ether were shaken up together and poured out into a small porcelain dish; the surface was dusted with lycopodium, and the ether-sponge presented to it: there was no action; the powder was not displaced or disturbed. The solution of ether was then boiled and filtered, and, when cold, the surface was again dusted with the powder. The ether-sponge now produced a repulsion of the powder, not so decidedly as with plain water, but still a good repulsion.

The solution of ether was also made to carry an oil-film. A drop of varnish formed an exquisite series of coloured rings, and the ether-sponge also displayed some very beautiful rings; but after a minute or two, when the adhesion between the solution and the film was complete, the ether-sponge was powerless.

It may also be mentioned that a vapour acts differently on the film according as it has a greater attraction for the water or for the film. If it has a strong attraction for the water, it will thin out and disperse the film. If its attraction is strong for the film, it will gather it up, thicken it, and deprive it of colour. Thus with a film of oil of lavender the ether scatters and disperses, while the benzole sponge thickens and attracts; in fact the benzole vapour condenses into little discs, which unite with the film. So also if a drop of oil of peppermint be placed on water, it spreads out into a honeycombed film displaying colour. If the ether-sponge be presented, the vapour pours down in a cataract and powerfully displaces the film (a very common effect of ether-vapour on films of the essential oils); whereas, if the turpentine sponge be held over it, the scattered parts of the film sail up to it, gather themselves together, and form a number of thickening lenticules.

I do not like to intrude further on your patience at present. Should this letter not disappoint the interest you have kindly expressed in this inquiry, I will trouble you with a second, and in the mean time subscribe myself,

Your attached friend,

CHARLES TOMLINSON.

King's College, London,
June 22, 1861.

XVIII. *On the Reduction of Observations of Underground Temperature; with Application to* Professor Forbes's *Edinburgh Observations, and the continued Calton Hill Series.* *By* Professor WILLIAM THOMSON, *F.R.S.*

[Concluded from p. 34.]

17. *APPLICATION to Thirteen Years' Observations* (1842–54) *at the Thermometric Station, Calton Hill.*—The observations on thermometers fixed by Professor Forbes at the different depths in the rock of Calton Hill, have been regularly continued weekly till the present time by the staff of the Royal Edinburgh Observatory, and regularly corrected to reduce to true temperatures of the bulbs, on the same system as before. Tables of these corrected observations, for the twelve years 1842 to 1854 inclusive, having been supplied to me through the kindness of Professor Piazzi Smyth, I have had the first five terms of the harmonic expression for each year determined in the following manner*:— In the first place, the observations were laid down graphically, and an interpolating curve drawn through the points, according to the method of Professor Forbes. The four curves thus obtained represent the history of the varying temperature, at the four different depths respectively, as completely and accurately as it can be inferred from the weekly observations. The space corresponding to each year was then divided into thirty-two equal parts (the first point of division being taken at the begining of the year), and the corresponding temperatures were taken from the curve. The coefficients of the double harmonic series (cosines and sines) for each year were calculated from these data, with the aid of the forms given by Mr. Archibald Smith, and published by the Board of Admiralty, for deducing the harmonic expression of the error of a ship's compass from observations on the thirty-two points. The general form of the harmonic expression being written thus—

$$V = A_0 + A_1 \cos 2\pi t + B_1 \sin 2\pi t + A_2 \cos 4\pi t + B_2 \sin 4\pi t + \&c.,$$

where V denotes the varying temperature to be expressed, and t the time, in terms of a year as unit. The following Table shows the results which were obtained, with the exception of the values of A_0:—

* The operations here described, involving, as may be conceived, no small amount of labour, were performed by Mr. D. M'Farlane, my laboratory assistant, and Mr. J. D. Everett, now Professor of Mathematics and Natural Philosophy in King's College, Windsor, N.S.

TABLE III.

Year.	Feet.	A_1.	B_1.	A_2.	B_2.	A_3.	B_3.	A_4.	B_4.
1842.	3	−6·10	5·00	+·01	+·25	+·60	+·06	+·23	−·71
	6	−2·85	−4·80	−·15	+·03	+·10	+·10	+·12	−·26
	12	+·34	2·73	−·12	−·13	−·08	−·04	+·01	−·04
	24	+·08	−·14	·00	−·07	−·02	−·04	−·01	−·02
1843.	3	−4·75	−5·11	+·17	+·91	+1·23	+·30	+·79	−·17
	6	−1·63	−4·38	−·20	+·61	+·45	+·42	+·32	+·30
	12	+·83	−2·04	−·18	−·08	−·05	+·17	−·03	+·10
	24	+·62	+·12	·00	−·02	−·01	−·01	·00	·00
1844.	3	−5·29	−4·53	−·05	+·70	+·74	+·71	+·08	+·49
	6	−2·11	−4·09	+·22	+·50	+·20	+·50	−·06	+·20
	12	+·52	−2·15	+·18	+·05	+·11	+·13	−·05	−·01
	24	+·59	−·02	−·03	−·02	·00	−·03	−·01	−·02
1845.	3	−5·17	−5·01	−·17	+·56	+·67	+·29	−·28	+·02
	6	−2·02	−4·38	+·07	+·30	·00	+·18	−·04	−·08
	12	+·63	−2·15	+·12	+·06	−·01	−·03	·00	+·02
	24	+·65	+·13	+·01	·00	+·01	+·02	+·01	+·02
1846.	3	−5·65	5·17	+·03	+1·05	−·86	+·64	+·00	−·49
	6	−2·37	−4·04	·38	+·44	−·63	−·39	−·11	−·22
	12	+·47	2·70	·30	·17	−·14	−·45	·00	−·07
	24	+·64	−·22	−·02	−·17	+·03	−·11	−·03	−·06
1847.	3	−5·36	−5·31	+·69	+·24	−·18	−·81	−·02	−·14
	6	−2·08	−4·58	+·18	+·32	+·11	−·39	−·05	−·04
	12	+·70	−2·37	−·03	+·17	+·12	+·14	+·03	+·02
	24	+·66	+·16	−·01	+·04	+·01	+·03	+·01	+·03
1848.	3	−5·83	−4·46	+·33	+·27	+·29	+·35	+·45	−·30
	6	−2·32	−4·16	+·13	+·27	+·02	+·23	+·28	+·09
	12	+·56	−2·15	+·04	+·16	−·01	+·09	+·04	+·11
	24	+·66	+·10	−·01	+·03	·00	+·02	−·01	+·01
1849.	3	−4·56	−4·41	+·05	+1·14	−·66	−·10	−·48	−·69
	6	−1·85	−3·97	−·20	+·45	−·28	−·15	+·01	−·25
	12	+·49	2·06	·23	+·04	+·04	−·06	+·09	−·05
	24	+·57	+·03	·00	·02	+·01	+·02	·00	+·01
1850.	3	−5·40	−4·50	−·12	+·70	−·54	−·82	−·15	−·42
	6	−2·13	−4·15	−·22	+·31	+·03	−·47	+·11	−·17
	12	+·17	−2·27	−·15	−·04	·10	−·05	+·04	+·01
	24	+·61	−·01	+·01	−·03	+·01	·00	−·01	−·01
1851.	3	−4·18	−4·53	+·12	+·96	−·09	+·31	+·22	+·18
	6	−1·65	−3·92	−·19	+·53	−·18	+·07	−·03	+·14
	12	+·61	−1·99	−·22	+·01	−·04	−·06	−·05	−·02
	24	+·56	+·02	+·01	−·05	·00	−·01	−·14	−·01
1852.	3	−4·92	−4·80	+·20	+1·32	+·64	−·24	−·46	+·31
	6	−1·87	−4·25	−·23	+·71	+·15	+·10	−·31	−·02
	12	+·54	−2·24	−·26	+·05	+·01	+·09	−·01	−·07
	24	+·61	−·03	−·12	−·07	−·01	−·04	·00	−·02
1853.	3	−5·08	−5·43	+·83	+·30	+·11	+·27	+·18	+·19
	6	−1·92	−4·57	+·38	+·41	−·05	+·17	+·06	+·13
	12	+·76	−3·15	−·01	+·21	−·01	·00	−·01	+·03
	24	+·62	+·18	−·39	+·03	·00	+·10	+·01	+·03
1854.	3	5·69	−4·56	−·61	+·53	·00	−·15	+·15	−·20
	6	−2·48	−4·27	−·50	−·01	·00	−·13	+·08	−·03
	12	+·42	−2·31	−·12	−·21	+·02	−·03	+·02	+·01
	24	+·03	−·03	+·02	−·02	·00	−·01	−·01	−·01
Average for 13 years, 1842 to 1854.	3	−5·236	−4·835	+·114	+·687	+·150	+·0778	+·05462	−·14846
	6	−2·122	−4·320	−·0838	+·375	−·00615	+·0185	+·02923	−·01615
	12	+·5415	−2·332	−·0985	+·00923	−·01846	−·00778	+·006154	+·003078
	24	+·6231	−·0200	−·0385	−·0285	−·00231	−·00462	−·01462	−·003846

The values which were found for A_0 should represent the annual mean temperatures. They differ slightly from the annual means shown in the Royal Observatory Report, which, derived as they are from a direct summation of all the weekly observations, must be more accurate. The variations, and the final average values of these annual means, present topics for investigation of the highest interest and importance, as I have remarked elsewhere (see British Association's Report, Section A, Glasgow, 1855); but as they do not belong to the special subject of the present paper, their consideration must be deferred to a future occasion.

18. *Theoretical Discussion.*—The mean value of the coefficients in the last line of the Table being obtained from so considerable a number of years, can be but very little influenced by irregularities from year to year, and must therefore correspond to harmonic functions for the different depths, which would express truly periodic variations of internal temperature consequent upon a continued periodical variation of temperature at the surface.

19. According to the principle of the superposition of thermal conductions, the difference between this continuous harmonic function of five terms for any one of the depths, and the actual temperature there at the corresponding time of each year, would be the real temperature consequent upon a certain real variation of superficial temperature. Hence the coefficients shown in the preceding Table afford the data, first by their mean values, to test the theory explained above for simple harmonic variations, and to estimate the conductivity of the soil or rock, as I propose now to do; and secondly, as I may attempt on a future occasion, to express analytically the residual variations which depend on the inequalities of climate from year to year, and to apply the mathematical theory of conduction to the non-periodic variations of internal temperature so expressed.

20. Let us accordingly now consider the complex harmonic functions corresponding to the mean coefficients of the preceding Table; and in the first place, let us reduce the double harmonic series in each case to series in each of which a single term represents the resultant simple harmonic variation of the period to which it corresponds, in the manner shown by the proposition and formulæ of § 3 above.

21. On looking to the annual and semiannual terms of the series so found, we see that their amplitudes diminish, and their epochs of maximum augment, with considerable regularity from the less to the greater depths. The following Table shows, for the annual terms, the logarithmic rate of diminution of the amplitudes, and the rate of retardation of the epoch between the points of observation in order of depth:—

Table IV.—Average of Thirteen Years, 1842 to 1854; Trap Rock of Calton Hill.

Depths below surface, in French feet.	Diminution of Napierian logarithm of amplitude, per French foot of descent.	Retardation of epoch in circular measure, per French foot of descent.
3 to 6 feet	·1310	·1233
6 to 12 „	·1163	·1140
12 to 24 „	·1121	·1145
3 to 24 „	·1160	·1156

22. The numbers here shown would all be the same if the conditions of uniformity supposed in the theoretical solution were fulfilled. Although, as in the previous comparisons, the agreement is on the whole better than might have been expected, there are certainly greater differences than can be attributed to errors of observation. Thus the means of the numbers in the two columns are for the three different intervals of depth in order as follows:—

	Mean deductions from amplitude and epoch.
3 to 6 feet	·127
6 to 12 „	·115
12 to 24 „	·113

numbers which seem to indicate an essential tendency to diminish at the greater depths. This tendency is shown very decidedly in each column separately; and it is also shown in each of the corresponding columns, in tables given above, of results derived from Professor Forbes's own series of a period of five years.

23. There can be no doubt that this discrepance is not attributable to errors of observation, and it must therefore be owing to deviation in the natural circumstances from those assumed for the foundation of the mathematical formula. In reality, none of the conditions assumed in Fourier's solution is rigorously fulfilled in the natural problem; and it becomes a most interesting subject for investigation to discover to what particular violation or violations of these conditions the remarkable and systematic difference discovered between the deductions from the formula and the results of observation is due. In the first place, the formula is strictly applicable only to periodic variations, and the natural variations of temperature are very far from being precisely periodic; but if we take the average annual variation through a sufficiently great number of years, it may be fairly presumed that irregularities from year to year will be eliminated: and that the discrepance we have now to explain does not depend on residual inequalities of this kind seems certain, from the fact that it exists in the average of Professor Forbes's first five years' series no less decidedly than in that of the period of thirteen years following.

24. For the true explanation we must therefore look either to inequalities (formal or physical) in the surface at the locality, or to inequalities of physical character of the rock below. It may be remarked, in the first place, that if the rates of diminution of logarithmic amplitude and of retardation of epoch, while less, as they both are, at the greater depths, remained exactly equal to one another, the conductivity must obviously be greater, and the specific heat less in the same proportion inversely, at the greater depths. For in that case, all that would be necessary to reconcile the results of observation with Fourier's formula, would be to alter the scale of measurement of depths so as to give a nominally constant rate of diminution of the logarithmic amplitude and of the retardation of epoch; and the physical explanation would be, that thicker strata at the greater depths, and thinner strata at the less depths (all of equal horizontal area), have all equal conducting powers and equal thermal capacities*.

25. Now in reality, a portion, but only a portion, of the discrepance may be done away with in this manner; for while the logarithmic amplitudes and the epochs each experience a somewhat diminished rate of variation per French foot of descent at the greater depths, this diminution is much greater for the former than for the latter; so that, although the mean rates per foot on the whole 21 feet are as nearly as possible equal for the two (being ·1160 for the logarithmic amplitudes, and ·1156 for the epoch), the rate of variation of the logarithmic amplitude exceeds that of the epoch by about 6 per cent. on the average of the stratum of 3 to 6 feet; and falls short of it by somewhat more than 2 per cent. in the lower stratum, 12 to 24 feet. To find how much of the discrepance is to be explained by the variation of conductivity and specific heat in inverse proportion to one another at the different depths, we may take the mean of the

* The "conducting power" of a solid plate is an expression of great convenience, which I define as the quantity of heat which it conducts per unit of time when its two surfaces are permanently maintained at temperatures differing by unity. In terms of this definition, the specific conductivity of a substance may be defined as the conducting power per unit area of a plate of unit thickness. The conducting power of a plate is calculated by multiplying the number which measures the specific conductivity of its substance by its area, and dividing by its thickness.

The *thermal capacity of a body* may be defined as the quantity of heat required to raise its mass a unit (or one degree) of temperature. The specific heat of a substance is the thermal capacity of a unit quantity of it, which may be either a unit of weight or a unit of bulk.

rates of variation of logarithmic amplitude and of epoch at each depth, and alter the scale of longitudinal reckoning downwards, so as to reduce the numerical measures of these rates to equality. This, however, we shall not do in either the five years' or the thirteen years' term, which we have hitherto considered separately, but for a harmonic annual variation representing the average of the whole eighteen years 1837 to 1854.

26. By taking for each depth the coefficients A_1, B_1 (not explicitly shown above), derived from the first five years' average, and multiplying by 5; taking similarly the coefficients A_1, B_1 for the succeeding thirteen years' average, and multiplying by 18; adding each of the former products to the corresponding one of the latter, and dividing by 18; we obtain, as the proper average for the whole eighteen years, the values shown in the following Table, in the columns headed A_1, B_1. The amplitudes and epochs shown in the next columns are deduced from these by the formulæ $\sqrt{(A_1^2+B_1^2)}$ and $\tan^{-1}\frac{B_1}{A_1}$ respectively:—

Table V.—Annual Harmonic Variation of Temperature in Calton Hill, from 1837 to 1844 inclusive.

Depths.	A_1 in degrees Fahr.	B_1 in degrees Fahr.	Amplitudes in degrees Fahr.	Epochs in degrees and minutes.
3 feet	$-5^\circ{\cdot}184$	$-4^\circ{\cdot}989$	$7^\circ{\cdot}1949$	$223^\circ\ 54'$
6 "	$-2{\cdot}080$	$-4{\cdot}416$	$4{\cdot}8812$	244 47
12 "	$+\ {\cdot}5961$	$-2{\cdot}3345$	$2{\cdot}4094$	284 19
24 "	$+\ {\cdot}0311$	$+\ {\cdot}0306$	${\cdot}6319$	362 47

From these, as before, for ten terms of five years and of thirteen years separately, we deduce the following:—

Table VI.—Average of Eighteen Years, 1837 to 1844; Trap Rock of Calton Hill.

Depths below surface in French feet.	Diminution of logarithmic amplitude, per French foot of descent.	Retardation of epoch in circular measure, per French foot of descent.
3 to 6 feet	·1286	·1215
6 to 12 "	·1177	·1150
12 to 24 "	·1115	·1141
3 to 24 "	·1157	·1154

27. Hence we have as final means, of effects on logarithmic amplitudes and on epochs, for the average annual variation on the whole period of eighteen years,—

1. From depth	3	feet to	6	feet . . .		·1250
2. "	6	"	12	" . . .		·1163
3. "	12	"	24	" . . .		·1128

If now, in accordance with the proposed plan, we measure depths, not in constant units of length, but in terms of thicknesses corresponding to equal conducting powers and thermal capacities, and if we continue to designate the thickness of the first stratum by its number 3 of French feet, our reckoning for the positions of the different thermometers will stand as follows:—

Table VII.

Thermometers numbered downwards.	Depths in true French feet below No. 1.	Depths in terms of conductive equivalents.
I.	0	0
II.	3	3
III.	9	$3+\frac{\cdot1163}{\cdot1250}\times6=8{\cdot}58$
IV.	21	$8{\cdot}58+\frac{\cdot1128}{\cdot1250}\times12=19{\cdot}41$

According to this way of reckoning depths, we have the following rates of variation of the logarithmic amplitudes, and of the epochs separately, reduced from the previously stated means for the whole period of eighteen years:—

Table VIII.

Portions of rock.	Rates of diminution of logarithmic amplitude per French foot, and conductive equivalents.	Rate of retardation of epoch per French foot, and conductive equivalents.
Between thermometers Nos. I. and II.	·1286	·1215
" " II. and III.	·1265	·1236
" " III. and IV.	·1236	·1264
Between thermometers Nos. I. and IV.	·1252	·1248

28. Comparing this Table with the preceding Table VI., we see, that the discrepancies are very much diminished; and we cannot doubt that the conductive power of the rock is less in the lower parts of the rock, and that the amount of the variation is approximately represented by Table VII. We have, however, in Table VIII. still too great discrepancies to allow us to consider variation in the value of kc as the only appreciable deviation from Fourier's conditions of uniformity.

29. In endeavouring to find whether these residual discre-

pancies are owing to variations of k and c not in inverse proportion one to the other, I have taken Fourier's equation

$$c\frac{dv}{dt}=k\frac{d^2v}{dx^2}+\frac{dk}{dx}\frac{dv}{dx},$$

where v denotes the temperature at time t, and at a distance x from an isothermal plane of reference (a horizontal plane through thermometer No. 1., for instance); k the conductivity, varying with x; and c the capacity for heat of a unit of volume, which may also vary with x. In this equation I have taken

$$v=a\epsilon^{-\mathrm{P}}\cos\left(\frac{2\pi t}{\mathrm{T}}-\mathrm{Q}\right),$$

where P and Q are functions of x, assumed so as to express, as nearly as may be, the logarithmic amplitudes, and the epochs, deduced from observation. I have thus obtained two equations of condition, from which I have determined k and c, as functions of x. The problem of finding what must be the conductivity and the specific heat at different depths below the surface, in order that, with all the other conditions of uniformity perfectly fulfilled, the annual harmonic variation may be exactly that which we have found on the average of the eighteen years' term at Calton Hill, is thus solved. The result is, however, far from satisfactory. The small variations in the values of P and Q which we have found in the representation of the observed temperatures require very large and seemingly unnatural variations in the values of k and c.

30. I can only infer that the residual discrepancies from Fourier's formula shown in Table VIII. are not with any probability attributable to variations of conductivity and specific heat in the rock, and conclude that they are to be explained by irregularities, physical and formal, in the surface. It is possible, indeed, that thermometric errors may have considerable influence, since there is necessarily some uncertainty in the corrections estimated for the temperatures of the different portions of the columns of liquid above the bulbs; and before putting much confidence in the discrepancies we have found as true expressions of the deviations in the natural circumstances from Fourier's conditions, a careful estimate of the probable or possible amount of error in the observed temperatures should be made. That even with perfect *data* of observation as great discrepancies should still be found in final reductions such as we have made, need not be unexpected when we consider the nature of the locality, which is described by Professor Forbes in the following terms:—

The position chosen for placing the thermometer was below the surface "in the Observatory enclosure on the Calton Hill, at

a height of 350 feet above the sea. The rock is a porphyritic trap, with a somewhat earthy basis, dull and tough fracture. *The exact position is a few yards east of the little transit house. There are also other buildings in the neighbourhood.* The ground rises slightly to the east, and *falls abruptly to the west at a distance of fifteen yards.* The immediate surface is flat, *partly covered with grass, partly with gravel**."

I have marked by italics those passages which describe circumstances such as it appears to me might account for the discrepancies in question.

31. *Application to Semiannual Harmonic Terms.*—The harmonic expressions given above (§ 15) for the average periodic variations for the three stations of Professor Forbes's original series of five years' observations, contain semiannual terms which are obviously not in accordance with theory. The retardations of epochs and the diminutions of amplitudes are, on the whole, too irregular to be reconcileable by any supposition as to the conductivities and specific heat of the soils and rocks involved, or as to the possible effects of irregularity of surface; and in two of the three stations the amplitude of the semiannual term is actually greater as found for the six-feet deep than for the three-feet deep thermometer, which is clearly an impossible result. The careful manner in which the observations have been made and corrected seems to preclude the supposition that these discrepancies, especially for the three-feet and six-feet thermometers, for which the amplitudes of the semiannual terms are from ·28° to ·74° (corresponding to variations of double those amounts, or from ·56° to 1°·48), can be attributed to errors in the *data*. It must be concluded, therefore, that the semiannual terms of those expressions do not represent any truly periodic elements of variation, and that they rather depend on irregularities of temperature in the individual years of the term of observation. Hence, until methods for investigating the conduction inwards of non-periodic variations of temperature are applied, we cannot consider that the special features of the progress of temperature during the five years' period at the three stations, from which our apparent semiannual terms have been derived, have been theoretically analysed. But, as we have seen, every irregularity depending on individual years is perfectly eliminated when the average annual variation over a sufficiently great number of years is taken. Hence it becomes interesting to examine particularly the semiannual terms for the eighteen years' average of the Calton Hill thermometers, which we now proceed to do.

* Professor Forbes "On the Temperature of the Earth," Trans. Roy. Soc. Edinb. 1846, p. 194.

32. Calculating as above (§ 26), for the coefficients A_1, B_1, the average values of A_2 and B_2, from Professor Forbes's results for his first five years' term, and from the averages for the next thirteen years shown in Table III. above, we find the values of A_2 and B_2 shown in the following Table. The amplitudes and epochs are deduced as usual by the formulæ $\sqrt{(A_2^2+B_2^2)}$ and $\tan^{-1}\frac{B_2}{A_2}$. These reductions I only make for the three-feet deep and the six-feet deep thermometers, since, for the two others, as may be judged by looking at the thirteen years' average shown in the former Table, the amounts of the semiannual variation do not exceed the probable errors in the data of observation sufficiently to allow us to draw any reliable conclusions from their apparent values.

Table IX.—Average Semiannual Harmonic Term, from Eighteen Years' Observations at Calton Hill.

Depths below surface, in French feet.	A_2 in degrees Fahr.	B_2 in degrees Fahr.	Amplitudes in degrees Fahr.	Epochs in degrees and minutes.
3 feet.	$^\circ$·1518	$^\circ$·5842	$^\circ$·604	75° 26′
6 feet.	·0461	·3911	·394	96 43

The ratio of diminution of the amplitude here is $\frac{\cdot 604}{\cdot 394}$, or 1·53, of which the Napierian logarithm is ·426. Dividing this by 3, we find

$$\cdot 142$$

as the rate of diminution of the logarithmic amplitude per French foot of descent.

The retardation of epoch shown is 21° 17′; and therefore the retardation per French foot of descent is 7° 6′, or, in circular measure,

$$\cdot 1239.$$

If the data were perfect for a periodical variation, and the conditions of uniformity supposed in Fourier's solution were fulfilled, these two numbers would agree, and each would be equal to $\sqrt{\frac{2\pi k}{c}}$. Hence, dividing them each by $\sqrt{2}$, we find

Apparent values of $\sqrt{\frac{\pi c}{k}}$.

·100 (by amplitudes).
·0877 (by epochs).

The true value of $\sqrt{\frac{\pi c}{k}}$ must, as we have seen, be ·116, to a very close degree of approximation.

33. When we consider the character of the reduction we have made, and remember that the data were such as to give no semblance of a theoretical agreement when the first five years' term of observations was taken separately, we may be well satisfied with the approach to agreement presented by these results, depending as they do on only eighteen years in all, and we may expect that, when the average is of a still larger term of observation, the discrepancies will be much diminished. In the mean time we may regard the semiannual term we have found for the three-feet deep thermometer as representing a true feature of the yearly vicissitude; and it will surely be interesting to find whether it is a constant feature for the locality of Edinburgh, to be reproduced on averages of subsequent terms of observation.

34. It may be remarked that the nearer to the equator is the locality, the greater relatively will be the semiannual term; that within the tropics the semiannual term may predominate, except at great depths; and that at the equator the tendency is for the annual term to disappear altogether, and to leave a semiannual term as the first in a harmonic expression of the yearly vicissitude of temperature. The facilities which underground observation affords for the analysis of periodic variations of temperature when the method of reduction which I have adopted is followed, will, it is to be hoped, induce those who have made similar observations in other localities to apply the same kind of analysis to their results; and it is much to be desired that the system of observing temperatures at two, if not more depths below the surface may be generally adopted at all meteorological stations, as it will be a most valuable means for investigating the harmonic composition of the annual vicissitudes.

III. *Deduction of Conductivities.*

35. Notwithstanding the difficulty we have seen must attend any attempt to investigate all the circumstances which must be understood in order to reconcile perfectly the observed results with theory, the general agreement which we have found is quite sufficient to allow us to form a very close estimate of the ratio of the conductivity of the rock to its specific heat per unit of bulk. Thus, according to the means deduced from the whole period of eighteen years' observation, the average rate of variation of the logarithmic amplitude of the annual term through the whole space of twenty-one feet is ·1157, and of the epoch of the same term, ·1154. The mean of these, or ·1156, can differ but very little

from the true average value of $\sqrt{\frac{\pi c}{k}}$ for the portion of rock between the extreme thermometers.

36. Dividing π by the square of the reciprocal of this number, we find 235·1 as the value of $\frac{k}{c}$, or, as we may call it, the conductivity of the rock in terms of the thermal capacity of a cubic foot of its own substance. In other words, we infer that all the heat conducted in a year (the unit of time) across each square foot of a plate one French foot thick, with its two sides maintained constantly at temperatures differing by 1°, would, if applied to raise the temperature of portions of the rock itself, produce a rise of 1° in 235 cubic feet. As it is difficult (although by no means impossible) to imagine circumstances in which the heat, regularly conducted through a stratum maintained, with its two sides, at perfectly constant temperatures, could be applied to *raise* the temperatures of other portions of the same substance, we may vary the statement of the preceding result, and obtain the following completely realizable illustration.

37. Let a large plate of the rock, everywhere one French foot thick, have every part of one of its sides (which, to avoid circumlocution, we shall call its lower side) maintained at one constant temperature, and let portions of homogeneous substance, at a temperature 1° lower, be continually placed in contact with the upper surface, and removed to be replaced by other homogeneous portions at the same lower temperature, as soon as the temperature of the matter actually thus applied rises in temperature by $\frac{1}{1000}$ of a degree. If this process is continued for a year, the whole quantity of the refrigerating matter thus used to carry away the heat conducted through the stratum must amount to 235,000 cubic feet for each square foot of area, which will be at the rate of ·00745 of a cubic foot per second. We may therefore imagine the process as effected by applying an extra stratum ·00745 of a foot thick every second of time. This extra stratum, after lying in contact for one second, will have risen in temperature by $\frac{1}{1000}$ of a degree. By means of the information contained in this apparently unpractical statement, many interesting problems may be practically solved, as I hope to show in a subsequent communication.

38. The value of $\sqrt{\frac{\pi c}{k}}$, derived from the whole eighteen years' period of observation (·1156), differs so little from that (·1154) found previously (§ 16) from Professor Forbes's observations and reductions of the first five of the years, that we may feel much confidence in the accuracy of the values ·1098 and ·06744, which, from his five years' data alone, we found (§ 16) for the corresponding constant with reference to the sand at the Experimental Garden and the sandstone of Craigleith Quarry. From them, calculating as above (§ 36), we find 260·5 and 690·7 as the values of $\frac{k}{c}$ for the terrestrial substances of these localities respectively,—results of which the meaning is illustrated by the statements of §§ 36 and 37.

39. To deduce the conductivities of the strata in terms of uniform thermal units, Professor Forbes had the "specific heats" of the substances determined experimentally by M. Regnault. The results, multiplied by the specific gravities, gave for the thermal capacities of portions of the three substances, in terms of that of an equal bulk of water, the values ·5283, ·3006, and ·4623 respectively. Now these must be the values of c if the thermal unit in which k is measured is the thermal capacity of a French cubic foot of water. Multiplying the values of $\frac{k}{c}$ found above by these values of c, we find for k the following values:—

Trap-rock of Calton Hill.	Sand of Experimental Gardens.	Sandstone of Craigleith.
124·2	78·31	319·3

The values found by Professor Forbes were -

111·2	82·6	298·3

Although many comparisons have been made between the conducting powers of different substances, scarcely any data as to thermal conductivity in absolute measure have been hitherto published, except these of Professor Forbes, and probably none approaching to their accuracy. The slightly different numbers to which we have been led by the preceding investigation are no doubt still more accurate.

40. To reduce these results to any other scale of linear measurement, we must clearly alter them in the inverse ratio of the square of the absolute lengths chosen for the units*. The

* Because the absolute amount of heat flowing through the plate across equal areas will be inversely as the thickness of the plate, and the effect of equal quantities of heat in raising the temperature of equal areas of the water will be inversely as the depth of the water. The same thing may be perhaps more easily seen by referring to the elementary definition of thermal conductivity (footnote to § 11 above). The absolute quantity of heat conducted across unit area of a plate of unit thickness, with its two sides maintained at temperatures differing by always the same amount, will be directly as the areas, and inversely as the thickness, and therefore simply as the absolute length chosen for unity. But the thermal unit in which these quantities are measured, being the capacity of a unit bulk of water, is

length of a French foot being 1·06575 of the British standard foot, we must therefore multiply the preceding numbers by 1·13581 to reduce them to convenient terms.

41. We may, lastly, express them in terms of the most common unit, which is the quantity of heat required to raise the temperature of a grain of water by 1°; and to do this we have only to multiply each of them by 7000 × 62·447, being the weight of a cubic foot in grains.

42. The following Table contains a summary of our results as to conductivity expressed in several different ways, one or other of which will generally be found convenient:—

TABLE X.—Thermal Conductivities of Edinburgh Strata, in British Absolute Units [Unit of Length, the English Foot].

Description of terrestrial substance.	Conductivities in terms of thermal capacity of unit bulk of substance $\left(\frac{k}{c}\right)$.			Conductivities in terms of thermal capacity of unit bulk of water (k).			Conductivities in terms of thermal capacity of one grain of water.
	Per ann.	Per 24 h.	Per second.	Per ann.	Per 24 h.	Per second.	Per second.
Trap-rock of Calton Hill.	267·0	·7310	·000008461	141·1	·3863	·000004471	1·9544
Sand of Experimental Gardens...	295·9	·8100	·000009375	88·9	·2435	·000002818	1·2319
Sandstone of Craigleith Quarry ...	784·5	2·1478	·00002486	362·7	·9929	·00001149	5·0225

43. The statements (§§ 36 and 37) by which the signification of $\frac{k}{c}$ has been defined and illustrated, require only to have *cubic feet of water* substituted for *cubic feet of rock*, in their calorimetric specifications, to be applicable similarly to define and illustrate the meaning of the conductivity denoted by k. The fluidity of the water allows a modified and somewhat simpler explanation, equivalent to that of § 36, to be now given as follows:—

44. If a long rectangular plate of rock one foot thick, in a position slightly inclined to the horizontal, have water one foot deep flowing over it in a direction parallel to its length, and if the lower surface of the plate be everywhere kept 1° higher in temperature than the upper, the water must flow at the rate of k times the length of the plate per unit of time in order that the heat conducted through the plate may raise it just 1° in temperature in its flow over the whole length. [It must be understood here that the plate becomes warmer, on the whole, under the lower parts of the stream of water, its upper surface being everywhere at the same temperature as the water in contact with it, while its lower surface is, by hypothesis, at a temperature 1° higher.] If, for instance, the plate be of Calton Hill trap-rock, the water must, according to the result we have found, flow at the rate of 141·1 times its length in a year, or of ·3863 of its length in twenty-four hours, to be raised just 1° in temperature in flowing over it. Thus, water one French foot deep, flowing over a plane bed of such rock at the rate of ·3863 of a mile in twenty-four hours, will in flowing one mile have its temperature raised 1° by heat conducted through the plate. The rates required to fulfil similar conditions for the sand of the Experimental Gardens and the sandstone of Craigleith Quarry are similarly found to be ·2435 of the length and ·9929 of the length in twenty-four hours.

directly as the cube of the unit length, and therefore the numbers expressing the quantities of heat compared will be inversely as the cubes of the lengths chosen for unity, and directly as these simple lengths; that is to say, finally, they will be inversely as the squares of these lengths.

XIX. *Chemical Notices from Foreign Journals.* *By* E. ATKINSON, *Ph.D., F.C.S.*

[Continued from p. 62.]

LOURENÇO* has succeeded in converting glycerine into propylic glycol, and glycol into ordinary alcohol. The formula of monohydrochloric glycerine only differs from that of propylic glycol by containing chlorine in the place of an atom of hydrogen. This relation, as well as that between monohydrochloric glycol and the corresponding monoatomic alcohol, is indicated in the following formulæ:—

$C^3 H^7 Cl O^2$	$C^3 H^8 O^2$
Monohydrochloric glycerine.	Propylic glycol.
$C^3 H^7 Cl O^4$	$C^3 H^8 O^4$
Monohydrochloric glycol.	Propylic alcohol.
$C^4 H^5 Cl O^4$	$C^4 H^6 O^4$
Hydrochloric glycol.	Alcohol.

By treating these hydrochloric ethers with nascent hydrogen, this chlorine is removed and replaced by hydrogen.

When monohydrochloric glycerine, diluted with its volume of water, was placed in contact with excess of sodium-amalgam, and the mixture left at the ordinary temperature, the amalgam was slowly decomposed with a slight disengagement of hydrogen, and formation of an abundant deposit of chloride of sodium.

* *Comptes Rendus*, May 20, 1861.