## New experimental test of Lorentz's theory of relativity

Chalmers W. Sherwin
17166 Pacato Way, San Diego, California 92128
(Received 4 September 1986)

A new experimental test of H. A. Lorentz's [Theory of Electrons (Columbia University Press, New York, 1909)] theory of relativity exploits a neglected concept of Lorentz, that the physical contraction of macroscopic matter moving with a velocity v, with respect to a postulated preferred inertial frame S (the ether), is caused by the relativistic shortening of the equilibrium lengths of the low-mass electronic bonds in a direction parallel to v. Following a change in the orientation of the bond with respect to v, this contraction generates what we call a transient Lorentzian stress. In all prior experiments in which a macroscopic structure is rotated with respect to a hypothesized v, this stress is so rapidly relieved that the macroscopic length adiabatically follows the length demanded by the bonds and no experimental consequences are observable, even with v as great as  $10^{-3}c$ . In the new experiment, a structure of length L is rotated at an angular frequency  $\omega_R$  about one end in a horizontal plane containing the postulated velocity v. At low rotation rates, when  $2\omega_R$  (the frequency of the transient Lorentzian stress) is small compared to  $\omega_V$  (the radial resonant vibration frequency of the rotating structure) both Einstein and Lorentz predict that its outer end should enscribe an exact circle, but, when  $2\omega_R$  approaches  $\omega_V$  (a requirement which causes very large stretching of the structure over its normal length) the Lorentz theory uniquely predicts that the transient stress does not have time to be fully relieved, and the outer end should enscribe an elliptical path which deviates from an exact circle by an amount  $\sim Lv^2/c^2$ , since the length of each atomic bond parallel to v changes by the factor  $(1-v^2/c^2)^{1/2}$ . A null result was observed for the case where the postulated velocity v is that of the frame in which the cosmic background radiation is isotropic.

#### I. INTRODUCTION

In his book *The Theory of Electronics*, published in 1909, H. A. Lorentz<sup>1</sup> completed his formulation of the theory of special relativity which agreed with all the experimental data available at that time. However, Einstein's theory<sup>2</sup> quickly became widely adopted, a situation which has continued even though the two theories continue to predict the same results for all experiments performed up to the present. The purpose of this paper is to describe a new experiment in which the two theories predict different results.

Lorentz's postulates may be paraphrased as follows.

- (1) There exists a preferred inertial reference frame S in which light propagates uniformily in all directions with the velocity c. (The "ether.")
- (2) The length of a rod moving uniformily in S with the velocity  $\mathbf{v}$  is physically contracted in a direction parallel to its motion by the factor  $(1-v^2/c^2)^{1/2}$ . (The Lorentz-Fitzgerald contraction.)
- (3) The frequency of a clock moving uniformly in S with a speed v is physically retarded by the factor  $(1-v^2/c^2)^{1/2}$ . [This hypothesis was newly added in 1909. Later, in 1922, Lorentz<sup>3</sup> noted that for the case of one particular type of clock, the rod-mirror clock, postulates (1) and (2) suffice to assure that it has the correct retardation factor independent of its orientation with respect to  $\mathbf{v}$ .]
- (4) The physical contraction of postulate (2) is caused by the contraction of the electron-mediated bonds in macroscopic matter. (Lorentz's spherical-shell model of the

electron exhibited the exact contraction property required, and he assumed that, in some not understood manner, the electrons were the agents of the bonding together of the more massive components of matter. We must recall that this hypothesis was necessarily vague since it predates even the Rutherford model of the atom.)

Lorentz and his contemporaries recognized that these postulates were equivalent to Einstein's formulation of the special theory of relativity, including conformity with the Lorentz transformation and the correct prediction of the results of all experimental tests performed up to that time. None the less, most of Lorentz's contemporaries rejected his theory. They regarded it as *ad hoc*, mechanism dependent, limited in scope to electromagnetic phenomena, and as we would say today, "inelegant" compared to Einstein's. Even Lorentz himself at the end of his lectures at the California Institute of Technology in 1922 seemed reluctantly to agree that Einstein's formulation might be preferable to his "old-fashioned road" based as it was on the concept of the ether and on the distinctiveness of space and time.<sup>4</sup>

A comprehensive survey of the subsequent development of ideas based on Lorentz's original theory has been given by Erlichson.<sup>5</sup> He noted that, over the years, a small minority of physicists have further analyzed and extended the scope of Lorentz's first three postulates, most notably Ives,<sup>6</sup> Builder,<sup>7</sup> and Prokhovnick.<sup>8</sup> Prokovnick has recently<sup>9</sup> simplified its postulational form and emphasizes its conceptural advantages. Also recently, Maciel and Tiomno<sup>10</sup> have proposed several new experiments involving light propagation on rotating structures which could dis-

tinguish the Lorentz ether theory from Einstein's special relativity.

Remarkably, none of the subsequent proponents of Lorentz's theory even mention, much less discuss, his idea that the relativistic contraction of the interatomic bonds might be the cause of the contraction of moving macroscopic rods, confidently predicted by both Einstein and Lorentz, but due to experimental limitations never observed, even today.

We make Lorentz's fourth postulate more explicit by assuming that, following a change in either the direction or magnitude of  $\mathbf{v}$ , a bond of length L between nuclei in macroscopic matter will adjust the equilibrium length (not the instantaneous length) of its component parallel to  $\mathbf{v}$  in a very short time—the order of L/c. This assumption is the basis of our prediction that a rapid change in  $\mathbf{v}$  will generate a transient stress in the bond, since the massive nuclei can not be displaced very quickly. This concept, which is consistent with the assumption that the Lorentz contraction is a *physical* phenomenon, is a key factor in defining the experiment to be described below.

# II. THE ASSOCIATION OF RELATIVISTIC LENGTH CONTRACTION WITH PHYSICAL STRUCTURE

Since the time of Lorentz there appears to have been with one exception—no consideration of his concept that the bonds between massive particles such as the nuclei of macroscopic matter might be the proximate physical cause of the relativistic contraction. The exception is P. W. Bridgeman. 11 In 1962 he noted that a lengthmeasuring device comprised of two equal, independent masses moved about by simultaneous, equal forces, as measured with instruments in S, can perform all the operations of length measurement. Thus, it is a bona fide measuring instrument and is operationally equivalent to a meter stick in S. However, when it is transported in a direction parallel to its length into a uniformly moving system S', it is measured by instruments in S to *not* be relativistically contracted. Bridgeman concluded that special relativity needs an additional postulate to the effect that the Lorentz transformation applies only when lengths are measured by bonded physical structures. In his summary he states, "when a meter stick is set into motion, interatomic forces are automatically called into play which produce the relativity shortening, but we were unable to specify further what the nature of these forces might be."11

Marzke and Wheeler<sup>12</sup> also used the concept of independent mass points to transfer a standard of length from one reference frame to another, but they restricted accelerations to a direction normal to the line between the masses, where there is no difference between physical rods and mass points.

It was an analysis similar to Bridgeman's that led the author to the realization that Lorentz's fourth postulate had never been tested experimentally under conditions that could reveal its existence, and that just possibly such a test might lead to a positive result.

In Sec. III we put aside ingrained habits of thought re-

garding space-time and analyze the experiment to be performed in terms of Lorentz's mechanistic concept that the relativistic change in the equilibrium length of the bonds in matter is basically a physical phenomenon which is caused either by a change in the magnitude of v or by a change in the orientation of the bonds with respect to v.

#### III. THEORY OF THE EXPERIMENT

In order to distinguish between Lorentz and Einstein we need conditions that will expose the transient Lorentzian stress to observation, if it exists.

While it is not practical to accelerate a macroscopic physical structure to a significant velocity in a time comparable to its period of vibration, it is possible to rotate a structure rapidly enough to expose the transient stress.

Consider a structure in the laboratory rotating about one end and assume the laboratory frame S', is moving with a constant velocity  $\mathbf{v}$  in the x direction with respect to the postulated preferred inertial frame S. At low rotation rates the length of the structure adiabatically follows the length demanded by the equilibrium lengths of the bonds and the outer end inscribes an exact circle, as measured by laboratory meter sticks, since they experience exactly the same dependence of length on angle. However, at high rotation rates, when the time required to rotate 90° becomes comparable to the period of vibration of the structure, the macroscopic length will not be able to exactly follow the "bond-equilibrium" length. This effect can be caused to occur at low rotation rates by adding a concentrated mass on the outer end of the structure to increase its vibrational period.

The experimental structure is comprised of a short rigid inner section of length  $R_2$ , slender elastic rod (or spring) of unstressed length  $R_1$ , mass  $m_s$ , and elastic constant  $k_s$ , and at the outer end, a concentrated mass  $m_A$ , where  $m_A \gg m_S$ .

When rotating at angular velocity  $\omega_R$ , the structure has a resonant radial vibration frequency

$$\omega_V = \sqrt{k/m}$$
,  $k = k_s + 3\omega_R^2$ ,  $m \simeq m_A + \frac{1}{2}m_S$ , (1)

and a total length

$$R = \frac{R_1 + R_2}{1 - (\omega_B / \omega_V)^2} \ . \tag{2}$$

Under typical operating conditions the length,  $R_0 = R - R_2$ , of the elastic rod is considerably stretched by centripetal force over its unstressed length  $R_1$ , but  $R_2$  is stretched a negligible amount due to its great rigidity. We consider the variation of length with angle of the elastic rod since it dominates the effects being analyzed.

The rod has n bonds along its length. When the rod is perpendicular to  $\mathbf{v}$ , each bond has an (centripetally stressed) equilibrium length  $L_0$ , and the equilibrium length of the rod  $R_0$  is  $nL_0$ .

When the rod is parallel to  $\mathbf{v}$ , assuming the Lorentz contraction applies to centripetally stressed bonds as well as to unstressed bonds, which must be true if the effect is truly a universal effect, each bond has an equilibrium length  $L_0(1-v^2/c^2)^{1/2}$  and the equilibrium length of the rod has the value  $nL_0(1-v^2/c^2)^{1/2}$ .

Let  $\theta$  be the angle between the rod and the x axis (and  $\mathbf{v}$ ), so, at constant rotation,  $\theta = \omega_R t$ . In general, for  $v \ll c$ , the equilibrium length  $L_E$  of a bond in a direction parallel to the rod length is

$$L_E \simeq L_0 [1 - (v^2/4c^2) - (v^2/4c^2)\cos(2\omega_R t)],$$
 (3)

and the equilibrium length of the rod is

$$R_E \simeq R_0 [1 - (v^2/4c^2) - (v^2/4c^2)\cos(2\omega_R t)]$$
 (4)

This is the length demanded at any time by the equilibrium length of the bonds in the direction parallel to the rod length. It has the following two components: a large constant component  $R_0(1-v^2/4c^2)$  and a small, time-varying component  $-R_0(v^2/4c^2)\cos(2\omega_R t)$ .

The radial motion of mass  $m_A$  is equivalent to that of a free-mass  $m_A$  on one end of a spring of fixed length  $R_0(1-v^2/4c^2)$ , which is being driven on the other end by an oscillating motion  $-R_0(v^2/4c^2)\cos(2\omega_R t)$ . When  $\omega_R$  is small enough, the inertial reaction of  $m_A$  is very small and its motion will exactly follow the full range of the oscillation on the driven end of the spring. However, at higher frequencies the distance between the driven end and  $m_A$  will not be constant. It will deviate from  $R_0(1-v^2/4c^2)$  by an amount r and will generate a restoring stress on the spring of amount -kr. Thus, the equation of motion of  $m_A$  is

$$m_A \frac{d^2}{dt^2} \left[ R_0 \left[ 1 - \frac{v^2}{4c^2} - \frac{v^2}{4c^2} \cos(2\omega_R t) \right] + r \right] = -kr$$
,

whose solution is

$$r = -\alpha_0 \frac{1}{(\omega_V / 2\omega_R)^2 - 1} \cos(2\omega_R t),$$

$$\alpha_0 = R_0 (v^2 / 4c^2), \quad v \ll c.$$
(6)

 $R_0 = R - R_2$  is essentially the average length of the centrifugally stretched spring, since  $r \ll R_0$ . r is the amount by which the path of  $m_A$  (at the outer end of the rotating structure) is predicted to deviate from a perfect circle of radius  $R = R_0 + R_2$ .

At low rotation rates, that is when  $\omega_V \gg 2\omega_R$ , Eq. (6) becomes

$$r \simeq -\alpha_0 (2\omega_R/\omega_V)^2 \cos(2\omega_R t) \ . \tag{7}$$

Thus, if an optical or microwave interferometer made with a highly rigid structure, for which the resonant longitudinal frequency is typically  $\sim 10^3$  Hz, is rotated at an angular rate of  $\sim 10^{-2}$  rotations/sec., then  $r \simeq 10^{-10} a_0$ . That is, the already small relativistic effect  $R_0 v^2 / 4c^2$  is multiplied by the factor  $10^{-10}$ . Thus, it appears that any effect due to the transient Lorentzian stress would be unobservable in all prior relativity experiments involving the slow rotation in space of such interferometers.

However, as Eq. (6) shows when  $2\omega_R \rightarrow \omega_V$ , the relativistic effect is no longer diminished but rather becomes greatly enhanced due to the resonance.

In the experiments described below the following numbers apply:

$$R_1 = 7.6 \text{ cm}, R_2 = 4.8 \text{ cm}, \omega_R = 2\pi \times 4.1/\text{sec},$$
  
 $\omega_V = 2\pi \times 10.1/\text{sec}.$ 

For these numbers the stretched length of the elastic structure  $R_0 = 10.05$  cm is 1.32 times its unstressed length  $R_1$ , as may be seen from Eq. (2). This degree of stretching, due to centripetal force, is far in excess of the elastic limit of solid rods. Thus, the elastic portion of the rotating structure used in the experiment is a low-mass, low-loss spring.

With the numbers given above, Eq. (6) becomes

$$r \simeq -(1.94)(R_0 v^2 / 4c^2) \cos(2\omega_R t),$$
  
 $R_0 = 10.05 \text{ cm}, \ \omega_R = 2\pi \times 4.1/\text{sec},$  (8)

so that we have an enhancement of the basic relativistic effect of a factor of 1.94. If  $v/c=10^3$ ,  $r_{\rm max}=48.7\times10^{-9}$  m, a magnitude about one-tenth the wavelength of light and readily measured using the equipment described in Sec. IV.

In deriving the equation of motion of  $m_A$  in Eq. (5), we neglected frictional losses. In the actual experiment, the Q value (which measures the damping rate of the spring's radial vibration) is measured to be approximately 1200. At the actual separation of the excitation frequency  $2\omega_R$ , from the radial resonance  $\omega_V$ , the deviation of the damped solution from the undamped solution is negligible

### IV. EXPERIMENTAL DESIGN

The experiment consists of rotating a structure containing a spring and an accelerometer on the outer end in a horizontal plane, and observing the output of the accelerometer [Columbia, Model 904, with a ratio of peak signal (mV) to peak acceleration (units of g, where  $1g = 9.80 \text{ m/sec}^2$ ) being 12.0] for the presence of a signal in a very narrow frequency band centered at  $2\omega_R$ . The amplitude of the signal should vary with sidereal time. Near the summer solstice at the latitude of the experiment (33° north), the direction of the velocity toward the region of space where the cosmic background radiation is isotropic<sup>13</sup> right ascension 11 h,  $+6^{\circ}\pm10^{\circ}$ , in the constellation of Leo) rises and sets almost east and west and in between comes very close to the zenith or nadir. Thus, the predicted signal at  $2\omega_R$  given by Eq. (8) should vary in amplitude from near zero when Leo is near the zenith or the nadir, to its maximum value 6 sidereal hours earlier or later, when Leo is near the horizon. Any signal at  $2\omega_R$  which is constant, or whose amplitude varies at any rate other than with a 12 sidereal hour period, is spurious.

Since the predicted signal corresponds to an ellipticity in the path of the accelerometer of the order of  $10^{-7}$  m, it follows that the central bearing controlling the rotation should either have an ellipticity which is much smaller than  $10^{-7}$  m, or be very constant. This requirement seemed to be beyond the accuracy of ordinary mechanical bearings, so the actual supporting bearing for the rotating structure is isolated from the point about which the spring and accelerometer are rotating by means of a long (0.61 m), slender (1.6 mm diam) "quill shaft," on which is hung a heavy aluminum disk (mass 20.7 kg). See Fig. 1. Be-

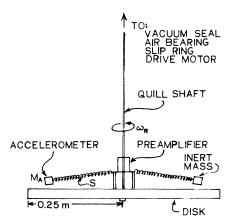


FIG. 1. Schematic drawing of the rotating structure used to test the hypothesis that the Lorentzian stress is a real, although transient, effect. The accelerometer of mass  $m_A$  (42.2 Gm) supported by the stretched spring rotates freely in a vacuum at the constant angular rate  $\omega_R$  about a precise center established by the heavy disk (20.7 kg) supported on a long (0.61 m) quill shaft of diameter 0.16 cm. The accelerometer output signal is conducted to the central axis by a fine-wire, coaxial spring S, amplified by the self-contained preamplifier (voltage gain  $10^2$ :1), transmitted to a mercury slip ring and then through a vacuum seal to an outside amplifier (voltage gain  $10^4$ :1). An identical spring and inert mass opposite to  $m_A$  preserves dynamic balance at different values of  $\omega_R$ .

cause of the flexibility of the quill shaft, the structure rotates about its own center of mass almost as if it were supported on a frictionless surface. Any ellipticity or other imperfections in the supporting (air-lubricated) bearing at the upper end of the quill shaft has a negligible influence on the actual center of rotation. To preserve dynamic balance at different rotational rates, a duplicate spring and compensating mass is attached opposite to the spring-accelerometer assembly.

When the rotating system starts from rest, the accelerometer and the compensating mass rest on the upper surface of the disk. An electric motor drive at the top of the quill shaft slowly accelerates the system to the point where the springs are substantially stretched and the masses are flying free.

The entire rotating structure is enclosed in a steel vacuum chamber to reduce aerodynamic drag and to prevent the aerodynamic excitation of the springs. Due to the vacuum and the air-bearing support, only a small fraction of a watt is needed to maintain a constant rotation rate.

A fine-wire, small helix diameter signal spring is stretched along the axis of the main supporting spring of the accelerometer, connecting the signal output of the accelerometer to the input to the preamplifier (voltage gain 100:1, with self-contained batteries) riding near the axis. The preamplifier is needed since the signal is in the microvolt range and even the mercury slip rings used to bring the accelerometer signal out are too noisy without prior amplification.

The rotation drive is comprised of a small variablespeed, servo-controlled electric motor coupled to the quill shaft by a belt, using pulleys whose diameters have nonintegral ratios. This is necessary since it was found that no amount of electrical shielding will totally suppress the spike noise from the commutator of the motor. With the nonintegral ratios, the signal processor reduces the motor noise to an insignificant level.

The outer portion of the disk is painted with 64 black and white sectors observed by a photocell to monitor angular speed and to control the servo drive. Two marks,  $180^{\circ}$  apart monitored by a second photocell located on an east-west line generate sampling pulses at exactly  $2\omega_R$  for use in the cross-correlator signal processor.

The signals from the accelerometer are amplified by a total voltage gain of  $1.0 \times 10^6$ , sampled and pulse stretched at the input to the cross correlator, smoothed by an *RC* filter of a time constant of 18 sec, and are displayed on an output microammeter which is visually observed.

The accelerometer-amplifier combination was independently calibrated using a separate test fixture which generated a known sinusoidal acceleration in the 8-Hz frequency range. The measured overall conversion factor of input acceleration to output voltage was  $12.0\times10^6$ -mV peak output per peak g input. This result agrees within 5% of the conversion factor based on the manufacturer's calibration of the accelerometer and the design gain of the amplifiers.

Only one source of spurious signal at  $2\omega_R$  was discovered. It was the second harmonic of a very constant signal at  $\omega_R$ . A series of tests showed that this signal  $\omega_R$ was caused by the magnetic induction of the supportspring-signal-spring assembly moving a residual magnetic field of a few tenths of a gauss inside the steel vacuum can, a conclusion that was confirmed by introducting a small magnet near the rotating system which greatly augmented the signal. Efforts to degauss the can were unsuccessful. The magnetic induction pickup in the support spring did not exactly cancel the opposite sign signal in the signal spring, and, since the magnetic field was not exactly uniform, a second harmonic signal at  $2\omega_R$  was generated. Fortunately, although the signal at  $2\omega_R$  had a magnitude about the same as that of the calculated Lorentzian "signal" at  $2\omega_R$ , it was very stable with time and was easily rejected.

The resonant radial vibration frequency  $\omega_V$  of the rotating structure is observed by causing a very small but sudden increase in pressure in the vacuum chamber. This shock-excites the radial vibration, permitting its frequency and its amplitude decay rate to be measured.

#### V. EXPERIMENTAL RESULTS

Several 24 sidereal hour runs ( $f_R = 4.1$  rotations sec) were made near the summer solstice with cross-correlator sampling pulses occuring when the accelerometer crosses an east-west line, and using the 18-sec RC integrating filter. The output meter of the cross correlator was read at intervals varying from a few minutes to 30 min. All runs gave the same results. The output meter, which responds only to a signal in a very narrow band centered at  $2f_R = 8.2$  Hz, showed an essentially constant value with sidereal time. See Fig. 2. The experimental data points

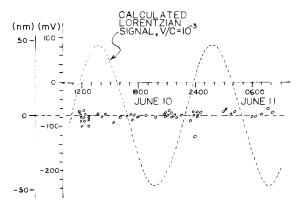


FIG. 2. Variation with solar time of the amplitude in millivolts of the accelerometer amplifier output signal in a very narrow band centered at  $2\omega_R$ . The accelerometer rotates in a horizontal plane at a time near the summer solstice when the constellation of Leo rises and sets almost east and west and approaches the nadir and zenith in between. The experimental points show that this signal is essentially constant with time. Using Eq. (8) of the text and the associated numerical values, where  $2\omega_R$  is approaching resonance with  $\omega_V$ , the radial vibration frequency, we plot the variation of this signal with sidereal time, as predicted by the Lorentzian hypothesis, for the case where  $v/c = 10^{-3}$ . The ordinate scale in nanometers shows the deviation of the calculated elliptical path of  $m_A$  from an exact circle radius of  $R_0 + R_2 = 0.1485$  m is 49 nm. This maximum deviation is approximately  $\pm 1/10$  a wavelength of light. If v/c were  $10^{-4}$ , the calculated Lorentzian deviation from an exact circle would be 100 times smaller, and the Lorentzian hypothesis would not be rejected by this experiment.

have an average constant value of -77 mV (at the output of the cross-correlator) with an rms scatter of 11.9 mV. The constant signal of -77 mV arises from magnetic induction in the residual magnetic field inside the vacuum can, as described in Sec. IV.

Several runs were made with the cross-correlator sampling pulses aligned in the north-south direction, with similar results. A series of experimental tests to identify the sources of the noise shown in Fig. 2 disclosed that of the 11.9-mV noise in the data, 6.3 mV was due to thermal noise, 0.3 mV was due of signal slip-ring noise and random electrical pick-up, 1.7 mV was generated by variations in the magnetic induction signal, and 3.3 mV was due to random variations in the accelerometer output signal. The latter two noise components appear to be generated principally by variations in the rotational speed arising from noise in the servo-speed control system.

The maximum amplitude of the calculated Lorentzian signal in Fig. 2 was determined using the second time derivative of Eq. (6). An analysis of the sources of experimental error involved in calculating this quantity (on the assumption that v/c is exactly  $10^{-3}$ ) shows that the amplitude of  $\pm 158$  mV has an estimated uncertainty of  $\pm 19\%$ . The largest single source of error is the calculation of the resonance factor between  $\omega_V$  and  $2\omega_R$ .

The data in Fig. 2 show that, if  $v/c = 10^{-3}$ , and v is directed near the ecliptic, the maximum of the calculated signal is 13 times rms noise, and the Lorentzian theory is rejected.

An analysis of possible improvements in experimental design indicated that the noise level could be reduced with some effort by a factor of about 1/30. In this case a postulated velocity with magnitude of  $v=10^{-4}c$ , or the earth's orbital speed, could be measured. However, there seems to be no other candidate for the velocity of the earth with respect to a hypothetical preferred inertial reference frame S, other than the reference frame in which the universal black-body radiation is isotropic, so more accurate measurements do not seem warrented.

The only way the Lorentzian theory could be made consistent with these data is to require that the length of a "floppy", very nonrigid structure (here, a low-mass spring with a heavy mass on one end) assumes its correct relativistic length appropriate to its angle with respect to v in a time small compared to its longitudinal resonance period. But this is inconsistent with any known concept of the physical contraction of a macroscopic structure.

<sup>&</sup>lt;sup>1</sup>H. A. Lorentz, *Theory of Electrons* (Dover, New York, 1952).

<sup>&</sup>lt;sup>2</sup>A. Einstein, Ann. Phys. (Paris) 17, 891 (1905).

<sup>&</sup>lt;sup>3</sup>H. A. Lorentz, *Problems of Modern Physics*, 1st ed. (Dover, New York, 1967), p. 92.

<sup>&</sup>lt;sup>4</sup>H. A. Lorentz, Problems of Modern Physics, Ref. 3, p. 212.

<sup>&</sup>lt;sup>5</sup>H. Erlichson, Am. J. Phys. **41**, 1,068 (1973).

<sup>&</sup>lt;sup>6</sup>H. E. Ives, Sci. Proc. R. Dublin Soc. **26**, 163 (1952). For numerous other references, see H. Erlichson, Ref. 5.

<sup>&</sup>lt;sup>7</sup>G. Builder, Aust. J. Phys. 11, 279 (1958); 11, 457 (1958); Philos. Sci. 26, 135 (1959).

<sup>8</sup>S. Prokhovnick, The Logic of Special Relativity (Cambridge University Press, New York, 1967).

<sup>&</sup>lt;sup>9</sup>S. Prokhovnick, *Light in Einstein's Universe* (Reidel, Massachusetts, 1985).

<sup>&</sup>lt;sup>10</sup>A. K. A. Maciel and J. Tiomno, Phys. Rev. Lett. 55, 143 (1985).

<sup>&</sup>lt;sup>11</sup>P. W. Bridgeman, A Sophisticate's Primer of Relativity (Wesleyan University Press, Middletown CT, 1962), p. 148.

<sup>&</sup>lt;sup>12</sup>R. F. Marzke and J. A. Wheeler, in Graviation and Relativity, edited by H. Chiu and W. F. Hoffman (Benjamin, New York, 1964).

<sup>&</sup>lt;sup>13</sup>G. F. Smoot, N. V. Gorenstein, and M. A. Muller, Phys. Rev. Lett. 39, 898 (1977).