Finally, comparative observations made simultaneously on the borders of the lake, within a few feet of the water, gave the following result:—

Temperature of the surface of the gravel		. . .	9°·90 C.
Temperature of the air 3 inches above the ground.			10·40
" "	6 feet	"	. 10·55
" "	15 feet	"	. 10·62

showing that the immediate neighbourhood of the water is sufficient to modify the results generally obtained on land.

The following conclusions may, I think, be safely drawn from the foregoing observations:—

1. The gradual increase of temperature occurring on ascending through the lower strata of the atmosphere, which appears constantly to prevail on land about and after sunset, is not apparent above a large surface of water.

2. The immediate vicinity of a large sheet of water is sufficient to modify to a considerable extent the effects of the nocturnal radiation of the earth, and thereby materially diminish the increase of temperature observed under ordinary circumstances on ascending above the surface of the ground.

3. One cannot help being struck by the great difference (amounting to between 2 and 3 Centigrade degrees) constantly observed between the temperature of the atmosphere a few feet above the ground, and that of the air at the same height above a large sheet of water.

III. *On Physical Lines of Force.* By J. C. Maxwell, *F.R.S., Professor of Natural Philosophy in King's College, London**.

Part III.—*The Theory of Molecular Vortices applied to Statical Electricity.*

In the first part of this paper† I have shown how the forces acting between magnets, electric currents, and matter capable of magnetic induction may be accounted for on the hypothesis of the magnetic field being occupied with innumerable vortices of revolving matter, their axes coinciding with the direction of the magnetic force at every point of the field.

The centrifugal force of these vortices produces pressures distributed in such a way that the final effect is a force identical in direction and magnitude with that which we observe.

In the second part‡ I described the mechanism by which these rotations may be made to coexist, and to be distributed according to the known laws of magnetic lines of force.

* Communicated by the Author.
† Phil. Mag. March 1861.
‡ Phil. Mag. April and May 1861.

I conceived the rotating matter to be the substance of certain cells, divided from each other by cell-walls composed of particles which are very small compared with the cells, and that it is by the motions of these particles, and their tangential action on the substance in the cells, that the rotation is communicated from one cell to another.

I have not attempted to explain this tangential action, but it is necessary to suppose, in order to account for the transmission of rotation from the exterior to the interior parts of each cell, that the substance in the cells possesses elasticity of figure, similar in kind, though different in degree, to that observed in solid bodies. The undulatory theory of light requires us to admit this kind of elasticity in the luminiferous medium, in order to account for transverse vibrations. We need not then be surprised if the magneto-electric medium possesses the same property.

According to our theory, the particles which form the partitions between the cells constitute the matter of electricity. The motion of these particles constitutes an electric current; the tangential force with which the particles are pressed by the matter of the cells is electromotive force, and the pressure of the particles on each other corresponds to the tension or potential of the electricity.

If we can now explain the condition of a body with respect to the surrounding medium when it is said to be "charged" with electricity, and account for the forces acting between electrified bodies, we shall have established a connexion between all the principal phenomena of electrical science.

We know by experiment that electric tension is the same thing, whether observed in statical or in current electricity; so that an electromotive force produced by magnetism may be made to charge a Leyden jar, as is done by the coil machine.

When a difference of tension exists in different parts of any body, the electricity passes, or tends to pass, from places of greater to places of smaller tension. If the body is a conductor, an actual passage of electricity takes place; and if the difference of tensions is kept up, the current continues to flow with a velocity proportional inversely to the resistance, or directly to the conductivity of the body.

The electric resistance has a very wide range of values, that of the metals being the smallest, and that of glass being so great that a charge of electricity has been preserved* in a glass vessel for years without penetrating the thickness of the glass.

Bodies which do not permit a current of electricity to flow through them are called insulators. But though electricity does

* By Professor W. Thomson.

not flow through them, electrical effects are propagated through them, and the amount of these effects differs according to the nature of the body; so that equally good insulators may act differently as dielectrics*.

Here then we have two independent qualities of bodies, one by which they allow of the passage of electricity through them, and the other by which they allow of electrical action being transmitted through them without any electricity being allowed to pass. A conducting body may be compared to a porous membrane which opposes more or less resistance to the passage of a fluid, while a dielectric is like an elastic membrane which may be impervious to the fluid, but transmits the pressure of the fluid on one side to that on the other.

As long as electromotive force acts on a conductor, it produces a current which, as it meets with resistance, occasions a continual transformation of electrical energy into heat, which is incapable of being restored again as electrical energy by any reversion of the process.

Electromotive force acting on a dielectric produces a state of polarization of its parts similar in distribution to the polarity of the particles of iron under the influence of a magnet†, and, like the magnetic polarization, capable of being described as a state in which every particle has its poles in opposite conditions.

In a dielectric under induction, we may conceive that the electricity in each molecule is so displaced that one side is rendered positively, and the other negatively electrical, but that the electricity remains entirely connected with the molecule, and does not pass from one molecule to another.

The effect of this action on the whole dielectric mass is to produce a general displacement of the electricity in a certain direction. This displacement does not amount to a current, because when it has attained a certain value it remains constant, but it is the commencement of a current, and its variations constitute currents in the positive or negative direction, according as the displacement is increasing or diminishing. The amount of the displacement depends on the nature of the body, and on the electromotive force; so that if h is the displacement, R the electromotive force, and E a coefficient depending on the nature of the dielectric,

$$R = -4\pi E^2 h;$$

and if r is the value of the electric current due to displacement,

$$r = \frac{dh}{dt}.$$

* Faraday, 'Experimental Researches,' Series XI.

† See Prof. Mossotti, "Discussione Analitica," *Memorie della Soc. Italiana* (Modena), vol. xxiv. part 2. p. 49.

These relations are independent of any theory about the internal mechanism of dielectrics; but when we find electromotive force producing electric displacement in a dielectric, and when we find the dielectric recovering from its state of electric displacement with an equal electromotive force, we cannot help regarding the phenomena as those of an elastic body, yielding to a pressure, and recovering its form when the pressure is removed.

According to our hypothesis, the magnetic medium is divided into cells, separated by partitions formed of a stratum of particles which play the part of electricity. When the electric particles are urged in any direction, they will, by their tangential action on the elastic substance of the cells, distort each cell, and call into play an equal and opposite force arising from the elasticity of the cells. When the force is removed, the cells will recover their form, and the electricity will return to its former position.

In the following investigation I have considered the relation between the displacement and the force producing it, on the supposition that the cells are spherical. The actual form of the cells probably does not differ from that of a sphere sufficiently to make much difference in the numerical result.

I have deduced from this result the relation between the statical and dynamical measures of electricity, and have shown, by a comparison of the electro-magnetic experiments of MM. Kohlrausch and Weber with the velocity of light as found by M. Fizeau, that the elasticity of the magnetic medium in air is the same as that of the luminiferous medium, if these two coexistent, coextensive, and equally elastic media are not rather one medium.

It appears also from Prop. XV. that the attraction between two electrified bodies depends on the value of E^2, and that therefore it would be less in turpentine than in air, if the quantity of electricity in each body remains the same. If, however, the *potentials* of the two bodies were given, the attraction between them would vary inversely as E^2, and would be greater in turpentine than in air.

Prop. XII.—To find the conditions of equilibrium of an elastic sphere whose surface is exposed to normal and tangential forces, the tangential forces being proportional to the sine of the distance from a given point on the sphere.

Let the axis of z be the axis of spherical coordinates.

Let ξ, η, ζ be the displacements of any particle of the sphere in the directions of x, y, and z.

Let p_{xx}, p_{yy}, p_{zz} be the stresses normal to planes perpendicular to the three axes, and let p_{yz}, p_{zx}, p_{xy} be the stresses of distortion in the planes yz, zx, and xy.

Let μ be the coefficient of cubic elasticity, so that if

$$p_{xx}=p_{yy}=p_{zz}=p,$$

$$p=\mu\left(\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz}\right). \quad . \quad . \quad . \quad . \quad (80)$$

Let m be the coefficient of rigidity, so that

$$p_{xx}-p_{yy}=m\left(\frac{d\xi}{dx}-\frac{d\eta}{dy}\right), \text{ \&c.} \quad . \quad . \quad . \quad . \quad . \quad (81)$$

Then we have the following equations of elasticity in an isotropic medium,

$$p_{xx}=(\mu-\tfrac{1}{3}m)\left(\frac{d\xi}{dx}+\frac{d\eta}{dy}+\frac{d\zeta}{dz}\right)+m\frac{d\xi}{dx}; \quad . \quad (82)$$

with similar equations in y and z, and also

$$p_{yz}=\frac{m}{2}\left(\frac{d\eta}{dz}+\frac{d\zeta}{dy}\right), \text{ \&c.} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (83)$$

In the case of the sphere, let us assume the radius $= a$, and

$$\xi=exz, \quad \eta=exy, \quad \zeta=f(x^2+y^2)+gz^2+d. \quad . \quad (84)$$

Then

$$\left.\begin{aligned} p_{xx}&=2(\mu-\tfrac{1}{3}m)(e+g)z+mez=p_{yy},\\ p_{zz}&=2(\mu-\tfrac{1}{3}m)(e+g)z+2mgz,\\ p_{yz}&=\frac{m}{2}(e+2f)y,\\ p_{zx}&=\frac{m}{2}(e+2f)z,\\ p_{xy}&=0. \end{aligned}\right\} \quad . \quad . \quad . \quad (85)$$

The equation of internal equilibrium with respect to z is

$$\frac{d}{dx}p_{zx}+\frac{d}{dy}p_{yz}+\frac{d}{dz}p_{zz}=0, \quad . \quad . \quad . \quad . \quad (86)$$

which is satisfied in this case if

$$m(e+2f+2g)+2(\mu-\tfrac{1}{3}m)(e+g)=0. \quad . \quad (87)$$

The tangential stress on the surface of the sphere, whose radius is a at an angular distance θ from the axis in plane xz,

$$\mathrm{T}=(p_{xx}-p_{zz})\sin\theta\cos\theta+p_{xz}(\cos^2\theta-\sin^2\theta) \quad . \quad . \quad (88)$$

$$=2m(e+f-g)a\sin\theta\cos^2\theta-\frac{ma}{2}(e+2f)\sin\theta. \quad . \quad (89)$$

In order that T may be proportional to $\sin\theta$, the first term must vanish, and therefore

$$g=e+f, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (90)$$

$$\mathrm{T}=-\frac{ma}{2}(e+2f)\sin\theta. \quad . \quad . \quad . \quad (91)$$

The normal stress on the surface at any point is

$$\mathrm{N}=p_{zz}\sin^2\theta+p_{yy}\cos^2\theta+2p_{xz}\sin\theta\cos\theta$$

$$=4(\mu-\tfrac{1}{3}m)(e+g)a\cos\theta+2ma\cos\theta\big((e+f)\sin^2\theta+g\cos^2\theta\big); \; (92)$$

or by (87) and (90),

$$\mathrm{N}=-ma(e+2f)\cos\theta. \quad . \quad . \quad . \quad . \quad (93)$$

The tangential displacement of any point is

$$t=\xi\cos\theta-\zeta\sin\theta=-(a^2f+d)\sin\theta. \quad . \quad . \quad . \quad (94)$$

The normal displacement is

$$n=\xi\sin\theta+\zeta\cos\theta=\big(a^2(e+f)+d\big)\cos\theta. \quad . \quad . \quad (95)$$

If we make

$$a^2(e+f)+d=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (96)$$

there will be no normal displacement, and the displacement will be entirely tangential, and we shall have

$$t=a^2e\sin\theta. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (97)$$

The whole work done by the superficial forces is

$$\mathrm{U}=\tfrac{1}{2}\Sigma(\mathrm{T}t)d\mathrm{S},$$

the summation being extended over the surface of the sphere.

The energy of elasticity in the substance of the sphere is

$$\mathrm{U}=\tfrac{1}{2}\Sigma\left(\frac{d\xi}{dx}p_{xx}+\frac{d\eta}{dy}p_{yy}+\frac{d\zeta}{dz}p_{zz}+\left(\frac{d\eta}{dz}+\frac{d\zeta}{dy}\right)p_{yz}+\left(\frac{d\zeta}{dx}+\frac{d\xi}{dz}\right)p_{zx}\right.$$
$$\left.+\left(\frac{d\xi}{dy}+\frac{d\eta}{dx}\right)p_{xy}\right)dV,$$

the summation being extended to the whole contents of the sphere.

We find, as we ought, that these quantities have the same value, namely

$$\mathrm{U}=-\tfrac{2}{3}\pi a^5me(e+2f). \quad . \quad . \quad . \quad . \quad (98)$$

We may now suppose that the tangential action on the surface arises from a layer of particles in contact with it, the particles being acted on by their own mutual pressure, and acting on the surfaces of the two cells with which they are in contact.

We assume the axis of z to be in the direction of maximum variation of the pressure among the particles, and we have to determine the relation between an electromotive force R acting on the particles in that direction, and the electric displacement h which accompanies it.

Prop. XIII.—To find the relation between electromotive force and electric displacement when a uniform electromotive force R acts parallel to the axis of z.

Take any element δS of the surface, covered with a stratum

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whose density is ρ, and having its normal inclined θ to the axes of z; then the tangential force upon it will be

$$\rho R\delta S \sin\theta = 2T\delta S, \qquad (99)$$

T being, as before, the tangential force on each side of the surface. Putting $\rho = \frac{1}{2\pi}$ as in equation (34)*, we find

$$R = -2\pi m a(e+2f). \qquad (100)$$

The displacement of electricity due to the distortion of the sphere is

$$\Sigma\delta S\tfrac{1}{2}\rho t \sin\theta \text{ taken over the whole surface}; \qquad (101)$$

and if h is the electric displacement per unit of volume, we shall have

$$\tfrac{4}{3}\pi a^3 h = \tfrac{2}{3}a^4 e, \qquad (102)$$

or

$$h = \frac{1}{2\pi} ae; \qquad (103)$$

so that

$$R = 4\pi^2 m \frac{e+2f}{e} h, \qquad (104)$$

or we may write

$$R = -4\pi E^2 h, \qquad (105)$$

provided we assume

$$E^2 = -\pi m \frac{e+2f}{e}. \qquad (106)$$

Finding e and f from (87) and (90), we get

$$E^2 = \pi m \frac{3}{1+\frac{5}{3}\frac{m}{\mu}}. \qquad (107)$$

The ratio of m to μ varies in different substances; but in a medium whose elasticity depends entirely upon forces acting between pairs of particles, this ratio is that of 6 to 5, and in this case

$$E^2 = \pi m. \qquad (108)$$

When the resistance to compression is infinitely greater than the resistance to distortion, as in a liquid rendered slightly elastic by gum or jelly,

$$E^2 = 3\pi m. \qquad (109)$$

The value of E^2 must lie between these limits. It is probable that the substance of our cells is of the former kind, and that we must use the first value of E^2, which is that belonging to

* Phil. Mag. April 1861.

a hypothetically "perfect" solid*, in which

$$5m = 6\mu, \qquad (110)$$

so that we must use equation (108).

Prop. XIV.—To correct the equations (9)† of electric currents for the effect due to the elasticity of the medium.

We have seen that electromotive force and electric displacement are connected by equation (105). Differentiating this equation with respect to t, we find

$$\frac{dR}{dt} = -4\pi E^2 \frac{dh}{dt}, \qquad (111)$$

showing that when the electromotive force varies, the electric displacement also varies. But a variation of displacement is equivalent to a current, and this current must be taken into account in equations (9) and added to r. The three equations then become

$$\left.\begin{aligned} p &= \frac{1}{4\pi}\left(\frac{d\gamma}{dy} - \frac{d\beta}{dz} - \frac{1}{E^2}\frac{dP}{dt}\right), \\ q &= \frac{1}{4\pi}\left(\frac{d\alpha}{dz} - \frac{d\gamma}{dx} - \frac{1}{E^2}\frac{dQ}{dt}\right), \\ r &= \frac{1}{4\pi}\left(\frac{d\beta}{dx} - \frac{d\alpha}{dy} - \frac{1}{E^2}\frac{dR}{dt}\right), \end{aligned}\right\} \qquad (112)$$

where p, q, r are the electric currents in the directions of x, y, and z; α, β, γ are the components of magnetic intensity; and P, Q, R are the electromotive forces. Now if e be the quantity of free electricity in unit of volume, then the equation of continuity will be

$$\frac{dp}{dx} + \frac{dq}{dy} + \frac{dr}{dz} + \frac{de}{dt} = 0. \qquad (113)$$

Differentiating (112) with respect to x, y, and z respectively, and substituting, we find

$$\frac{de}{dt} = \frac{1}{4\pi E^2}\frac{d}{dt}\left(\frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz}\right); \qquad (114)$$

whence

$$e = \frac{1}{4\pi E^2}\left(\frac{dP}{dx} + \frac{dQ}{dy} + \frac{dR}{dz}\right), \qquad (115)$$

the constant being omitted, because $e=0$ when there are no electromotive forces.

Prop. XV.—To find the force acting between two electrified bodies.

The energy in the medium arising from the electric displace-

* See Rankine "On Elasticity," Camb. and Dub. Math. Journ. 1851.
† Phil. Mag. March 1861.

ments is

$$U = -\Sigma\tfrac{1}{2}(Pf+Qg+Rh)\delta V, \quad . \quad . \quad . \quad (116)$$

where P, Q, R are the forces, and f, g, h the displacements. Now when there is no motion of the bodies or alteration of forces, it appears from equations (77)* that

$$P = -\frac{d\Psi}{dx}, \quad Q = -\frac{d\Psi}{dy}, \quad R = -\frac{d\Psi}{dz}; \quad . \quad (118)$$

and we know by (105) that

$$P = -4\pi E^2 f, \quad Q = -4\pi E^2 g, \quad R = -4\pi E^2 h; \quad . \quad (119)$$

whence

$$U = \frac{1}{8\pi E^2}\Sigma\left(\overline{\frac{d\Psi}{dx}}\Big|^2 + \overline{\frac{d\Psi}{dy}}\Big|^2 + \overline{\frac{d\Psi}{dz}}\Big|^2\right)\delta V. \quad . \quad (120)$$

Integrating by parts throughout all space, and remembering that Ψ vanishes at an infinite distance,

$$U = -\frac{1}{8\pi E^2}\Sigma\Psi\left(\frac{d^2\Psi}{dx^2} + \frac{d^2\Psi}{dy^2} + \frac{d^2\Psi}{dz^2}\right)\delta V; \quad (121)$$

or by (115),

$$U = \tfrac{1}{2}\Sigma(\Psi e)\delta V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (122)$$

Now let there be two electrified bodies, and let e_1 be the distribution of electricity in the first, and Ψ_1 the electric tension due to it, and let

$$e_1 = \frac{1}{4\pi E^2}\left(\frac{d^2\Psi_1}{dx^2} + \frac{d^2\Psi_1}{dy^2} + \frac{d^2\Psi_1}{dz^2}\right). \quad . \quad . \quad . \quad (123)$$

Let e_2 be the distribution of electricity in the second body, and Ψ_2 the tension due to it; then the whole tension at any point will be $\Psi_1 + \Psi_2$, and the expansion for U will become

$$U = \tfrac{1}{2}\Sigma(\Psi_1 e_1 + \Psi_2 e_2 + \Psi_1 e_2 + \Psi_2 e_1)\delta V. \quad . \quad . \quad (124)$$

Let the body whose electricity is e_1 be moved in any way, the electricity moving along with the body, then since the distribution of tension Ψ_1 moves with the body, the value of $\Psi_1 e_1$ remains the same.

$\Psi_2 e_2$ also remains the same; and Green has shown (Essay on Electricity, p. 10) that $\Psi_1 e_2 = \Psi_2 e_1$, so that the work done by moving the body against electric forces

$$W = \delta U = \delta\Sigma(\Psi_2 e_1)\delta V. \quad . \quad . \quad . \quad . \quad . \quad (125)$$

And if e_1 is confined to a small body,

$$W = e_1 \delta\Psi_2,$$

* Phil. Mag. May 1861.

or

$$F dr = e_1 \frac{d\Psi_2}{dr} dr, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (126)$$

where F is the resistance and dr the motion.

If the body e_2 be small, then if r is the distance from e_2, equation (123) gives

$$\Psi_2 = E^2\frac{e_2}{r};$$

whence

$$F = -E^2\frac{e_1 e_2}{r^2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (127)$$

or the force is a repulsion varying inversely as the square of the distance.

Now let η_1 and η_2 be the same quantities of electricity measured statically, then we know by definition of electrical quantity

$$F = -\frac{\eta_1\eta_2}{r^2}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (128)$$

and this will be satisfied provided

$$\eta_1 = Ee_1 \text{ and } \eta_2 = Ee_2; \quad . \quad . \quad . \quad . \quad . \quad (129)$$

so that the quantity E previously determined in Prop. XIII. is the number by which the electrodynamic measure of any quantity of electricity must be multiplied to obtain its electrostatic measure.

That electric current which, circulating round a ring whose area is unity, produces the same effect on a distant magnet as a magnet would produce whose strength is unity and length unity placed perpendicularly to the plane of the ring, is a unit current; and E units of electricity, measured statically, traverse the section of this current in one second,—these units being such that any two of them, placed at unit of distance, repel each other with unit of force.

We may suppose either that E units of positive electricity move in the positive direction through the wire, or that E units of negative electricity move in the negative direction, or, thirdly, that $\frac{1}{2}$E units of positive electricity move in the positive direction, while $\frac{1}{2}$E units of negative electricity move in the negative direction at the same time.

The last is the supposition on which MM. Weber and Kohlrausch* proceed, who have found

$$\tfrac{1}{2}E = 155{,}370{,}000{,}000, \quad . \quad . \quad . \quad . \quad (130)$$

the unit of length being the millimetre, and that of time being one second, whence

$$E = 310{,}740{,}000{,}000. \quad . \quad . \quad . \quad . \quad (131)$$

* *Abhandlungen der König. Sächsischen Gesellschaft*, vol. iii. (1857), p. 260.

Prop. XVI.—To find the rate of propagation of transverse vibrations through the elastic medium of which the cells are composed, on the supposition that its elasticity is due entirely to forces acting between pairs of particles.

By the ordinary method of investigation we know that

$$V=\sqrt{\frac{m}{\rho}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (132)$$

where m is the coefficient of transverse elasticity, and ρ is the density. By referring to the equations of Part I., it will be seen that if ρ is the density of the matter of the vortices, and μ is the "coefficient of magnetic induction,"

$$\mu=\pi\rho; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (133)$$

whence

$$\pi m=V^2\mu; \quad . \quad . \quad . \quad . \quad . \quad . \quad (134)$$

and by (108),

$$E=V\sqrt{\mu}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (135)$$

In air or vacuum $\mu=1$, and therefore

$$\left.\begin{aligned} V&=E,\\ &=310{,}740{,}000{,}000 \text{ millimetres per second},\\ &=193{,}088 \text{ miles per second}.\end{aligned}\right\} \quad . \quad (136)$$

The velocity of light in air, as determined by M. Fizeau*, is 70,843 leagues per second (25 leagues to a degree) which gives

$$\begin{aligned} V&=314{,}858{,}000{,}000 \text{ millimetres}\\ &=195{,}647 \text{ miles per second}. \quad . \quad . \quad . \quad . \quad (137)\end{aligned}$$

The velocity of transverse undulations in our hypothetical medium, calculated from the electro-magnetic experiments of MM. Kohlrausch and Weber, agrees so exactly with the velocity of light calculated from the optical experiments of M. Fizeau, that we can scarcely avoid the inference that *light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena.*

Prop. XVII.—To find the electric capacity of a Leyden jar composed of any given dielectric placed between two conducting surfaces.

Let the electric tensions or potentials of the two surfaces be Ψ_1 and Ψ_2. Let S be the area of each surface, and θ the distance between them, and let e and $-e$ be the quantities of electricity

* *Comptes Rendus*, vol. xxix. (1849), p. 90. In Galbraith and Haughton's 'Manual of Astronomy,' M. Fizeau's result is stated at 169,944 geographical miles of 1000 fathoms, which gives 193,118 statute miles; the value deduced from aberration is 192,000 miles.

on each surface; then the capacity

$$C=\frac{e}{\Psi_1-\Psi_2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (138)$$

Within the dielectric we have the variation of Ψ perpendicular to the surface

$$=\frac{\Psi_1-\Psi_2}{\theta}.$$

Beyond either surface this variation is zero.

Hence by (115) applied at the surface, the electricity on unit of area is

$$\frac{\Psi_1-\Psi_2}{4\pi E^2\theta}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (139)$$

and we deduce the whole capacity of the apparatus,

$$C=\frac{S}{4\pi E^2\theta}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (140)$$

so that the quantity of electricity required to bring the one surface to a given tension varies directly as the surface, inversely as the thickness, and inversely as the square of E.

Now the coefficient of induction of dielectrics is deduced from the capacity of induction-apparatus formed of them; so that if D is that coefficient, D varies inversely as E^2, and is unity for air. Hence

$$D=\frac{V^2}{V_1^{\,2}\mu}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (141)$$

where V and V_1 are the velocities of light in air and in the medium. Now if i is the index of refraction, $\frac{V}{V_1}=i$, and

$$D=\frac{i^2}{\mu}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (142)$$

so that the inductive power of a dielectric varies directly as the square of the index of refraction, and inversely as the magnetic inductive power.

In dense media, however, the optical, electric, and magnetic phenomena may be modified in different degrees by the particles of gross matter; and their mode of arrangement may influence these phenomena differently in different directions. The axes of optical, electric, and magnetic properties will probably coincide; but on account of the unknown and probably complicated nature of the reactions of the heavy particles on the ætherial medium, it may be impossible to discover any general numerical relations between the optical, electric, and magnetic ratios of these axes.

It seems probable, however, that the value of E, for any given axis, depends upon the velocity of light whose vibrations are parallel to that axis, or whose plane of polarization is perpendicular to that axis.

In a uniaxal crystal, the axial value of E will depend on the velocity of the extraordinary ray, and the equatorial value will depend on that of the ordinary ray.

In "positive" crystals, the axial value of E will be the least and in negative the greatest.

The value of D_1, which varies inversely as E^2, will, *cæteris paribus*, be greatest for the axial direction in positive crystals, and for the equatorial direction in negative crystals, such as Iceland spar. If a spherical portion of a crystal, radius $=a$, be suspended in a field of electric force which would act on unit of electricity with force $=I$, and if D_1 and D_2 be the coefficients of dielectric induction along the two axes in the plane of rotation, then if θ be the inclination of the axis to the electric force, the moment tending to turn the sphere will be

$$\frac{3}{2}\frac{(D_1-D_2)}{(2D_1+1)(2D_2+1)}I^2a^3\sin 2\theta, \quad . \quad . \quad . \quad (143)$$

and the axis of greatest dielectric induction (D_1) will tend to become parallel to the lines of electric force.

IV. *On the Direction of the Joints in the Faces of Oblique Arches. By* G. B. Airy, *Esq., Astronomer Royal*.*

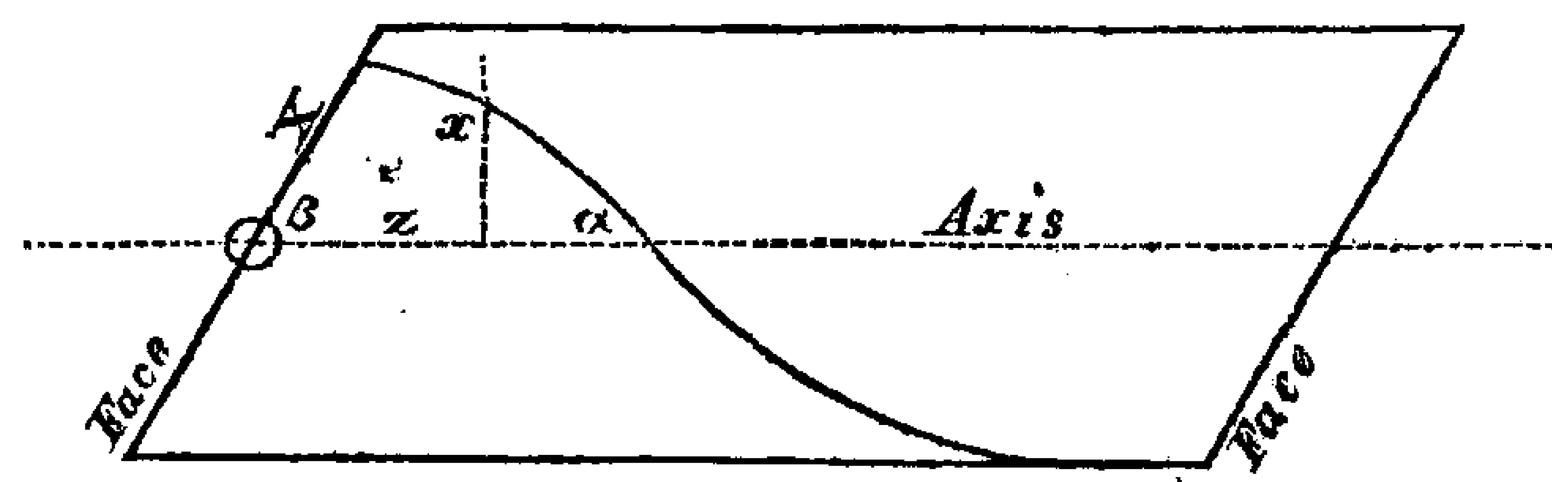
MY attention was lately called to the following passage in Mr. Buck's 'Essay on Oblique Bridges,' 2nd edition, p. 7. "After having had several drawings of the faces of oblique arches made on a large scale and projected with great exactitude, we observed that the following remarkable property exists. If the lines, which are the chords of the small curves forming the joints in the face of the arch, be produced, they will all meet in one point O, below the axis of the cylinder; and this property was found to hold even when the obliquity is so great as to depress the point O out of the cylinder altogether." The author then determines the point O by geometrical calculations for the joints at the spring of the arch, and, as far as I can perceive, makes use of this empirical theorem for determining the directions of all the other face-joints.

The theorem is perfectly correct; and the discovery of it bears testimony to the accuracy with which the author's plans must have been drawn, in a process of rather difficult geometry, and to the care with which they have been examined. The theorem, moreover, is true in the utmost generality, as regards the extent

* Communicated by the Author.

of the arc of cylinder (applying even when the entire barrel or cylinder is built in spiral courses as for an oblique arch); and as regards the relation between the angle which the face-plane makes with the axis, and the angle which the direction of the spiral courses makes with the axis (no condition whatever being required for either of them). Only, as the joints are slightly curved, it is proper to suppose that the stones are not very deep, and that the geometrical direction used is a tangent to the lower part of the curve of each: the middle of each joint-curve might be used equally well, but the resulting point of convergence would be slightly altered.

The theorem may thus be investigated by the processes of analytical geometry.



Let the diagram represent the horizontal plan of the oblique arch, the curved line being the projection of the spiral in which one of the longitudinal joints meets the cylindrical intrados, or a concentric cylinder (as that which passes through the middle of the stones' depth). Let O be the origin of coordinates, z the ordinate parallel to the axis, of any point in the helical surface which forms the longitudinal joint; x the horizontal ordinate transversal to the axis, of the same point, x being not necessarily terminated in the curved line; y the vertical ordinate, its foot being in the horizontal plane passing through the axis of the cylinder. And let r be the distance of the same point from the axis of the cylinder; θ the inclination of r to the vertical. Also let α be the angle at which the spiral intersects the ridge-line of the cylinder, β the angle at which the face of the arch cuts the same line. Then

$$x=r\sin\theta,\ y=r\cos\theta, \text{ and } \frac{x}{y}=\tan\theta.$$

Now if H be the value of θ in the helical surface when $z=0$ (H having a different numerical value for every different helix, and being the characteristic of the particular longitudinal joint under consideration), θ will $=H-n.z$; where n is a constant depending on the slope of the spiral, to be expressed more conveniently hereafter. It will be remarked here that the attribution of a constant value to H implies that, in any section of the helical sur-