process as it takes place in nature, dispenses alike with hypothetical radicals and residues, both of which are, however, convenient for the purposes of notation. In the selection of a typical form, to which a great number of species may be referred, hydrogen or water merits the preference from its simplicity, and from the important part which it plays in the generation of species. Water and carbonic anhydride are both so directly concerned in the generation of the bodies in the carbon series, that either may be assumed as the type; but we prefer to regard C^2O^4, like the other anhydrides, as only a derivative of the type of water, and eventually of the hydrogen type.

These views were first put forward by myself in 1848, when I expressed the opinion that they were destined to form "the basis of a true natural system of chemical classification;" and it was only after having opposed them for four years to those of Gerhardt, that this chemist, in June 1852, renounced his views, and without any acknowledgment adopted my own*. Already in 1851, Williamson, in a paper read before the British Association, had developed the ideas on the water type to which Wurtz refers above; and to him the English editor of Gmelin's 'Handbook' ascribes the theory. The notion of condensed types, and of H^2 as the primal type, was not, so far as I am aware, brought forward by either of these, and remained unnoticed until resuscitated by Wurtz in 1855, seven years after I had first announced it, and one year after my reclamation, published in the American Journal of Science, in March 1854.

My claims have not, however, been overlooked by Dr Wolcott Gibbs. In an essay on the polyacid bases, he remarks that in a previous paper he had attributed the theory of water types to Gerhardt and Williamson, and adds, "In this I find I have not done justice to Mr. T. Sterry Hunt, to whom is exclusively due the credit of having first applied the theory to the so-called oxygen acids and to the anhydrides, and in whose earlier papers may be found the germs of most of the ideas on classification usually attributed to Gerhardt and his disciples†." It will be seen, from what precedes, that I not only applied the theory, as Dr. Gibbs remarks, but, except so far as Laurent's suggestion goes, invented it and published it in all its details some years before it was accepted by a single chemist.

In conclusion, I have only to ask that future historians will do justice to the memory of Auguste Laurent, and will ascribe to whom it is due the credit of having given to the science a theory which has exercised such an important influence on modern che-

* *Ann. de Chim. et de Phys.* [3] vol. xxxvii. p. 285.
† Proceedings of the American Association, Baltimore, May 1858, p. 197.

mical speculation and research, remembering that my own publications on the subject, which cover the whole ground, were some years earlier than those of Williamson, Gerhardt, Wurtz, or Kolbe.

Montreal, January 1861.

IV. *On the Reduction of Observations of Underground Temperature; with Application to* Professor Forbes's *Edinburgh Observations, and the continued Calton Hill Series.* *By* Professor William Thomson, F.R.S.*

1. *Analysis of Periodic Variations.*

1. EVERY purely periodical function is, as is well known, expressible by means of a series of constant coefficients multiplying sines and cosines of the independent variable with a constant factor and its multiples. This important truth was arrived at by an admirable piece of mathematical analysis, called for by Daniel Bernoulli, partially given by La Grange, and perfected by Fourier.

2. To simplify my references to the mathematical propositions of this theory, I shall commence by laying down the following definitions:—

Def. 1. A simple harmonic function is a function which varies as the sine or cosine of the independent variable, or of an angle varying in simple proportion with the independent variable. The harmonic curve is the well-known name applied to the graphic representation, on the ordinary Cartesian system, of what I am now defining as a simple harmonic function. It is the form of a string vibrating in such a manner as to give the simplest and smoothest possible character of sound; and, in this case, the displacement of each particle of the string is a harmonic function of the time, besides being a harmonic function of the distance of its position of equilibrium from either end of the string. The sound in this case may be called a perfect unison.

Def. 2. The argument of a simple harmonic function is the angle to the sine or cosine of which it is proportional.

Cor. The argument of a harmonic function is equal to the independent variable multiplied by a constant factor, with a constant added; that is to say, it may be any linear function of the independent variable.

Def. 3. When time is the independent variable, the epoch is

* From the Transactions of the Royal Society of Edinburgh, vol. xxii. part 2. Communicated by the Author.

the interval which elapses from the era of reckoning till the function first acquires a maximum value. The augmentation of argument corresponding to that interval will be called "the epoch in angular measure," or simply "the epoch" when no ambiguity can exist as to what is meant.

Def. 4. The period of a simple harmonic function is the augmentation which the independent variable must receive to increase the argument by a circumference.

Cor. If c denote the coefficient of the independent variable in the argument, the period is equal to $\frac{2\pi}{c}$. Thus if T denote the period, ϵ the epoch in angular measure, and t the independent variable, the argument proper for a cosine is

$$\frac{2\pi t}{T} - \epsilon;$$

and the argument for a sine,

$$\frac{2\pi t}{T} - \epsilon + \frac{\pi}{2}.$$

3. *Composition and Resolution of Simple Harmonic Functions of one Period.*

Prop. The sum of any two simple harmonic functions of one period is equal to one simple harmonic function whose amplitude is the diagonal of a parallelogram described upon lines drawn from one point to lengths equal to the amplitudes of the given functions, at angles measured from a fixed line of reference equal to their epochs, and whose epoch is the inclination of the same diagonal to the same line of reference.

Cor. 1. If A, A′ be the amplitudes of two simple harmonic functions of equal period, and ϵ, ϵ' their epochs, that is to say, if $A\cos(mt-\epsilon)$, $A'\cos(mt-\epsilon')$ be two simple harmonic functions, the one simple harmonic function equal to their sum has for its amplitude and its epoch the following values respectively:—

$$\text{(amplitude)} \quad \{(A\cos\epsilon + A'\cos\epsilon')^2 + (A\sin\epsilon + A'\sin\epsilon')^2\}^{\frac{1}{2}},$$
$$\text{or } \{A^2 + 2AA'\cos(\epsilon'-\epsilon) + A'^2\}^{\frac{1}{2}};$$

$$\text{(epoch)} \qquad \tan^{-1}\frac{A\sin\epsilon + A'\sin\epsilon'}{A\cos\epsilon + A'\cos\epsilon'}.$$

Cor. 2. Any number of simple harmonic functions, of equal period, added together, are equivalent to a single harmonic function of which the amplitude and epoch are derived from the amplitude and epochs of the given functions, in the same manner as the magnitude and inclination to a fixed line of reference, of the resultant of any number of forces in one plane, are derived from the magnitudes and the inclinations to the same line of reference of the given forces.

Cor. 3. The physical principle of the superposition of sounds being admitted, any number of simple unisons of one period coexisting, produce one simple unison of the same period, of which the intensity (measured by the square of the amplitude) and the epoch are determined in the manner just specified.

Cor. 4. The sum of any number of simple harmonic functions of one period vanishes for every argument, if it vanishes for any two arguments not differing by a semicircumference, or by some multiple of a semicircumference.

Cor. 5. The co-existence of perfect unisons may constitute perfect silence.

Cor. 6. A simple harmonic function of any epoch may be resolved into the sum of two whose epochs are respectively zero and a quarter period, and whose amplitudes are respectively equal to the value of the given function for the arguments zero and a quarter period respectively.

4. *Complex Harmonic Functions.*—Harmonic functions of different periods added can never produce a simple harmonic function. If their periods are commensurable, their sum may be called a complex harmonic function.

Cor. A complex harmonic function is the proper expression for a perfect harmony in music.

5. *Expressibility of Arbitrary Functions by Trigonometrical series.*

Prop. A complex harmonic function, with a constant term added, is the proper expression, in mathematical language, for any arbitrary periodic function.

6. *Investigation of the Trigonometrical Series expressing an Arbitrary Function.*—Any arbitrary periodic function whatever being given, the amplitudes and epochs of the terms of a complex harmonic function, which shall be equal to it for every value of the independent variable, may be investigated by the "method of indeterminate coefficients," applied to determine an infinite number of coefficients from an infinite number of equations of condition, by the assistance of the integral calculus as follows:—

Let $F(t)$ denote the function, and T its period. We must suppose the value of $F(t)$ known for every value of t, from $t=o$ to $t=T$. Let M_0 denote the constant term, and let M_1, M_2, M_3, &c. denote the amplitudes, and ϵ_1, ϵ_2, ϵ_3, &c. the epochs of the successive terms of the complex harmonic functions by which it is to be expressed; that is to say, let these constants be such that

$$(Ft)=M_0+M_1\cos\left(\frac{2\pi t}{T}-\epsilon_1\right)+M_2\cos\left(\frac{4\pi t}{T}-\epsilon_2\right)$$
$$+M_3\cos\left(\frac{6\pi t}{T}-\epsilon_3\right)+\&c.$$

Then, expanding each cosine by the ordinary formula, and assuming

$$M_1\cos\epsilon_1=A_1,\quad M_2\cos\epsilon_2=A_2,\quad \&c.,$$
$$M_1\sin\epsilon_1=B_1,\quad M_2\sin\epsilon_2=B_2,\quad \&c.,$$

we have

$$F(t)=A_0+A_1\cos\frac{2\pi t}{T}+A_2\cos\frac{4\pi t}{T}+A_3\cos\frac{6\pi t}{T}+\&c.,$$
$$+B_1\sin\frac{2\pi t}{T}+B_2\sin\frac{4\pi t}{T}+B_3\sin\frac{6\pi t}{T}+\&c.$$

Multiplying each member by $\cos\frac{2i\pi t}{T}dt$, where i denotes o or any integer, and integrating from $t=o$ to $t=T$, we have

$$\int_o^T F(t)\cos\frac{2i\pi t}{T}dt=A_i\int_o^T\left(\cos\frac{2i\pi t}{T}\right)^2dt,$$
$$=A_i\times\tfrac{1}{2}T,\text{ when } i \text{ is any integer};$$

or

$$=A_0\times T,\text{ when } i=0.$$

Hence

$$A_0=\frac{1}{T}\int_o^T F(t)dt,$$
$$A_i=\frac{2}{T}\int_o^T F(t)\cos\frac{2i\pi t}{T}dt;$$

and similarly we find

$$B=\frac{2}{T}\int_o^T F(t)\sin\frac{2i\pi t}{T}dt:$$

equations by which the coefficients in the double series of sines and cosines are expressed in terms of the values of the function supposed known from $t=o$ to $t=T$. The amplitudes and epochs of the single harmonic terms of the chief period and its submultiples are calculated from them, according to the following formula

$$\tan\epsilon_i=\frac{B}{A_i};\quad M_i=(A_i^2+B_i^2)^{\frac{1}{2}}$$

(or for logarithmic calculation,

$$M_i=A_i\sec\epsilon_i).$$

The preceding investigation is sufficient as a solution of the problem, to find a complex harmonic function expressing a given arbitrary periodic function, when once we are assured that the problem is possible; and when we have this assurance, it proves that the resolution is *determinate*, that is to say, that no other complex harmonic function than the one we have found can satisfy the conditions. For a thorough and most interesting analysis of the subject, supplying all that is wanting to complete the investigation, and giving admirable views of the problem from all sides, the reader is referred to Fourier's delightful treatise. A concise and perfect synthetical investigation of the harmonic expression of an arbitrary periodic function is to be found in Poisson's *Théorie Mathématique de la Chaleur*, chap. vii.

II. *Periodic Variations of Terrestrial Temperature.*

7. If the whole surface of the earth were at each instant of uniform temperature, and if this temperature were made to vary as a perfectly periodic function of the time, the temperature at any internal point must ultimately come to vary also as a periodic function of the time, with the same period, whatever may have been the initial distribution of temperature throughout the whole. Fourier's principles show how the periodic variation of internal temperature is to be conceived as following, with diminished amplitude and retarded phase, from the varying temperature at the surface supposed given: and by his formulæ the precise law according to which the amplitude would diminish and the phase would be retarded, for points more and more remote from the surface, if the figure were truly spherical and the substance homogeneous, is determined.

8. The largest application of this theory to the earth as a whole is to the analysis of imaginable secular changes of temperature, with at least thousands of millions of years for a period. In such an application, it would be necessary to take into account the spherical figure of the earth as a whole. Periodic variations at the surface with any period less than a million* of years will,

* A periodic variation of external temperature of one million years' period would give variations of temperature within the earth sensible to one thousand times greater depths than a similar variation of one year's period. Now the ordinary annual variation is reduced to $\frac{1}{20}$th of its superficial

at points below the surface, give rise to variations of temperature not appreciably influenced by the general curvature, and sensibly agreeing with what would be produced if the surface were an infinite plane, except insofar as they are modified by superficial irregularities. Hence Fourier's formulæ for an infinite solid, bounded on one side by an infinite plane, of which the temperature is made to vary arbitrarily, contain the proper analysis for diurnal or annual variations of terrestrial temperature, unless a theory of the effect of inequalities of surface (upon which no investigator has yet ventured) is aimed at.

9. The effect of diurnal variations of temperature becomes insensible at so small a distance below the surface, that in most localities irregularities of soil and drainage must prevent any very satisfactory theoretical treatment of their inward progression and extinction from being carried out. At depths exceeding three feet below the surface, all periodic effects of daily variations of temperature become insensible in most soils, and the observable changes are those due to a daily average, varying from day to day. If now the annual variation of temperature were truly periodic, a complex harmonic function could be determined to represent for all time the temperature at three feet or any greater depth. But in reality the annual variation is very far from recurring in a perfectly periodic manner, since there are both great differences in the annual average temperatures, and never-ceasing irregularities in the progress of the variation within each year. A full theory of the consequent variations of temperature propagated downwards, must include the consideration of non-periodic changes; but the most convenient first step is that which I propose to take in the present communication, in which the average annual variations for groups of years will be discussed according to the laws to which periodic variations are subject.

10. The method which Fourier has given for treating this and other similar problems is founded on the principle of the independent superposition of thermal conductions. This principle holds rigorously in nature, except insofar as the conductivity or

amount at a depth of 25 French feet, and is scarcely sensible at a depth of 50 French feet (being there reduced, in such rock as that of Calton Hill, to $\frac{1}{400}$). Hence, at a depth of 50,000 French feet, or about ten English miles, a variation having one million years for its period would be reduced to $\frac{1}{400}$. If the period were ten thousand million years, the variation would similarly be reduced to $\frac{1}{400}$ at 1000 miles' depth, and would be to some appreciable extent affected by the spherical figure of the whole earth, although to only a very small extent, since there would be comparatively but very little change of temperature (less than $\frac{1}{20}$ of the superficial amount) beyond the first layer of 500 miles' thickness.

the specific heat of the conducting substance may vary with the changes of temperature to which it is subjected; and it may be accepted with very great confidence in the case with which we are now concerned, as it is not at all probable that either the conductivity or the specific heat of the rock or soil can vary at all sensibly under the influence of the greatest changes of temperature experienced in their natural circumstances; and, indeed, the only cause we can conceive as giving rise to sensible change in these physical qualities is the unequal percolation of water, which we may safely assume to be confined in ordinary localities to depths of less than three feet below the surface. The particular mode of treatment which I propose to apply to the present subject consists in expressing the temperature at any depth as a complex harmonic function of the time, and considering each term of this function separately, according to Fourier's formulæ for the case of a simple harmonic variation of temperature, propagated inwards from the surface. The laws expressed by these formulæ may be stated in general terms as follows.

11. *Fourier's Solution stated**.—If the temperature at any point of an infinite plane, in a solid extending infinitely in all directions, be subjected to a simple harmonic variation, the temperature throughout the solid on each side of this plane will follow everywhere according to the simple harmonic law, with epochs retarded equally, and with amplitudes diminished in a constant proportion for equal augmentations of distance. The retardation of epoch expressed in circular measure (arc divided by radius) is equal to the diminution of the Napierian logarithm of the amplitude; and the amount of each per unit of distance is equal to $\sqrt{\frac{\pi c}{\mathrm{T}k}}$, if c denote the capacity for heat of a unit bulk of the substance, and k its conductivity †.

12. Hence, if the complex harmonic functions expressing the varying temperature at two different depths be determined, and each term of the first be compared with the corresponding term of the second, the value of $\sqrt{\frac{\pi c}{\mathrm{T}k}}$ may be determined either by dividing the difference of the Napierian logarithms of the amplitudes, or the difference of the epochs by the distance between the points. The comparison of each term in the one series with the

* For the mathematical demonstration of this solution, see Note appended to Professor Everett's paper, which follows the present article in the Transactions.

† That is to say, the quantity of heat conducted per unit of time across a unit area of a plate of unit thickness, with its two surfaces permanently maintained at temperatures differing by unity.

corresponding term in the other series gives us, therefore, two determinations of the value of $\sqrt{\frac{\pi c}{k}}$, which should agree perfectly, if (1) the data were perfectly accurate, if (2) the isothermal surfaces throughout were parallel planes, and if (3) the specific heat and conductivity of the soil were everywhere and always constant.

As these conditions are not strictly fulfilled in any natural application, the first thing to be done in working out the theory is to test how far the different determinations agree, and to judge accordingly of the applicability of the theory in the circumstances. If the test thus afforded prove satisfactory, the value of the conductivity in absolute measure may be deduced from the result with the aid of a separate experimental determination of the specific heat.

13. The method thus described differs from that followed by Professor Forbes, in substituting the separate consideration of separate terms of the complex harmonic function for the examination of the whole variation unanalysed, which he conducted according to the plan laid down by Poisson.

This plan consists in using the formulæ for a simple harmonic variation, as approximately applicable to the actual variation. At great depths the amplitudes of the second and higher terms of the complex harmonic function become so much reduced as not sensibly to influence the variation, which is consequently there expressed with sufficient accuracy by a single harmonic term of yearly period; but at even the greatest depths for which continuous observations have actually been made, the second (or semi-annual) term has a very sensible influence, and the third and fourth terms are by no means without effect on the variations at three feet and six feet from the surface. A close agreement with theory is therefore not to be expected, until the method of analysis which I now propose is applied. It may be added that in the theoretical reductions hitherto made, either by Professor Forbes or others, the amplitudes of the variations for the different depths have alone been compared, and the very interesting conclusion of theory, as to the relation between the absolute amount of retardation of phase and the diminution of amplitude for any increase of depth, has remained untested.

14. In Professor Forbes's paper *, the very difficult operations which he had performed for effecting the construction and the sinking of the thermometers, and the determination of the corrections to be applied to obtain the true temperatures of the earth at the different depths from the readings of the scales graduated on their stems protruding above the surface, are fully described. The results of five years' observations—1837 to 1842—are given, along with most interesting graphical representations and illustrations. A process of graphic interpolation, for estimating the temperatures at times intermediate between those of the observations, is applied for the purpose of obtaining data from which the complex harmonic functions expressing the temperatures actually observed for the different depths are determined. I am thus indebted to Professor Forbes for the mode of procedure (described below) which I have myself followed in expressing the variations of temperature during the succeeding thirteen years for the Calton Hill station (where alone the observations were continued). The only variation from his process which I have made is, that, instead of taking twelve points of division for the yearly period, I have taken thirty-two, with a view to obtaining a more perfect representation of all the features of the observed variations, and a more exact average for the principal terms, especially the annual and the semi-annual terms of the complex harmonic function expressing them.

* "Account of some Experiments on the Temperature of the Earth at different Depths and in different Soils near Edinburgh," Transactions of the Royal Society of Edinburgh, vol. xvi. part 2. Edinburgh, 1846.

15. *Application of the General Theory to Five Years' Observations* (1837 to 1842) *at Professor Forbes's three Thermometric Stations.*—The first application which I made of the analytical theory explained above, was to the harmonic terms which Professor Forbes had found for expressing the average annual progressions of temperature during the five years' term of observations at the three stations. These terms (which I have recalculated to get their values true to a greater number of significant figures), with alterations of notation which I have found convenient for the analytical expressions, are as follows:—

Three Feet below Surface.

Observatory . . . $45{\cdot}49+7{\cdot}39\cos 2\pi(t-{\cdot}63\ )+0{\cdot}362\cos 2\pi(2t-{\cdot}669)$
Experimental Gardens. $46{\cdot}13+9{\cdot}00\cos 2\pi(t-{\cdot}616)+0{\cdot}737\cos 2\pi(2t-{\cdot}183)$
Craigleith . . . . $45{\cdot}88+8{\cdot}16\cos 2\pi(t-{\cdot}617)+0{\cdot}284\cos 2\pi(2t-{\cdot}154)$

Six Feet below Surface.

Observatory . . . $45{\cdot}86+5{\cdot}06\cos 2\pi(t-{\cdot}686)+0{\cdot}433\cos 2\pi(2t-{\cdot}731)$
Experimental Gardens. $46{\cdot}42+6{\cdot}66\cos 2\pi(t-{\cdot}665)+0{\cdot}501\cos 2\pi(2t-{\cdot}182)$
Craigleith . . . . $45{\cdot}92+6{\cdot}16\cos 2\pi(t-{\cdot}649)+0{\cdot}368\cos 2\pi(2t-{\cdot}305)$

Twelve Feet below Surface.

Observatory . . . $46{\cdot}36+2{\cdot}44\cos 2\pi(t-{\cdot}799)+0{\cdot}075\cos 2\pi(2t-{\cdot}833)$
Experimental Gardens $46{\cdot}76+3{\cdot}38\cos 2\pi(t-{\cdot}782)+0{\cdot}230\cos 2\pi(2t-{\cdot}390)$
Craigleith . . . . $45{\cdot}92+4{\cdot}22\cos 2\pi(t-{\cdot}713)+0{\cdot}067\cos 2\pi(2t-{\cdot}819)$

Twenty-four Feet below Surface.

Observatory . . . $46·87+0·655\cos 2\pi(t-1·013)$
Experimental Gardens $47·09+0·920\cos 2\pi(t-\ ·986)$
Craigleith . . . . $46·07+1·940\cos 2\pi(t-\ ·849)$

The semi-annual terms in these equations present so great irregularities (those for the Calton Hill station, for instance, showing a greater amplitude at 6 feet depth than at 3 feet), that no satisfactory result can be obtained by including them in the theoretical discussion on which we are now about to enter. We shall see later, however, that when an average for the whole period of eighteen years for the Calton Hill station is taken, the semi-annual terms are, for the 3 feet and 6 feet depths, in fair agreement with theory; and for the two greater depths are as small as is necessary for the verification of the theory, and so small as not to be much influenced by errors of observation and of reduction, or of "corrections" for temperature of the thermometer tubes. For the present, we attend exclusively to the annual terms. The amplitudes and epochs of these terms, extracted from the preceding equations, are shown in the following Table:—

Table I. Annual Harmonic Variations of Temperature.

Depths below surface in French feet.	Calton Hill.			Experimental Garden.			Craigleith Quarry.		
	Amplitudes in degrees Fahr.	Epochs of maximum.		Amplitudes in degrees Fahr.	Epochs of maximum.		Amplitudes in degrees Fahr.	Epochs of maximum.	
		In degrees and minutes.	In months and days.		In degrees and minutes.	In months and days.		In degrees and minutes.	In months and days.
Feet.									
3	7·386	226° 52′	Aug. 19	9·063	221° 46′	Aug. 13	8·069	220° 0′	Aug. 14
6	5·063	247 5	Sept. 8	6·661	239 20	31	6·148	233 43	26
12	2·455	287 30	Oct. 19	3·408	281 27	Oct. 13	4·216	256 42	Sept. 17
24	0·655	365 6	Jan. 6	0·920	355 0	Dec. 27	1·836	305 46	Nov. 7

By taking the differences of the Napierian logarithms of the amplitudes, and the differences of epochs reduced to circular measure (arc divided by radius), thus shown for the different depths, and dividing each by the corresponding difference of depths, we find the following numbers:—

Table II. Rates of Logarithmic Diminution in Amplitude, and of Retardation in Epoch, of Annual Harmonic Variations Downwards.

[illegible]	Calton Hill.		Experimental Garden.		Craigleith Quarry.	
	[illegible]	[illegible]	Rate of diminution of Napierian logarithm of amplitude per foot of descent.	Rate of retardation of epoch in circular measure per foot of descent.	Rate of diminution of Napierian logarithm of amplitude per foot of descent.	Rate of retardation of epoch in circular measure per foot of descent.
[illegible]	[illegible]	·1170	·1004	·1163	·09372	·06599
[illegible]	[illegible]	·1170	·1130	·1193	·06304	·06690
[illegible]	[illegible]	·1190	·1084	·1062	·06476	·06690
[illegible]	[illegible]	·1140	·1082	·1114	·06841	·06648

All the numbers here shown for each station would be equal, if the conditions of uniformity supposed in the theoretical solution were fulfilled. The discrepancies are, with the exception of one of the numbers for Craigleith Quarry, on the whole small, smaller, indeed, than might be expected when the very great deviations of the true circumstances from the theoretical conditions are considered. The mean results over the 21 feet, shown in the last line, present very remarkable agreements,—the numbers derived from amplitudes being identical with that derived from epochs for the Calton Hill station, while the differences between the corresponding numbers for the two other stations are in each case only about three per cent. Taking that one number for the first station, and the mean of the slightly differing numbers derived from amplitudes and from epochs respectively for the second and third, we have undoubtedly very accurate determinations of the value of $\sqrt{\frac{\pi c}{k}}$ for the three stations, which are as follows:—

Calton Hill trap rock.	Experimental Garden sand.	Craigleith Quarry sandstone.
$\sqrt{\frac{\pi c}{k}}=·1154$	$\sqrt{\frac{\pi c}{k}}=·1098$	$\sqrt{\frac{\pi c}{k}}=·06744$

A continuation of the observations at Calton Hill not only leads, as we shall see, to almost identical results, both by diminution of amplitude and by retardation, on the whole 21 feet, but also reproduces some of the features of discrepance presented by the progress of the variation through the intermediate depths, and therefore confirms the general accuracy of the preceding results, for all the stations, so far as it might be questioned because of only five years' observations having been available. Further consideration of these results, and deduction of the conductivities of the different portions of the earth's crust involved, are deferred until after we have taken into account the further data for Calton Hill, to the reduction of which we now proceed.

[To be continued.]

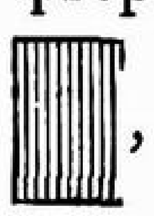
V. *Meteorological Charts.* By Francis Galton, *Esq.**

[With a Plate.]

WHEN contemporary meteorological reports from numerous stations are printed one after another in a column (such as we may see in newspapers and certain foreign publications), they present no picture to the reader's mind. Lists of this description are therefore insufficient to do more than supply data which meteorological students must protract as they best can, upon a map, in some notation intelligible to themselves, at a considerable expense of labour and artistic skill.

It is needless to enlarge upon the serious obstacle which the necessity of doing this opposes to the pursuit of meteorology. It has sufficed to convert what might be a very popular science into a laborious and difficult study. We require means of printing, not lists of dry figures, but actual charts which should record meteorological observations pictorially and geographically, without sacrificing detail. It is then in the belief that an attempt I have just made to supply this desideratum might interest some of your readers, and perhaps lead to useful suggestions, that I forward the accompanying chart. (Plate II). It has been printed with moveable types, which I designed and caused to be cast; and I am much indebted to Mr. W. Spottiswoode, who printed it, for his aid in carrying out my ideas. The map simply incorporates the newspaper data of the day to which it refers, and was printed, not with any scientific object, but solely for the purpose of experiment.

Explanation of the Symbols.

The shade signifies cloud, of an amount proportional to its depth. The types with lines round them, , stand for rain. Cloud types have been interpolated where observations were

* Communicated by the Author.

wanting. The horseshoes show the direction of the wind current: thus,  means wind *from* the west. An included spot , or line , or cross , respectively signify that the wind is gentle, moderate, or strong; where neither dot, line, nor cross are inserted, the force of the wind is unknown. Thermometrical data are expressed by figures, printed below the wind symbols. The first two figures of each set stand for the height of the ordinary thermometer, and the last figure (in a different type) for the difference between this and the thermometer with a wetted bulb. To save confusion of figures, barometer heights are not inserted on the face of the present map; but lines of equal barometric pressure have been deduced from the existing observations, and the places where lines corresponding to each integral one-tenth of an inch cut the marginal columns, have been marked. Thus a straight line joining the pair of figures, 29·7, is approximately the line of that pressure.

I do not consider the types here employed as forming a complete series. An additional shade for cloud is especially wanted.

It will be observed that no space would be lost by this mode of representation, supposing we possessed observations corresponding to every type space of the map.

42 Rutland Gate, S.W.

VI. *On the Curves situate on a Surface of the Second Order.* By A. Cayley, *Esq.**

A SURFACE of the second order has on it a double system of generating lines, real or imaginary; and any two generating lines of the first kind form with any two generating lines of the second kind a skew quadrangle. If the equations of the planes containing respectively the first and second, second and third, third and fourth, fourth and first sides of the quadrangle are $x=0$, $y=0$, $z=0$, $w=0$, and if the constant multipliers which are implicitly contained in x, y, z, w respectively are suitably determined, then the equation of the surface of the second order (or say for shortness the quadric surface) is $xw-yz=0$.

Assume $\frac{y}{x}=\frac{\mu}{\lambda}$, $\frac{z}{x}=\frac{\nu}{\rho}$, then $\frac{\mu}{\lambda}$, $\frac{\nu}{\rho}$, or say $(\lambda, \mu, \nu, \rho)$, may be regarded as the coordinates of a point on the quadric surface; we in fact have $x:y:z:w=1:\frac{\mu}{\lambda}:\frac{\nu}{\rho}:\frac{\mu\nu}{\lambda\rho}$, or what is the same

* Communicated by the Author.