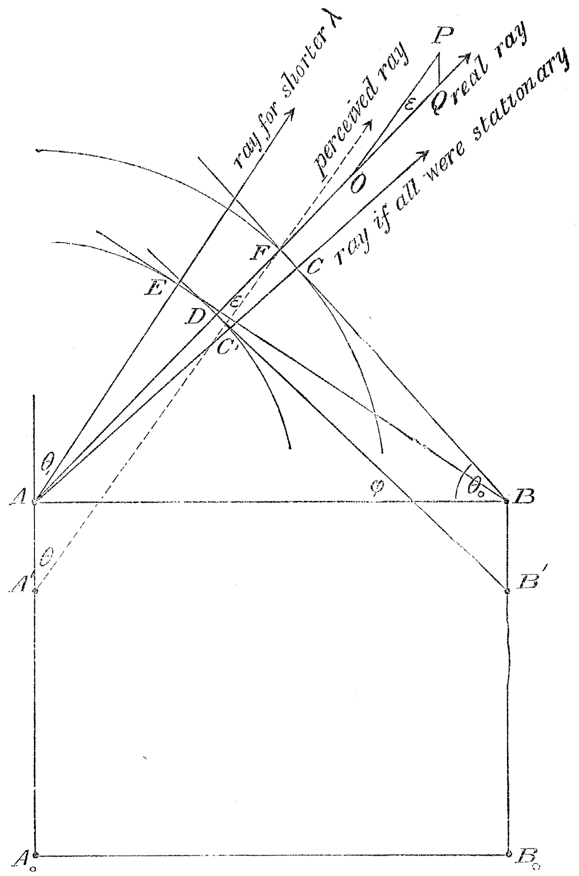
Effect of Motion on Diffraction Grating.

56. To avoid any confusion about motion relatively to source, and the alteration of wave-length thus caused, it will be best to abandon our usual convenient plan of letting the ether move, and attend explicitly to the motion of the grating with its telescope and observer; all else being stationary.

Fig. 14.



Details of Doppler effect with moving grating, AB, and telescope OP.

Consider a plane wave, A_0B_0, advancing through a stationary medium with ordinary velocity, V, towards a stationary grating. Let $A_0A = AC = \lambda_0$ be an ordinary wave-length, while $AB = s$ is the width of one complete element of the grating; then BC is a wave-front, and AC is the ray, inclined to the normal to grating at angle $CBA = \theta_0$.

Now let the grating advance with velocity v to meet the wave a distance $BB' = AA' = CC'$ in one period; the disturbance B_0 will only have to go as far as B′, and the disturbance A only as far as C′; so drawing a tangent B′D to the sphere of radius AC′, we get the wave-front appropriate to moving grating; AD is the ray, inclined to the normal to grating at angle $DB'A' = \phi$.

Now θ_0 and ϕ are very nearly equal; showing that diffraction really does depend on wave-length simply, in spite of motion of grating, so far as minutiæ of the first order are concerned.

But then this direction, ϕ, will not be the direction appreciated by the observer; for the motion of his telescope will cause ordinary aberration, since his motion is partially across the diffracted rays.

Not to confuse the figure, I indicate the telescope OP further along the ray. While the light is travelling along it its eyepiece will have time to move to Q, such that

$$\frac{PQ}{OQ} = \frac{v}{V} = \frac{AA'}{AF} = \alpha.$$

Hence, A′F is parallel to the axis of the telescope which receives the ray, and may be called the apparent or perceived ray. The angle at which it is inclined to the grating-normal may be called θ.

Now θ is less than θ_0, and is very nearly the same as if wave-length had been really shortened to AD, instead of AC.

Draw BE a tangent to the AD circle, and we get the wave-front appropriate to this shortened wave and a stationary grating; while AE is the ray belonging thereto, the inclination of which to the grating-normal we may call θ_1. [$\sin \theta_1 = (1 - \alpha) \sin \theta_0$.]

Now, plainly, AE and A′F are very nearly parallel, but not quite; there is a second-order difference between θ and θ_1, which may be readily calculated.

Perhaps the simplest way of displaying the result is to introduce the aberration angle POQ $= \epsilon$ (such that $\sin \epsilon = \alpha \sin \theta$) and to write

$$\sin \theta_1 = \sin \phi - \alpha \cos \phi \sin \theta_0;$$

whereas

$$\sin \theta = \sin \phi \cos \epsilon - \alpha \cos \phi \sin \theta.$$

(Or one might write $\cot \theta = \cot \phi + \alpha \operatorname{cosec} \phi$.)

The difference between the apparent ray and the shortened wave-length ray is approximately

$$(\theta_1 - \theta) \cos \theta = \sin \phi (1 - \cos \epsilon) - \alpha^2 \cos \phi \sin \theta_0,$$

or

$$\theta - \theta_1 = \alpha^2 \tan \theta (\cos \phi - \tfrac{1}{2} \sin \theta \sin \phi),$$

and is probably quite too small to be detected.

The point of the whole thing is that a grating has *the same real effect whether moving or stationary*, but that the motion of the observing telescope causes an *aberration* which necessitates very nearly the same alteration of its direction as if the waves were really shortened in simple proportion to the motion. The Doppler effect caused by motion of observer is, therefore, essentially a case of common aberration.

Now, as there is no hypothesis or difficulty whatever about the aberrational effect of a moving telescope, all that has been said of a grating applies, at least broadly, to a prism.

Effect of Motion on the Dispersion of a Prism.

57. The deviating power of a prism depends on its relative refractive index with respect to the surrounding medium; hence, in this sense, its deviation is certainly affected by the length of the waves with which it is supplied.

Its dispersive power, however, is not a superficial, but a deep-seated, phenomenon, depending on its internal structure; and, since no variation of outside medium can affect *internal* wave-length, the dispersive power of a prism may be assumed constant for given waves. It follows that the dispersion caused by a given prism, immersed in different media, is simply proportional to the mean deviation in each case for given kind of light.

But what about the effect of motion?

If only we can assume that the prism interferes with the ether as little as the grating has been supposed to do, then all that has been said of the grating remains true of the prism. If we supposed the prism to modify the free ether inside it, we should have to modify this statement. On the hypothesis of FRESNEL, however, the free ether is not supposed to be affected; and experiments directed to test the matter, by ascertaining the effect of prism chasing a source at the same speed, have resulted in finding this effect zero, in accordance with the above statement. Hence it must be allowed that a Doppler effect observed by a prism depends *really* on wave-length, but *apparently* on frequency, just as is the case with a grating.

It must be noticed that the observation of a Doppler effect by a prism depends entirely on dispersion; *i.e.*, on waves of different length being affected differently. But prisms can be constructed whose dispersion is corrected and neutralized. Such achromatic prisms, if perfectly achromatic, will treat waves of all sizes alike; and, accordingly, the shortening of the waves from a moving source will not produce any effect. Achromatic prisms will behave to terrestrial and to extra-terrestrial sources, *i.e.*, to relatively stationary and relatively moving sources, in the same way.

ARAGO used an achromatic prism on a star when he showed that refractive index was unaffected by motion of the earth.* In criticising ARAGO's experiment adversely, MASCART forgets this, and thinks he ought to have perceived a Doppler effect. MASCART used a terrestrial source and an ordinary dispersive prism, when he experienced the same negative result. MAXWELL sent light both ways through his prism,

* BABINET, 'Comptes Rendus,' vol. 9, p. 774 (1839). ARAGO, 'Ann. de Chim. et de Phys.' (3), vol. 37, p. 180 (1853). MAXWELL, 'Phil. Trans.' (1868), vol. 158, p. 532; also 'Ency. Brit.' article "Ether." HOEK, 'Archives Néerlandaises' (1869), vol. 4, p. 443. MASCART, 'Annales de l'Ecole Normale,' vols. 1 and 3 (1872 and 1874); Professor MASCART here describes a large number of negative experiments which he has made as to the effect of motion on most of the phenomena of optics.

and therefore neutralized all refraction, except what was entirely caused by motion, when he proved that this latter was *nil.*

BABINET, HOEK, and MASCART, all tried a modified form of the same experiment in an interferential manner, and likewise got a negative result.

58. If we wish to follow out the ether motion through a prism into greater detail we can say:—Let the prism advance with velocity v to meet the waves, and let the ether in it be carried forward with velocity kv, then the virtual velocity of the light towards the prism is $V + v$, and inside the prism is

$$\frac{V + v}{\mu} - kv;$$

hence, on ordinary notions of refraction the new index will be

$$\mu' = \frac{(V + v)\,\mu}{(V + v) - k\mu v};$$

or

$$\frac{\mu}{\mu'} = 1 - k\mu\alpha,$$

where

$$\alpha = v/(V + v) = d\lambda/\lambda,$$

or

$$d\mu = k\mu^2\alpha,$$

which, on FRESNEL'S hypothesis, equals $(\mu^2 - 1)\,\alpha$.

This seems to give a sort of theory of dispersion for the case:—

$$\frac{d\mu}{\mu^2 - 1} = \frac{d\lambda}{\lambda},$$

or

$$\frac{\mu - 1}{\mu + 1} = A\lambda^2,$$

or

$$\mu = \frac{c^2 + \lambda^2}{c^2 - \lambda^2}.$$

Interference Effects with Rays at Different Angles to Ether Drift. Effects of Normal Reflexion. Further Discussion of the Theory of Mr. MICHELSON'S *Experiment.*

59. The experiment of MICHELSON, already referred to in § 25, has to do with the effect of a plane mirror sending a ray straight back upon itself. Consider the aspect of the mirror necessary to do this; first, for the case of a moving source in a stationary medium (fig. 15).

Let S_1 be initial position of the source, throwing off a wave-front to M, and itself moving to S, so that

$$S_1S = vT_1; \text{ and } S_1M = r_1 = VT_1.$$

Let SM now be reflected at M so as to travel to S_2, and reach it the same time as source, then

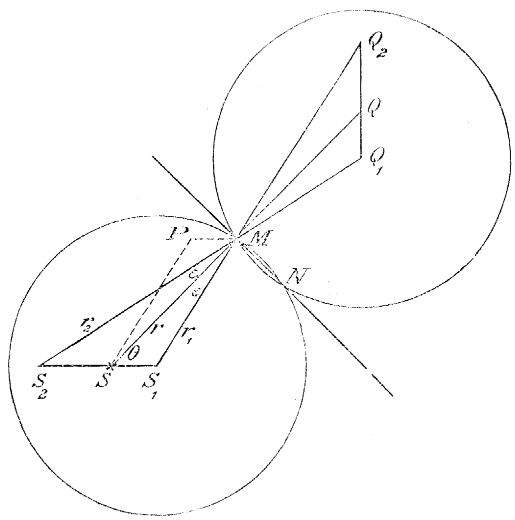
$$SS_2 = vT_2; \text{ and } MS_2 = r_2 = VT_2.$$

Hence

$$\frac{S_1S}{SS_2} = \frac{T_1}{T_2} = \frac{r_1}{r_2},$$

wherefore SM, bisects the angle S_1MS_2, or the angles of incidence and reflexion are precisely equal, and the required mirror is normal to SM, but is not tangential to the wave-front.

Fig. 15.



Details of normal reflexion with moving source or moving medium.

Let Q_1, Q, Q_2 be the images in this mirror of S_1, S, S_2, and with centre Q_1 construct a circle of radius QM cutting the wave-front in N; MN is the mirror able to throw a light ray back on the moving source.

A stationary observing telescope will observe the source along Q_2M; one moving with the source will observe it along QM, that is, in its true direction at moment of vision. The colour will change by the amount log r_1/r, as already said; and, as for any possible interference effect, the fringes will shift by an amount depending on

$$\frac{T_1 + T_2}{2T} = \frac{r_1 + r_2}{2r} = \frac{\cos \epsilon}{1 - \alpha^2} = \frac{\sqrt{(1 - \alpha^2 \sin^2 \theta)}}{1 - \alpha^2},$$

which gives very approximately

$$T_1 + T_2 - 2T \approx \alpha^2 T \, (1 + \cos^2 \theta).$$

There is, therefore, always a lag of phase caused by the motion, which cannot be made negative, or even zero, but which is a minimum when the motion is across the line of light, and a maximum when along it; being, indeed, twice as great for motion along as it is for motion across. Supplementary angles give the same effect.

One may express the fact by saying that the virtual distance the light has to go is S_1Q_2, or S_2Q_1, instead of SQ.

Case when the Mirror Moves too.

60. It is observed that in this investigation the mirror has been supposed stationary with respect to the medium, it is therefore possible, if the mirror is moving at the same rate as the source, *e.g.*, if they were both fixed and the medium streaming past both, that the circumstances may be a little different; because since the whole of the wave-front does not strike quite simultaneously, there may be time for some effect to occur during its period of contact, short though of course it is. Not even for normal incidence is the time of impact of a finite portion of a wave infinitesimal; for even when the source is infinitely distant (or when a collimating lens is used) it has to be remembered that the waves are not normal to the rays in a moving medium, and that, accordingly, when the incidence of the ray is normal and the medium movement not normal, the wave is inclined to the surface, a minute, but possibly important, angle.

But in MICHELSON'S arrangement the ray is not exactly normal on the tangentially moving mirror, but is inclined so that the mirror is precisely parallel to the wave-front; and so the time of contact is nothing on either mirror.

The statement of theory, therefore, proceeds as follows, without apparent error.

Let S be and remain the position of the source, and let a mirror MN (fig. 15) be arranged normal to SM, so as to send a ray back upon itself, when everything is stationary, in time 2T.

It is required to find if any tilt must be given to the mirror to send the ray back upon itself when the medium is moving, and whether a different time will be taken in the journey.

While light travels from S to M the wave-front's centre has drifted to S_1, and, accordingly, it strikes the mirror obliquely and is reflected as if coming from Q_1; hence it would travel towards S_2 but for the drift. The drift will carry it precisely back to S if $SS_2 = vT_2$, T_2 being the time of the return journey; just as $SS_1 = vT_1$, when T_1 is the time of the outward journey.

Hence, no tilt whatever is required by the mirror, but it reflects the light back upon itself just as when the medium was stationary, and the distance really travelled is exactly the same as it was, viz., $2r$. The velocity of the light is, however, different on the out and in journeys, for

$$V_1 = V \cos \epsilon - v \cos \theta,$$
$$V_2 = V \cos \epsilon + v \cos \theta,$$

so

$$T_1 + T_2 = \frac{r}{V_1} + \frac{r}{V_2} = \frac{2T \cos \epsilon}{1 - \alpha^2} = 2T \frac{\sqrt{(1 - \alpha^2 \sin^2 \theta)}}{1 - \alpha^2},$$

as before (§ 59).

61. In the actual experiment, as performed for instance by MICHELSON, it is natural to use a collimator and plane waves, and since his null result is very surprising and remarkable, it may be as well to examine whether the introduction of the lens pro-

duces any disturbance. At first I thought the lens and glass slabs used by him might possibly be the cause of his failure to get any result, because the ray across the motion travels through the glass obliquely, while along the motion it travels normally. But a little consideration shows that both along and across the motion the effect of the glass would be to increase the lag in a simply proportional manner to the previous lag. And calculation gives as the time of the journey, when a total thickness z of glass is interposed, using FRESNEL'S theory that the speed of the ether inside the glass is $1/\mu^2$ of what it is outside,

$$T_1 = \frac{r - z}{V \cos \epsilon + v \cos \theta} + \frac{z}{\frac{V}{\mu} \cos \epsilon + \frac{v}{\mu^2} \cos \theta},$$

and T_2 = corresponding expression with v negative.

So

$$T_1 + T_2 = \frac{2(r - z) \cos \epsilon}{V(1 - \alpha^2)} + \frac{2\mu z \cos \epsilon}{V\left(1 - \frac{\alpha^2}{\mu^2}\right)} = \frac{2T \cos \epsilon}{1 - \alpha^2} + \frac{2\mu z \cos \epsilon}{1 - \alpha^2}\left(\frac{1}{1 - \frac{\alpha^2}{\mu^2}} - \frac{1}{1 - \alpha^2}\right)$$

where

$$T = \frac{r + (\mu - 1) z}{V};$$

wherefore the effect of introducing glass is to increase the lag, but not quite so much as by the equivalent of the extra distance thus virtually added, the second term in the above expression being negative; but the diminution is independent of direction, except when fourth powers of aberration magnitude are attended to. Neglecting these, the effect of the glass is merely to cause, in addition to the lag naturally to be expected, an extra term, independent of direction, of this value:

$$-\frac{2\mu z}{V}\left(1 - \frac{1}{\mu^2}\right)\alpha^2.$$

MICHELSON'S *Interference Experiment in a Different Medium.*

62. Indeed, the simplest plan would be to consider the effect of immersing the whole arrangement in a different medium. It is merely to change the light velocity V to V/μ, and its mechanical velocity v to v/μ^2 the ethereal velocity inside it. Consequently α becomes α/μ.

The aberration angle ϵ changes to ϵ', such that

$$\sin \epsilon' = \frac{\alpha}{\mu} \sin \theta,$$

and the lag

$$\frac{2r \cos \epsilon}{V(1-\alpha^2)} \quad \text{becomes} \quad \frac{2\mu r \cos \epsilon'}{V(1-\alpha^2/\mu^2)},$$

approximately μ times as great as before. But although this is the case, *the extra lag caused by motion* is not so great inside the medium as it was *in vacuo*, for

$$\frac{2\mu r \cos \epsilon'}{V\left(1-\frac{\alpha^2}{\mu^2}\right)} - \frac{2r \cos \epsilon}{V(1-\alpha^2)} = \frac{2r \cos \epsilon}{V(1-\alpha^2)} \left(\frac{\mu \cos \epsilon' (1-\alpha^2)}{\left(1-\frac{\alpha^2}{\mu^2}\right) \cos \epsilon} - 1\right),$$

or, approximately,

$$= \frac{2(\mu-1) r \cos \epsilon}{V(1-\alpha^2)} - \frac{2\mu r}{V}\left(1-\frac{1}{\mu^2}\right)\alpha^2.$$

[The conclusion here is that whatever may be the effect of a dense medium it is independent of θ, and therefore can have nothing to say to MICHELSON'S experiment, which entirely depends on a difference between what can be observed with $\theta = 0$ and $\theta = 90°$.—July, 1893.]

The Laws of Reflexion and Refraction as modified by Motion.

63. It is necessary now to enter on the somewhat thorny question as to the effect of motion upon the laws of reflexion and refraction. FRESNEL by considering some special cases satisfied himself that no discrepancy need be expected on his version of the undulatory theory; and Sir GEORGE STOKES, examining the question in a more general manner in 1846, proved that, at least as far as the first order of minutiæ, the laws were obeyed in spite of any relative motion between mirror and medium (motion of source has obviously nothing to do with it, unless it affects the shape of the incident wave). And the long continued use of artificial horizons by astronomers shows that there has been no practical doubt on the subject, at least as far as reflexion is concerned.

But these statements do not by any means exhaust the subject; the law of reflexion is *not* precisely obeyed in a moving medium, and recently MICHELSON has proposed to utilize the theoretical error (which has never yet been practically realized) as a fresh method of attacking the problem of the relative motion of the ether and the earth.

I propose, therefore, to enter upon it, and I must confess that though the results are easily stated, they have given me much trouble to be sure of, and I have found a good many mare's nests by the way.

The reasoning for reflexion and refraction is much the same, and I attend more pronouncedly to reflexion because without assuming FRESNEL'S theory as to the motion of ether inside dense matter we have no guide to what shall happen in refraction;

and although the theory has been to a certain extent, and with fairly high accuracy, verified, yet it can hardly be yet said to have a secure rational basis.

In a drifting medium we must draw a clear distinction between waves and rays; the laws obeyed by one need not be obeyed by the other, for they are inclined to each other, and may become differently inclined after reflexion or refraction.

Now it is pretty plain that if motion is to have any effect upon these aberration angles, the rays must be differently inclined to the direction of drift; and on the other hand, if motion is to affect the reflexion of waves, that it must act during the period of contact of a wave with the reflecting surface; so that if a wave comes down plumb it will rebound as it comes, because its time of contact is then infinitesimal and no finite motion could cause any disturbance. But even in this case of normal incidence the law of reflexion need not be obeyed for rays, for they are not normal to the waves, and will be differently inclined to the direction of drift, unless indeed the latter be either normal or tangential.

64. The following are statements which I will afterwards justify:—

(1) The planes of incidence and reflexion are always the same.

(2) The angles of incidence and reflexion, as measured between rays and normal to surface, in general differ.

(3) If the mirror is stationary and medium moving, they differ by a quantity depending principally on the square of aberration magnitude, *i.e.*, by one part in a hundred million, and a stationary telescope would be able to observe the effect, if it were delicate enough.

(4) If the medium is stationary and mirror moving, the angles differ by a quantity depending principally on the first power of aberration magnitude, *i.e.*, by one part in ten thousand, but a telescope moving with the mirror will not be able to observe this large effect; for the apparent (or commonplace) aberration caused by the motion of the receiver will obliterate the odd powers and leave only the even powers of the aberration, so that the observed effect should be the same as in case 3.

(5) As regards the angles which the reflected and incident waves make with the surface, *i.e.*, as to the obedience to the law of reflexion shown by *waves* instead of by *rays*, in case 3 the angles differ by an amount depending on the first order of aberration, but in case 4 they only differ by the square of this quantity.

(6) At grazing incidence the ordinary laws are accurately obeyed by the rays as observed, and at normal incidence the error is a maximum.

(7) The ordinary laws are obeyed whenever the direction of motion is tangential or normal to the mirror.

(8) In general the shape of the incident waves is not precisely preserved after reflexion, so that, when spherical waves impinge on a mirror in a moving medium, the reflected waves from a plane mirror diverge from a sort of caustic instead of from a point, and the position of the image varies (but almost infinitesimally) with the position of the observer. In other words, such a mirror acts to a parallel beam as if

slightly tilted, to a divergent beam as if slightly curved. But either effect, as observable in the result, is almost hopelessly small.

(9) Similar statements are true for refraction.

65. In considering a plane mirror in a drifting medium it is very tempting to image the direction of drift of successive wave centres (fig. 4); in which case everything will be symmetrical, and the law of reflexion will be obeyed altogether, by both waves and rays, in the simplest possible manner. But a little thought shows that this is illegitimate, for it would make the reflected waves assisted in their progress by the reflected drift just as much as the incident waves are assisted; whereas they are really travelling in the teeth of the wind, their progress being impeded and their wave-length shortened just as much as the incident waves are helped and lengthened (or of course *vice versâ*). Plainly the drift is not reflected, but must be supposed to act on the waves emitted by the image exactly as it acts on the waves emitted by the source.

Another tempting thing to do is to start a system of waves from source and its ordinary image simultaneously, both subject to precisely the same drift velocity, one being the incident, the other the reflected system. But applying this, and taking a pair of waves intersecting at any one point of the mirror, it will be found they have not travelled the same distance to get there, nor have they taken the same time, and the drift of their centres has been different. Moreover, they do not intersect at a second point of the same wave, and, in fact, the system behind the mirror is not in any sense the image of the front set.

The really essential thing is that the phase of the reflected wave shall be identical with that of its incident exciter at the point of contact with the mirror, and accordingly that the time of virtual journey from any point to be considered as an image is to be equal to the time of journey from the corresponding point of the source. Nothing less direct or more geometrical than this seems satisfactory, so it had better be applied in its usual Huyghenian baldness. At the same time a little caution is necessary in using HUYGHENS' construction in a moving medium, for the centre of the elementary waves does not remain at the point of incidence, but drifts away, as in fig. 4, and the construction has to remember this, or it will go wrong.

Laws of Reflexion and Refraction in a Drifting Medium.

66. Since the direction of drift need not be in the plane of incidence, it will be convenient to resolve it into two components, respectively in and perpendicular to that plane, and consider them separately.

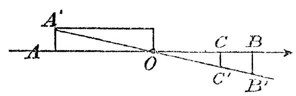
Component of Drift Perpendicular to Plane of Incidence.

The perpendicular component is very easily disposed of, as was shown by Sir GEORGE STOKES.* For looking down the normal to the mirror we shall see the

* 'Math. and Phys. Papers,' vol. 1, p. 144.

incident and reflected rays AO and OB, incident at the point O, affected by the drift in such a way that their direction is A′O and OB′ respectively; but they are still both in a plane normal to the surface, and they still make equal angles with the surface.

Fig. 16.



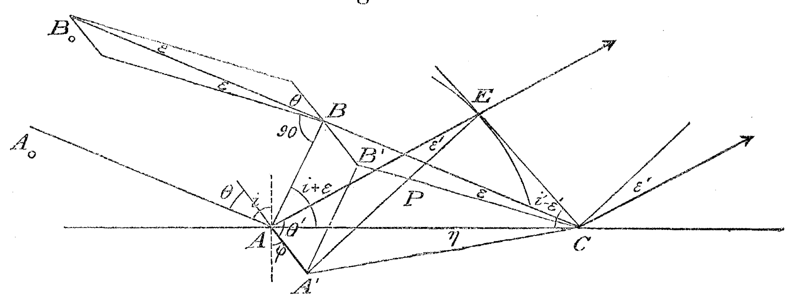
And as regards refraction, an equal length of the refracted ray will be foreshortened into $OC = OB/\mu$; and the drift inside the medium being, according to FRESNEL'S theory, BB'/μ^2, will in the longer time available carry C to C′, such that OC′B′ remains a straight line; accordingly, the plane of the refracted ray is still the plane of incidence. Hence the perpendicular component of the drift is wholly ineffective, and we have only to consider the component in the plane of incidence; call it v.

Component of Drift in Plane of Incidence.

Consider a plane wave A_0B_0 travelling, with an aberration angle ϵ between its normal and its direction of advance caused by a drift in direction AA′, towards a plane mirror, at a pace compounded of the speed of light and of the drift, viz., $V \cos \epsilon + v \cos \theta$.

Let incident and reflected rays make angles i and i' respectively with the mirror-normal, and θ and θ' with the line of drift, so that if ϕ is the inclination of the drift direction to the mirror-normal, $\theta = i - \phi$, $\pi - \theta' = i' + \phi$.

Fig. 17.



The incident and reflected waves will be inclined to the surface by the angles $i + \epsilon$, $i' - \epsilon'$, respectively.

When the wave AB strikes the mirror, A becomes a source of radiation, and B travels on to C, with a velocity compounded of BB′ and B′C. By the time B arrives at C the radiation from A will have spread a distance equal to B′C through the medium, and the centre of a wave with this radius will have had time to drift with the moving medium to A′, a distance equal to BB′; hence, drawing the semicircle A′B′EC, and choosing on it a point E, such that A′E = B′C, we have the direction of the reflected ray, viz., AE, and of the reflected wave CE.

67. It is easy to see that the triangle PA′C is isosceles, and accordingly that the angle A′CA is equal to half the difference of the inclinations of incident and reflected waves to the mirror surface ; *i.e.*, calling this angle η,

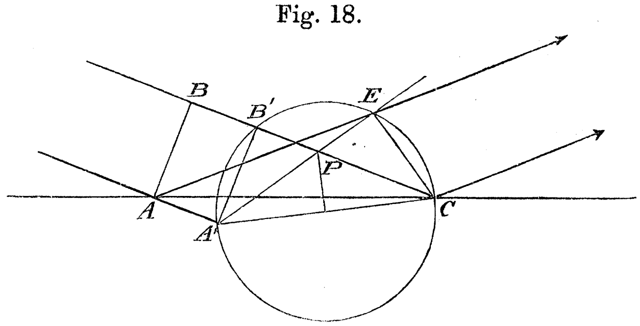
$$2\eta = (i + \epsilon) - (i' - \epsilon'),$$

or

$$\eta = \frac{\epsilon + \epsilon'}{2} + \frac{i - i'}{2},$$

hence the *wave* is reflected precisely as if the mirror were rotated through the angle η and there were no drift ; the angle of virtual rotation being very approximately the mean of the aberration angles.

Fig. 18.



The first approximation to its value is

$$\eta = \alpha \sin i \cos \phi ;$$

it practically vanishes, therefore, for normal incidence and for tangential drift.

Further, as regards the change of width of the beam or distance between the rays, it is apparent that measured along the wave-surface it is the same, because EC = A′B′ = AB ; so measured perpendicularly it changes in the ratio of $\cos \epsilon : \cos \epsilon'$ before and after reflexion.

68. It is not to be supposed that the *ray* is reflected after this manner ; and, in fact, we shall find that the error of ray-reflexion, or difference between angle of incidence and reflexion, $i - i'$, is exceedingly small.

To determine this difference, and the whole circumstances of the problem, we write down the following equations, obvious from figure 17 :

$$\frac{\sin \epsilon}{\sin \theta} = \frac{\mathrm{BB'}}{\mathrm{B'C}} = \frac{v}{\mathrm{V}} = \alpha = \frac{\mathrm{AA'}}{\mathrm{A'E}} = \frac{\sin \epsilon'}{\sin \theta'},$$

$$\theta = i - \phi, \qquad \theta' = \pi - (i' + \phi).$$

Also, for the time of journey of the wave from position AB to EC,

$$t = \frac{\mathrm{BB'}}{v} = \frac{\mathrm{B'C}}{\mathrm{V}} = \frac{\mathrm{BC}}{\mathrm{V}\cos \epsilon + v \cos \theta} = \frac{\mathrm{AE}}{\mathrm{V}\cos \epsilon' + v\cos \theta'} = \frac{\mathrm{A'E}}{\mathrm{V}} = \frac{\mathrm{AA'}}{v}.$$

Lastly

$$\frac{BC}{AC} = \frac{\sin(i+\epsilon)}{\cos\epsilon}; \qquad \frac{AE}{AC} = \frac{\sin(i'-\epsilon')}{\cos\epsilon'}.$$

These solve the problem, and they may be conveniently worked on the following lines—

$$\sin\epsilon = \alpha\sin(i-\phi); \qquad \sin\epsilon' = \alpha\sin(i'+\phi),$$

$$\frac{\sin i' - \cos i'\tan\epsilon'}{\sin i + \cos i\tan\epsilon} = \frac{AE}{BC} = \frac{V\cos\epsilon' + v\cos\theta'}{V\cos\epsilon + v\cos\theta} = \frac{\cos\epsilon' - \alpha\cos(i'+\phi)}{\cos\epsilon + \alpha\cos(i-\phi)} = \frac{\cos\epsilon - \alpha\cos(i-\phi)}{\cos\epsilon' + \alpha\cos(i'+\phi)},$$

the last equality being added for convenience, and being true because

$$\cos^2\epsilon - \alpha^2\cos^2\theta = 1 - \alpha^2.$$

Therefore, exactly,

$$\sin i\cos\epsilon - \tfrac{1}{2}\alpha^2\cos i\sin 2(i-\phi)\sec\epsilon = \sin i'\cos\epsilon' - \tfrac{1}{2}\alpha^2\cos i'\sin 2(i'+\phi)\sec\epsilon',$$

whence, expanding $\cos\epsilon$ and neglecting α^4,

$$\sin i - \sin i' = \alpha^2\cos^3 i\sin 2\phi,$$

or

$$i - i' = \alpha^2\cos^2 i\sin 2\phi.$$

The discrepancy between the angles of incidence and reflexion (which I call for brevity the error of reflexion) is therefore exactly expressible in even powers of aberration magnitude, and no part of it reverses with the reversal of the ray. It vanishes for grazing incidence, and is a maximum for normal incidence (at which I am somewhat surprised). It vanishes both for tangential and for normal drift, being a maximum when the medium drifts at 45° to the mirror.

The maximum possible value of the error of reflexion is α^2, or 10^{-8} of a radian, or $0''\cdot00205$, or $\frac{1}{500}$th of a second of arc; an amount which, although equivalent to an error of only 15 inches in the circumference of the earth, it is perhaps possible to detect; especially if, as Mr. MICHELSON suggests, it be increased by multiple reflexion. Indeed, it strikes me as perhaps the simplest way of examining into the motion of the ether near the earth.

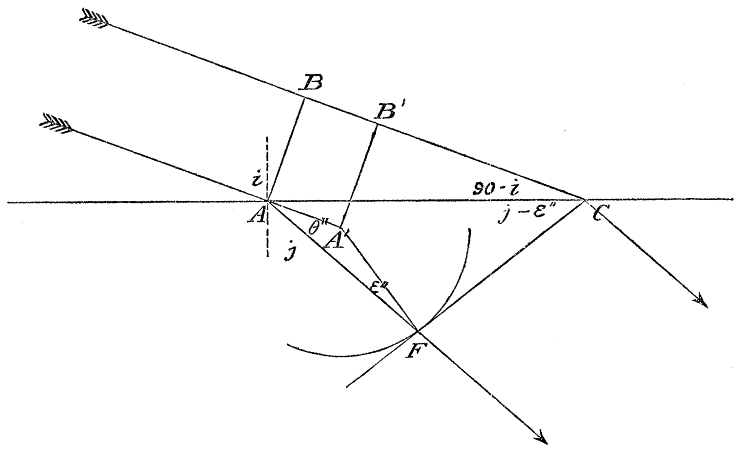
Refraction in a Drifting Medium.

69. The reasoning for refraction is precisely of the same kind, and there needs nothing more but to write down the equations, putting V/μ everywhere instead of V, v/μ^2 instead of v, and consequently α/μ instead of α.

It is best thus to assume FRESNEL'S theory, and leave observation to point out any deviation from it that may be existent.

A separate figure may save confusion; and though the general case is easily drawn (like fig. 17) a special case serves better for illustration, and I depict the case of drift along incident ray.

Fig. 19.



θ'' and ϵ'' are the angles between refracted ray and the drift direction and wave-normal respectively; the angle of refraction, defined as usual, may be called j; so that

$$\theta'' = \phi - j,$$

$$\sin \epsilon'' = \frac{\alpha}{\mu} \sin \theta'',$$

$$t = \frac{\mathrm{AA}'}{v/\mu^2} = \frac{\mathrm{A'F}}{\mathrm{V}/\mu} = \frac{\mathrm{AF}}{\mathrm{V}/\mu \cos \epsilon'' + v/\mu^2 \cos \theta''},$$

$$\frac{\mathrm{AF}}{\mathrm{AC}} = \frac{\sin (j - \epsilon'')}{\cos \epsilon''}.$$

These are the equations to be used in conjunction with the previous set, and so it follows that

$$\frac{\sin j - \cos j \tan \epsilon''}{\sin i + \cos i \tan \epsilon} = \frac{\mathrm{AF}}{\mathrm{BC}} = \frac{1}{\mu} \frac{\cos \epsilon'' + \alpha/\mu \cos \theta''}{\cos \epsilon + \alpha \cos \theta} = \frac{1}{\mu} \frac{\cos \epsilon - \alpha \cos (i - \phi)}{\cos \epsilon'' - \alpha/\mu \cos (j - \phi)}.$$

Wherefore

$$\sin i \cos \epsilon - \alpha \sin \phi - \alpha \tan \epsilon \cos i \cos (i - \phi)$$
$$= \mu \left\{ \sin j \cos \epsilon'' - \frac{\alpha}{\mu} \sin \phi + \frac{\alpha}{\mu} \tan \epsilon'' \cos j \cos (j - \phi) \right\}.$$

Or

$$\sin i \cos \epsilon - \mu \sin j \cos \epsilon'' = \tfrac{1}{2} \alpha^2 \left\{ \cos i \sin 2 (i - \phi) - \frac{1}{\mu} \cos j \sin 2 (j - \phi) \right\}.$$

Which shows that the difference between $\sin i$ and $\mu \sin j$, or the error of refraction, is likewise of the second order of aberration magnitude, *i.e.*, ordinarily speaking, nil;

its value being easily obtainable if ever wanted. The displacement of Fraunhofer lines due to the Sun's rotation is a small thing to detect with a prism spectroscope, but this effect of motion on terrestrial sources, if it is ever to be seen, is 660 times smaller.

70. It may, perhaps, be well to check over our results by the less geometrical method employed by Sir GEORGE STOKES, viz., by expressing the fact that the intersection of the three waves (incident, reflected, and refracted) with the mirror is a joint intersection, and runs along the mirror at a pace which can easily be written down (viz., AC/t); for the wave advances through the medium at a speed V, and the medium helps it along with a component of its drift velocity $v \cos(\theta + \epsilon)$; so the total speed of the joint wave intersection as it runs along the mirror is

$$\frac{V + v \cos(\theta + \epsilon)}{\sin(i + \epsilon)},$$

which it is easy to see is precisely the same as what we should have obtained by attending to rays and to the figure, viz.,

$$\frac{AC}{t} = \frac{V \cos \epsilon + v \cos \theta}{\sin(i + \epsilon)/\cos \epsilon}.$$

So the equations for reflection and refraction can be written down at once, thus:

$$\frac{V + v \cos(i - \phi + \epsilon)}{\sin(i + \epsilon)} = \frac{V - v \cos(i' + \phi - \epsilon')}{\sin(i' - \epsilon')} = \frac{V/\mu - v/\mu^2 \cos(j - \phi - \epsilon'')}{\sin(j - \epsilon'')},$$

together with the values of the aberration angles, obtained, say, by resolving the wave and drift velocities perpendicular to the ray, or resultant direction of advance, and expressing the fact that they must neutralize each other;

$$\frac{v}{V} = \frac{\sin \epsilon}{\sin(i - \phi)} = \frac{\sin \epsilon'}{\sin(i' + \phi)} = \frac{\mu \sin \epsilon''}{\sin(j - \phi)}.$$

These two sets of equations contain the entire solution, and of course μ may be written μ_2/μ_1 if it is a question of passage from one medium to another instead of from vacuum to a medium; the V and v then expressing speeds in first medium.

71. In Sir GEORGE AIRY'S* beautifully performed and described experiment of the value of the coefficient of aberration measured by a zenith sector full of water, there should, we see, on FRESNEL'S theory, have been a slight discrepancy, but one wholly too small to observe with the various inaccuracies inseparable from star-light. If it is to be detected it must be with light from a terrestrially fixed source. The obser-

'Phil. Mag.,' iv., vol. 43, p. 310.

vations of HOEK and others, performed with terrestrial light, aimed only at disproving KLINKERFUES' notion that an error proportional to the first power was to be expected, and did not aim at the immense delicacy needed to observe α^2.

Wave-length as altered by Reflexion.

72. Since the laws of reflexion are so closely obeyed, an image in a mirror will practically appear just the same whether the medium is stationary or not, and, accordingly, the image may be treated as the virtual source for all questions relating to wave-length and Doppler effect, and the waves coming from that image will in general be affected by the drift otherwise than are the waves coming from the source, because the direction is different.

For instance, sunshine strikes the earth perpendicularly to its motion, but reflected sunshine may coincide with the direction of motion, and, in that case, will have to travel against (or with) the ether wind precisely as if it came from a terrestrial source, and its wave-length will be affected as already reckoned; in other words, thinking of a mirror moving with the orbital motion of the earth only, considered as circular, the image of the Sun moves as if attached to the mirror (not at twice the rate), and, accordingly, reflected sunshine behaves as regards wave-length precisely as if it were coming from a terrestrial source. [More generally (*i.e.*, including eccentricity of orbit and aberration) reflected light *as seen by an observer moving with the mirror* appears in every respect like direct light.]

For irregular reflexion, *e.g.*, from white paper or from the Moon or a planet, these things can be treated as being themselves the sources.

Change of Phase caused by Reflexion in a Moving Medium.

73. Now consider the phase as affected by reflexion.

Consider the two parallel rays A and B, in fig. 17, distant b from each other, and let B lag initially by an amount $b \tan \epsilon$ behind A (§ 67), then, after reflexion, the distance apart has changed so that $b/\cos \epsilon = b'/\cos \epsilon' = c$ say, and the lag is $b' \tan \epsilon'$.

Hence the gain of lag by reflexion, $b' \tan \epsilon' - b \tan \epsilon$

$$= c(\sin \epsilon' - \sin \epsilon),$$
$$= -2\alpha c \cos \frac{i+i'}{2} \sin\left(\phi - \frac{i-i'}{2}\right),$$

which, very approximately,

$$= -2\alpha c \cos i \sin \phi.$$

For normal incidence and tangential drift it has its maximum value, $2\alpha b$.

Now whatever the initial lag may be, and it may be arbitrary, the final lag will differ from it by this same amount; and if the rays, instead of being parallel, are coincident in path, then no difference in phase is caused by reflexion.

Change of Energy at Reflexion from Moving Mirror

74. When reflexion takes place from a moving (receding) mirror, there is some work done on the mirror by reason of the intrinsic pressure of light. Calling the energy per unit volume $e + e'$, the energy of the incident light per second is Ve, and of the reflected Ve', the pressure is $e + e'$, and the work done per second $(e + e')\,v$.

So

$$\mathrm{V}e - \mathrm{V}e' = (e + e')\,v,$$

or

$$e\,(1 - \alpha) = e'\,(1 + \alpha).$$

Or consider the mirror fixed and medium moving with source away from it, the speed of incident light is $\mathrm{V} - v$, of the reflected is $\mathrm{V} + v$, but no work is done; so

$$(\mathrm{V} - v)\,e = (\mathrm{V} + v)\,e'.$$

Wherefore, on either mode of consideration, the energy of the reflected light is from a receding mirror less, from an advancing mirror greater, than that of the incident light, in the ratio

$$\frac{e'}{e} = \frac{1 - \alpha}{1 + \alpha}.$$

Possible Effect of Light Pressure in Astronomy.

75. Light energy per unit volume on Mount Whitney, as determined by LANGLEY, amounted to 67 microbarads, or, say, in outside space, three quarters of 10^{-4} ergs per cubic centimetre; giving a pressure of the same number of dynes per sq. centim.

This pressure on the Moon is withdrawn during eclipses, but, although equal to the ordinary weight of 10,000 tons or so, it is too small to make sensible perturbations, as it could only push the Moon $\frac{1}{40}$th inch in a fortnight.

On a small body, however, it may become comparable with gravitation.*

On a small-enough dust particle, such as may be in tails of comets, the light pressure and gravitative attraction of the Sun might balance. I make the size about 1 micron diameter for a sphere of the density of water, at any distance. Anything smaller than this would be repelled, and would get up an excessive velocity in time.

[* I find that FITZGERALD made a communication years ago to the Royal Dublin Society on this subject.]

Direction of Motion of a place on the Earth.

76. Of all the motions to which the earth is subject its orbital motion is the largest, and is the most important for aberrational effects; but two others must not be overlooked, since they may introduce secular variations into the amount of those effects, viz., the diurnal rotation and the motion of the system through space.

The speed of the motion of the system is only approximately known, but it is estimated at 10·9 miles a second, or 1·75 million C.G.S., and its direction is completely specified by stating a point among the fixed stars.

The speed of the diurnal rotation is $\frac{\text{equatorial circumference}}{\text{1 sidereal day}} \times \cos$ latitude, or very small compared with that of light, and its direction is simply from west to east. It causes a variation in the observed total aberration, amounting to nearly 2 per cent. at the equator.

Both these motions are steady.

The orbital motion is not quite constant in speed, and goes through the whole plane cycle of directions, but its average value may be stated as $\frac{1}{10,000}$ that of light, and its direction is sufficiently expressed for practical purposes by saying that it is in the plane of the ecliptic, and at right angles to the Sun's direction. For instance, a half-moon is roughly in the line of the earth's orbital motion. We are moving as if going away from an increasing half-moon or towards a decreasing half-moon. Another way of putting the matter, is that at midnight the annual and diurnal motions approximately agree in direction, at midday they are opposed. At the epoch of the solstices the agreement is good, *i.e.*, the orbital motion at a solstice is from east to west at noon, from west to east at midnight; and at no time of the year is the error of this statement of very great practical import, for even at the equinoxes 91·7 per cent. of the motion is in the direction stated.

A clock might easily be made to point out the direction of orbital motion. By starlight it is never difficult to realize it, for there are usually planets enough to make the ecliptic manifest, and there is no difficulty in estimating whereabouts the Sun is. Hold a twenty-four hour watch in the plane of the ecliptic with its noon line pointing west, and its hour hand will constantly indicate the direction of the earth's orbital motion. The only difficulty is knowing where the plane of ecliptic is. Consider a terrestrial globe with its axis tilted $23\frac{1}{2}°$, and rotating by internal mechanism once in twenty-four hours. The plane of ecliptic is horizontal, and the direction of motion will be given by a pointer revolving once a year in a horizontal plane, or, more simply, by the appropriate radius of a horizontal card with 365 days of the year written round its circumference. With that alone, however, it would be a little puzzling to compare this slowly changing direction with the position of any given locality on the rotating earth. The whole might be turned by hand till the required locality came to the top, with the axis in the meridian, and then the pointer would agree with the direction of motion; or, more simply, there need be no globe at all, but simply a polar axis revolving once a day opposite to the earth, and carrying with it a dial with the names of the months and days recorded round its circumference, set on the axis at an obliquity of $23\frac{1}{2}°$, and adjusted once for all to coincide with the ecliptic. The date on the card will then point out the line of motion. The clock, if kept to G.M.T., would never give the motion more erroneously than a small correction analogous to the equation of time. By giving the card one step forward every 29th of February, it could be kept right until the whole thing wanted that $\frac{1}{260}$th part of a rotation per century about a vertical axis which precession demands. [My assistant, Mr. E. E. ROBINSON, has connected a clock through a HOOKE'S joint with a pointer which moves so as very fairly to indicate the direction of orbital motion at any instant.]

Electrical methods of detecting Motion through Ether.

77. It might perhaps appear possible that electrical methods may succeed in showing a first-order effect of terrestrial motion, since charged bodies in motion repel each other with modified force.

It is not possible to control or vary it except by combining the above several kinds of movement, and FITZGERALD has suggested a plan of observing whatever effect may be caused by the alternate agreement and disagreement between the earth's orbital motion and the solar system's proper motion: say by measuring the attraction of charged parallel plates at intervals of six months.

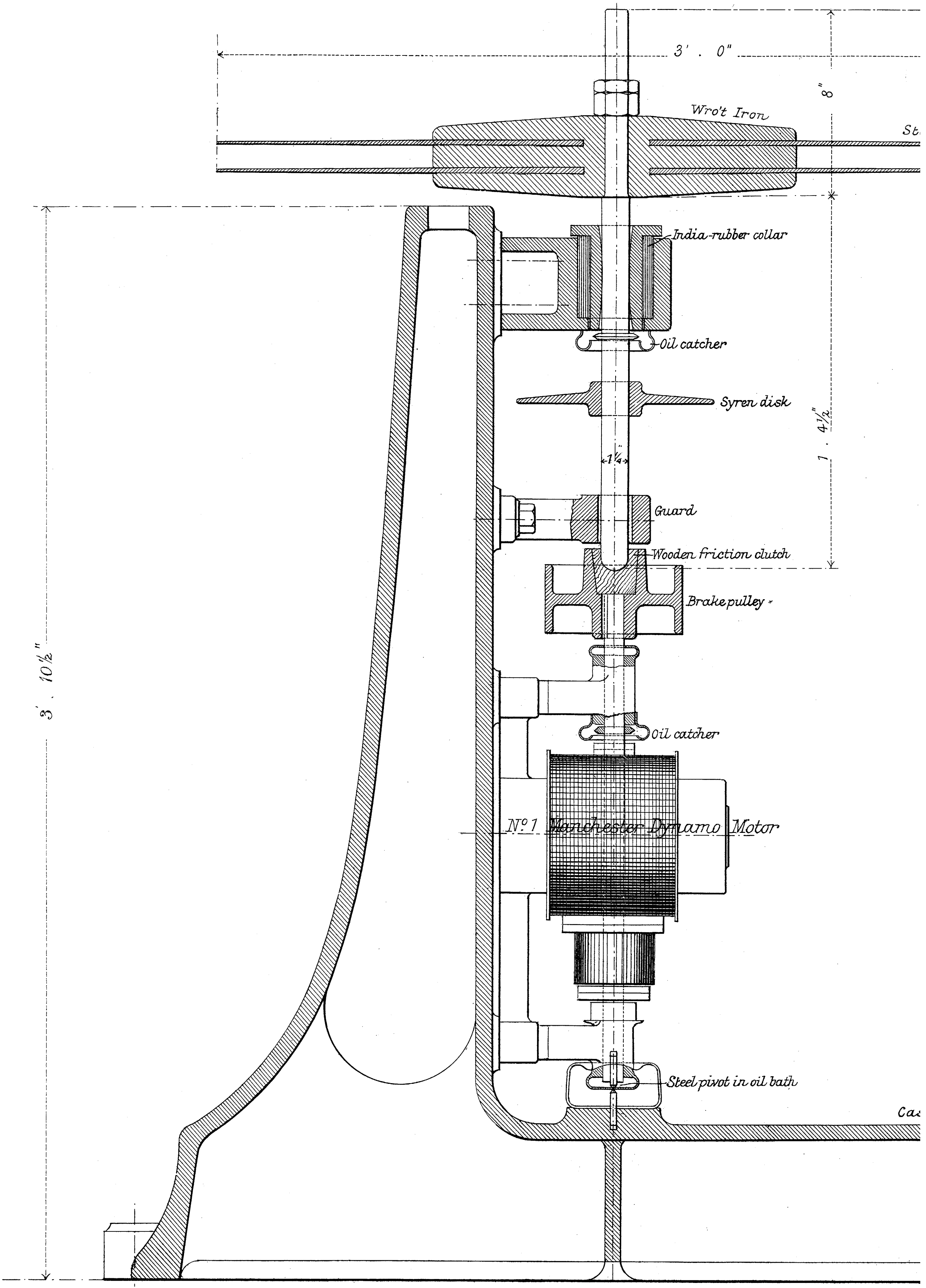
But, inasmuch as the force between charged bodies is independent of the direction of their motion, or (otherwise) because the electrical attraction between parallel moving charges depends on the product of their velocities, it must be the second-order of aberration magnitude that is really involved.

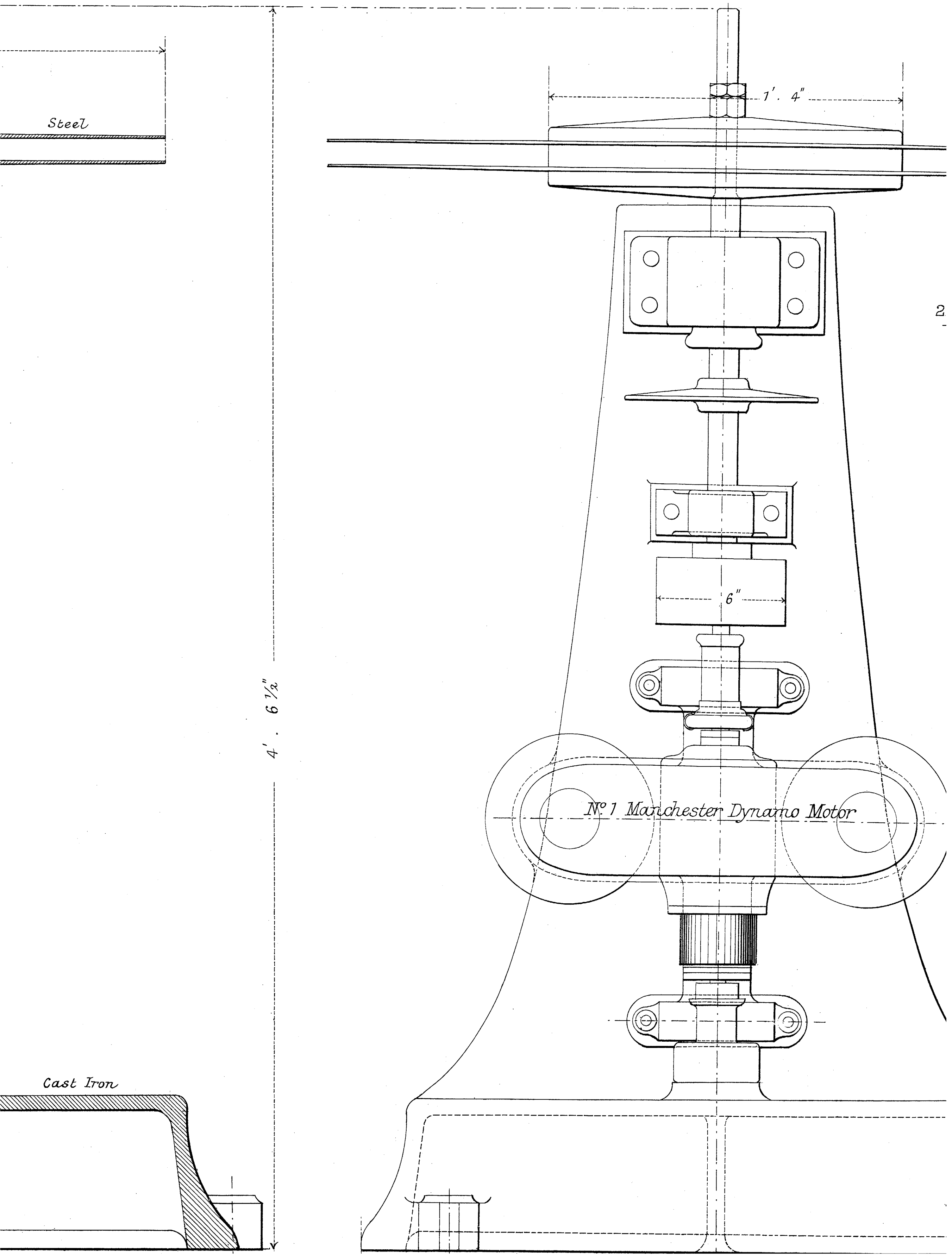
DESCRIPTION OF PLATES 31 AND 32.

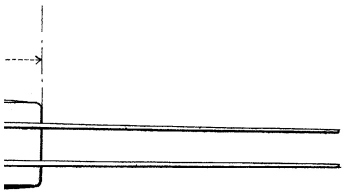
Plate 31. Details of optical frame, showing the mode of supporting the mirrors, both the silvered and the semi-transparent.

Plate 32. Details of whirling machine, showing the pair of steel disks, 1 yard in diameter, driven by an electric motor.

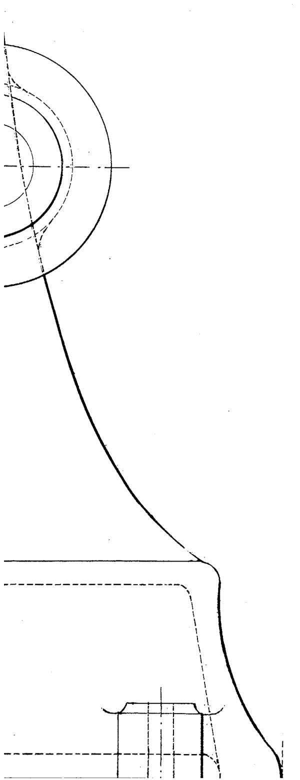
Lodge.

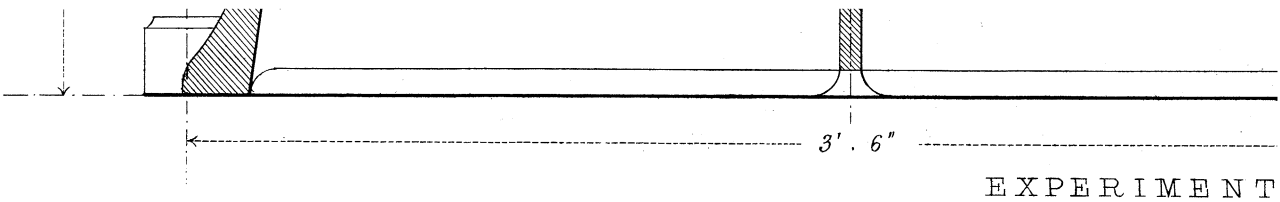

3' . 0"
8"
Wro't Iron
India-rubber collar
Oil catcher
Syren disk
1 . 4½"
1¼"
Guard
Wooden friction clutch
Brake pulley
Oil catcher
No 1 Manchester Dynamo Motor
Steel pivot in oil bath
3' . 10½"


Steel
1'. 4"
6"
4'. 6½"
No. 1 Manchester Dynamo Motor
Cast Iron

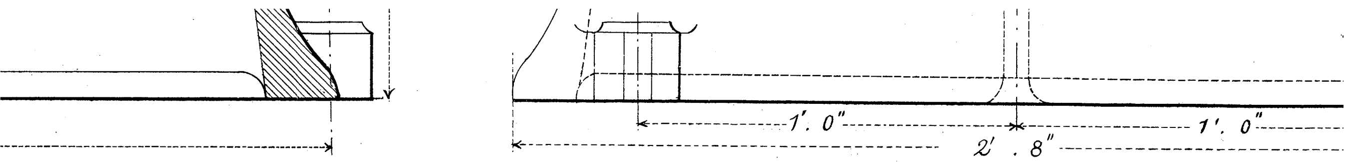


Scale
2 in. = 1 Foot

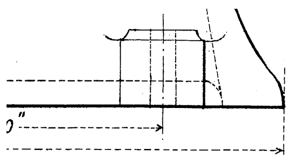




EXPERIMENT



ENTAL WHIRLING MACHINE.



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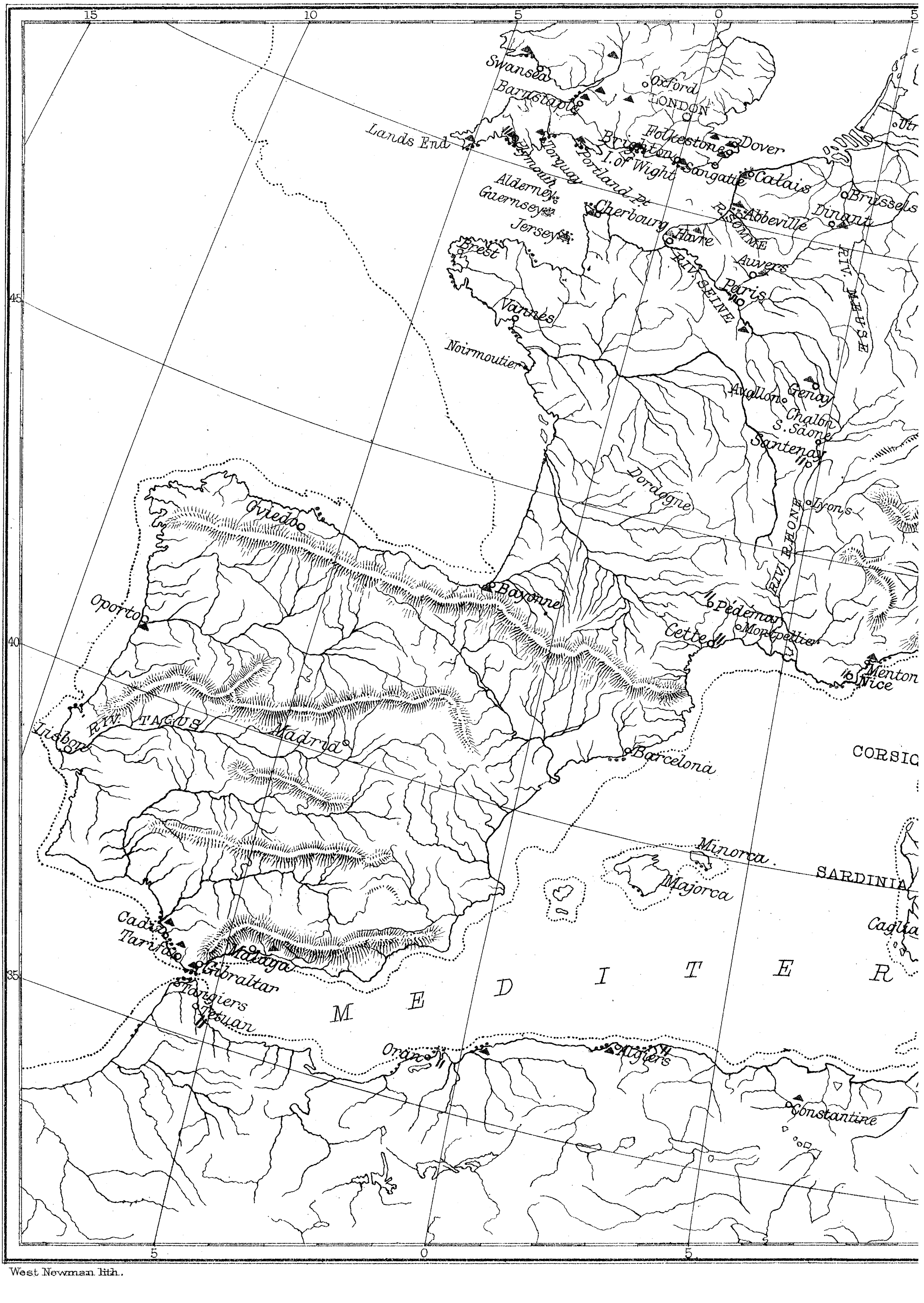

West Newman lith.

MAP OF
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Pisa
Mentone
Nice
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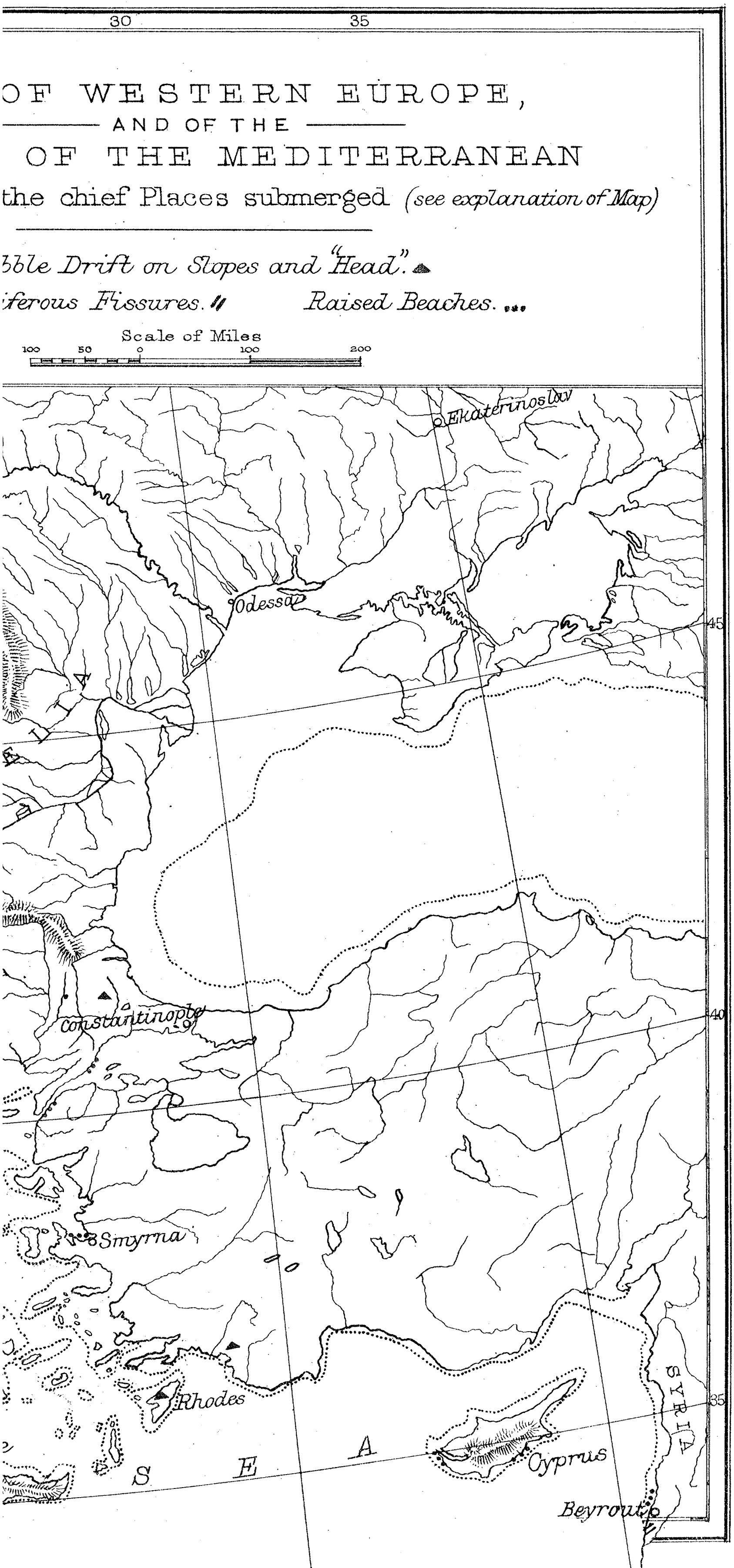
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OF WESTERN EUROPE,
AND OF THE
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West Newman lith.


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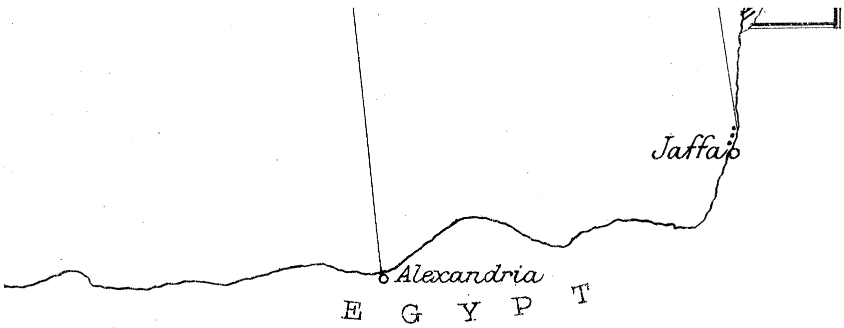
Jaffa
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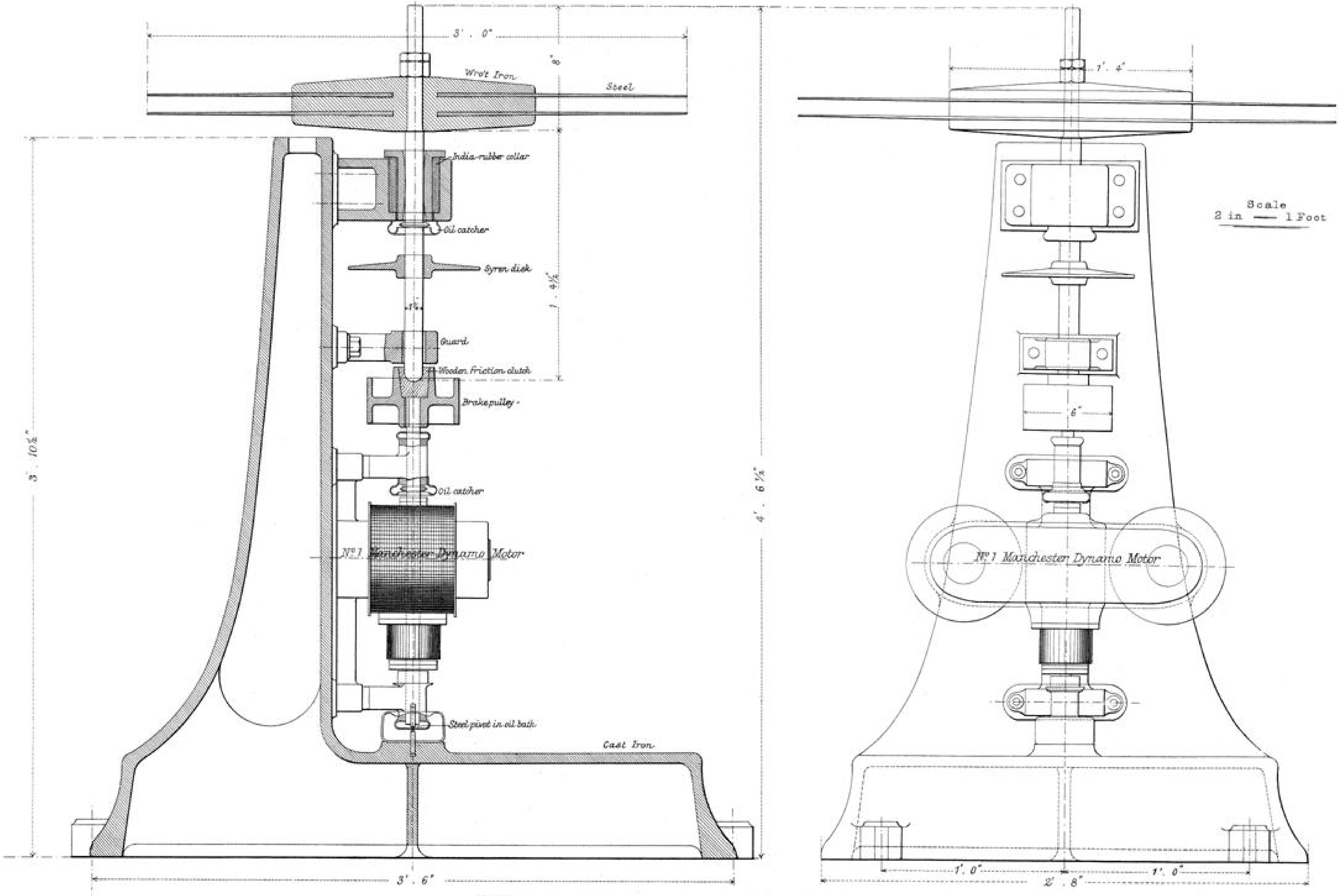

EXPERIMENTAL WHIRLING MACHINE.

West, Newman lith.

West Newman lith.

West Newman lith.


MAP OF W
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ROUMELIA
RIV. DANUBE
Genoa
Spezzia
Pisa
Mentone
Nice
Leghorn
Bastia
CORSICA
Rome
Naples
Capri
SARDINIA
Cagliari
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Kerkenna I.
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Crete
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OF WESTERN EUROPE,
AND OF THE
OF THE MEDITERRANEAN
the chief Places submerged (see explanation of Map)
bble Drift on Slopes and "Head".
ferous Fissures.
Raised Beaches.
Scale of Miles
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Ekaterinoslav
Odessa
Constantinople
Smyrna
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Cyprus
Beyrout
SYRIA
S E A
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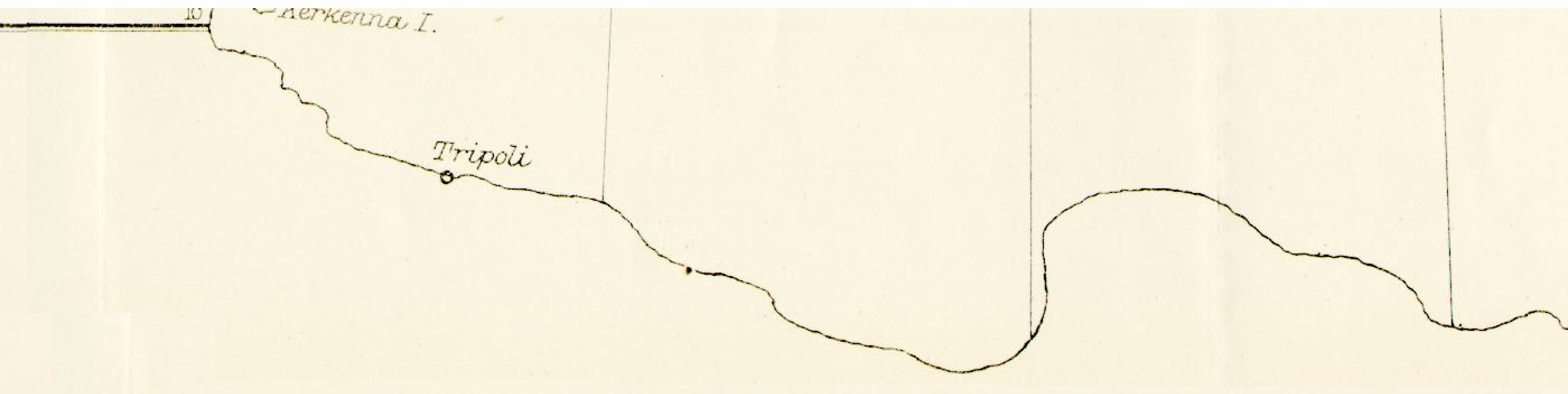
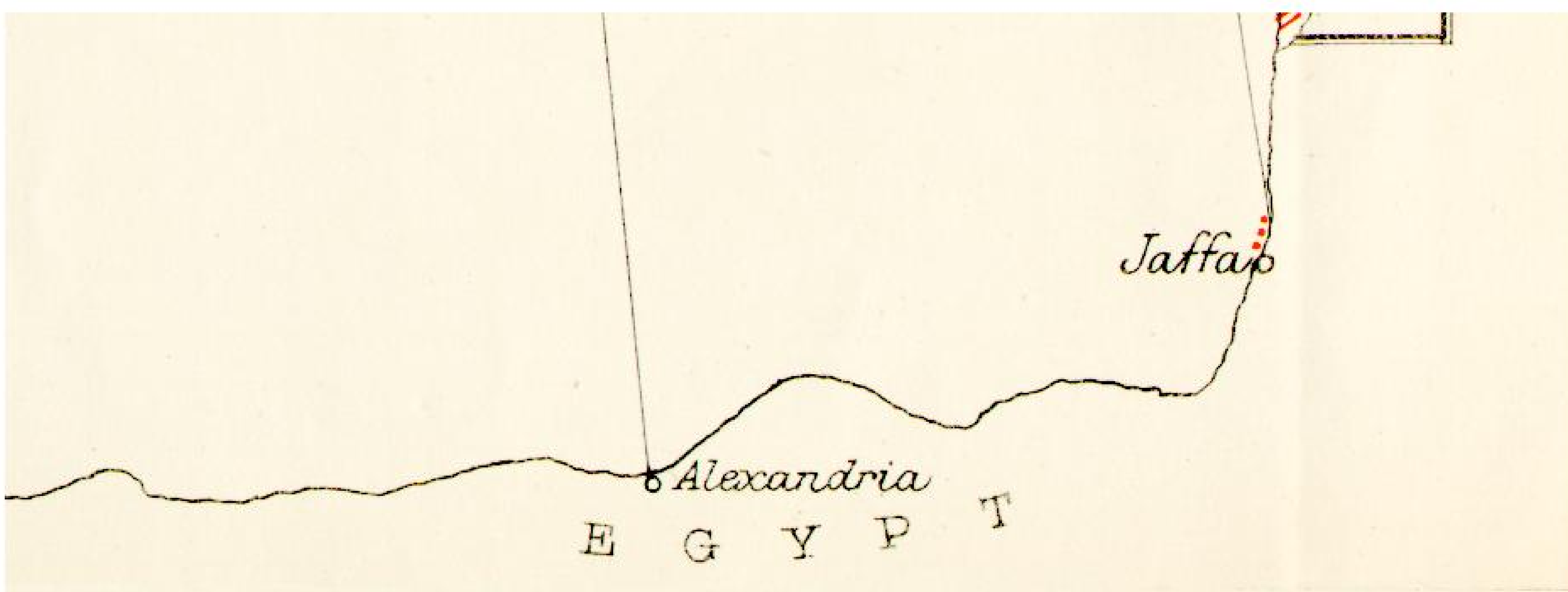
West Newman lith.


Kerkenna I.
Tripoli


Jaffa
Alexandria
EGYPT