

May 4, 1854.

Colonel SABINE, R.A., Treas. and V.P., in the Chair.

In accordance with the Statutes, the Chairman read the following list of Candidates recommended by the Council to the Society for election :—

George James Allman, M.D.	Robert Mallet, Esq.
Edward William Brayley, Esq.	Charles May, Esq.
Alexander Bryson, M.D.	Capt. Thomas E. L. Moore, R.N.
J. Lockhart Clarke, Esq.	Captain Richard Strachey.
Joseph Dickinson, M.D.	Robert Dundas Thomson, M.D.
Ronald Campbell Gunn, Esq.	Samuel Charles Whitbread, Esq.
Robert Hunt, Esq.	William Crawford Williamson,
John Bennet Lawes, Esq.	Esq.

The following papers were read :—

- I. "Account of Researches in Thermo-electricity." By Professor W. THOMSON of Glasgow, F.R.S. Received April 20, 1854.

§ I. *On the Thermal Effects of Electric Currents in Unequally Heated Conductors.*

Theoretical considerations (communicated in December 1851 to the Royal Society of Edinburgh), founded on observations which had been made regarding the law of thermo-electric force in an unequally heated circuit of two metals, led me to the conclusion that an electric current must exercise a convective effect on heat in a homogeneous metallic conductor of which different parts are kept at different temperatures. A special application of the reasoning to the case of a compound circuit of copper and iron was made, and it is repeated here because of the illustration it affords of the mechanical principles on which the general reasoning is founded.

Becquerel discovered that if one junction of copper and iron, in a circuit of the two metals, be kept at an ordinary atmospheric temperature, while the other is raised gradually to a red or white heat, a current first sets from copper to iron through the hot junction, increasing in strength only as long as the temperature is below about 300° Cent.; and becoming feebler with farther elevations of temperature until it ceases, and a current actually sets in the contrary direction when a high red heat is attained. Many experimenters have professed themselves unable to verify this extraordinary discovery, but the description which M. Becquerel gives of his experiments leaves no room for the doubts which some have thrown upon his conclusion, and establishes the thermo-electric inversion between iron and copper, not as a singular case (extraordinary and unexpected as it appeared), but as a phenomenon to be looked for between any two metals, when tried through a sufficient range of temperature, especially any two which lie near one another in the thermo-electric series for ordinary temperatures. M. Regnault has verified M. Becquerel's conclusion so far, in finding that the strength of the current in a circuit of copper and iron wire did not increase sensibly for elevations of temperature above 240° Cent., and began to diminish when the temperature considerably exceeded this limit; but the actual inversion observed by M. Becquerel is required to show that the diminution of strength in the current is due to a real falling off in the electromotive force, and not to the increased resistance known to be produced by an elevation of temperature.

From Becquerel's discovery it follows that, for temperatures below a certain limit, which, for particular specimens of copper and iron wire, I have ascertained, by a mode of experimenting described below, to be 280° Cent., copper is on the negative side of iron in the thermo-electric series, and on the positive side for higher temperatures; and at the limiting temperature copper and iron are thermo-electrically neutral to one another. It follows, according to the general mechanical theory of thermo-electric currents referred to above, that electricity passing from copper to iron causes the absorption or the evolution of heat according as the temperature of the metals is below or above the neutral point; but neither evolution nor absorption of heat, if the temperature be precisely that of neutrality (a conclusion which I have already partially verified by

experiment). Hence, if in a circuit of copper and iron, one junction be kept about 280° , that is, at the neutral temperature, and the other at any lower temperature, a thermo-electric current will set from copper to iron through the hot, and from iron to copper through the cold junction; causing the evolution of heat at the latter, and the raising of weights too if it be employed to work an electro-magnetic engine, but not causing the absorption of any heat at the hot junction. Hence there must be an absorption of heat at some part or parts of the circuit consisting solely of one metal or of the other, to an amount equivalent to the heat evolved at the cold junction, together with the thermal value of any mechanical effects produced in other parts of the circuit. The locality of this absorption can only be where the temperatures of the single metals are non-uniform, since the thermal effect of a current in any homogeneous uniformly heated conductor is always an evolution of heat. Hence there must be on the whole an absorption of heat, caused by the current in passing from cold to hot in copper, and from hot to cold in iron. When a current is forced through the circuit against the thermo-electric force, the same reasoning establishes an evolution of heat to an amount equivalent to the sum of the heat that would be then taken in at the cold junction, and the value in heat of the energy spent by the agency (chemical or of any other kind) by which the electromotive force is applied. The aggregate reversible thermal effect, thus demonstrated to exist in the unequally heated portions of the two metals, might be produced in one of the metals alone, or (as appears more natural to suppose) it may be the sum or difference of effects experienced by the two. Adopting as a matter of form the latter supposition, without excluding the former possibility, we may assert that either there is absorption of heat by the current passing from hot to cold in the copper, and evolution, to a less extent, in the iron of the same circuit; or there is absorption of heat produced by the current from hot to cold in the iron, and evolution of heat to a less amount in the copper; or there must be absorption of heat in each metal, with the reverse effect in each case when the current is reversed. The reversible effect in a single metal of non-uniform temperature may be called a convection of heat; and to avoid circumlocution, I shall express it, that the vitreous electricity carries heat with it, or that the specific heat of vitreous electricity is positive,

when this convection is in the nominal "direction of the current," and I shall apply the same expressions to "resinous electricity" when the convection is against the nominal direction of the current. It is established then that one or other of the following three hypotheses must be true :—

Vitreous electricity carries heat with it in an unequally heated conductor whether of copper or iron; but more in copper than in iron.

Or Resinous electricity carries heat with it in an unequally heated conductor whether of copper or iron; but more in iron than in copper.

Or Vitreous electricity carries heat with it in an unequally heated conductor of copper, and Resinous electricity carries heat with it in an unequally heated conductor of iron.

Immediately after communicating this theory to the Royal Society of Edinburgh, I commenced trying to ascertain by experiment which of the three hypotheses is the truth, as Theory with only thermo-electric data could not decide between them. I had a slight bias in favour of the first rather than the second, in consequence of the positiveness which, after Franklin, we habitually attribute to the vitreous electricity, and a very strong feeling of the improbability of the third. With the able and persevering exertions of my assistant, Mr. M^cFarlane, applied to the construction of various forms of apparatus and to assist me in conducting experiments, the research has been carried on, with little intermission, for more than two years. Mr. Robert Davidson, Mr. Charles A. Smith, and other friends have also given much valuable assistance during the greater part of this time, in the different experimental investigations of which results are now laid before the Royal Society. Only nugatory results were obtained until recently from multiplied and varied experiments both on copper and iron conductors; but the theoretical anticipation was of such a nature that no want of experimental evidence could influence my conviction of its truth. About four months ago, by means of a new form of apparatus, I ascertained that *resinous electricity carries heat with it in an unequally heated iron conductor*. A similar equally sensitive arrangement showed no result for copper. The second hypothesis might then have been expected to hold; but to ascertain the truth with certainty I have

continued ever since, getting an experiment on copper nearly every week with more and more sensitive arrangements, and at last, in two experiments, I have made out with certainty, that *vitreous electricity carries heat with it in an unequally heated copper conductor*.

The third hypothesis is thus established : a most unexpected conclusion I am willing to confess.

I intend to continue the research, and I hope not only to ascertain the nature of the thermal effects in other metals, but to determine its amount in absolute measure in the most important cases, and to find how it varies, if at all, with the temperature ; that is, to determine the character (positive or negative) and the value of the specific heat, varying or not with the temperature, of the unit of current electricity in various metals.

§ II. *On the Law of Thermo-electric Force in an unequally heated circuit of two Metals.*

A general relation between the specific heats of electricity in two different metals, and the law of thermo-electric force, in a circuit composed of them according to the temperatures of their junctions, was established in the communication to the Royal Society of Edinburgh referred to above, and was expressed by an equation* which may now be simplified by the thermometric assumption

$$t = \frac{J}{\mu} ;$$

(μ denoting Carnot's function, J Joule's equivalent, and t the temperature measured from an absolute zero, about $273\frac{1}{2}^{\circ}$ Cent. below the freezing-point,) since this assumption defines a system of *thermometry in absolute measure*, which the experimental researches recently made by Mr. Joule and myself establish as not differing sensibly from the scale of the air-thermometer between ordinary limits. The equation, when so modified, takes the following form :—

$$F = J \left\{ \frac{\Theta_S}{S}(S - T) + \int_T^S \mathfrak{S} \left(1 - \frac{T}{t} \right) dt \right\},$$

where \mathfrak{S} denotes the excess of the specific heat of electricity in the metal through which the current goes from cold to hot above the specific heat of the same electricity in the other metal, at the tem-

* See Proceedings R.S.E. Dec. 1851, or Philosophical Magazine 1852.

perature t ; F the thermo-electric force in the circuit when the two junctions are kept at the temperatures S and T respectively, of which the former is the higher; and Θ_s the amount of heat absorbed per unit of electricity crossing the hot junction. The following relation (similarly simplified in form) was also established:—

$$\mathfrak{S} = \frac{\Theta}{t} - \frac{d\Theta}{dt}.$$

These relations show how important it is towards the special object of determining the specific heats of electricity in metals, to investigate the law of electromotive force in various cases, and to determine the thermal effect of electricity in passing from one metal to another at various temperatures. Both of these objects of research are therefore included in the general investigation of the subject.

The only progress I have as yet made in the last-mentioned branch of the inquiry, has been to demonstrate experimentally that there is a cooling or heating effect produced by a current between copper and iron at an ordinary atmospheric temperature according as it passes from copper to iron or from iron to copper, in verification of a theoretical conclusion mentioned above: but I intend shortly to extend the verification of theory to a demonstration that reverse effects take place between those metals at a temperature above their neutral point of about 280° Cent.; and I hope also to be able to make determinations in absolute measure of the amount of the Peltier effect for a given strength of current between various pairs of metals.

With reference to laws of electromotive force in various cases, I have commenced by determining the order of several specimens of metals in the thermo-electric series, and have ascertained some very curious facts regarding varieties in this series which exist at different temperatures. In this I have only followed Becquerel's remarkable discovery, from which I had been led to the reasoning and experimental investigation regarding copper and iron described above. My way of experimenting has been to raise the temperature first of one junction as far as the circumstances admit, keeping the other cold, and then to raise the temperature of the other gradually, and watch the indications of a galvanometer during the whole process. When an inversion of the current is noticed, the changing temperature is

brought back till the galvanometer shows no current ; and then (by a process quite analogous to that followed by Mr. Joule and Dr. Lyon Playfair in ascertaining the temperature at which water is of maximum density) the temperatures of the two junctions are approximated, the galvanometer always being kept as near zero as possible. When the difference between any two temperatures on each side of the neutral point which give no current is not very great, their arithmetical mean will be the neutral temperature. A regular deviation of the mean temperature from the true neutral temperature is to be looked for with wide ranges, and a determination of it would show the law according to which the difference of the specific heat of electricity in the two metals varies with the temperatures ; but I have not even as yet ascertained with certainty the existence of such a deviation in any particular case. The following is a summary of the principal results I have already obtained in this department of the subject.

The metals tried being,—three platinum wires (P_1 the thickest, P_2 the thinnest, and P_3 one of intermediate thickness), brass wires (B), a lead wire (L'), slips of sheet lead (L), copper wires (C), and iron wire (I), I find that the specimens experimented on stand thermo-electrically at different temperatures in the order shown in the following Table, and explained in the heading by reference to bismuth and antimony, or to the terms “negative” and “positive” as often used :—

Temp. Cent.	Bismuth “Negative.”	Antimony “Positive.”
−20	... P_3 ... c P_2 P_1I.....
0	... P_3 ... l' P_2C P_1I.....
37	... P_3 b ... $\{L/P_2\}$	C... P_1I.....
64	... P_3 P_2 ... b ... l' $\{CP_1\}$I.....
130	... P_3 P_2 $\{BP_1\}$...L... C.....I.....
140	... P_3 P_2 P_1 ... $\{BL\}$CI.....
280	... P_3 P_2 P_1	b ... $\{CI\}$
300	... P_3 P_2 P_1	bI.....C

It must be added, by way of explanation, that the bracket enclosing the symbols of any two of the metallic specimens indicates that they are neutral to one another at the corresponding temperature, and the arrow-head below one of them shows the direction in which it is changing its place with reference to the other, in the series, as the temperature is raised. When there is any doubt as to a position as shown in the Table, the symbol of the metal is a small letter instead of a capital.

The rapidity with which copper changes its place among some of the other metals (the platinum and iron) is very remarkable. Brass also changes its place in the same direction possibly no less rapidly than copper; and lead changes its place also in the same direction but certainly less rapidly than brass, which after passing the thick platinum wire (P_1) at 130° Cent. passes the lead at 140° , the lead itself having probably passed the thick platinum at some temperature a little below 130° *.

The conclusion as regards specific heats of electricity in the different metals, from the equation expressing thermo-electric force given above, is that the specific heat of vitreous electricity is greater in each metal passing another from left to right in the series as the temperature rises than in the metal it passes: thus in particular,—

The specific heat of vitreous electricity is greater in copper than in platinum or in iron; greater in brass than in platinum or in lead; and greater in lead than in platinum.

It is probable enough from the results regarding iron and copper mentioned above, that the specific heat of vitreous electricity is positive in brass; very small positive, or else negative, in platinum, perhaps of about the same value as in iron. It will not be difficult to test these speculations either by direct experiment on the convective effects of electric currents in the different metals, or by comparative measurements of thermo-electric forces for various temperatures in circuits of the metals, and I trust to be able to do so before long.

§ III. *On Thermo-electricity in crystalline metals, and in metals in a state of mechanical strain.*

Having recently been occupied with an extension of the mechani-

* I have since found that it does pass the thick platinum, at the temperature 118° . [May 16, 1854.]

cal theory to the phenomena of thermo-electricity in crystalline metals, I have been led to experimental investigation on this branch of the subject. The difficulty of obtaining actual metallic crystals of considerable dimensions made it desirable to imitate crystalline structure in various ways. The analogies of the crystalline optical properties which have been observed in transparent solids, in a state of strain, and of the crystalline structure as regards magnetic induction which Dr. Tyndall's remarkable experiments show to be produced not only in bismuth but in wax, thick paste of flour, and "the pith of fresh rolls," by pressure, made it almost certain that pressure or tension on a mass of metal would give it the thermo-electric properties of a crystal. The only case which I have as yet had time to try, verifies this anticipation. I have found that copper wire stretched by a weight bears to similar copper wire unstretched, exactly the thermo-electric relation which Svanberg discovered in a bar cut equatorially from a crystal of bismuth or antimony compared with a bar cut axially from a crystal of the same metal. Thus I found that:—

If part of a circuit of copper wire be stretched by a considerable force and the remainder left in its natural condition, or stretched by a less force, and if either extremity of the stretched part be heated, *a current sets from the stretched to the unstretched part through the hot junction*: and if the wire be stretched and unstretched on the two sides of the heated part alternately, the current is reversed (as far as I have been able yet to test, instantaneously) with each change the tension.

I intend to make similar experiments on other metallic wires; also to try the effect of transverse as well as of longitudinal tension on slips of sheet metal *with their ends at different temperatures*, when placed longitudinally in an electric circuit; and the effects of oblique tension on slips of metal similarly placed in a circuit, but kept with their ends at the same temperature and their *lateral edges unequally heated*. I have no doubt of being able so to verify every thermo-electric characteristic of crystalline structure, in metals in a state of strain.

Glasgow College, March 30, 1854.

P.S. April 19, 1854.—I have today found by experiment that iron
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wire when stretched by a considerable force bears a thermo-electric relation to unstretched iron wire, the opposite of that which I had previously discovered in the case of copper wire; and I have ascertained that when the wire is alternately stretched and unstretched on the two sides of a heated part the current is reversed along with the change of tension, always passing from the unstretched to the stretched part, through the hot locality.

I hope before the end of the present Session to have a complete account of all the experiments of which the results are stated above, ready to communicate to the Royal Society.

II. "An Introductory Memoir upon Quantics." By ARTHUR CAYLEY, Esq., F.R.S. Received April 20, 1854.

The subject of Quantics is defined as the entire subject of rational and integral functions, and of the equations and loci to which these give rise, but the memoir relates principally to the functions called quantics; a quantic being in fact a rational and integral function, homogeneous in regard to a set of facients (x, y, \dots) , or more generally homogeneous in regard to each of several such sets separately. A quantic of the degrees m, m', \dots in the sets (x, y, \dots) (x', y', \dots) &c. is represented by a notation such as

$$(*) (x, y, \dots)^m (x', y', \dots)^{m'}, \dots,$$

where the mark $*$ is considered as indicative of the absolute generality of the quantic. The coefficients of the different terms of the quantic may be either mere numerical multiples of single letters or elements, such as a, b, c, \dots , or else functions (in general rational and integral functions) of such elements; this explains the meaning of the expression the elements of a quantic. The theory leads to the discussion of the derivatives called covariants. Of these covariants a very general definition is given as follows, viz. considering the quantic $(*) (x, y, \dots)^m (x', y', \dots)^{m'}, \dots$, and selecting any two facients of the same set, *e. g.* the facients x, y , it is remarked that there is always an operation upon the elements tantamount as regards the quantic to the operation $x d_y$, viz. if we differentiate with respect to each element, multiply by proper functions of the elements and add, the