the electrodes. Floats may be put in all the compartments, except those containing the platinum plates, in which the liquid is too much agitated by gaseous bubbles due to electrolysation; and on viewing these floats with a glass, the displacements I have described become sensible much earlier. In the intermediate compartments the liquid remains stationary for several hours; but after a certain time the liquid begins to rise in the compartments towards the positive pole, and to fall in those towards the negative pole. I shall mention but one precaution which must not be neglected in these experiments, namely, that the diaphragms must be as equal as possible.

In a second series of experiments I closed one end of each of two glass tubes with a porcelain diaphragm fixed with mastic. Each of these tubes was then placed in a glass vessel, and both vessels and tubes were filled to the same height with well-water. The same current passed through both tubes, in each case passing from the water in the vessel to that in the tube, the only difference being in the position of the platinum electrodes, which in the one case were very near the diaphragm, while in the other they were placed at the greatest possible distance from it. Under these circumstances I invariably found that the electric endosmose made its appearance much sooner, and with much greater intensity in the first case than in the second.

I shall not stop to discuss the consequences of these experiments, since they appear to me to be obvious, and to prove that the phenomenon in question is no other than that mentioned above, that is to say, a case of endosmose produced by changes in the composition of the liquid in contact with the two electrodes. I should mention here that the liquid round the positive electrode always acquires an acid reaction, while that round the negative electrode becomes alkaline, and that these effects are produced even when distilled water is employed. I did not content myself with the ancient experiments of Dutrochet, which prove that there is a current of endosmose from an acid liquid to water, from water to an alkaline liquid, and from an acid to an alkaline liquid. I repeated the experiment with the two liquids which had been in contact with the electrodes as described above, sometimes making use of both of the liquids, sometimes testing each of them separately with pure water. I invariably found that there was endosmose from the liquid that had been in contact with the positive electrode to pure water, and from pure water to the liquid that had been in contact with the negative electrode. It appears therefore that the conditions for the production of ordinary endosmose are undoubtedly present in the phenomenon called electric endosmose. I should, however, observe that the amount of displacement by endosmose is much less when the liquids which have been in contact with the electrodes are experimented on simply without any electric current, and that it is hardly perceptible in the case of electrolysed distilled water. Without attempting to explain all the phenomena of electric endosmose, it seems natural to suppose that the presence of electricity, and the peculiar state in which the elements of electrolysation are produced, give to these products properties which influence the effect of endosmose, and which cease with the cessation of the current.—*Comptes Rendus*, Dec. 1860.

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XXV. *On Physical Lines of Force.* By J. C. MAXWELL, *Professor of Natural Philosophy in King's College, London**.

PART I.—*The Theory of Molecular Vortices applied to Magnetic Phenomena.*

IN all phenomena involving attractions or repulsions, or any forces depending on the relative position of bodies, we have to determine the *magnitude* and *direction* of the force which would act on a given body, if placed in a given position.

In the case of a body acted on by the gravitation of a sphere, this force is inversely as the square of the distance, and in a straight line to the centre of the sphere. In the case of two attracting spheres, or of a body not spherical, the magnitude and direction of the force vary according to more complicated laws. In electric and magnetic phenomena, the magnitude and direction of the resultant force at any point is the main subject of investigation. Suppose that the direction of the force at any point is known, then, if we draw a line so that in every part of its course it coincides in direction with the force at that point, this line may be called a *line of force*, since it indicates the direction of the force in every part of its course.

By drawing a sufficient number of lines of force, we may indicate the direction of the force in every part of the space in which it acts.

Thus if we strew iron filings on paper near a magnet, each filing will be magnetized by induction, and the consecutive filings will unite by their opposite poles, so as to form fibres, and these fibres will *indicate* the direction of the lines of force. The beautiful illustration of the presence of magnetic force afforded by this experiment, naturally tends to make us think of

* Communicated by the Author.

the lines of force as something real, and as indicating something more than the mere resultant of two forces, whose seat of action is at a distance, and which do not exist there at all until a magnet is placed in that part of the field. We are dissatisfied with the explanation founded on the hypothesis of attractive and repellent forces directed towards the magnetic poles, even though we may have satisfied ourselves that the phenomenon is in strict accordance with that hypothesis, and we cannot help thinking that in every place where we find these lines of force, some physical state or action must exist in sufficient energy to produce the actual phenomena.

My object in this paper is to clear the way for speculation in this direction, by investigating the mechanical results of certain states of tension and motion in a medium, and comparing these with the observed phenomena of magnetism and electricity. By pointing out the mechanical consequences of such hypotheses, I hope to be of some use to those who consider the phenomena as due to the action of a medium, but are in doubt as to the relation of this hypothesis to the experimental laws already established, which have generally been expressed in the language of other hypotheses.

I have in a former paper* endeavoured to lay before the mind of the geometer a clear conception of the relation of the lines of force to the space in which they are traced. By making use of the conception of currents in a fluid, I showed how to draw lines of force, which should indicate by their number the amount of force, so that each line may be called a unit-line of force (see Faraday's 'Researches,' 3122); and I have investigated the path of the lines where they pass from one medium to another.

In the same paper I have found the geometrical significance of the "Electrotonic State," and have shown how to deduce the mathematical relations between the electrotonic state, magnetism, electric currents, and the electromotive force, using mechanical illustrations to assist the imagination, but not to account for the phenomena.

I propose now to examine magnetic phenomena from a mechanical point of view, and to determine what tensions in, or motions of, a medium are capable of producing the mechanical phenomena observed. If, by the same hypothesis, we can connect the phenomena of magnetic attraction with electromagnetic phenomena and with those of induced currents, we shall have found a theory which, if not true, can only be proved to be erroneous by experiments which will greatly enlarge our knowledge of this part of physics.

* See a paper "On Faraday's Lines of Force," Cambridge Philosophical Transactions, vol. x. part 1.

The mechanical conditions of a medium under magnetic influence have been variously conceived of, as currents, undulations, or states of displacement or strain, or of pressure or stress.

Currents, issuing from the north pole and entering the south pole of a magnet, or circulating round an electric current, have the advantage of representing correctly the geometrical arrangement of the lines of force, if we could account on mechanical principles for the phenomena of attraction, or for the currents themselves, or explain their continued existence.

Undulations issuing from a centre would, according to the calculations of Professor Challis, produce an effect similar to attraction in the direction of the centre; but admitting this to be true, we know that two series of undulations traversing the same space do not combine into one resultant as two attractions do, but produce an effect depending on relations of *phase* as well as intensity, and if allowed to proceed, they diverge from each other without any mutual action. In fact the mathematical laws of attractions are not analogous in any respect to those of undulations, while they have remarkable analogies with those of currents, of the conduction of heat and electricity, and of elastic bodies.

In the Cambridge and Dublin Mathematical Journal for January 1847, Professor William Thomson has given a "Mechanical Representation of Electric, Magnetic, and Galvanic Forces," by means of the displacements of the particles of an elastic solid in a state of strain. In this representation we must make the angular displacement at every point of the solid proportional to the magnetic force at the corresponding point of the magnetic field, the direction of the axis of rotation of the displacement corresponding to the direction of the magnetic force. The absolute displacement of any particle will then correspond in magnitude and direction to that which I have identified with the electrotonic state; and the relative displacement of any particle, considered with reference to the particle in its immediate neighbourhood, will correspond in magnitude and direction to the quantity of electric current passing through the corresponding point of the magneto-electric field. The author of this method of representation does not attempt to explain the origin of the observed forces by the effects due to these strains in the elastic solid, but makes use of the mathematical analogies of the two problems to assist the imagination in the study of both.

We come now to consider the magnetic influence as existing in the form of some kind of pressure or tension, or, more generally, of *stress* in the medium.

Stress is action and reaction between the consecutive parts of a body, and consists in general of pressures or tensions different in different directions at the same point of the medium.

The necessary relations among these forces have been investigated by mathematicians; and it has been shown that the most general type of a stress consists of a combination of three principal pressures or tensions, in directions at right angles to each other.

When two of the principal pressures are equal, the third becomes an axis of symmetry, either of greatest or least pressure, the pressures at right angles to this axis being all equal.

When the three principal pressures are equal, the pressure is equal in every direction, and there results a stress having no determinate axis of direction, of which we have an example in simple hydrostatic pressure.

The general type of a stress is not suitable as a representation of a magnetic force, because a line of magnetic force has direction and intensity, but has no third quality indicating any difference between the *sides* of the line, which would be analogous to that observed in the case of polarized light*.

We must therefore represent the magnetic force at a point by a stress having a single axis of greatest or least pressure, and all the pressures at right angles to this axis equal. It may be objected that it is inconsistent to represent a line of force, which is essentially dipolar, by an axis of stress, which is necessarily isotropic; but we know that *every* phenomenon of action and reaction is isotropic in its *results*, because the effects of the force on the bodies between which it acts are equal and opposite, while the nature and origin of the force may be dipolar, as in the attraction between a north and a south pole.

Let us next consider the mechanical effect of a state of stress symmetrical about an axis. We may resolve it, in all cases, into a simple hydrostatic pressure, combined with a simple pressure or tension along the axis. When the axis is that of greatest pressure, the force along the axis will be a pressure. When the axis is that of least pressure, the force along the axis will be a tension.

If we observe the lines of force between two magnets, as indicated by iron filings, we shall see that whenever the lines of force pass from one pole to another, there is *attraction* between those poles; and where the lines of force from the poles avoid each other and are dispersed into space, the poles *repel* each other, so that in both cases they are drawn in the direction of the resultant of the lines of force.

It appears therefore that the stress in the axis of a line of magnetic force is a *tension*, like that of a rope.

If we calculate the lines of force in the neighbourhood of two gravitating bodies, we shall find them the same in direction as

* See Faraday's 'Researches,' 3252.

those near two magnetic poles of the same name; but we know that the mechanical effect is that of attraction instead of repulsion. The lines of force in this case do not run between the bodies, but avoid each other, and are dispersed over space. In order to produce the effect of attraction, the stress along the lines of gravitating force must be a *pressure*.

Let us now suppose that the phenomena of magnetism depend on the existence of a tension in the direction of the lines of force, combined with a hydrostatic pressure; or in other words, a pressure greater in the equatorial than in the axial direction: the next question is, what mechanical explanation can we give of this inequality of pressures in a fluid or mobile medium? The explanation which most readily occurs to the mind is that the excess of pressure in the equatorial direction arises from the centrifugal force of vortices or eddies in the medium having their axes in directions parallel to the lines of force.

This explanation of the cause of the inequality of pressures at once suggests the means of representing the dipolar character of the line of force. Every vortex is essentially dipolar, the two extremities of its axis being distinguished by the direction of its revolution as observed from those points.

We also know that when electricity circulates in a conductor, it produces lines of magnetic force passing through the circuit, the direction of the lines depending on the direction of the circulation. Let us suppose that the direction of revolution of our vortices is that in which vitreous electricity must revolve in order to produce lines of force whose direction within the circuit is the same as that of the given lines of force.

We shall suppose at present that all the vortices in any one part of the field are revolving in the same direction about axes nearly parallel, but that in passing from one part of the field to another, the direction of the axes, the velocity of rotation, and the density of the substance of the vortices are subject to change. We shall investigate the resultant mechanical effect upon an element of the medium, and from the mathematical expression of this resultant we shall deduce the physical character of its different component parts.

Prop. I.—If in two fluid systems geometrically similar the velocities and densities at corresponding points are proportional, then the differences of pressure at corresponding points due to the motion will vary in the duplicate ratio of the velocities and the simple ratio of the densities.

Let l be the ratio of the linear dimensions, m that of the velocities, n that of the densities, and p that of the pressures due to the motion. Then the ratio of the *masses* of corresponding portions will be l^3n, and the ratio of the velocities acquired in traversing similar parts of the systems will be m; so that l^3mn is the ratio of the momenta acquired by similar portions in traversing similar parts of their paths.

The ratio of the surfaces is l^2, that of the forces acting on them is l^2p, and that of the times during which they act is $\frac{l}{m}$; so that the ratio of the impulse of the forces is $\frac{l^3p}{m}$, and we have now

$$l^3mn=\frac{l^3p}{m},$$

or

$$m^2n=p;$$

that is, the ratio of the pressures due to the motion (p) is compounded of the ratio of the densities (n) and the duplicate ratio of the velocities (m^2), and does not depend on the linear dimensions of the moving systems.

In a circular vortex, revolving with uniform angular velocity, if the pressure at the axis is p_0, that at the circumference will be $p_1=p_0+\frac{1}{2}\rho v^2$, where ρ is the density and v the velocity at the circumference. The *mean pressure* parallel to the axis will be

$$p_0+\tfrac{1}{4}\rho v^2=p_2.$$

If a number of such vortices were placed together side by side with their axes parallel, they would form a medium in which there would be a pressure p_2 parallel to the axes, and a pressure p_1 in any perpendicular direction. If the vortices are circular, and have uniform angular velocity and density throughout, then

$$p_1-p_2=\tfrac{1}{4}\rho v^2.$$

If the vortices are not circular, and if the angular velocity and the density are not uniform, but vary according to the same law for all the vortices,

$$p_1-p_2=C\rho v^2,$$

where ρ is the mean density, and C is a numerical quantity depending on the distribution of angular velocity and density in the vortex. In future we shall write $\frac{\mu}{4\pi}$ instead of $C\rho$, so that

$$p_1-p_2=\frac{1}{4\pi}\mu v^2, \quad \ldots \ldots \quad (1)$$

where μ is a quantity bearing a constant ratio to the density, and v is the linear velocity at the circumference of each vortex.

A medium of this kind, filled with molecular vortices having their axes parallel, differs from an ordinary fluid in having different pressures in different directions. If not prevented by properly arranged pressures, it would tend to expand laterally. In so doing, it would allow the diameter of each vortex to expand

and its velocity to diminish in the same proportion. In order that a medium having these inequalities of pressure in different directions should be in equilibrium, certain conditions must be fulfilled, which we must investigate.

Prop. II.—If the direction-cosines of the axes of the vortices with respect to the axes of x, y, and z be l, m, and n, to find the normal and tangential stresses on the coordinate planes.

The actual stress may be resolved into a simple hydrostatic pressure p_1 acting in all directions, and a simple tension p_1-p_2, or $\frac{1}{4\pi}\mu v^2$, acting along the axis of stress.

Hence if p_{xx}, p_{yy}, and p_{zz} be the normal stresses parallel to the three axes, considered positive when they tend to increase those axes; and if p_{yz}, p_{zx}, and p_{xy} be the tangential stresses in the three coordinate planes, considered positive when they tend to increase simultaneously the symbols subscribed, then by the resolution of stresses*,

$$p_{xx}=\frac{1}{4\pi}\mu v^2l^2-p_1$$

$$p_{yy}=\frac{1}{4\pi}\mu v^2m^2-p$$

$$p_{zz}=\frac{1}{4\pi}\mu v^2n^2-p_1$$

$$p_{yz}=\frac{1}{4\pi}\mu v^2mn$$

$$p_{zx}=\frac{1}{4\pi}\mu v^2nl$$

$$p_{xy}=\frac{1}{4\pi}\mu v^2lm.$$

If we write

$$\alpha=vl, \quad \beta=vm, \text{ and } \gamma=vn,$$

then

$$\left.\begin{array}{ll} p_{xx}=\frac{1}{4\pi}\mu\alpha^2-p_1 & p_{yz}=\frac{1}{4\pi}\mu\beta\gamma \\ p_{yy}=\frac{1}{4\mu}\mu\beta^2-p_1 & p_{zx}=\frac{1}{4\pi}\mu\gamma\alpha \\ p_{zz}=\frac{1}{4\pi}\mu\gamma^2-p_1 & p_{xy}=\frac{1}{4\pi}\mu\alpha\beta. \end{array}\right\} \quad (2)$$

Prop. III.—To find the resultant force on an element of the medium, arising from the variation of internal stress.

* Rankine's 'Applied Mechanics,' art. 106.

We have in general, for the force in the direction of x per unit of volume by the law of equilibrium of stresses*,

$$X=\frac{d}{dx}p_{xx}+\frac{d}{dy}p_{xy}+\frac{d}{dz}p_{xz} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

In this case the expression may be written

$$X=\frac{1}{4\pi}\left\{\frac{d(\mu\alpha)}{dx}\alpha+\mu\alpha\frac{d\alpha}{dx}-4\pi\frac{dp_1}{dx}+\frac{d(\mu\beta)}{dy}\alpha+\mu\beta\frac{d\alpha}{dy}\right.$$
$$\left.+\frac{d(\mu\gamma)}{dz}\alpha+\mu\gamma\frac{d\alpha}{dz}\right\}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Remembering that $\alpha\frac{d\alpha}{dx}+\beta\frac{d\beta}{dx}+\gamma\frac{d\gamma}{dx}=\frac{1}{2}\frac{d}{dx}(\alpha^2+\beta^2+\gamma^2)$, this becomes

$$X=\alpha\frac{1}{4\pi}\left(\frac{d}{dx}(\mu\alpha)+\frac{d}{dy}(\mu\beta)+\frac{d}{dz}(\mu\gamma)\right)+\frac{1}{8\pi}\mu\frac{d}{dx}(\alpha^2+\beta^2+\gamma^2)$$
$$-\mu\beta\frac{1}{4\pi}\left(\frac{d\beta}{dx}-\frac{d\alpha}{dy}\right)+\mu\gamma\frac{1}{4\pi}\left(\frac{d\alpha}{dz}-\frac{d\gamma}{dx}\right)-\frac{dp_1}{dx}. \quad . \quad . \quad . \quad (5)$$

The expressions for the forces parallel to the axes of y and z may be written down from analogy.

We have now to interpret the meaning of each term of this expression.

We suppose α, β, γ to be the components of the force which would act upon that end of a unit magnetic bar which points to the north.

μ represents the magnetic inductive capacity of the medium at any point referred to air as a standard. $\mu\alpha$, $\mu\beta$, $\mu\gamma$ represent the quantity of magnetic induction through unit of area perpendicular to the three axes of x, y, z respectively.

The total amount of magnetic induction through a closed surface surrounding the pole of a magnet, depends entirely on the strength of that pole; so that if $dx\,dy\,dz$ be an element, then

$$\left(\frac{d}{dx}\mu\alpha+\frac{d}{dy}\mu\beta+\frac{d}{dz}\mu\gamma\right)dx\,dy\,dz=4\pi m\,dx\,dy\,dz, \quad . \quad (6)$$

which represents the total amount of magnetic induction outwards through the surface of the element $dx\,dy\,dz$, represents the amount of "imaginary magnetic matter" within the element, of the kind which points north.

The *first term* of the value of X, therefore,

$$\alpha\frac{1}{4\pi}\left(\frac{d}{dx}\mu\alpha+\frac{d}{dy}\mu\beta+\frac{d}{dz}\mu\gamma\right), \quad . \quad . \quad . \quad . \quad (7)$$

may be written

$$\alpha m, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

* Rankine's 'Applied Mechanics,' art. 116.

where α is the intensity of the magnetic force, and m is the amount of magnetic matter pointing north in unit of volume.

The physical interpretation of this term is, that the force urging a north pole in the positive direction of x is the product of the intensity of the magnetic force resolved in that direction, and the strength of the north pole of the magnet.

Let the parallel lines from left to right in fig. 1 represent a field of magnetic force such as that of the earth, *s n* being the direction from south to north. The vortices, according to our hypothesis, will be in the direction shown by the arrows in fig. 3, that is, in a plane perpendicular to the lines of force, and revolving in the direction of the hands of a watch when observed from *s* looking towards *n*. The parts of the vortices above the plane of the paper will be moving towards *e*, and the parts below that plane towards *w*.

We shall always mark by an arrow-head the direction in which we must look in order to see the vortices rotating in the direction of the hands of a watch. The arrow-head will then indicate the *northward* direction in the magnetic field, that is, the direction in which that end of a magnet which points to the north would set itself in the field.

Now let A be the end of a magnet which points north. Since it repels the north ends of other magnets, the lines of force will be directed *from* A outwards in all directions. On the north side the line A D will be in the *same* direction with the lines of the magnetic field, and the velocity of the vortices will be *increased*. On the south side the line A C will be in the opposite direction, and the velocity of the vortices will be diminished, so that the lines of force are more powerful on the north side of A than on the south side.

We have seen that the mechanical effect of the vortices is to produce a tension along their axes, so that the resultant effect

Fig. 1.

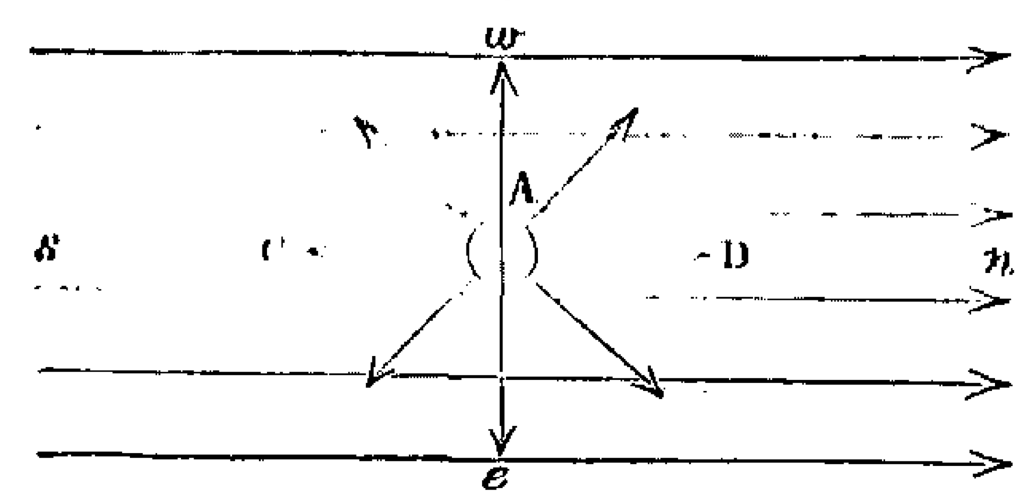


Fig. 2.

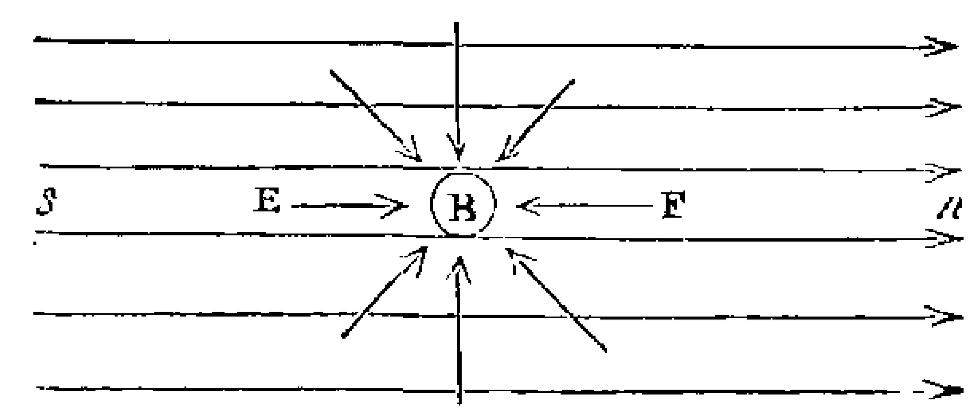
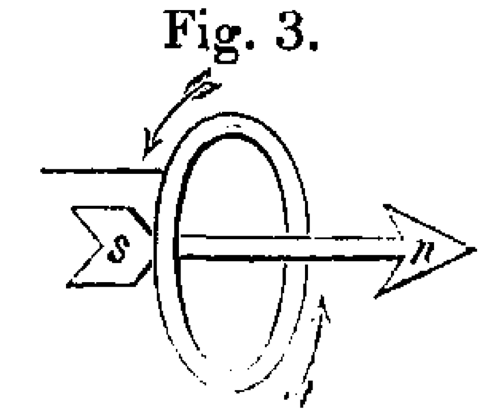


Fig. 3.



on A will be to pull it more powerfully towards D than towards C; that is, A will tend to move to the north.

Let B in fig. 2 represent a south pole. The lines of force belonging to B will tend *towards* B, and we shall find that the lines of force are rendered stronger towards E than towards F, so that the effect in this case is to urge B towards the south.

It appears therefore that, on the hypothesis of molecular vortices, our first term gives a mechanical explanation of the force acting on a north or south pole in the magnetic field.

We now proceed to examine the second term,

$$\frac{1}{8\pi}\mu\frac{d}{dx}(\alpha^2+\beta^2+\gamma^2).$$

Here $\alpha^2+\beta^2+\gamma^2$ is the square of the intensity at any part of the field, and μ is the magnetic inductive capacity at the same place. Any body therefore placed in the field will be urged *towards places of stronger magnetic intensity* with a force depending partly on its own capacity for magnetic induction, and partly on the rate at which the square of the intensity increases.

If the body be placed in a fluid medium, then the medium, as well as the body, will be urged towards places of greater intensity, so that its hydrostatic pressure will be increased in that direction. The resultant effect on a body placed in the medium will be the *difference* of the actions on the body and on the portion of the medium which it displaces, so that the body will tend to or from places of greatest magnetic intensity, according as it has a greater or less capacity for magnetic induction than the surrounding medium.

In fig. 4 the lines of force are represented as converging and becoming more powerful towards the right, so that the magnetic tension at B is stronger than at A, and the body A B will be urged to the right. If the capacity for magnetic induction is greater in the body than in the surrounding medium, it will move to the right, but if less it will move to the left.

Fig. 4.

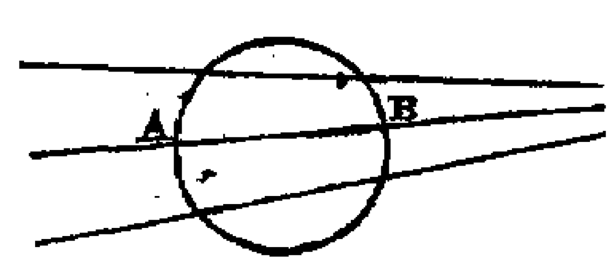
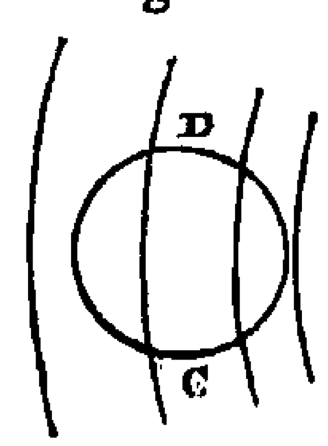


Fig. 5.



We may suppose in this case that the lines of force are converging to a magnetic pole, either north or south, on the right hand.

In fig. 5 the lines of force are represented as vertical, and becoming more numerous towards the right. It may be shown that if the force increases towards the right, the lines of force will be curved towards the right. The effect of the magnetic tensions will then be to draw any body towards the right with a force depending on the excess of its inductive capacity over that of the surrounding medium.

We may suppose that in this figure the lines of force are those surrounding an electric current perpendicular to the plane of the paper and on the right hand of the figure.

These two illustrations will show the mechanical effect on a paramagnetic or diamagnetic body placed in a field of varying magnetic force, whether the increase of force takes place along the lines or transverse to them. The form of the second term of our equation indicates the general law, which is quite independent of the direction of the lines of force, and depends solely on the manner in which the force *varies* from one part of the field to another.

We come now to the third term of the value of X,

$$-\mu\beta\frac{1}{4\pi}\left(\frac{d\beta}{dx}-\frac{d\alpha}{dy}\right).$$

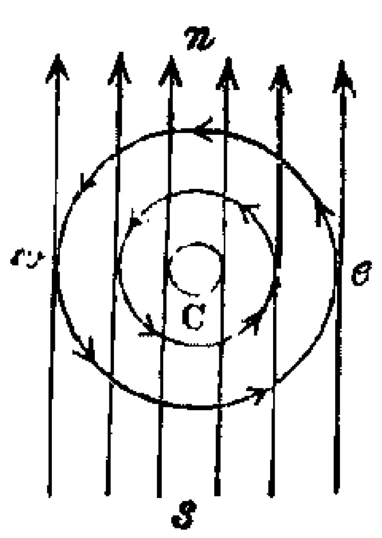
Here $\mu\beta$ is, as before, the quantity of magnetic induction through unit of area perpendicular to the axis of y, and $\frac{d\beta}{dx}-\frac{d\alpha}{dy}$ is a quantity which would disappear if $\alpha dx+\beta dy+\gamma dz$ were a complete differential, that is, if the force acting on a unit north pole were subject to the condition that no work can be done upon the pole in passing round any closed curve. The quantity represents the work done on a north pole in travelling round unit of area in the direction from $+x$ to $+y$ parallel to the plane of xy. Now if an electric current whose strength is r is traversing the axis of z, which, we may suppose, points vertically upwards, then, if the axis of x is east and that of y north, a unit north pole will be urged round the axis of z in the direction from x to y, so that in one revolution the work done will be $=4\pi r$. Hence $\frac{1}{4\pi}\left(\frac{d\beta}{dx}-\frac{d\alpha}{dy}\right)$ represents the *strength of an electric current parallel to z* through unit of area; and if we write

$$\frac{1}{4\pi}\left(\frac{d\gamma}{dy}-\frac{d\beta}{dz}\right)=p,\quad \frac{1}{4\pi}\left(\frac{d\alpha}{dz}-\frac{d\gamma}{dx}\right)=q,\quad \frac{1}{4\pi}\left(\frac{d\beta}{dx}-\frac{d\alpha}{dy}\right)=r, \qquad (9)$$

then p, q, r will be the quantity of electric current per unit of area perpendicular to the axes of x, y, and z respectively.

The physical interpretation of the third term of X, $-\mu\beta r$, is that if $\mu\beta$ is the quantity of magnetic induction parallel to y, and r the quantity of electricity flowing in the direction of z, the element will be urged in the direction of $-x$, transversely to the direction of the current and of the lines of force; that is, an *ascending* current in a field of force magnetized towards the *north* would tend to move *west*.

Fig. 6.



To illustrate the action of the molecular vortices, let $s\,n$ be the direction of magnetic force in the field, and let C be the section of an ascending magnetic current perpendicular to the paper. The lines of force due to this current will be circles drawn in the opposite direction from that of the hands of a watch; that is, in the direction $n\,w\,s\,e$. At e the lines of force will be the sum of those of the field and of the current, and at w they will be the difference of the two sets of lines; so that the vortices on the east side of the current will be more powerful than those on the west side. Both sets of vortices have their equatorial parts turned towards C, so that they tend to expand towards C, but those on the east side have the greatest effect, so that the resultant effect on the current is to urge it towards the *west*.

The fourth term,

$$+\mu\gamma\frac{1}{4\pi}\left(\frac{d\alpha}{dz}-\frac{d\gamma}{dx}\right), \text{ or } +\mu\gamma q, \quad . \quad . \quad . \quad (10)$$

may be interpreted in the same way, and indicates that a current q in the direction of y, that is, to the north, placed in a magnetic field in which the lines are vertically upwards in the direction of z, will be urged towards the *east*.

The fifth term,

$$-\frac{dp_1}{dx}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

merely implies that the element will be urged in the direction in which the hydrostatic pressure p_1 diminishes.

We may now write down the expressions for the components of the resultant force on an element of the medium per unit of volume, thus:

$$X=\alpha m+\frac{1}{8\pi}\mu\frac{d}{dx}(v^2)-\mu\beta r+\mu\gamma q-\frac{dp_1}{dx}, \quad . \quad . \quad . \quad (12)$$

$$Y=\beta m+\frac{1}{8\pi}\mu\frac{d}{dy}(v^2)-\mu\gamma p+\mu\alpha r-\frac{dp_1}{dy}, \quad . \quad . \quad . \quad (13)$$

$$Z=\gamma m+\frac{1}{8\pi}\mu\frac{d}{dz}(v^2)-\mu\alpha q+\mu\beta p-\frac{dp_1}{dz}. \quad . \quad . \quad . \quad (14)$$

The first term of each expression refers to the force acting on magnetic poles.

The second term to the action on bodies capable of magnetism by induction.

The third and fourth terms to the force acting on electric currents.

And the fifth to the effect of simple pressure.

Before going further in the general investigation, we shall consider equations (12, 13, 14,) in particular cases, corresponding to those simplified cases of the actual phenomena which we seek to obtain in order to determine their laws by experiment.

We have found that the quantities p, q, and r represent the resolved parts of an electric current in the three coordinate directions. Let us suppose in the first instance that there is *no electric current*, or that p, q, and r vanish. We have then by (9),

$$\frac{d\gamma}{dy}-\frac{d\beta}{dz}=0, \quad \frac{d\alpha}{dz}-\frac{d\gamma}{dx}=0, \quad \frac{d\beta}{dx}-\frac{d\alpha}{dy}=0, \quad . \quad (15)$$

whence we learn that

$$\alpha dx+\beta dy+\gamma dz=d\phi \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

is an exact differential of ϕ, so that

$$\alpha=\frac{d\phi}{dx}, \quad \beta=\frac{d\phi}{dy}, \quad \gamma=\frac{d\phi}{dz}: \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

μ is proportional to the density of the vortices, and represents the "capacity for magnetic induction" in the medium. It is equal to 1 in air, or in whatever medium the experiments were made which determined the powers of the magnets, the strengths of the electric currents, &c.

Let us suppose μ constant, then

$$m=\frac{1}{4\pi}\left(\frac{d}{dx}(\mu\alpha)+\frac{d}{dy}(\mu\beta)+\frac{d}{dz}(\mu\gamma)\right)$$
$$=\frac{1}{4\pi}\mu\left(\frac{d^2\phi}{dx^2}+\frac{d^2\phi}{dy^2}+\frac{d^2\phi}{dz^2}\right) \quad . \quad . \quad . \quad . \quad . \quad (18)$$

represents the amount of imaginary magnetic matter in unit of volume. That there may be no resultant force on that unit of volume arising from the action represented by the first term of equations (12, 13, 14), we must have $m=0$, or

$$\frac{d^2\phi}{dx^2}+\frac{d^2\phi}{dy^2}+\frac{d^2\phi}{dz^2}=0. \quad . \quad . \quad . \quad . \quad (19)$$

Now it may be shown that equation (19), if true within a given space, implies that the forces acting within that space are such as would result from a distribution of centres of force beyond that space, attracting or repelling inversely as the square of the distance.

Hence the lines of force in a part of space where μ is uniform, and where there are no electric currents, must be such as would result from the theory of "imaginary matter" acting at a distance. The assumptions of that theory are unlike those of ours, but the results are identical.

Let us first take the case of a single magnetic pole, that is, one end of a long magnet, so long that its other end is too far off to have a perceptible influence on the part of the field we are considering. The conditions then are, that equation (18) must be fulfilled at the magnetic pole, and (19) everywhere else. The only solution under these conditions is

$$\phi = -\frac{m}{\mu}\frac{1}{r}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

where r is the distance from the pole, and m the strength of the pole.

The repulsion at any point on a unit pole of the same kind is

$$\frac{d\phi}{dr} = \frac{m}{\mu}\frac{1}{r^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

In the standard medium $\mu=1$; so that the repulsion is simply $\frac{m}{r^2}$ in that medium, as has been shown by Coulomb.

In a medium having a greater value of μ (such as oxygen, solutions of salts of iron, &c.) the attraction, on our theory, ought to be *less* than in air, and in diamagnetic media (such as water, melted bismuth, &c.) the attraction between the same magnetic poles ought to be *greater* than in air.

The experiments necessary to demonstrate the difference of attraction of two magnets according to the magnetic or diamagnetic character of the medium in which they are placed, would require great precision, on account of the limited range of magnetic capacity in the fluid media known to us, and the small amount of the difference sought for as compared with the whole attraction.

Let us next take the case of an electric current whose quantity is C, flowing through a cylindrical conductor whose radius is R, and whose length is infinite as compared with the size of the field of force considered.

Let the axis of the cylinder be that of z, and the direction of the current positive, then within the conductor the quantity of current per unit of area is

$$r = \frac{C}{\pi R^2} = \frac{1}{4\pi}\left(\frac{d\beta}{dx} - \frac{d\alpha}{dy}\right); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

so that within the conductor

$$\alpha = -2\frac{C}{R^2}y, \quad \beta = 2\frac{C}{R^2}x, \quad \gamma = 0. \quad . \quad . \quad . \quad . \quad (23)$$

Beyond the conductor, in the space round it,

$$\phi = 2C\tan^{-1}\frac{y}{x}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

$$\alpha = \frac{d\phi}{dx} = -2C\frac{y}{x^2+y^2}, \quad \beta = \frac{d\phi}{dy} = 2C\frac{x}{x^2+y^2}, \quad \gamma = \frac{d\phi}{dz} = 0. \quad (25)$$

If $\rho = \sqrt{x^2+y^2}$ is the perpendicular distance of any point from the axis of the conductor, a unit north pole will experience a force $= \frac{2C}{\rho}$, tending to move it round the conductor in the direction of the hands of a watch, if the observer view it in the direction of the current.

Let us now consider a current running parallel to the axis of z in the plane of xz at a distance ρ. Let the quantity of the current be c', and let the length of the part considered be l, and its section s, so that $\frac{c'}{s}$ is its strength per unit of section. Putting this quantity for ρ in equations (12, 13, 14), we find

$$X = -\mu\beta\frac{c'}{s}$$

per unit of volume; and multiplying by ls, the volume of the conductor considered, we find

$$X = -\mu\beta c' l$$
$$= -2\mu\frac{Cc'l}{\rho}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

showing that the second conductor will be attracted towards the first with a force inversely as the distance.

We find in this case also that the amount of attraction depends on the value of μ, but that it varies directly instead of inversely as μ; so that the attraction between two conducting wires will be greater in oxygen than in air, and greater in air than in water.

We shall next consider the nature of electric currents and electromotive forces in connexion with the theory of molecular vortices.