Michelson–Morley experiment revisited*

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Abstract

The idea of the Michelson–Morley experiment is theoretically reanalyzed. Elementary arguments are put forward indicating that, rigorously, conclusions following from the experiment impose rather weak constraints on the angular dependence of the speed of light.

According to XIX-century physics light was supposed to propagate in the aether, a mysterious medium devised especially for this purpose. As light was to travel with respect to the aether with a fixed speed, an experiment was suggested to detect the dependence of the speed on the direction in a moving frame of the laboratory on the Earth. The experiment was proposed and the first time performed by Albert Michelson (a Nobel laureate, American physicist born in Poland) [1]. The experiment is continually being repeated, known as the Michelson–Morley (MM) experiment, with ever increasing accuracy and improved technical realization (see, e.g., [2]). The result is always the same, negative, i.e. no dependence of the speed of light on the direction has been detected, at least, this is a standard (but not rigorous) conclusion. As a consequence, the speed of light is the same in each inertial frame in any direction, and no aether exists.

An elementary theoretical analysis of the MM experiment proposed in this paper yields a general solution. In other words, we show that the standard, negative result of the MM experiment imposes only very mild constrains on the angular dependence (anisotropy) of the speed of light, too mild to exclude the idea of the aether at all. It is not our intention to revitalize the idea of the aether but only we would like to quantify the freedom allowed by the MM experiment.

*On the occasion of the *World Year of Physics 2oo5*.

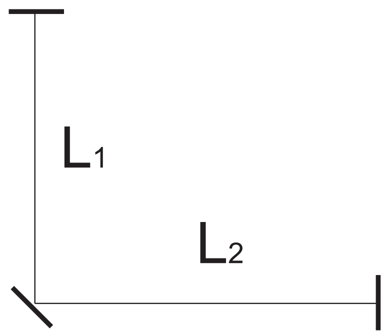

Figure 1: The Michelson–Morley interferometer.

Let us briefly recall the idea of the MM experiment. In any, traditional or modern, version of the experiment we compare the differences of the time of the travel of light in two orthogonal directions (vertical and horizontal, say) in two positions (primary and final) differing by 90°. To arrive at our result we assume full generality. Therefore, let us introduce the following notation (see Fig. 1):

$c_+^\perp$ — forward vertical speed of light,
$c_-^\perp$ — backward vertical speed of light,
$c_+^\shortparallel$ — forward horizontal speed of light,
$c_-^\shortparallel$ — backward horizontal speed of light,

and

L_1 — primarily vertical route,
L_2 — primarily horizontal route.

In primary position, the vertical travel of light takes

$$t_1 = \frac{L_1}{c_+^\perp} + \frac{L_1}{c_-^\perp}, \tag{1}$$

whereas in horizontal direction

$$t_2 = \frac{L_2}{c_+^\shortparallel} + \frac{L_2}{c_-^\shortparallel}. \tag{2}$$

The difference is

$$\Delta t = t_2 - t_1 = L_2\left(\frac{1}{c_+^\shortparallel} + \frac{1}{c_-^\shortparallel}\right) - L_1\left(\frac{1}{c_+^\perp} + \frac{1}{c_-^\perp}\right). \tag{3}$$

After rotation, we have

$$\Delta t' = L_2\left(\frac{1}{c_+^\perp} + \frac{1}{c_-^\perp}\right) - L_1\left(\frac{1}{c_+^\shortparallel} + \frac{1}{c_-^\shortparallel}\right). \tag{4}$$

The change one could possibly observe is of the form

$$\Delta t^* = \Delta t' - \Delta t = (L_1 + L_2)\left[\left(\frac{1}{c_+^\perp} + \frac{1}{c_-^\perp}\right) - \left(\frac{1}{c_+^{\shortparallel}} + \frac{1}{c_-^{\shortparallel}}\right)\right]. \tag{5}$$

The negative result of the MM experiment formally means $\Delta t^* = 0$. (Our analysis is purely theoretical, and we are not going to engage into the debate whether the equality $\Delta t^* = 0$ is experimentally well-established or not.) A very (in fact, too) simplified but common argumentation says that the equality $\Delta t^* = 0$ implies $c_+^\perp = c_-^\perp = c_+^{\shortparallel} = c_-^{\shortparallel}$.

A bit less simplified, and also very common, argumentation presumes a strictly defined form of $c_\pm^\perp$, $c_\pm^{\shortparallel}$ following from geometrical analysis (the Pythagorean theorem and geometrical addition of velocities) of the movement of inertial systems with respect to the aether, i.e.

$$c_\pm^\perp = \sqrt{c^2 - v^2} \tag{6}$$

and

$$c_\pm^{\shortparallel} = c \mp v, \tag{7}$$

where c — speed of light with respect to the aether, v — velocity of the inertial system with respect to the aether. The possibility (6)-(7) is also excluded by virtue of the experimental fact $\Delta t^* = 0$, where Δt^* is defined by Eq. (5). But there is infinitely many other possibilities consistent with Δt^* defined by Eq. (5) equal to zero. The aim of the paper is to bring this fact to the reader's attention.

We can rewrite the equation $\Delta t^* = 0$ with Δt^* defined by Eq. (5) in the following form

$$z_+^\perp + z_-^\perp = z_+^{\shortparallel} + z_-^{\shortparallel}, \tag{8}$$

where for simplicity we use inverses of the speeds, $z \equiv 1/c$. Eq. (8) is a functional equation with continuum of solutions.

Let us consider the two-dimensional case, first. The simplest way to solve the problem in two dimensions is to analyze a picture (see, Fig. 2). The length of the vector $\vec{z}$ in Fig. 2 corresponds to the inverse of the speed of light in the direction of $\vec{z}$. Thus, the circle corresponds to a constant (independent of the direction) speed of light. Since Eq. (8) is a single equation with four unknowns, the three unknowns are arbitrary and determine the fourth one. For example, the three coordinates: of the black dot, of the intersections of the solid curve with positive y and positive x axes respectively are arbitrary, and they determine the coordinate of the white dot satisfying Eq. (8). Analogously, the whole solid curve in Fig. 2 is practically arbitrary (it should only be unique as a continuous function of polar coordinates) and determines the (dotted) segment with negative coordinates x and y. As a side remark, we observe that changing the angle between the arms of the MM interferometer changes the dotted segment.

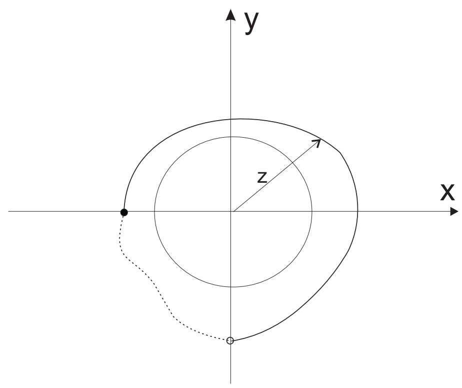

Figure 2: A general solution of the two-dimensional problem, i.e., a solution of the functional equation (8).

Namely, the angle between the vectors $\vec{z}$ pointing at the black dot and the white dot, respectively, is equal to the greater angle between the arms of the interferometer. In principle, the whole curve could be discontinuous in two dotted points. But we can easily avoid this possibility by appropriate deformation of the primary solid curve. We could also require the mirror symmetry of the curve. For example, the axis of the symmetry could be interpreted as the direction of the movement of the laboratory frame, in the spirit of the aether philosophy. A general solution would be determined by an arbitrary solid curve (see Fig. 3) starting at the black dot (as in Fig. 2) and terminating at the intersection with the line $y = x$. Mirror reflection with respect to the line $y = x$ reproduces the rest of the curve except the last quarter, which should by constructed according to (8) as described earlier in this paragraph. One can easily check that, thanks to the mirror symmetry, this time, the whole curve is automatically continuous provided the primary segment is continuous.

Up to now we have been considering an unphysical two-dimensional construction. It appears that in three dimensions constraints are a bit stronger, and the three-dimensional case is qualitatively different. Therefore, we present now an explicit three-dimensional analysis. We will determine directional dependence of the speed of light consistent with MM-type experiments. Thus, roughly, we are interested in non-constant (continuous) functions on a sphere S^2, $z(x) \neq const$, defining (the inverse of) the speed of light in the direction associated to $x \in S^2$, i.e., if we place the interferometer in the center of the interior of S^2, the values of the all four z's ($z_\pm^\perp$, $z_\pm^\parallel$) are given by the values of the function $z(x)$ for x belonging to the points of S^2 corresponding to appropriate axes of the interferometer. The only constraint for the values of $z(x)$

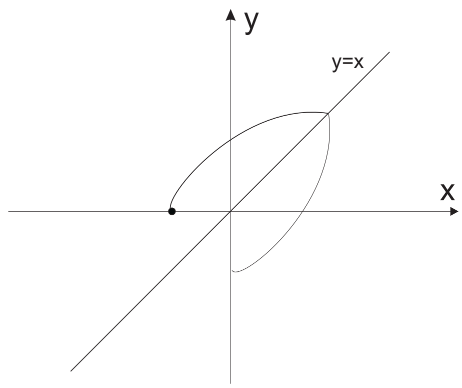

Figure 3: A general axially symmetric two-dimensional solution.

is given by Eq. (8). For any configuration of the interferometer two z's correspond to the upper hemisphere of S^2 and the other two to the lower one. The two upper points uniquely determine the positions of the other two. Therefore, we can confine ourselves to consideration of the two points on the upper hemisphere. We can project the upper hemisphere onto a two-dimensional disc D^2. Let us now define an auxiliary function on D^2, $\bar{z}(y)$, $y \in D^2$, where informally $\bar{z} = z_+ + z_-$, i.e. the value of $\bar{z}$ is the sum of the opposite z's. It is easy to show that $\bar{z}$ has to be a constant function on D^2. It indirectly follows from Eq. (8) by virtue of transitivity. Namely, we can connect arbitrary two points B and C on D^2, and compare the corresponding values of $\bar{z}$ using an additional auxiliary point A, and next apply Eq. (8) to the both pairs (see Fig. 4). The pair B, C does not, in general, corresponds to a position of the interferometer, but the both pairs B, A and C, A, by construction, do. Since $\bar{z}(B) = \bar{z}(A)$ and $\bar{z}(C) = \bar{z}(A)$, then $\bar{z}(B) = \bar{z}(C)$, and $\bar{z} = const$, say $\bar{z} = 2c_0^{-1}$, with c_0 positive.

The constancy of $\bar{z}$ is a strong restriction, not having a counterpart in two dimensions. To decipher this restriction let us now consider an auxiliary function z_+ on D^2 corresponding to one of z's entering the sum defining $\bar{z}$, say z_+, then $z_- = \bar{z} - z_+$. The continuous function z_+ is almost arbitrary and the only restriction, of topological nature, is coming from the fact that the sum of the values of z_+ on opposite sides of the boundary of D^2 should be equal $2c_0^{-1}$.

This simple informal discussion could also be summarized in more mathematical terms. First of all, "in the first approximation", we can observe that the constant function $\bar{z}$ on D^2 is actually a constant function on a two-dimensional real projective surface $\mathbb{R}P^2$. It should seem acceptable, because identification of opposite points of S^2 provides $\mathbb{R}P^2$ by definition, or in other words, we are interested in functions on a

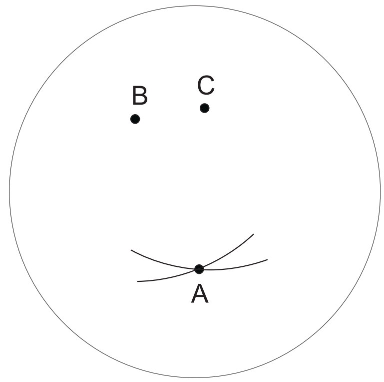

Figure 4: The "angular distance" between B and A, as well as between C and A equals $\pi/2$ because the both pairs correspond to possible positions of the interferometer.

set of rays rather than on a set of directions. For further convenience, we shift the function down to zero, which is a kind of additive normalization. What is less obvious, we claim that we now deal with a twisted real linear bundle $\mathcal{B}$ over the base manifold $\mathbb{R}P^2$. The "shifted" $\bar{z}$ denoted as $\tilde{\bar{z}}$ is a zero cross-section of $\mathcal{B}$, whereas the (shifted) function z_+ denoted as $\tilde{z}_+$ becomes an arbitrary continuous cross-section of $\mathcal{B}$. The form (the twist) of the bundle $\mathcal{B}$ follows from the observation that the values of $\tilde{z}_+$ on opposite sides of the boundary of D^2 (at the identified points) should be opposite, i.e., the non-trivial element of the discrete group $\mathbb{Z}_2$, coming from the corresponding principal bundle, acts in the fiber $\mathbb{R}$ (Fig. 5). To exclude negative speeds of light, and consequently negative times of the travel of light, we can limit the real values of $\tilde{z}$'s to the interval $|\tilde{z}| \leq c_0^{-1}$, and we can speak on the interval bundle $\bar{\bar{\mathcal{B}}}$ instead of the linear bundle $\mathcal{B}$. Recapitulating, we could state that all solutions of the problem (angular-dependent speeds of light consistent with the null-effect of the MM experiment) are parameterized by cross-sections of the non-trivial bundle $\bar{\bar{\mathcal{B}}}$. Obviously, non-constant solutions (non-zero cross-sections) do exist. We could also choose an axially symmetric solution on demand. Still the simplest possibility corresponding to Eq. (6) and Eq. (7) is excluded, but some mild deformations, e.g. expressed in the framework of the Mansouri–Sexl test theory with the parameters $a^2 = b^2\left(1-v^2\right)^2$, $d^2 = b^2\left(1-v^2\right)$ [3], are allowable.

In concluding remarks we refer, generally, to existent literature. In numerous experimentally inclined papers the negative result of the MM experiment is often over interpreted as a proof of the isotropy of the speed of light (see, e.g., the second paragraph in [2]). Instead, in more theoretically inclined literature it is often emphasized

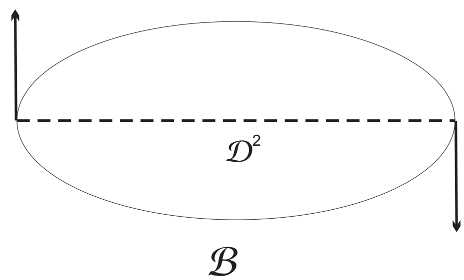

Figure 5: The non-trivial element of $\mathbb{Z}_2$ acts at the identified points on opposite sides of the boundary of D^2.

that in the context of the MM experiment (and not only) one should speak about the two-way speed of light rather than about the one-way one [4]. In fact, our result that cross-sections of the bundle $\overline{\overline{\mathcal{B}}}$ parameterize the admissible one-way speeds of light is a refinement of the above-mentioned statement. The two-dimensional case is even less restrictive than three-dimensional one (richer in a sense) but as unphysical is less interesting.

This work is supported by the Polish Ministry of Scientific Research and Information Technology under the grant No. PBZ/MIN/008/P03/2003 and by the University of Lodz.

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[4] See, e.g.:
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