the first step towards a rational is the discovery of an empirical law of the rotations, in which such an element as the inclination of the rotation axis to the plane of revolution (casily calculable except for the two innermost and two outermost planets) would evidently be involved.

Such a law seems pointed to by the regularity of the decrease of the rotations, when the angular, instead of the linear velocities or times are considered. The respective angular velocities of rotation of the inner family are ·29811, ·26902, ·26181, and ·25879; and of Jupiter and Saturn, ·63313 and ·59907 respectively.

15. The attempts I have made to discover the law of the planetary rotations have had as yet no complete result*. But the following incidental observation with regard to the angular velocities of revolution and the distances may perhaps be worth noting towards such a theory of the formation of the system as above alluded to.

By Kepler's third law,

$$P=cD^{\frac{3}{2}}\,;\ \text{whence}\ \frac{V}{D}\ \text{or}\ \omega=c'\frac{1}{D^{\frac{3}{2}}}.$$

But under this law there might, in comparing successive velocities and distances, be found relations of inequality *ad infinitum*. The actual relations may, however, be thus expressed:—The angular velocities of revolution and the distances are in inverse geometrical progressions with inverse differences, except the innermost planet of each family.

To say that the distances are in geometrical progression, each nearer planet being half the distance of the next more remote†, or that the angular velocities of revolution are in geometrical progression, each nearer planet revolving with twice the velocity of the next more remote, would be very far from accurate; but it seems interesting to observe, as by this law, that when the distance of a planet is *more* than twice that of the next inner, its angular velocity of rotation is *less* than half that of the next inner, and *vice versâ*. And that the only exceptions to this rule should be the innermost planet of each family, viz. Mercury and Jupiter, appears significant.

* The results of an approximative formula were given in a paper "On a general Law of Rotation applied to the Planets," read by me at the Oxford Meeting of the British Association, June 1860.

† See Humboldt's remarks on the Law of Bode, or rather of Titius. Cosmos, vol. iii. pp. 319, 320.

Mercury and Venus.

$$D=36{,}298051=\tfrac{1}{2}\times 68{,}631843+1{,}982129,$$

$$\frac{V}{D}=0\cdot0029760=2\times0\cdot0011651+0\cdot0006468.$$

Venus and Earth.

$$D=68{,}631843\ =\tfrac{1}{2}\times 94{,}885000\ +21{,}189343,$$

$$\frac{V}{D}=0\cdot00116510=2\times0\cdot00071676-0\cdot00026842.$$

Earth and Mars.

$$D=94{,}885000\ =\tfrac{1}{2}\times 144{,}575333+22{,}587334,$$

$$\frac{V}{D}=0\cdot00071676=2\times0\cdot00038108-0\cdot00004540.$$

Jupiter and Saturn.

$$D=493{,}654546\ =\tfrac{1}{2}\times 905{,}087708\ +41{,}110692,$$

$$\frac{V}{D}=0\cdot000060411=2\times0\cdot000024332+0\cdot0000117470.$$

Saturn and Uranus.

$$D=905{,}087708\ =\tfrac{1}{2}\times 1820{,}020075\ -4{,}922329,$$

$$\frac{V}{D}=0\cdot000024332=2\times0\cdot0000085313+0\cdot0000072598.$$

Uranus and Neptune.

$$D=1820{,}020075\ =\tfrac{1}{2}\times 2849{,}991384\ +395{,}024383,$$

$$\frac{V}{D}=0\cdot0000085313=2\times0\cdot0000043542-0\cdot0000001871\,*.$$

6 Stone Buildings, Lincoln's Inn,
March 1861.

XLIV. *On Physical Lines of Force.* By J. C. Maxwell, *Professor of Natural Philosophy in King's College, London*†.

[With a Plate.]

Part II.—*The Theory of Molecular Vortices applied to Electric Currents.*

WE have already shown that all the forces acting between magnets, substances capable of magnetic induction, and electric currents, may be mechanically accounted for on the sup-

* This fifteenth paragraph may be taken as an abstract of my paper "On the Revolutional Velocities and Distances of the Planets," read before the Royal Astronomical Society, Jan. 11, 1861.

† Communicated by the Author.

position that the surrounding medium is put into such a state that at every point the pressures are different in different directions, the direction of least pressure being that of the observed lines of force, and the difference of greatest and least pressures being proportional to the square of the intensity of the force at that point.

Such a state of stress, if assumed to exist in the medium, and to be arranged according to the known laws regulating lines of force, will act upon the magnets, currents, &c. in the field with precisely the same resultant forces as those calculated on the ordinary hypothesis of direct action at a distance. This is true independently of any particular theory as to the *cause* of this state of stress, or the mode in which it can be sustained in the medium. We have therefore a satisfactory answer to the question, "Is there any mechanical hypothesis as to the condition of the medium indicated by lines of force, by which the observed resultant forces may be accounted for?" The answer is, the lines of force indicate the direction of *minimum pressure* at every point of the medium.

The second question must be, "What is the mechanical cause of this difference of pressure in different directions?" We have supposed, in the first part of this paper, that this difference of pressures is caused by molecular vortices, having their axes parallel to the lines of force.

We also assumed, perfectly arbitrarily, that the direction of these vortices is such that, on looking along a line of force from south to north, we should see the vortices revolving in the direction of the hands of a watch.

We found that the velocity of the circumference of each vortex must be proportional to the intensity of the magnetic force, and that the density of the substance of the vortex must be proportional to the capacity of the medium for magnetic induction.

We have as yet given no answers to the questions, "How are these vortices set in rotation?" and "Why are they arranged according to the known laws of lines of force about magnets and currents?" These questions are certainly of a higher order of difficulty than either of the former; and I wish to separate the suggestions I may offer by way of provisional answer to them, from the mechanical deductions which resolved the first question, and the hypothesis of vortices which gave a probable answer to the second.

We have, in fact, now come to inquire into the physical connexion of these vortices with electric currents, while we are still in doubt as to the nature of electricity, whether it is one substance, two substances, or not a substance at all, or in what way it differs from matter, and how it is connected with it.

We know that the lines of force are affected by electric currents, and we know the distribution of those lines about a current; so that from the force we can determine the amount of the current. Assuming that our explanation of the lines of force by molecular vortices is correct, why does a particular distribution of vortices indicate an electric current? A satisfactory answer to this question would lead us a long way towards that of a very important one, "What is an electric current?"

I have found great difficulty in conceiving of the existence of vortices in a medium, side by side, revolving in the same direction about parallel axes. The contiguous portions of consecutive vortices must be moving in opposite directions; and it is difficult to understand how the motion of one part of the medium can coexist with, and even produce, an opposite motion of a part in contact with it.

The only conception which has at all aided me in conceiving of this kind of motion is that of the vortices being separated by a layer of particles, revolving each on its own axis in the opposite direction to that of the vortices, so that the contiguous surfaces of the particles and of the vortices have the same motion.

In mechanism, when two wheels are intended to revolve in the same direction, a wheel is placed between them so as to be in gear with both, and this wheel is called an "idle wheel." The hypothesis about the vortices which I have to suggest is that a layer of particles, acting as idle wheels, is interposed between each vortex and the next, so that each vortex has a tendency to make the neighbouring vortices revolve in the same direction with itself.

In mechanism, the idle wheel is generally made to rotate about a *fixed* axle; but in epicyclic trains and other contrivances, as, for instance, in Siemens's governor for steam-engines*, we find idle wheels whose centres are capable of motion. In all these cases the motion of the centre is the half sum of the motions of the circumferences of the wheels between which it is placed. Let us examine the relations which must subsist between the motions of our vortices and those of the layer of particles interposed as idle wheels between them.

Prop. IV.—To determine the motion of a layer of particles separating two vortices.

Let the circumferential velocity of a vortex, multiplied by the three direction-cosines of its axis respectively, be α, β, γ, as in Prop. II. Let l, m, n be the direction-cosines of the normal to any part of the surface of this vortex, the outside of the surface being regarded positive. Then the components of the velocity of the particles of the vortex at this part of its surface will be

* See Goodeve's 'Elements of Mechanism,' p. 118.

$$n\beta - m\gamma \quad \text{parallel to } x,$$
$$l\gamma - n\alpha \quad \text{parallel to } y,$$
$$m\alpha - l\beta \quad \text{parallel to } z.$$

If this portion of the surface be in contact with another vortex whose velocities are α', β', γ', then a layer of very small particles placed between them will have a velocity which will be the mean of the superficial velocities of the vortices which they separate, so that if u is the velocity of the particles in the direction of x,

$$u = \tfrac{1}{2}m(\gamma' - \gamma) - \tfrac{1}{2}n(\beta' - \beta), \quad . \quad . \quad . \quad . \quad (27)$$

since the normal to the second vortex is in the opposite direction to that of the first.

Prop. V.—To determine the whole amount of particles transferred across unit of area in the direction of x in unit of time.

Let x_1, y_1, z_1 be the coordinates of the centre of the first vortex, x_2, y_2, z_2 those of the second, and so on. Let V_1, V_2, &c. be the volumes of the first, second, &c. vortices, and V the sum of their volumes. Let dS be an element of the surface separating the first and second vortices, and x, y, z its coordinates. Let ρ be the quantity of particles on every unit of surface. Then if p be the whole quantity of particles transferred across unit of area in unit of time in the direction of x, the whole momentum parallel to x of the particles within the space whose volume is $\overline{V}$ will be $\overline{V}p$, and we shall have

$$\overline{V}p = \Sigma u\rho dS, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

the summation being extended to every surface separating any two vortices within the volume $\overline{V}$.

Let us consider the surface separating the first and second vortices. Let an element of this surface be dS, and let its direction-cosines be l_1, m_1, n_1 with respect to the first vortex, and l_2, m_2, n_2 with respect to the second; then we know that

$$l_1 + l_2 = 0, \quad m_1 + m_2 = 0, \quad n_1 + n_2 = 0. \quad . \quad . \quad (29)$$

The values of α, β, γ vary with the position of the centre of the vortex; so that we may write

$$\alpha_2 = \alpha_1 + \frac{d\alpha}{dx}(x_2 - x_1) + \frac{d\alpha}{dy}(y_2 - y_1) + \frac{d\alpha}{dz}(z_2 - z_1), \quad . \quad (30)$$

with similar equations for β and γ.

The value of u may be written:—

$$\begin{aligned} u = {} & \frac{1}{2}\frac{d\gamma}{dx}\Big(m_1(x-x_1) + m_2(x-x_2)\Big) \\ & + \frac{1}{2}\frac{d\gamma}{dy}\Big(m_1(y-y_1) + m_2(y-y_2)\Big) + \frac{1}{2}\frac{d\gamma}{dz}\Big(m_1(z-z_1) + m_2(z-z_2)\Big) \\ & - \frac{1}{2}\frac{d\beta}{dx}\Big(n_1(x-x_1) + n_2(x-x_2)\Big) - \frac{1}{2}\frac{d\beta}{dy}\Big(n_1(y-y_1) + n_2(y-y_2)\Big) \\ & - \frac{1}{2}\frac{d\beta}{dz}\Big(n_1(z-z_1) + n_1(z-z_2)\Big). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31) \end{aligned}$$

In effecting the summation of $\Sigma u\rho dS$, we must remember that round any closed surface $\Sigma l dS$ and all similar terms vanish; also that terms of the form $\Sigma l y dS$, where l and y are measured in different directions, also vanish; but that terms of the form $\Sigma l x dS$, where l and x refer to the same axis of coordinates, do not vanish, but are equal to the volume enclosed by the surface. The result is

$$\overline{V}p = \frac{1}{2}\rho\left(\frac{d\gamma}{dy} - \frac{d\beta}{dz}\right)(V_1 + V_2 + \&c.); \quad . \quad . \quad (32)$$

or dividing by $\overline{V} = V_1 + V_2 + \&c.$,

$$p = \frac{1}{2}\rho\left(\frac{d\gamma}{dy} - \frac{d\beta}{dz}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

If we make

$$\rho = \frac{1}{2\pi}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

then equation (33) will be identical with the first of equations (9), which give the relation between the quantity of an electric current and the intensity of the lines of force surrounding it.

It appears therefore that, according to our hypothesis, an electric current is represented by the transference of the moveable particles interposed between the neighbouring vortices. We may conceive that these particles are very small compared with the size of a vortex, and that the mass of all the particles together is inappreciable compared with that of the vortices, and that a great many vortices, with their surrounding particles, are contained in a single complete molecule of the medium. The particles must be conceived to roll without sliding between the vortices which they separate, and not to touch each other, so that, as long as they remain within the same complete molecule, there is no loss of energy by resistance. When, however, there is a general transference of particles in one direction, they must pass from one molecule to another, and in doing so, may experience resistance, so as to waste electrical energy and generate heat.

Now let us suppose the vortices arranged in a medium in any arbitrary manner. The quantities $\frac{d\gamma}{dy}-\frac{d\beta}{dz}$, &c. will then in general have values, so that there will at first be electrical currents in the medium. These will be opposed by the electrical resistance of the medium; so that, unless they are kept up by a continuous supply of force, they will quickly disappear, and we shall then have $\frac{d\gamma}{dy}-\frac{d\beta}{dz}=0$, &c.; that is, $\alpha dx+\beta dy+\gamma dz$ will be a complete differential (see equations (15) and (16)); so that our hypothesis accounts for the distribution of the lines of force.

In Plate V. fig. 1, let the vertical circle E E represent an electric current flowing from copper C to zinc Z through the conductor E E′, as shown by the arrows.

Let the horizontal circle M M′ represent a line of magnetic force embracing the electric circuit, the north and south directions being indicated by the lines S N and N S.

Let the vertical circles V and V′ represent the molecular vortices of which the line of magnetic force is the axis. V revolves as the hands of a watch, and V′ the opposite way.

It will appear from this diagram, that if V and V′ were contiguous vortices, particles placed between them would move downwards; and that if the particles were forced downwards by any cause, they would make the vortices revolve as in the figure. We have thus obtained a point of view from which we may regard the relation of an electric current to its lines of force as analogous to the relation of a toothed wheel or rack to wheels which it drives.

In the first part of the paper we investigated the relations of the statical forces of the system. We have now considered the connexion of the motions of the parts considered as a system of mechanism. It remains that we should investigate the dynamics of the system, and determine the forces necessary to produce given changes in the motions of the different parts.

Prop. VI.—To determine the actual energy of a portion of a medium due to the motion of the vortices within it.

Let α, β, γ be the components of the circumferential velocity, as in Prop. II., then the actual energy of the vortices in unit of volume will be proportional to the density and to the square of the velocity. As we do not know the distribution of density and velocity in each vortex, we cannot determine the numerical value of the energy directly; but since μ also bears a constant though unknown ratio to the mean density, let us assume that the energy

in unit of volume is

$$E=C\mu(\alpha^2+\beta^2+\gamma^2),$$

where C is a constant to be determined.

Let us take the case in which

$$\alpha=\frac{d\phi}{dx},\quad \beta=\frac{d\phi}{dy},\quad \gamma=\frac{d\phi}{dz}. \quad . \quad . \quad . \quad (35)$$

Let

$$\phi=\phi_1+\phi_2, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

and let

$$\frac{\mu}{4\pi}\left(\frac{d^2\phi_1}{dx^2}+\frac{d^2\phi_1}{dy^2}+\frac{d^2\phi_2}{dz^2}\right)=m_1 \text{ and } \frac{\mu}{4\pi}\left(\frac{d^2\phi_2}{dx^2}+\frac{d^2\phi_2}{dy^2}+\frac{d^2\phi_2}{dz^2}\right)=m_2; \quad (37)$$

then ϕ_1 is the potential at any point due to the magnetic system m_1, and ϕ_2 that due to the distribution of magnetism represented by m_2. The actual energy of all the vortices is

$$E=\Sigma C\mu(\alpha^2+\beta^2+\gamma^2)dV, \quad . \quad . \quad . \quad . \quad (38)$$

the integration being performed over all space.

This may be shown by integration by parts (see Green's 'Essay on Electricity,' p. 10) to be equal to

$$E=-4\pi C\Sigma(\phi_1m_1+\phi_2m_2+\phi_1m_2+\phi_2m_1)dV. \quad . \quad (39)$$

Or since it has been proved (Green's 'Essay,' p. 10) that

$$\Sigma\phi_1m_2dV=\Sigma\phi_2m_1dV,$$

$$E=-4\pi C(\phi_1m_1+\phi_2m_2+2\phi_1m_2)dV. \quad . \quad . \quad . \quad (40)$$

Now let the magnetic system m_1 remain at rest, and let m_2 be moved parallel to itself in the direction of x through a space δx; then, since ϕ_1 depends on m_1 only, it will remain as before, so that ϕ_1m_1 will be constant; and since ϕ_2 depends on m_2 only, the distribution of ϕ_2 about m_2 will remain the same, so that ϕ_2m_2 will be the same as before the change. The only part of E that will be altered is that depending on $2\phi_1m_2$, because ϕ_1 becomes $\phi_1+\frac{d\phi_1}{dx}\delta x$ on account of the displacement. The variation of actual energy due to the displacement is therefore

$$\delta E=-4\pi C\Sigma\left(2\frac{d\phi_1}{dx}m_2\right)dV\,\delta x. \quad . \quad . \quad (41)$$

But by equation (12), the work done by the mechanical forces on m_2 during the motion is

$$\delta W=\Sigma\left(\frac{d\phi_1}{dx}m_2dV\right)\delta x; \quad . \quad . \quad . \quad . \quad . \quad (42)$$

and since our hypothesis is a purely mechanical one, we must have by the conservation of force,

$$\delta E+\delta W=0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

that is, the loss of energy of the vortices must be made up by work done in moving magnets, so that

$$-4\pi C\Sigma\left(2\frac{d\phi_1}{dx}m_2 dV\right)\delta x+\Sigma\left(\frac{d\phi_1}{dx}m_2 dV\right)\delta x=0,$$

or

$$C=\frac{1}{8\pi}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

so that the energy of the vortices in unit of volume is

$$\frac{1}{8\pi}\mu(\alpha^2+\beta^2+\gamma^2); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

and that of a vortex whose volume is V is

$$\frac{1}{8\pi}\mu(\alpha^2+\beta^2+\gamma^2)V. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46)$$

In order to produce or destroy this energy, work must be expended on, or received from, the vortex, either by the tangential action of the layer of particles in contact with it, or by change of form in the vortex. We shall first investigate the tangential action between the vortices and the layer of particles in contact with them.

***Prop.* VII.—To find the energy spent upon a vortex in unit of time by the layer of particles which surrounds it.**

Let P, Q, R be the forces acting on unity of the particles in the three coordinate directions, these quantities being functions of x, y, and z. Since each particle touches two vortices at the extremities of a diameter, the reaction of the particle on the vortices will be equally divided, and will be

$$-\frac{1}{2}P, \quad -\frac{1}{2}Q, \quad -\frac{1}{2}R$$

on each vortex for unity of the particles; but since the superficial density of the particles is $\frac{1}{2\pi}$ (see equation (34)), the forces on unit of surface of a vortex will be

$$-\frac{1}{4\pi}P, \quad -\frac{1}{4\pi}Q, \quad -\frac{1}{4\pi}R.$$

Now let dS be an element of the surface of a vortex. Let the direction-cosines of the normal be l, m, n. Let the coordinates of the element be x, y, z. Let the component velocities of the surface be u, v, w. Then the work expended on that element of surface will be

$$\frac{dE}{dt}=-\frac{1}{4\pi}(Pu+Qv+Rw)dS. \quad . \quad . \quad . \quad . \quad (47)$$

Let us begin with the first term, $PudS$. P may be written

$$P_0+\frac{dP}{dx}x+\frac{dP}{dy}y+\frac{dP}{dz}z, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (48)$$

and

$$u=n\beta-m\gamma.$$

Remembering that the surface of the vortex is a closed one, so that

$$\Sigma nxdS=\Sigma mxdS=\Sigma nydS=\Sigma mzdS=0,$$

and

$$\Sigma mydS=\Sigma nzdS=V,$$

we find

$$\Sigma PudS=\left(\frac{dP}{dz}\beta-\frac{dP}{dy}\gamma\right)V, \quad . \quad . \quad . \quad . \quad (49)$$

and the whole work done on the vortex in unit of time will be

$$\frac{dE}{dt}=-\frac{1}{4\pi}\Sigma(Pu+Qv+Rw)dS$$

$$=\frac{1}{4\pi}\left\{\alpha\left(\frac{dQ}{dz}-\frac{dR}{dy}\right)+\beta\left(\frac{dR}{dx}-\frac{dP}{dz}\right)+\gamma\left(\frac{dP}{dy}-\frac{dQ}{dx}\right)\right\}V. \quad (50)$$

Prop. VIII.—To find the relations between the alterations of motion of the vortices, and the forces P, Q, R which they exert on the layer of particles between them.

Let V be the volume of a vortex, then by (46) its energy is

$$E=\frac{1}{8\pi}\mu(\alpha^2+\beta^2+\gamma^2)V, \quad . \quad . \quad . \quad . \quad . \quad (51)$$

and

$$\frac{dE}{dt}=\frac{1}{4\pi}\mu V\left(\alpha\frac{d\alpha}{dt}+\beta\frac{d\beta}{dt}+\gamma\frac{d\gamma}{dt}\right). \quad . \quad . \quad (52)$$

Comparing this value with that given in equation (50), we find

$$\alpha\left(\frac{dQ}{dz}-\frac{dR}{dy}-\mu\frac{d\alpha}{dt}\right)+\beta\left(\frac{dR}{dx}-\frac{dP}{dz}-\mu\frac{d\beta}{dt}\right)$$
$$+\gamma\left(\frac{dP}{dy}-\frac{dQ}{dx}-\mu\frac{d\gamma}{dt}\right)=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad (53)$$

This equation being true for all values of α, β, and γ, first let β and γ vanish, and divide by α. We find

Similarly,

and

$$\left.\begin{aligned}\frac{dQ}{dz}-\frac{dR}{dy}&=\mu\frac{d\alpha}{dt},\\ \frac{dR}{dx}-\frac{dP}{dz}&=\mu\frac{d\beta}{dt},\\ \frac{dP}{dy}-\frac{dQ}{dx}&=\mu\frac{d\gamma}{dt}.\end{aligned}\right\}\quad\ldots\ldots\quad(54)$$

From these equations we may determine the relation between the alterations of motion $\frac{d\alpha}{dt}$, &c. and the forces exerted on the layers of particles between the vortices, or, in the language of our hypothesis, the relation between changes in the state of the magnetic field and the electromotive forces thereby brought into play.

In a memoir "On the Dynamical Theory of Diffraction" (Cambridge Philosophical Transactions, vol. ix. part 1, section 6), Professor Stokes has given a method by which we may solve equations (54), and find P, Q, and R in terms of the quantities on the right-hand of those equations. I have pointed out* the application of this method to questions in electricity and magnetism.

Let us then find three quantities F, G, H from the equations

$$\left.\begin{aligned}\frac{dG}{dz}-\frac{dH}{dy}&=\mu\alpha,\\ \frac{dH}{dx}-\frac{dF}{dz}&=\mu\beta,\\ \frac{dF}{dy}-\frac{dG}{dx}&=\mu\gamma,\end{aligned}\right\}\quad\ldots\ldots\ldots\quad(55)$$

with the conditions

$$\frac{1}{4\pi}\left(\frac{d}{dx}\mu\alpha+\frac{d}{dy}\mu\beta+\frac{d}{dz}\mu\gamma\right)=m=0,\quad .\quad(56)$$

and

$$\frac{dF}{dx}+\frac{dG}{dy}+\frac{dH}{dz}=0.\quad\ldots\ldots\ldots\quad(57)$$

Differentiating (55) with respect to t, and comparing with (54), we find

$$P=\frac{dF}{dt},\quad Q=\frac{dG}{dt},\quad R=\frac{dH}{dt}.\quad\ldots\quad(58)$$

* Cambridge Philosophical Transactions, vol. x. part 1. art. 3, "On Faraday's Lines of Force."

We have thus determined three quantities, F, G, H, from which we can find P, Q, and R by considering these latter quantities as the rates at which the former ones vary. In the paper already referred to, I have given reasons for considering the quantities F, G, H as the resolved parts of that which Faraday has conjectured to exist, and has called the *electrotonic state*. In that paper I have stated the mathematical relations between this electrotonic state and the lines of magnetic force as expressed in equations (55), and also between the electrotonic state and electromotive force as expressed in equations (58). We must now endeavour to interpret them from a mechanical point of view in connexion with our hypothesis.

We shall in the first place examine the process by which the lines of force are produced by an electric current.

Let A B, Pl. V. fig. 2, represent a current of electricity in the direction from A to B. Let the large spaces above and below A B represent the vortices, and let the small circles separating the vortices represent the layers of particles placed between them, which in our hypothesis represent electricity.

Now let an electric current from left to right commence in A B. The row of vortices *g h* above A B will be set in motion in the opposite direction to that of a watch. (We shall call this direction +, and that of a watch −.) We shall suppose the row of vortices *k l* still at rest, then the layer of particles between these rows will be acted on by the row *g h* on their lower sides, and will be at rest above. If they are free to move, they will rotate in the negative direction, and will at the same time move from right to left, or in the opposite direction from the current, and so form an *induced* electric current.

If this current is checked by the electrical resistance of the medium, the rotating particles will act upon the row of vortices *k l*, and make them revolve in the positive direction till they arrive at such a velocity that the motion of the particles is reduced to that of rotation, and the induced current disappears. If, now, the primary current A B be stopped, the vortices in the row *g h* will be checked, while those of the row *k l* still continue in rapid motion. The momentum of the vortices beyond the layer of particles *p q* will tend to move them from left to right, that is, in the direction of the primary current; but if this motion is resisted by the medium, the motion of the vortices beyond *p q* will be gradually destroyed.

It appears therefore that the phenomena of induced currents are part of the process of communicating the rotatory velocity of the vortices from one part of the field to another.

[To be continued.]

Here the near coincidence of the results in the first and third columns shows that the relation between k and T may be approximately expressed by the formula

$$k = 14{\cdot}15\,T^{\frac{1}{3}}, \text{ or } T = \left(\frac{k}{14{\cdot}15}\right)^3. \quad . \quad . \quad . \quad (7)$$

Hastings, April 1, 1861.

LI. *On Physical Lines of Force.* *By* J. C. Maxwell, *Professor of Natural Philosophy in King's College, London.*

[With a Plate.]

Part II.—*The Theory of Molecular Vertices applied to Electric Currents.*

[Concluded from p. 291.]

As an example of the action of the vortices in producing induced currents, let us take the following case:—Let B, Pl. V. fig. 3, be a circular ring, of uniform section, lapped uniformly with covered wire. It may be shown that if an electric current is passed through this wire, a magnet placed within the coil of wire will be strongly affected, but no magnetic effect will be produced on any external point. The effect will be that of a magnet bent round till its two poles are in contact.

If the coil is properly made, no effect on a magnet placed outside it can be discovered, whether the current is kept constant or made to vary in strength; but if a conducting wire C be made to *embrace* the ring any number of times, an electromotive force will act on that wire whenever the current in the coil is made to vary; and if the circuit be *closed,* there will be an actual current in the wire C.

This experiment shows that, in order to produce the electromotive force, it is not necessary that the conducting wire should be placed in a field of magnetic force, or that lines of magnetic force should pass through the substance of the wire or near it. All that is required is that lines of force should pass through the circuit of the conductor, and that these lines of force should vary in quantity during the experiment.

In this case the vortices, of which we suppose the lines of magnetic force to consist, are all within the hollow of the ring, and outside the ring all is at rest. If there is no conducting circuit embracing the ring, then, when the primary current is made or broken, there is no action outside the ring, except an instantaneous pressure between the particles and the vortices which they separate. If there is a continuous conducting circuit embracing the ring, then, when the primary current is made, there will be a current in the opposite direction through C; and when it is broken, there will be a current through C in the same direction as the primary current.

We may now perceive that induced currents are produced when the electricity yields to the electromotive force,—this force, however, still existing when the formation of a sensible current is prevented by the resistance of the circuit.

The electromotive force, of which the components are P, Q, R, arises from the action between the vortices and the interposed particles, when the velocity of rotation is altered in any part of the field. It corresponds to the pressure on the axle of a wheel in a machine when the velocity of the driving wheel is increased or diminished.

The electrotonic state, whose components are F, G, H, is what the electromotive force would be if the currents, &c. to which the lines of force are due, instead of arriving at their actual state by degrees, had started instantaneously from rest with their actual values. It corresponds to the *impulse* which would act on the axle of a wheel in a machine if the actual velocity were suddenly given to the driving wheel, the machine being previously at rest.

If the machine were suddenly stopped by stopping the driving wheel, each wheel would receive an impulse equal and opposite to that which it received when the machine was set in motion.

This impulse may be calculated for any part of a system of mechanism, and may be called the *reduced momentum* of the machine for that point. In the varied motion of the machine, the actual force on any part arising from the variation of motion may be found by differentiating the reduced momentum with respect to the time, just as we have found that the electromotive force may be deduced from the electrotonic state by the same process.

Having found the relation between the velocities of the vortices and the electromotive forces when the centres of the vortices are at rest, we must extend our theory to the case of a fluid medium containing vortices, and subject to all the varieties of fluid motion. If we fix our attention on any one elementary portion of a fluid, we shall find that it not only travels from one place to another, but also changes its form and position, so as to be elongated in certain directions and compressed in others, and at the same time (in the most general case) turned round by a displacement of rotation.

These changes of form and position produce changes in the velocity of the molecular vortices, which we must now examine.

The alteration of form and position may always be reduced to three simple extensions or compressions in the direction of three rectangular axes, together with three angular rotations about any set of three axes. We shall first consider the effect of three simple extensions or compressions.

Prop. IX.—To find the variations of α, β, γ in the parallelopiped x, y, z when x becomes $x+\delta x$; $y, y+\delta y$; and $z, z+\delta z$; the volume of the figure remaining the same.

By Prop. II. we find for the work done by the vortices against pressure,

$$\delta W = p_1 \delta(xyz) - \frac{\mu}{4\pi}(\alpha^2 yz\delta x + \beta^2 zx\delta y + \gamma^2 xy\delta z); \quad (59)$$

and by Prop. VI. we find for the variation of energy,

$$\delta E = \frac{\mu}{4\pi}(\alpha\delta\alpha + \beta\delta\beta + \gamma\delta\gamma)xyz. \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

The sum $\delta W + \delta E$ must be zero by the conservation of energy, and $\delta(xyz)=0$, since xyz is constant; so that

$$\alpha\left(\delta\alpha - \alpha\frac{\delta x}{x}\right) + \beta\left(\delta\beta - \beta\frac{\delta y}{y}\right) + \gamma\left(\delta\gamma - \gamma\frac{\delta z}{z}\right) = 0. \quad (61)$$

In order that this should be true independently of any relations between α, β, and γ, we must have

$$\delta\alpha = \alpha\frac{\delta x}{x}, \quad \delta\beta = \beta\frac{\delta y}{y}, \quad \delta\gamma = \gamma\frac{\delta z}{z}. \quad . \quad . \quad . \quad . \quad . \quad (62)$$

Prop. X.—To find the variations of α, β, γ due to a rotation θ_1 about the axis of x from y to z, a rotation θ_2 about the axis of y from z to x, and a rotation θ_3 about the axis of z from x to y.

The axis of β will move away from the axis of x by an angle θ_3; so that β resolved in the direction of x changes from 0 to $-\beta\theta_3$.

The axis of γ approaches that of x by an angle θ_2; so that the resolved part of γ in direction x changes from 0 to $\gamma\theta_2$.

The resolved part of α in the direction of x changes by a quantity depending on the second power of the rotations, which may be neglected. The variations of α, β, γ from this cause are therefore

$$\delta\alpha = \gamma\theta_2 - \beta\theta_3, \quad \delta\beta = \alpha\theta_3 - \gamma\theta_1, \quad \delta\gamma = \beta\theta_1 - \alpha\theta_2. \quad (63)$$

The most general expressions for the distortion of an element produced by the displacement of its different parts depend on the nine quantities

$$\frac{d}{dx}\delta x, \ \frac{d}{dy}\delta x, \ \frac{d}{dz}\delta x; \ \frac{d}{dx}\delta y, \ \frac{d}{dy}\delta y, \ \frac{d}{dz}\delta y; \ \frac{d}{dx}\delta z, \ \frac{d}{dy}\delta z, \ \frac{d}{dz}\delta z;$$

and these may always be expressed in terms of nine other quantities, namely, three simple extensions or compressions,

$$\frac{\delta x'}{x'}, \quad \frac{\delta y'}{y'}, \quad \frac{\delta z'}{z'}$$

along three axes properly chosen, x', y', z', the nine direction-cosines of these axes with their six connecting equations, which are equivalent to three independent quantities, and the three rotations $\theta_1, \theta_2, \theta_3$ about the axes of x, y, z.

Let the direction-cosines of x' with respect to x, y, z be l_1, m_1, n_1, those of y', l_2, m_2, n_2, and those of z', l_3, m_3, n_3; then we find

$$\left.\begin{aligned} \frac{d}{dx}\delta x &= l_1^2\frac{\delta x'}{x'} + l_2^2\frac{\delta y'}{y'} + l_3^2\frac{\delta z'}{z'}, \\ \frac{d}{dy}\delta x &= l_1 m_1\frac{\delta x'}{x'} + l_2 m_2\frac{\delta y'}{y'} + l_3 m_3\frac{\delta z'}{z'} - \theta_3, \\ \frac{d}{dz}\delta x &= l_1 n_1\frac{\delta x'}{x'} + l_2 n_2\frac{\delta y'}{y'} + l_3 n_3\frac{\delta z'}{z'} + \theta_2, \end{aligned}\right\} \quad . \quad . \quad (64)$$

with similar equations for quantities involving δy and δz.

Let α', β', γ' be the values of α, β, γ referred to the axes of x', y', z'; then

$$\left.\begin{aligned} \alpha' &= l_1\alpha + m_1\beta + n_1\gamma, \\ \beta' &= l_2\alpha + m_2\beta + n_2\gamma, \\ \gamma' &= l_3\alpha + m_3\beta + n_3\gamma. \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (65)$$

We shall then have

$$\delta\alpha = l_1\delta\alpha' + l_2\delta\beta' + l_3\delta\gamma' + \gamma\theta_2 - \beta\theta_3, \quad . \quad . \quad . \quad (66)$$

$$= l_1\alpha'\frac{\delta x'}{x'} + l_2\beta'\frac{\delta y'}{y'} + l_3\gamma'\frac{\delta z'}{z'} + \gamma\theta_2 - \beta\theta_3. \quad . \quad (67)$$

By substituting the values of α', β', γ', and comparing with equations (64), we find

$$\delta\alpha = \alpha\frac{d}{dx}\delta x + \beta\frac{d}{dy}\delta x + \gamma\frac{d}{dz}\delta x \quad . \quad . \quad . \quad . \quad (68)$$

as the variation of α due to the change of form and position of the element. The variations of β and γ have similar expressions.

Prop. XI.—To find the electromotive forces in a moving body.

The variation of the velocity of the vortices in a moving element is due to two causes—the action of the electromotive forces, and the change of form and position of the element. The whole variation of α is therefore

$$\delta\alpha = \frac{1}{\mu}\left(\frac{dQ}{dz} - \frac{dR}{dy}\right)\delta t + \alpha\frac{d}{dx}\delta x + \beta\frac{d}{dy}\delta x + \gamma\frac{d}{dz}\delta x. \quad . \quad (69)$$

But since α is a function of x, y, z and t, the variation of α may be also written

$$\delta\alpha = \frac{d\alpha}{dx}\delta x + \frac{d\alpha}{dy}\delta y + \frac{d\alpha}{dz}\delta z + \frac{d\alpha}{dt}\delta t. \quad . \quad . \quad . \quad . \quad (70)$$

Equating the two values of $\delta\alpha$ and dividing by δt, and remembering that in the motion of an incompressible medium

$$\frac{d}{dx}\frac{dx}{dt}+\frac{d}{dy}\frac{dy}{dt}+\frac{d}{dz}\frac{dz}{dt}=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

and that in the absence of free magnetism

$$\frac{d\alpha}{dx}+\frac{d\beta}{dy}+\frac{d\gamma}{dz}=0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (72)$$

we find

$$\frac{1}{\mu}\left(\frac{dQ}{dz}-\frac{dR}{dy}\right)+\gamma\frac{d}{dz}\frac{dx}{dt}-\alpha\frac{d}{dz}\frac{dz}{dt}-\alpha\frac{d}{dy}\frac{dy}{dt}+\beta\frac{d}{dy}\frac{dx}{dt}$$
$$+\frac{d\gamma}{dz}\frac{dx}{dt}-\frac{d\alpha}{dz}\frac{dz}{dt}-\frac{d\alpha}{dy}\frac{dy}{dt}+\frac{d\beta}{dy}\frac{dx}{dt}-\frac{d\alpha}{dt}=0. \quad . \quad (73)$$

Putting

$$\alpha=\frac{1}{\mu}\left(\frac{dG}{dz}-\frac{dH}{dy}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (74)$$

and

$$\frac{d\alpha}{dt}=\frac{1}{\mu}\left(\frac{d^2G}{dz\,dt}-\frac{d^2H}{dy\,dt}\right), \quad . \quad . \quad . \quad . \quad . \quad (75)$$

where F, G, and H are the values of the electrotonic components for a fixed point of space, our equation becomes

$$\frac{d}{dz}\left(Q+\mu\gamma\frac{dx}{dt}-\mu\alpha\frac{dz}{dt}-\frac{dG}{dt}\right)-\frac{d}{dy}\left(R+\mu\alpha\frac{dy}{dt}-\mu\beta\frac{dx}{dt}-\frac{dH}{dt}\right)=0. \quad (76)$$

The expressions for the variations of β and γ give us two other equations which may be written down from symmetry. The complete solution of the three equations is

$$\left.\begin{aligned} P&=\mu\gamma\frac{dy}{dt}-\mu\beta\frac{dz}{dt}+\frac{dF}{dt}-\frac{d\Psi}{dx}, \\ Q&=\mu\alpha\frac{dz}{dt}-\mu\gamma\frac{dx}{dt}+\frac{dG}{dt}-\frac{d\Psi}{dy}, \\ R&=\mu\beta\frac{dx}{dt}-\mu\alpha\frac{dy}{dt}+\frac{dH}{dt}-\frac{d\Psi}{dz}. \end{aligned}\right\} \quad . \quad . \quad . \quad (77)$$

The first and second terms of each equation indicate the effect of the motion of any body in the magnetic field, the third term refers to changes in the electrotonic state produced by alterations of position or intensity of magnets or currents in the field, and Ψ is a function of x, y, z, and t, which is indeterminate as far as regards the solution of the original equations, but which may always be determined in any given case from the circumstances of the problem. The physical interpretation of Ψ is, that it is the *electric tension* at each point of space.

The physical meaning of the terms in the expression for the electromotive force depending on the motion of the body, may be made simpler by supposing the field of magnetic force uniformly magnetized with intensity α in the direction of the axis of x. Then if l, m, n be the direction-cosines of any portion of a linear conductor, and S its length, the electromotive force resolved in the direction of the conductor will be

$$e=S(Pl+Qm+Rn), \quad . \quad . \quad . \quad . \quad . \quad . \quad (78)$$

or

$$e=S\mu\alpha\left(m\frac{dz}{dt}-n\frac{dy}{dt}\right), \quad . \quad . \quad . \quad . \quad . \quad . \quad (79)$$

that is, the product of $\mu\alpha$, the quantity of magnetic induction over unit of area multiplied by $S\left(m\frac{dz}{dt}-n\frac{dy}{dt}\right)$, the area swept out by the conductor S in unit of time, resolved perpendicular to the direction of the magnetic force.

The electromotive force in any part of a conductor due to its motion is therefore measured by the *number* of lines of magnetic force which it crosses in unit of time; and the total electromotive force in a closed conductor is measured by the change of the number of lines of force which pass through it; and this is true whether the change be produced by the motion of the conductor or by any external cause.

In order to understand the mechanism by which the motion of a conductor across lines of magnetic force generates an electromotive force in that conductor, we must remember that in Prop. X. we have proved that the change of form of a portion of the medium containing vortices produces a change of the velocity of those vortices; and in particular that an extension of the medium in the direction of the axes of the vortices, combined with a contraction in all directions perpendicular to this, produces an increase of velocity of the vortices; while a shortening of the axis and bulging of the sides produces a diminution of the velocity of the vortices.

This change of the velocity of the vortices arises from the internal effects of change of form, and is independent of that produced by external electromotive forces. If, therefore, the change of velocity be prevented or checked, electromotive forces will arise, because each vortex will press on the surrounding particles in the direction in which it tends to alter its motion.

Let A, fig. 4, represent the section of a vertical wire moving in the direction of the arrow from west to east, across a system of lines of magnetic force running north and south. The curved lines in fig. 4 represent the lines of fluid motion about the wire, the wire being regarded as stationary, and the fluid as having a motion relative to it. It is evident that, from this figure, we can trace the variations of form of an element of the fluid, as the form of the element depends, not on the absolute motion of the whole system, but on the relative motion of its parts.

In front of the wire, that is, on its east side, it will be seen that as the wire approaches each portion of the medium, that portion is more and more compressed in the direction from east to west, and extended in the direction from north to south; and since the axes of the vortices lie in the north and south direction, their velocity will continually tend to increase by Prop. X., unless prevented or checked by electromotive forces acting on the circumference of each vortex.

We shall consider an electromotive force as positive when the vortices tend to move the interjacent particles *upwards* perpendicularly to the plane of the paper.

The vortices appear to revolve as the hands of a watch when we look at them from south to north; so that each vortex moves upwards on its west side, and downwards on its east side. In front of the wire, therefore, where each vortex is striving to increase its velocity, the electromotive force upwards must be greater on its west than on its east side. There will therefore be a continual increase of upward electromotive force from the remote east, where it is zero, to the front of the moving wire, where the upward force will be strongest.

Behind the wire a different action takes place. As the wire moves away from each successive portion of the medium, that portion is extended from east to west, and compressed from north to south, so as to tend to diminish the velocity of the vortices, and therefore to make the upward electromotive force greater on the east than on the west side of each vortex. The upward electromotive force will therefore increase continually from the remote west, where it is zero, to the back of the moving wire, where it will be strongest.

It appears, therefore, that a vertical wire moving eastwards will experience an electromotive force tending to produce in it an upward current. If there is no conducting circuit in connexion with the ends of the wire, no current will be formed, and the magnetic forces will not be altered; but if such a circuit exists, there will be a current, and the lines of magnetic force and the velocity of the vortices will be altered from their state previous to the motion of the wire. The change in the lines of force is shown in fig. 5. The vortices in front of the wire, instead of merely producing pressures, actually increase in velocity, while those behind have their velocity diminished, and those at the sides of the wire have the direction of their axes altered; so that the final effect is to produce a force acting on the wire as a resistance to its motion. We may now recapitulate the assumptions we have made, and the results we have obtained.

(1) Magneto-electric phenomena are due to the existence of matter under certain conditions of motion or of pressure in every part of the magnetic field, and not to direct action at a distance between the magnets or currents. The substance producing these effects may be a certain part of ordinary matter, or it may be an æther associated with matter. Its density is greatest in iron, and least in diamagnetic substances; but it must be in all cases, except that of iron, very rare, since no other substance has a large ratio of magnetic capacity to what we call a vacuum.

(2) The condition of any part of the field, through which lines of magnetic force pass, is one of unequal pressure in different directions, the direction of the lines of force being that of least pressure, so that the lines of force may be considered lines of tension.

(3) This inequality of pressure is produced by the existence in the medium of vortices or eddies, having their axes in the direction of the lines of force, and having their direction of rotation determined by that of the lines of force.

We have supposed that the direction was that of a watch to a spectator looking from south to north. We might with equal propriety have chosen the reverse direction, as far as known facts are concerned, by supposing resinous electricity instead of vitreous to be positive. The effect of these vortices depends on their density, and on their velocity at the circumference, and is independent of their diameter. The density must be proportional to the capacity of the substance for magnetic induction, that of the vortices in air being 1. The velocity must be very great, in order to produce so powerful effects in so rare a medium.

The size of the vortices is indeterminate, but is probably very small as compared with that of a complete molecule of ordinary matter*.

(4) The vortices are separated from each other by a single layer of round particles, so that a system of cells is formed, the partitions being these layers of particles, and the substance of each cell being capable of rotating as a vortex.

(5) The particles forming the layer are in *rolling contact* with both the vortices which they separate, but do not rub against each other. They are perfectly free to roll between the vortices

* The angular momentum of the system of vortices depends on their average diameter; so that if the diameter were sensible, we might expect that a magnet would behave as if it contained a revolving body within it, and that the existence of this rotation might be detected by experiments on the free rotation of a magnet. I have made experiments to investigate this question, but have not yet fully tried the apparatus.

and so to change their place, provided they keep within one *complete molecule* of the substance; but in passing from one molecule to another they experience resistance, and generate irregular motions, which constitute heat. These particles, in our theory, play the part of electricity. Their motion of translation constitutes an electric current, their rotation serves to transmit the motion of the vortices from one part of the field to another, and the tangential pressures thus called into play constitute electromotive force. The conception of a particle having its motion connected with that of a vortex by perfect rolling contact may appear somewhat awkward. I do not bring it forward as a mode of connexion existing in nature, or even as that which I would willingly assent to as an electrical hypothesis. It is, however, a mode of connexion which is mechanically conceivable, and easily investigated, and it serves to bring out the actual mechanical connexions between the known electro-magnetic phenomena; so that I venture to say that any one who understands the provisional and temporary character of this hypothesis, will find himself rather helped than hindered by it in his search after the true interpretation of the phenomena.

The action between the vortices and the layers of particles is in part tangential; so that if there were any slipping or differential motion between the parts in contact, there would be a loss of the energy belonging to the lines of force, and a gradual transformation of that energy into heat. Now we know that the lines of force about a magnet are maintained for an indefinite time without any expenditure of energy; so that we must conclude that wherever there is tangential action between different parts of the medium, there is no motion of slipping between those parts. We must therefore conceive that the vortices and particles roll together without slipping; and that the interior strata of each vortex receive their proper velocities from the exterior stratum without slipping, that is, the angular velocity must be the same throughout each vortex.

The only process in which electro-magnetic energy is lost and transformed into heat, is in the passage of electricity from one molecule to another. In all other cases the energy of the vortices can only be diminished when an equivalent quantity of mechanical work is done by magnetic action.

(6) The effect of an electric current upon the surrounding medium is to make the vortices in contact with the current revolve so that the parts next to the current move in the same direction as the current. The parts furthest from the current will move in the opposite direction; and if the medium is a conductor of electricity, so that the particles are free to move in any direction, the particles touching the outside of these vortices will

be moved in a direction contrary to that of the current, so that there will be an induced current in the opposite direction to the primary one.

If there were no resistance to the motion of the particles, the induced current would be equal and opposite to the primary one, and would continue as long as the primary current lasted, so that it would prevent all action of the primary current at a distance. If there is a resistance to the induced current, its particles act upon the vortices beyond them, and transmit the motion of rotation to them, till at last all the vortices in the medium are set in motion with such velocities of rotation that the particles between them have no motion except that of rotation, and do not produce currents.

In the transmission of the motion from one vortex to another, there arises a force between the particles and the vortices, by which the particles are pressed in one direction and the vortices in the opposite direction. We call the force acting on the particles the electromotive force. The reaction on the vortices is equal and opposite, so that the electromotive force cannot move any part of the medium as a whole, it can only produce currents. When the primary current is stopped, the electromotive forces all act in the opposite direction.

(7) When an electric current or a magnet is moved in presence of a conductor, the velocity of rotation of the vortices in any part of the field is altered by that motion. The force by which the proper amount of rotation is transmitted to each vortex, constitutes in this case also an electromotive force, and, if permitted, will produce currents.

(8) When a conductor is moved in a field of magnetic force, the vortices in it and in its neighbourhood are moved out of their places, and are changed in form. The force arising from these changes constitutes the electromotive force on a moving conductor, and is found by calculation to correspond with that determined by experiment.

We have now shown in what way electro-magnetic phenomena may be imitated by an imaginary system of molecular vortices. Those who have been already inclined to adopt an hypothesis of this kind, will find here the conditions which must be fulfilled in order to give it mathematical coherence, and a comparison, so far satisfactory, between its necessary results and known facts. Those who look in a different direction for the explanation of the facts, may be able to compare this theory with that of the existence of currents flowing freely through bodies, and with that which supposes electricity to act at a distance with a force depending on its velocity, and therefore not subject to the law of conservation of energy.

The facts of electro-magnetism are so complicated and various, that the explanation of any number of them by several different hypotheses must be interesting, not only to physicists, but to all who desire to understand how much evidence the explanation of phenomena lends to the credibility of a theory, or how far we ought to regard a coincidence in the mathematical expression of two sets of phenomena as an indication that these phenomena are of the same kind. We know that partial coincidences of this kind have been discovered; and the fact that they are only partial is proved by the divergence of the laws of the two sets of phenomena in other respects. We may chance to find, in the higher parts of physics, instances of more complete coincidence, which may require much investigation to detect their ultimate divergence.

Note.—Since the first part of this paper was written, I have seen in Crelle's *Journal* for 1859, a paper by Prof. Helmholtz on Fluid Motion, in which he has pointed out that the lines of fluid motion are arranged according to the same laws as the lines of magnetic force, the path of an electric current corresponding to a line of axes of those particles of the fluid which are in a state of rotation. This is an additional instance of a *physical analogy*, the investigation of which may illustrate both electro-magnetism and hydrodynamics.

LII. *Remarks on* Mr. Cayley's *Note.* *By* G. B. Jerrard*.

DESIGNATING by u, v two rational n-valued homogeneous functions of the roots of the equation

$$x^m+\Lambda_1 x^{m-1}+\Lambda_2 x^{m-2}+\ldots+\Lambda_m=0,$$

we find by Lagrange's theory that

$$\left.\begin{aligned} v&=\mu_{n-1}+\mu_{n-2}\,u+\mu_{n-3}\,u^2+\ldots+\mu_0\,u^{n-1}\\ u&=\nu_{n-1}+\nu_{n-2}\,v+\nu_{n-3}\,v^2+\ldots+\nu_0\,v^{n-1}\end{aligned}\right\};\quad\ldots\quad (e)$$

in which $\mu_{n-1}, \mu_{n-2}, \ldots\mu_0, \nu_{n-1}, \nu_{n-2}, \ldots\nu_0$ are symmetrical functions of the roots of the original equation in x; and u, v depend separately on two equations of the nth degree

$$u^n+\alpha_1 u^{n-1}+\alpha_2 u^{n-2}+\ldots+\alpha_n=0,\quad\ldots\quad (\mathrm{U})$$

$$v^n+\beta_1 v^{n-1}+\beta_2 v^{n-2}+\ldots+\beta_n=0,\quad\ldots\quad (\mathrm{V})$$

$\alpha_1, \alpha_2, \ldots\alpha_n, \beta_1, \beta_2, \ldots\beta_n$ being, as well as $\mu_{n-1}, \ldots\nu_0$, symmetrical functions of the roots of the equation in x.

I ought to observe that any coefficient, μ_{n-s}, in the equation

* Communicated by the Author.

(e_1) may take the form

$$\frac{\mathrm{M}_{n-s}}{\mathrm{D}},$$

M_{n-s}, D being expressive of whole functions, and D, which remains constant while M_{n-s} successively becomes $\mathrm{M}_{n-1}, \mathrm{M}_{n-2}, \ldots\mathrm{M}_0$, being such as not to vanish except when (U) has equal roots. We find in fact from the researches of Lagrange that

$$\mathrm{D}=\mathrm{F}(u_1)\ \mathrm{F}(u_2)\ldots\mathrm{F}(u_n),$$

where $\mathrm{F}(u)=nu^{n-1}+(n-1)\alpha_1 u^{n-2}+(n-2)\alpha_2 u^{n-3}+\ldots+\alpha_{n-1}$; $u_1, u_2, \ldots u_n$ denoting the n roots of the equation (U).

Of the meaning of the analogous expression

$$\frac{\mathrm{N}_{n-s}}{\mathrm{D}'}$$

which obtains in (e_2) for ν_{n-s}, it is needless to speak. Indeed, having found one of the two equations (e), say (e_1), we may in general deduce the other, (e_2), from it by the method of the highest common divisor.

Let us now examine the following extract from Mr. Cayley's paper in the last Number (that for March) of the Philosophical Magazine.

"Writing," he says, "with Mr. Cockle and Mr. Harley,

$$\tau=x_\alpha x_\beta+x_\beta x_\gamma+x_\gamma x_\delta+x_\delta x_\epsilon+x_\epsilon x_\alpha,$$

$$\tau'=x_\alpha x_\gamma+x_\gamma x_\epsilon+x_\epsilon x_\beta+x_\beta x_\delta+x_\delta x_\alpha,$$

then $(\tau+\tau'$ is a symmetrical function of all the roots, and it must be excluded; but) $(\tau-\tau')^2$ or $\tau\tau'$ are each of them 6-valued functions of the form in question, and either of these functions is linearly connected with the Resolvent Product. In Lagrange's general theory of the solution of equations, if

$$f\iota=x_1+\iota x_2+\iota^2 x_3+\iota^3 x_4+\iota^4 x_5,$$

then the coefficients of the equation the roots whereof are $(f\iota)^5$, $(f\iota^2)^5$, $(f\iota^3)^5$, $(f\iota^4)^5$, and in particular the last coefficient $(f\iota f\iota^2 f\iota^3 f\iota^4)^5$, are determined by an equation of the sixth degree; and this last coefficient is a perfect fifth power, and its fifth root, or $f\iota f\iota^2 f\iota^3 f\iota^4$, is the function just referred to as the Resolvent Product.

"The conclusion from the foregoing remarks is, that *if the equation for* W *has the above property of the rational expressibility of its roots,* the equation of the sixth order resulting from Lagrange's general theory has the same property."

Here the question arises, Is it certain that $f\iota f\iota^2 f\iota^3 f\iota^4$ can, by