may be reduced to white, and made identical with that of paper illuminated with white light. The wave-lengths of the component colours were then measured by observing the angle of diffraction through a grating. It was found that the colours from red to green-yellow ($\lambda=2082$) were complementary to colours ranging from green-blue ($\lambda=1818$) to violet, and that the colours between green-yellow and green-blue have no homogeneous complementaries, but must be neutralized by mixtures of red and violet.

M. HELMHOLTZ also gives a provisional diagram of the curve formed by the spectrum on NEWTON'S diagram, for which his experiments did not furnish him with the complete data.

* POGGENDORFF'S Annalen, Band lxxxvii. (Philosophical Magazine, 1852, December.)

† POGGENDORFF'S Annalen, Band lxxxix. (Philosophical Magazine, 1854, April.) ‡ Ibid. Band xciv.

Accounts of experiments by myself on the mixture of artificial colours by rapid rotation, may be found in the Transactions of the Royal Society of Edinburgh, vol. xxi. pt. 2 (1855); in an appendix to Professor George Wilson's work on Colour-Blindness; in the Report of the British Association for 1856, p. 12; and in the Philosophical Magazine, July 1857, p. 40. These experiments show that, for the normal eye, there are three, and only three, elements of colour, and that in the colour-blind one of these is absent. They also prove that chromatic observations may be made, both by normal and abnormal eyes, with such accuracy, as to warrant the employment of the results in the calculation of colour-equations, and in laying down colour-diagrams by Newton's rule.

The first instrument which I made (in 1852) to examine the mixtures of the colours of the spectrum was similar to that which I now use, but smaller, and it had no constant light for a term of comparison. The second was $6\frac{1}{2}$ feet long, made in 1855, and showed *two* combinations of colour side by side. I have now succeeded in making the mixture much more perfect, and the comparisons more exact, by using white reflected light, instead of the second compound colour. An apparatus in which the light passes through the prisms, and is reflected back again in nearly the same path by a concave mirror, was shown by me to the British Association in 1856. It has the advantage of being portable, and need not be more than half the length of the other, in order to produce a spectrum of equal length. I am so well satisfied with the working of this form of the instrument, that I intend to make use of it in obtaining equations from a greater variety of observers than I could meet with when I was obliged to use the more bulky instrument. It is difficult at first to get the observer to believe that the compound light can ever be so adjusted as to appear to his eyes identical with the white light in contact with it. He has to learn what adjustments are necessary to produce the requisite alteration under all circumstances, and he must never be satisfied till the two parts of the field are identical in colour and illumination. To do this thoroughly, implies not merely good eyes, but a power of judging as to the exact nature of the difference between two very pale and nearly identical tints, whether they differ in the amount of red, green, or blue, or in brightness of illumination.

In the following paper I shall first lay down the mathematical theory of Newton's diagram, with its relation to Young's theory of the colour-sensation. I shall then describe the experimental method of mixing the colours of the spectrum, and determining the wave-lengths of the colours mixed. The results of my experiments will then be given, and the chromatic relations of the spectrum exhibited in a system of colour-equations, in Newton's diagram, and in three curves of intensity, as in Brewster's diagram. The differences between the results of two observers will then be discussed, showing on what they depend, and in what way such differences may affect the vision of persons otherwise free from defects of sight.

§ III. *Mathematical Theory of* NEWTON'S *Diagram of Colours.*

NEWTON'S diagram is a plane figure, designed to exhibit the relations of colours to each other.

Every point in the diagram represents a colour, simple or compound, and we may conceive the diagram itself so painted, that every colour is found at its corresponding point. Any colour, differing only in quantity of illumination from one of the colours of the diagram, is referred to it as a unit, and is measured by the ratio of the illumination of the given colour to that of the corresponding colour in the diagram. In this way the *quantity* of a colour is estimated. The resultant of mixing any two colours of the diagram is found by dividing the line joining them inversely as the quantity of each; then, if the sum of these quantities is unity, the resultant will have the illumination as well as the colour of the point so found; but if the sum of the components is different from unity, the *quantity* of the resultant will be measured by the sum of the components.

This method of determining the position of the resultant colour is mathematically identical with that of finding the centre of gravity of two weights, and placing a weight equal to their sum at the point so found. We shall therefore speak of the resultant tint as the sum of its components placed at their centre of gravity.

By compounding this resultant tint with some other colour, we may find the position of a mixture of three colours, at the centre of gravity of its components; and by taking these components in different proportions, we may obtain colours corresponding to every part of the triangle of which they are the angular points. In this way, by taking any three colours we should be able to construct a triangular portion of NEWTON'S diagram by painting it with mixtures of the three colours. Of course these mixtures must be made to correspond with optical mixtures of light, not with mechanical mixtures of pigments.

Let us now take any colour belonging to a point of the diagram outside this triangle. To make the centre of gravity of the three weights coincide with this point, one or more of the weights must be made negative. This, though following from mathematical principles, is not capable of direct physical interpretation, as we cannot exhibit a negative colour.

The equation between the three selected colours, x, y, z, and the new colour u, may in the first case be written

$$u=x+y+z, \quad \ldots \quad (1.)$$

x, y, z being the quantities of colour required to produce u. In the second case suppose that z must be made negative,

$$u=x+y-z. \quad \ldots \quad (2.)$$

As we cannot realize the term $-z$ as a negative colour, we transpose it to the other side of the equation, which then becomes

$$u+z=x+y, \quad \ldots \quad (3.)$$

which may be interpreted to mean, that the resultant tint, $u+z$, is identical with the resultant, $x+y$. We thus find a mixture of the new colour with one of the selected colours, which is chromatically equivalent to a mixture of the other two selected colours.

When the equation takes the form

$$u = x - y - z, \quad \ldots \ldots \ldots \quad (4.)$$

two of the components being negative, we must transpose them thus,

$$u + y + z = x, \quad \ldots \ldots \ldots \quad (5.)$$

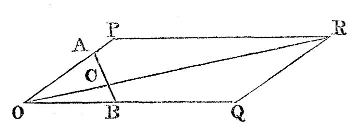
which means that a mixture of certain proportions of the new colour and two of the three selected, is chromatically equivalent to the third. We may thus in all cases find the relation between any three colours and a fourth, and exhibit this relation in a form capable of experimental verification; and by proceeding in this way we may map out the positions of all colours upon NEWTON'S diagram. Every colour in nature will then be defined by the position of the corresponding colour in the diagram, and by the ratio of its illumination to that of the colour in the diagram.

§ IV. *Method of representing Colours by Straight Lines drawn from a Point.*

To extend our ideas of the relations of colours, let us form a new geometrical conception by the aid of solid geometry.

Let us take as origin any point not in the plane of the diagram, and let us draw lines through this point to the different points of the diagram; then the direction of any of these lines will depend upon the position of the point of the diagram through which it passes, so that we may take this line as the representative of the corresponding colour on the diagram.

In order to indicate the *quantity* of this colour, let it be produced beyond the plane of the diagram in the same ratio as the given colour exceeds in illumination the colour on the diagram. In this way every colour in nature will be represented by a line drawn through the origin, whose *direction* indicates the *quality* of the colour, while its *length* indicates its *quantity*.



Let us find the resultant of two colours by this method. Let O be the origin and AB be a section of the plane of the diagram by that of the paper. Let OP, OQ be lines representing colours, A, B the corresponding points in the diagram; then the quantity of P will be $\frac{OP}{OA} = p$, and that of Q will be $\frac{OQ}{OB} = q$. The resultant of these will be represented in the diagram by the point C, where AC : CB :: $q : p$, and the quantity of the resultant will be $p+q$, so that if we produce OC to R, so that OR$=(p+q)$OC, the line OR will represent the resultant of OP and OQ in direction and magnitude. It is easy to prove, from this construction, that OR is the diagonal of the parallelogram of which OP and OQ are two sides. It appears therefore that if colours are represented in quantity and quality by the magnitude and direction of straight lines, the rule for the composition of colours is identical with that for the composition of forces in mechanics. This analogy has been well brought out by Professor GRASSMANN in POGGENDORFF'S 'Annalen,' Bd. lxxxix.

We may conceive an arrangement of actual colours in space founded upon this construction. Suppose each of these radiating lines representing a given colour to be itself illuminated with that colour, the brightness increasing from zero at the origin to unity, where it cuts the plane of the diagram, and becoming continually more intense in proportion to the distance from the origin. In this way every colour in nature may be matched, both in quality and quantity, by some point in this coloured space.

If we take any three lines through the origin as axes, we may, by coordinates parallel to these lines, express the position of any point in space. That point will correspond to a colour which is the resultant of the three colours represented by the three coordinates.

This system of coordinates is an illustration of the resolution of a colour into three components. According to the theory of YOUNG, the human eye is capable of three distinct primitive sensations of colour, which, by their composition in various proportions, produce the sensations of actual colour in all their varieties. Whether any kinds of light have the power of exciting these primitive sensations separately, has not yet been determined.

If colours corresponding to the three primitive sensations can be exhibited, then all colours, whether produced by light, disease, or imagination, are compounded of these, and have their places within the triangle formed by joining the three primaries. If the colours of the pure spectrum, as laid down on the diagram, form a triangle, the colours at the angles *may* correspond to the primitive sensations. If the curve of the spectrum does not reach the angles of the circumscribing triangle, then no colour in the spectrum, and therefore no colour in nature, corresponds to any of the three primary sensations.

The only data at present existing for determining the primary colours, are derived from the comparison of observations of colour-equations by colour-blind, and by normal eyes. The colour-blind equations differ from the others by the non-existence of one of the elements of colour, the relation of which to known colours can be ascertained. It appears, from observations made for me by two colour-blind persons*, that the elementary sensation which they do not possess is a red approaching to crimson, lying beyond both vermilion and carmine. These observations are confirmed by those of Mr. POLE, and by others which I have obtained since. I have hopes of being able to procure a set of colour-blind equations between the colours of the spectrum, which will indicate the missing primary in a more exact manner.

The experiments which I am going to describe have for their object the determination of the position of the colours of the spectrum upon NEWTON'S diagram, from actual observations of the mixtures of those colours. They were conducted in such a way, that in every observation the judgment of the observer was exercised upon two parts of an illuminated field, one of which was so adjusted as to be chromatically identical with the other, which, during the whole series of observations, remained of one constant

* Transactions of the Royal Society of Edinburgh, vol. xxi. pt. 2. p. 286.

intensity of white. In this way the effects of subjective colours were entirely got rid of, and all the observations were of the same kind, and therefore may claim to be equally accurate; which is not the case when comparisons are made between bright colours of different kinds.

The chart of the spectrum, deduced from these observations, exhibits the colours arranged very exactly along two sides of a triangle, the extreme red and violet forming doubtful portions of the third side. This result greatly simplifies the theory of colour, if it does not actually point out the three primary colours themselves.

§ V. *Description of an Instrument for making definite Mixtures of the Colours of the Spectrum.*

The experimental method which I have used consists in forming a combination of three colours belonging to different portions of the spectrum, the quantity of each being so adjusted that the mixture shall be white, and equal in intensity to a given white. Fig. 1, Plate I. represents the instrument for making the observations. It consists of two tubes, or long boxes, of deal, of rectangular section, joined together at an angle of about 100°.

The part AK is about 5 feet long, 7 inches broad, and 4 deep; KN is about 2 feet long, 5 inches broad, and 4 deep; BD is a partition parallel to the side of the long box. The whole of the inside of the instrument is painted black, and the only openings are at the end AC, and at E. At the angle there is a lid, which is opened when the optical parts have to be adjusted or cleaned.

At E is a fine vertical slit; L is a lens; at P there are two equilateral prisms. The slit E, the lens L, and the prisms P are so adjusted, that when light is admitted at E a *pure spectrum* is formed at AB, the extremity of the long box. A mirror at M is also adjusted so as to reflect the light from E along the narrow compartment of the long box to BC. See fig. 3.

At AB is placed the contrivance shown in fig. 2, Plate I. A′B′ is a rectangular frame of brass, having a rectangular aperture of 6×1 inches. On this frame are placed six brass sliders, XYZ. Each of these carries a knife-edge of brass in the plane of the surface of the frame.

These six moveable knife-edges form three slits, XYZ, whicn may be so adjusted as to coincide with any three portions of the pure spectrum formed by light from E. The intervals behind the sliders are closed by hinged shutters, which allow the sliders to move without letting light pass between them.

The inner edge of the brass frame is graduated to twentieths of an inch, so that the position of any slit can be read off. The breadth of the slit is ascertained by means of a wedge-shaped piece of metal, 6 inches long, and tapering to a point from a breadth of half an inch. This is gently inserted into each slit, and the breadth is determined by the distance to which it enters, the divisions on the wedge corresponding to the 200th of an inch difference in breadth, so that the unit of breadth is ·005 inch.

Now suppose light to enter at E, to pass through the lens, and to be refracted by the two prisms at P; a pure spectrum, showing FRAUNHOFER'S lines, is formed at AB, but only that part is allowed to pass which falls on the three slits XYZ. The rest is stopped by the shutters. Suppose that the portion falling on X belongs to the red part of the spectrum; then, of the white light entering at E, only the red will come through the slit X. If we were to admit red light at X it would be refracted to E, by the principle in Optics, that the course of any ray may be reversed. If, instead of red light, we were to admit white light at X, still only red light would come to E; for all other light would be either more or less refracted, and would not reach the slit at E. Applying the eye at the slit E, we should see the prism P uniformly illuminated with red light, of the kind corresponding to the part of the spectrum which falls on the slit X when white light is admitted at E.

Let the slit Y correspond to another portion of the spectrum, say the green; then, if white light is admitted at Y, the prism, as seen by an eye at E, will be uniformly illuminated with green light; and if white light be admitted at X and Y simultaneously, the colour seen at E will be a compound of red and green, the proportions depending on the breadth of the slits and the intensity of the light which enters them. The third slit, Z, enables us to combine any three kinds of light in any given proportions, so that an eye at E shall see the face of the prism at P uniformly illuminated with the colour resulting from the combination of the three. The position of these three rays in the spectrum is found by admitting the light at E, and comparing the position of the slits with the position of the principal fixed lines; and the breadth of the slits is determined by means of the wedge.

At the same time white light is admitted through BC to the mirror of black glass at M, whence it is reflected to E, past the edge of the prism at P, so that the eye at E sees through the lens a field consisting of two portions, separated by the edge of the prism; that on the left hand being compounded of three colours of the spectrum refracted by the prism, while that on the right hand is white light reflected from the mirror. By adjusting the slits properly, these two portions of the field may be made equal, both in colour and brightness, so that the edge of the prism becomes almost invisible.

In making experiments, the instrument was placed on a table in a room moderately lighted, with the end AB turned towards a large board covered with white paper, and placed in the open air, so as to be uniformly illuminated by the sun. In this way the three slits and the mirror M were all illuminated with white light of the same intensity, and all were affected in the same ratio by any change of illumination; so that if the two halves of the field were rendered equal when the sun was under a cloud, they were found nearly correct when the sun again appeared. No experiments, however, were considered good unless the sun remained uniformly bright during the whole series of experiments.

After each set of experiments light was admitted at E, and the position of the fixed lines D and F of the spectrum was read off on the scale at AB. It was found that after the instrument had been some time in use these positions were invariable, showing that the eye-hole, the prisms, and the scale might be considered as rigidly connected.

§ VI. *Method of determining the Wave-length corresponding to any point of the Spectrum on the Scale* AB.

Two plane surfaces of glass were kept apart by two parallel strips of goldbeaters' leaf, so as to enclose a stratum of air of nearly uniform thickness. Light reflected from this stratum of air was admitted at E, and the spectrum formed by it was examined at AB by means of a lens. This spectrum consists of a large number of bright bands, separated by dark spaces at nearly uniform intervals, these intervals, however, being considerably larger as we approach the violet end of the spectrum.

The reason of these alternations of brightness is easily explained. By the theory of NEWTON'S rings, the light reflected from a stratum of air consists of two parts, one of which has traversed a path longer than that of the other, by an interval depending on the thickness of the stratum and the angle of incidence. Whenever the interval of retardation is an exact multiple of a wave-length, these two portions of light destroy each other by interference; and when the interval is an odd number of half wave-lengths, the resultant light is a maximum.

In the ordinary case of NEWTON'S rings, these alternations depend upon the varying thickness of the stratum; while in this case a pencil of rays of different wave-lengths, but all experiencing the same retardation, is analysed into a spectrum, in which the rays are arranged in order of their respective wave-lengths. Every ray whose wave-length is an exact submultiple of the retardation will be destroyed by interference, and its place will appear dark in the spectrum; and there will be as many dark bands seen as there are rays whose wave-lengths fulfil this condition.

If, then, we observe the positions of the dark bands on the scale AB, the wave-lengths corresponding to these positions will be a series of submultiples of the retardation.

Let us call the first dark band visible on the red side of the spectrum zero, and let us number them in order 1, 2, 3, &c. towards the violet end. Let N be the number of undulations corresponding to the band zero which are contained in the retardation R; then if n be the number of any other band, $N+n$ will be the number of the corresponding wave-lengths in the retardation, or in symbols,

$$R=(N+n)\lambda. \qquad (6.)$$

Now observe the position of two of FRAUNHOFER'S fixed lines with respect to the dark bands, and let n_1, n_2 be their positions expressed in the number of bands, whole or fractional, reckoning from zero. Let λ_1, λ_2 be the wave-lengths of these fixed lines as determined by FRAUNHOFER, then

$$R=(N+n_1)\lambda_1=(N+n_2)\lambda_2, \qquad (7.)$$

whence

$$N=\frac{n_2\lambda_2-n_1\lambda_1}{\lambda_1-\lambda_2}=\frac{(n_2-n_1)}{\lambda_1-\lambda_2}\lambda_2-n_1, \qquad (8.)$$

and

$$R=\frac{n_2-n_1}{\lambda_1-\lambda_2}\lambda_1\lambda_2. \quad (9.)$$

Having thus found N and R, we may find the wave-length corresponding to the dark band n from the formula

$$\lambda=\frac{R}{N+n}. \quad (10.)$$

In my experiments the line D corresponded with the seventh dark band, and F was between the 15th and 16th, so that $n_2=15\cdot7$. Here then for D,

$$\left.\begin{array}{ll} n_1=7 & \lambda_1=2175 \\ n_2=15\cdot7 & \lambda_2=1794 \end{array}\right\}\text{in Fraunhofer's measure,} \quad (11.)$$

and for F,

whence we find $N=34,\ R=89175.$ (12.)

There were 22 bands visible, corresponding to 22 different positions on the scale AB, as determined 4th August, 1859.

TABLE I.

Band.	Scale.	Band.	Scale.	Band.	Scale.
$n=$ 1	17	$n=$ 9	36	$n=$ 16	57
2	19	10	39	17	61
3	$21\frac{1}{4}$	11	42	18	65
4	$23\frac{1}{2}$	12	45	19	69
5	26	13	48	20	73
6	$28\frac{1}{2}$	14	51	21	77
7	31	15	54	22	82
8	$33\frac{1}{2}$				

Sixteen equidistant points on the scale were chosen for standard colours in the experiments to be described. The following Table gives the reading on the scale AB, the value of $N+n$, and the calculated wave-length for each of these:—

TABLE II.

Scale.	$(N+n)$.	Wave-length.	Colour.
20	36·4	2450	Red.
24	38·3	2328	Scarlet.
28	39·8	2240	Orange.
32	41·4	2154	Yellow.
36	42·9	2078	Yellow-green.
40	44·3	2013	Green.
44	45·7	1951	Green.
48	47·0	1879	Bluish green.
52	48·3	1846	Blue-green.
56	49·6	1797	Greenish blue.
60	50·8	1755	Blue.

TABLE II. (continued.)

Scale.	(N+*n*).	Wave-length.	Colour.
64	51·8	1721	Blue.
68	52·8	1688	Blue.
72	53·7	1660	Indigo.
76	54·7	1630	Indigo.
80	55·6	1604	Indigo.

Having thus selected sixteen distinct points of the spectrum on which to operate, and determined their wave-lengths and apparent colours, I proceeded to ascertain the mathematical relations between these colours in order to lay them down on NEWTON'S diagram. For this purpose I selected three of these as points of reference, namely, those at 24, 44, and 68 of the scale. I chose these points because they are well separated from each other on the scale, and because the colour of the spectrum at these points does not appear to the eye to vary very rapidly, either in hue or brightness, in passing from one point to another. Hence a small error of position will not make so serious an alteration of colour at these points, as if we had taken them at places of rapid variation; and we may regard the amount of the illumination produced by the light entering through the slits in these positions as sensibly proportional to the breadth of the slits.

(24) corresponds to a bright scarlet about one-third of the distance from C to D; (44) is a green very near the line E; and (68) is a blue about one-third of the distance from F to G.

§ VII. *Method of Observation.*

The instrument is turned with the end AB towards a board, covered with white paper, and illuminated by sunlight. The operator sits at the end AB, to move the sliders, and adjust the slits; and the observer sits at the end E, which is shaded from any bright light. The operator then places the slits so that their centres correspond to the three standard colours, and adjusts their breadths till the observer sees the prism illuminated with pure white light of the same intensity with that reflected by the mirror M. In order to do this, the observer must tell the operator what difference he observes in the two halves of the illuminated field, and the operator must alter the breadth of the slits accordingly, always keeping the centre of each slit at the proper point of the scale. The observer may call for more or less red, blue or green; and then the operator must increase or diminish the width of the slits X, Y, and Z respectively. If the variable field is darker or lighter than the constant field, the operator must widen or narrow all the slits in the same proportion. When the variable part of the field is nearly adjusted, it often happens that the constant white light from the mirror appears tinged with the complementary colour. This is an indication of what is required to make the resemblance of the two parts of the field of view perfect. When no difference can be detected between the two parts of the field, either in colour or in brightness, the observer must look away for some time, to relieve the strain on the eye, and then look again. If the eye thus refreshed still judges the two parts of the field to be equal, the observation may be considered complete, and the operator must measure the breadth of each slit by means of the wedge, as before described, and write down the result as a colour-equation, thus—

Oct. 18, J. $18\cdot5(24)+27(44)+37(68)=W^{*}$. . . . . (13.)

This equation means that on the 18th of October the observer J. (myself) made an observation in which the breadth of the slit X was 18·5, as measured by the wedge, while its centre was at the division (24) of the scale; that the breadths of Y and Z were 27 and 37, and their positions (44) and (68); and that the illumination produced by these slits was exactly equal, in my estimation as an observer, to the constant white W.

The position of the slit X was then shifted from (24) to (28), and when the proper adjustments were made, I found a second colour-equation of this form—

Oct. 18, J. $16(28)+21(44)+37(68)=W$. . . . . (14.)

Subtracting one equation from the other and remembering that the figures in brackets are merely symbols of position, not of magnitude, we find

$$16(28)=18\cdot5(24)+6(44), \quad \ldots \quad (15.)$$

showing that (28) can be made up of (24) and (44), in the proportion of 18·5 to 6.

In this way, by combining each colour with two standard colours, we may produce a white equal to the constant white. The red and yellow colours from (20) to (32) must be combined with green and blue, the greens from (36) to (52) with red and blue, and the blues from (56) to (80) with red and green.

The following is a specimen of an actual series of observations made in this way by another observer (K.):—

Table III.

Oct. 13, 1859. Observer (K).

(X).	(Y).	(Z).	
$18\frac{1}{2}(24)$	$+32\frac{1}{2}(44)$	$+32\ (68)$	$=W^{*}$.
$17\frac{1}{2}(24)$	$+32\frac{1}{2}(44)$	$+63\ (80)$	$=W$.
$18\ (24)$	$+32\frac{1}{2}(44)$	$+35\ (72)$	$=W$.
$19\ (24)$	$+32\ (44)$	$+31\frac{1}{2}(68)$	$=W^{*}$.
$19\ (24)$	$+30\frac{1}{2}(44)$	$+35\ (64)$	$=W$.
$20\ (24)$	$+23\ (44)$	$+39\ (60)$	$=W$.
$21\ (24)$	$+14\ (44)$	$+58\ (56)$	$=W$.
$22\ (24)$	$+62\ (52)$	$+11\ (68)$	$=W$.
$22\ (24)$	$+42\ (48)$	$+29\frac{1}{2}(68)$	$=W$.
$19\ (24)$	$+31\frac{1}{2}(44)$	$+33\ (68)$	$=W^{*}$.
$16\ (24)$	$+28\ (40)$	$+32\frac{1}{2}(68)$	$=W$.
$6\ (24)$	$+27\ (36)$	$+32\frac{1}{2}(68)$	$=W$.
$23\ (32)$	$+11\frac{1}{2}(44)$	$+32\frac{1}{2}(68)$	$=W$.
$17\ (28)$	$+26\ (44)$	$+32\frac{1}{2}(68)$	$=W$.
$20\ (24)$	$+33\frac{1}{2}(44)$	$+32\frac{1}{2}(68)$	$=W^{*}$.
$46\ (20)$	$+33\ (44)$	$+30\ (68)$	$=W$.

The equations marked with an asterisk (*) are those which involve the three standard colours, and since every other equation must be compared with them, they must be often repeated.

The following Table contains the *means* of four sets of observations by the same observer (K):—

TABLE IV. (K.)

$$44{\cdot}3(20)+31{\cdot}0(44)+27{\cdot}7(68)=W.$$
$$16{\cdot}1(28)+25{\cdot}6(44)+30{\cdot}6(68)=W.$$
$$22{\cdot}0(32)+12{\cdot}1(44)+30{\cdot}6(68)=W.$$
$$6{\cdot}4(24)+25{\cdot}2(36)+31{\cdot}3(68)=W.$$
$$15{\cdot}3(24)+26{\cdot}0(40)+30{\cdot}7(68)=W.$$
$$19{\cdot}8(24)+35{\cdot}0(46)+30{\cdot}2(68)=W.$$
$$21{\cdot}2(24)+41{\cdot}4(48)+27{\cdot}0(68)=W.$$
$$22{\cdot}0(24)+62{\cdot}0(52)+13{\cdot}0(68)=W.$$
$$21{\cdot}7(24)+10{\cdot}4(44)+61{\cdot}7(56)=W.$$
$$20{\cdot}5(24)+23{\cdot}7(44)+40{\cdot}5(60)=W.$$
$$19{\cdot}7(24)+30{\cdot}3(44)+33{\cdot}7(64)=W.$$
$$18{\cdot}0(24)+31{\cdot}2(44)+32{\cdot}3(72)=W.$$
$$17{\cdot}5(24)+30{\cdot}7(44)+44{\cdot}0(76)=W.$$
$$18{\cdot}3(24)+33{\cdot}2(44)+63{\cdot}7(80)=W.$$

§ VIII. *Determination of the Average Error in Observations of different kinds.*

In order to estimate the degree of accuracy of these observations, I have taken the differences between the values of the three standard colours as originally observed, and their means as given by the above Table. The sum of all the errors of the red (24) from the means, was 31·1, and the number of observations was 42, which gives the average error ·74.

The sum of errors in green (44) was 48·0, and the number of observations 31, giving a mean error 1·55.

The sum of the errors in blue (68) was 46·9, and the number of observations 35, giving a mean error 1·16.

It appears therefore that in the observations generally, the average error does not exceed 1·5; and therefore the results, if confirmed by several observations, may safely be trusted to that degree of accuracy.

The equation between the three standard colours was repeatedly observed, in order to detect any alteration in the character of the light, or any other change of condition which would prevent the observations from being comparable with one another; and also because this equation is used in the reduction of all the others, and therefore requires to be carefully observed. There are twenty observations of this equation, the mean of which gives

$$18{\cdot}6(24)+31{\cdot}4(44)+30{\cdot}5(68)=W^{*} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16.)$$

as the standard equation.

We may use the twenty observations of this equation as a means of determining the relations between the errors in the different colours, and thus of estimating the accuracy of the observer in distinguishing colours.

The following Table gives the result of these operations, where R stands for (24), G for (44), and B for (68):—

TABLE V.—Mean Errors in the Standard Equation.

(R)= ·54	(G−B)=·99	(G+B)=2·31	$\sqrt{G^2+B^2}=1{\cdot}67$
(G)=1·22	(B−R)=·85	(B+R)=1·59	$\sqrt{B^2+R^2}=1{\cdot}26$
(B)=1·15	(R−G)=·86	(R+G)=1·57	$\sqrt{R^2+G^2}=1{\cdot}33$
(R+G+B)=2·67		$\sqrt{R^2+G^2+B^2}=1{\cdot}76$	

The first column gives the mean difference between the observed value of each of the colours and the mean of all the observations. The second column shows the average error of the observed *differences* between the values of the standards, from the mean value of those differences. The third column shows the average error of the *sums* of two standards, from the mean of such sums. The fourth column gives the square root of the sum of the squares of the quantities in the first column. I have also given the average error of the sum of R, G and B, from its mean value, and the value of $\sqrt{R^2+G^2+B^2}$.

It appears from the first column that the red is more accurately observed than the green and blue.

§ IX. *Relative Accuracy in Observations of Colour and of Brightness.*

If the errors in the different colours occurred perfectly independent of each other, then the probable mean error in the sum or difference of any two colours would be the square root of the sum of their squares, as given in the fourth column. It will be seen, however, that the number in the second column is always less, and that in the third always greater, than that in the fourth; showing that the errors are not independent of each other, but that positive errors in any colour coincide more often with positive than with negative errors in another colour. Now the *hue* of the resultant depends on the *ratios* of the components, while its *brightness* depends on their sum. Since, therefore, the difference of two colours is always more accurately observed than their sum, variations of *colour* are more easily detected than variations in *brightness*, and the eye appears to be a more accurate judge of the identity of colour of the two parts of the field than of their equal illumination. The same conclusion may be drawn from the value of the mean error of the sum of the three standards, which is 2·67, while the square root of the sum of the squares of the errors is 1·76.

§ X. *Reduction of the Observations.*

By eliminating W from the equations of page 71 by means of the standard equation, we obtain equations involving each of the fourteen selected colours of the spectrum, along with the three standard colours; and by transposing the selected colour to one side of the equation, we obtain its value in terms of the three standards. If any of the terms of these equations are negative, the equation has no physical interpretation as it stands, but by transposing the negative term to the other side it becomes positive, and then the equation may be verified.

The following Table contains the values of the fourteen selected tints in terms of the standards. To avoid repetition, the symbols of the standard colours are placed at the head of each column.

Table VI.

Observer (K.).	(24.)	(44.)	(68.)
44·3(20)=	18·6	+ 0·4	+ 2·8
16·1(28)=	18·6	+ 5·8	− 0·1
22·0(32)=	18·6	+19·3	− 0·1
25·2(36)=	12·2	+31·4	− 0·8
26·0(40)=	3·3	+31·4	− 0·2
35·0(46)=	− 1·2	+31·4	+ 0·3
41·4(48)=	− 2·6	+31·4	+ 3·5
62·0(52)=	− 3·4	+31·4	+17·5
61·7(56)=	− 3·1	+21·0	+30·5
40·5(60)=	− 1·9	+ 7·7	+30·5
33·7(64)=	− 1·1	+ 1·1	+30·5
32·3(72)=	+ 0·6	+ 0·2	+30·5
44·0(76)=	+ 1·1	+ 0·7	+30·5
63·7(80)=	+ 0·3	− 1·8	+30·5

From these equations we may lay down a chart of the spectrum on Newton's diagram by the following method:—Take any three points, A, B, C, and let A represent the standard colour (24), B (44), and C (68). Then, to find the position of any other colour, say (20), divide AC in P so that (18·6)AP=(2·8)PC, and then divide BP in Q so that (18·6+2·8)PQ=(0·4)QB. At the point Q the colour corresponding to (20) must be placed. In this way the diagram of fig. 4. Plate I. has been constructed from the observations of all the colours.