might be produced by a smaller rate of change through a greater space; thus an addition of about 0·5 to the specific gravity of a sphere of 1 mile in diameter, or of 0·33 to one of 10 miles diameter, would have nearly the same effect, and it cannot be contended that these are improbably large.

Should the experiment ever be repeated, it would be desirable to swing the pendulum at one (at least) intermediate station between the top and bottom of the shaft, by which means any error of this kind might be approximately eliminated. In the mean time I think there are hardly sufficient grounds for impugning the correctness of the value of E (5·67) deduced by the late Francis Baily from his carefully conducted repetition of the Cavendish experiment.

IV. "On Practical Methods for rapid Signalling by the Electric Telegraph." By Prof. W. Thomson, F.R.S. Received November 14, 1856.

I am at present engaged in working out various practical applications of the formulæ communicated some time ago in a short article on the "Theory of the Electric Telegraph" (Proceedings, May 17, 1855), and I hope to be able very soon to lay the results in full before the Royal Society. In the mean time, as the project of an Atlantic Telegraph is at this moment exciting much interest, I shall explain shortly a telegraphic system to which, in the course of this investigation, I have been led, as likely to give nearly the same rapidity of utterance by a submarine one-wire cable of ordinary lateral dimensions between Ireland and Newfoundland, as is attained on short air or submarine lines by telegraphic systems in actual use.

Every system of working the electric telegraph must comprehend (1) a plan of operating at one extremity, (2) a plan of observing at the other, and (3) a code of letter-signals. These three parts of the system which I propose will be explained in order,—I. for long submarine lines, and II. for air or short submarine lines.

I. *Proposed telegraphic system for long submarine lines.*

1. *Plan of operating.*—This consists in applying a regulated gal-

vanic battery to give, during a limited time, a definite variation of electric potential determined by theory, so as to fulfil the condition of producing an electric effect at the other extremity, which, after first becoming sensible, rises very rapidly to a maximum, then sinks as rapidly till it becomes again, and continues, insensible.

The principle followed is that pointed out by Fourier, by which we see, that, when the wire is left with both ends uninsulated after any electrical operations whatever have been performed upon it, the distribution of electric potential through it will very quickly be reduced to a harmonic law, with an amplitude falling in equal proportions during equal intervals of time. Unless the electric operations fulfil a certain condition, this ulterior distribution is according to the simple harmonic law (that is, is proportional to the sine of the distance from either extremity, the whole length being reckoned as 180°). The condition which I propose to fulfil is, that the coefficient of the simple harmonic term in the expression for the electrical potential shall vanish. Then, according to Fourier, the distribution will very much more quickly wear into one following a double harmonic law (that is, the sine of the distance from one extremity, the whole length being reckoned as 360°). In this state of electrification the two halves of the wire on each side of its middle point, being symmetrically and oppositely electrified, will discharge into one another, as well as into the earth at their remote extremities; each will be like a single wire of half-length, with the simple harmonic distribution; and the wire will, on the whole, be discharged as fast as a wire of half the length, or four times as fast as a wire of the whole length, after an ordinary electrification. There is considerable latitude as to the mode of operating so as to fulfil this condition, but the theoretical investigation is readily available for finding the best way of fulfilling it in practice. The result, as I have tested by actual calculation of the electric pulse at the remote end, is most satisfactory. The calculations, and curves exhibiting the electric pulse in a variety of cases, will, I trust, very soon be laid before the Royal Society.

The time and law of operations being once fixed upon, a mechanical contrivance of the simplest kind will give the means of directing a regulated galvanic battery to perform it with exactness, and to any stated degree (positive or negative) of strength. Complete plans of all details I have ready to describe when wanted, and shall very soon be able to state exactly the battery power required for a cable of stated dimensions.

2. *Plan of observation for receiving a message.*—The instrument which I propose is Helmholtz's galvanometer, with or without modification. The time of vibration of the suspended magnet, and the efficiency of the copper damper, will be so arranged, that during the electric pulse the suspended magnet will turn from its position of equilibrium into a position of maximum deflection, and will fall back to rest in its position of equilibrium. The possibility of fulfilling these conditions is obvious from the form of the curve I have found to represent the electric pulse. The observer will watch through a telescope the image of a scale reflected from the polished side of the magnet, or from a small mirror carried by the magnet, and he will note the letter or number which each maximum deflection brings into the middle of his field of view.

3. *Code of letter-signals.*—The most obvious way of completing a telegraphic system on the plans which have been described, is to have the twenty-six letters of the alphabet written on the scale of which the image in the suspended mirror is observed, and to arrange thirteen positive and thirteen negative strengths of electric operation, which will give deflections, positive or negative, bringing one or other of these letters on the reflected scale into the centre of the field of view. But it would be bad economy to give the simple signals to rare letters, and to require double or triple signals for double and triple combinations of frequent occurrence. Besides, by the plans which I have formed, it will, I believe, be easy to make much more than thirteen different positive and thirteen different negative strengths of electric operation, giving unmistakeably different degrees of deflection; and if so, then many of the most frequent double and triple combinations, as well as all the twenty-six letters of the alphabet singly, might be made by simple signals. But it is also possible (although I believe highly improbable), that in practice only three or four, or some number less than thirteen, of unmistakeably different deflections could be produced in the galvanometer at one end by electric operations performed on the other extremity. If so, the whole twenty-six letters could not each have a simple signal, and double signals would have to be chosen for the less frequent letters.

Experience must show what number of perfectly distinct simple signals can be made, and I have scarcely a doubt but that it will be much more than twenty-six. Then it will be easy to invent a letter code which will use these signals with the best economy for the language in which the message is to be delivered. Towards this object I have commenced collecting statistics showing the relative frequency with which the different simple letters, and various combinations of simple letters, occur in the English language, and I must soon have information enough to guide in choosing the best code for a given number of simple signals.

The investigation leading to a measurement of the electro-magnetic unit of electricity in terms of the electro-static unit, published since the commencement of the present year by Kohlrausch and Weber, has given all that is required to deduce from Weber's own previous experiments the measurement of the electric conductivity of copper wire in terms of the proper kind of unit for the telegraph problem. The data required for estimating the rapidity of action in a submarine wire of stated dimensions would be completed by a determination of the specific inductive capacity of gutta percha, or better still, a direct experiment on the electro-static capacity of a yard or two cut from the cable itself. I have estimated the retardations of various electric pulses, and the practicable rate of transmitting messages by cables 2400 miles long, and of certain ordinary lateral dimensions, on the assumption that the specific inductive capacity of gutta-percha, measured as Faraday did that of sulphur, shell-lac, &c., is 2, from which it probably does not differ much. These estimates have been published elsewhere (Athenæum, Oct. 1856), and I shall not repeat them until I can along with them give a table of estimates for cables of various dimensions, with the uncertainty as to the physical property of gutta-percha either done away with by experiment, or taken strictly into account.

II. *Plan for rapid self-recording signals by air wires and short submarine cables.*

The consideration of the preceding plans has led me to think of a system of working air lines, and short submarine lines, by which great rapidity of utterance, considerably greater I believe than any hitherto practised, may be attained. I have no doubt but that on this system five or six distinct letters per second, or sixty words per minute, may be readily delivered through air lines and submarine lines up to 100 miles, or perhaps even considerably more, of length, and recorded by a self-acting apparatus, which I shall describe in a communication I hope to make to the Royal Society before its next meeting.

V. "On Practical Methods for Rapid Signalling by the Electric Telegraph." (Second communication.) By Prof. W. Thomson, F.R.S. Received December 11, 1856.

I. *Further remarks on proposed method for great distances.*

Since my former communication on this subject I have worked out the determination of operations performed at one extremity of a submarine wire, so adjusted, that when the other extremity is kept constantly uninsulated, the subsidence of the electricity in the wire shall follow the *triple harmonic law* (that is to say, the electrical potential shall ultimately vary along the wire in proportion to the sine of the distance from either end, one-third of the length of the wire being taken as 180°). The condensation of the electrical pulse at the receiving extremity, due to such operations, is of course considerably greater than that which is obtained from operations leading only to *the double harmonic* as described in my last communication; but experience will be necessary to test whether or not the precision of adjustment in the operations required to obtain the advantages which the theory indicates, can be attained in practice when so high a degree of condensation is aimed at. The theory shows exactly what amount and duration of residual charge in the wire would result from stated deviations from perfect accuracy in the adjustments of the operations; but it cannot be known for certain, without actual trial, within what limit such deviations can be kept in practice. From Weber's experiments on the electric conductivity of copper, and from measurements which I have made on specimens of the cable now in process of manufacture for the Atlantic telegraph, I think it highly probable that, with an alphabet of twenty letters, one letter could be delivered every two seconds between Newfoundland and Ireland (which would give, without any condensed code, six words per minute) on the general plan which I explained in my last communication; and that no higher battery power than from 150 to 200 small cells of Daniell's (perhaps even considerably less) would be required. Whether or not this system may ultimately be found preferable to the very simple and undoubtedly practicable method of telegraphing invented by Mr. Wildman Whitehouse, can scarcely be decided until one or both methods shall have been tested on a cable of the dimensions of the Atlantic cable, either actually submerged or placed in perfectly similar inductive circumstances.

II. *Method for telegraphing through submarine or subterranean lines of not more than* 500 *miles length.*

The plan which I have proposed to describe for rapid signalling through shorter wires, has one characteristic in common with the plan I have already suggested for the Atlantic telegraph; namely, that of using different strengths of current for different signals.

But in lines of less than 500 miles, condensed pulses, such as have been described, may be made to follow one another more rapidly than to admit of being read off by an observer watching the image of a scale in a suspended mirror; and a new plan of receiving and recording the indications becomes necessary.

Of various plans which I have considered, the following seems most likely to prove convenient in practice.

Several small steel magnets (perhaps each about half an inch long) are suspended horizontally by fine threads or wires at different positions in the neighbourhood of a coil of which one end is connected with the line wire and the other with the earth. Each of these magnets is held in a position deflected from the magnetic meridian by two stops on which its ends press; and two other small stops of platinum wire are arranged to prevent it from turning through more than a very small angle when actuated by any deflecting force making it leave the first position. When a current passing through the coil produces this effect on any one of these magnets, it immediately strikes the last-mentioned stops, and so completes a circuit through a local battery and makes a mark on prepared electro-chemical paper. For each suspended magnet there is a separate style, but of course one battery is sufficient for the whole printing process. One set of the different suspended magnets are so adjusted, that a current in one direction of any strength falling short of a certain limit makes only one of them move; that a current in the same direction, of strength exceeding this limit but falling short of another limit, moves another also of the suspended magnets; and so on for a succession of different limits of strength of current in one direction. The remaining set of suspended magnets are adjusted to move with different strengths of current in the other direction through the coil. Without experience it is impossible to say how many gradations of strength could be conveniently arranged to be thus distinguished unmistakeably. I have no doubt, however, that very moderate applications of electric resources would give at least three different strengths of current in each direction, which could with ease and certainty be distinguished from one another by the test which the suspended magnets afford. Thus, a signal of six varieties —one letter of an alphabet of six—could be recorded by almost instantaneous movements of six suspended magnets, making one, two or three marks by one set of three styles, or one, two or three marks by another set of three styles, placed all six beside one another, pressing on a slip of electro-chemical paper drawn by clockwork, as in the Morse instrument.

In subterranean or submarine lines of less than 100 miles length, it would be easy, by means of simple battery applications, followed by connexions with the earth, or by means of simple electro-magnetic impulses at one end of the wire, to give ten or twelve of such signals per second without any confusion of utterance at the other end. The confusion of utterance which would be experienced in working thus through longer lines would be easily done away with, in any length up to 500 miles, by following up each battery application with a reverse application for a shorter time, or by following up each electro-magnetic impulse by a weaker reverse impulse, so as approximately to fulfil the condition (described in my former communication), of reducing the subsidence of the electrification in the wire to the double harmonic form. It would, I believe, be readily practicable to send distinctly five or six such signals per second (each a distinct letter of an alphabet of six) through a wire of 500 miles length in a submarine cable of ordinary dimensions. To perform the electrical operations required for sending a message on this system, mechanism might be had recourse to, and, by the use of perforated slips, as in Bain's and other systems, it would be easy to work from twelve to twenty of the six-fold varied signals per second through lines of less than 100 miles length. Operating by the hand is, however, I believe, generally preferred for ordinary telegraphing; and no such speed as the last-mentioned could be attained even by a skilful operator working with both hands. Six distinct letters or signs of an alphabet of thirty, could, however, I believe, be delivered per second by the two hands working on a key-board with twelve keys (perhaps like those of a pianoforte), provided the keys are so arranged as to fulfil the following conditions:—

(1) That by simply striking once any one of a first set of six of the keys, an electric operation of one or other of the six varieties shall be made twice, the second time commencing at a definite interval (perhaps $\frac{1}{12}$th of a second) later than the first.

(2) That by striking one or other of the remaining six keys at the same time, or very nearly at the same time, as one of the first set, the second operation of the double electric signal will be that corresponding to the key of the second set which is struck, instead of being a mere repetition of the operation corresponding to the key of the first set.

It would certainly be easy to make a key-board to fulfil these conditions with the aid of some clockwork power. Then by arranging the thirty-six permutations and doubles of the six simple signals to represent an alphabet of thirty-six letters and signs, an experienced operator would have to direct his mind to only six different letters per second, while executing them by six double operations with his fingers. That it would be possible to work by hand at this rate there can be no doubt, when we consider the marvels of rapid execution so commonly attained by practice on the pianoforte; and it appears not improbable that in regular telegraphic work, practised operators of ordinary skill could perform from four to six letters with ease per second, or from forty to sixty words per minute, on lines of not more than 100 miles length. The six signals per second, which, according to the preceding estimate, could be distinctly conveyed by a submerged wire of 500 miles in length, could of course be easily performed by the hand, with the aid of a key-board and clockwork power adapted to make the double operations for giving rapid subsidence of electricity in the wire when any one key is touched, and to let the different strengths of current, in one direction or the other, be produced by the different keys. Thus without a condensed code, thirty words per minute could be telegraphed through subterranean or submarine lines of 500 miles; and from thirty to fifty or sixty words per minute through such lines, of lengths of from 500 miles to 100 miles.

The rate of from fifty to sixty words per minute could be attained through almost any length of air line, were it not for the defects of insulation to which such lines are exposed. If the imperfection of the insulation remained constant, or only varied slowly from day to day with the humidity of the atmosphere, the method I have indicated might probably, with suitable adjustments, be made successful; and I think it possible that it may be found to answer for air lines of hundreds of miles' length. But in a short air line, the strengths of the currents received, at one extremity, from graduated operations performed at the other, might suddenly, in the middle of a message, become so much changed as to throw all the indications into confusion, in consequence of a shower of rain, or a trickling of water along a spider's web.

VI. "On the Equation of Laplace's Functions," &c. By W. F. Donkin, M.A., F.R.S., F.R.A.S., Savilian Professor of Astronomy, Oxford. Received December 3, 1856.

(Abstract.)

The equation $\frac{d^2u}{dx^2}+\frac{d^2u}{dy^2}+\frac{d^2u}{dz^2}=0$, when transformed by putting $x=r\sin\theta\cos\phi$, $y=r\sin\theta\sin\phi$, $z=r\cos\theta$, may be written in the form

$$\left\{\left(\sin\theta\frac{d}{d\theta}\right)^2+\left(\frac{d}{d\phi}\right)^2+(\sin\theta)^2 r\frac{d}{dr}\left(r\frac{d}{dr}+1\right)\right\}u=0; \quad (1)$$

and if $u=u_0+u_1r+u_2r^2+\ldots+u_nr^n+\ldots$, we find on substituting this value in (1), and equating to zero the coefficient of r^n, that u_n satisfies the equation

$$\left\{\left(\sin\theta\frac{d}{d\theta}\right)^2+\left(\frac{d}{d\phi}\right)^2+n(n+1)(\sin\theta)^2\right\}u_n=0, \quad . \quad . \quad (2)$$