caused by currents of air of different velocities thus produced, he was enabled to arrive at a measure of the velocities in tubes placed in a still atmosphere, as described in his former paper.

The author in that paper pointed out a correspondence between the variations of force in the upward currents of atmospheric air in the tubes and variations in the humidity of the atmosphere, and expressed his belief that the variations were attributable in great measure to the varying hygrometric conditions of the atmosphere.

In further proof of this position, he has appended two tables, showing that both natural and artificial increase of atmospheric humidity are accompanied by increase in the velocity of the rotations, and that in each case increase of humidity is attended by increase of velocity, independent of temperature.

III. "On the Thermal Effects of Fluids in Motion." By J. P. Joule, Esq., F.R.S., and Professor W. Thomson, F.R.S.

Received May 23, 1856.

On the Temperature of Solids exposed to Currents of Air.

In examining the thermal effects experienced by air rushing through narrow passages, we have found, in various parts of the stream, very decided indications of a lowering of temperature (see Phil. Trans. June 1853), but never nearly so great as theoretical considerations at first led us to expect, in air forced by its own pressure into so rapid motion as it was in our experiments. The theoretical investigation is simply as follows:—Let P and V denote the pressure and the volume of a pound of the air moving very slowly up a wide pipe towards the narrow passage. Let p and v denote the pressure and the volume per pound in any part of the narrow passage, where the velocity is q. Let also $e-\text{E}$ denote the difference of intrinsic energies of the air per pound in the two situations. Then the equation of mechanical effect is

$$\frac{q^2}{2g}=(\text{PV}-pv)+(\text{E}-e),$$

since the first member is the mechanical value of the motion, per pound of air; the first bracketed term of the second member is the excess of work done in pushing it forward, above the work spent by it in pushing forward the fluid immediately in advance of it in the narrow passage; and the second bracketed term is the amount of intrinsic energy given up by the fluid in passing from one situation to the other.

Now, to the degree of accuracy to which air follows Boyle's and Gay-Lussac's laws, we have

$$pv = \frac{t}{T} PV,$$

if t and T denote the temperatures of the air in the two positions reckoned from the absolute zero of the air-thermometer. Also, to about the same degree of accuracy, our experiments on the temperature of air escaping from a state of high pressure through a porous plug, establish Mayer's hypothesis as the thermo-dynamic law of expansion; and to this degree of accuracy we may assume the intrinsic energy of a mass of air to be independent of its density when its temperature remains unaltered. Lastly, Carnot's principle, as modified in the dynamical theory, shows that a fluid which fulfils those three laws must have its capacity for heat in constant volume constant for all temperatures and pressures,—a result confirmed by Regnault's direct experiments to a corresponding degree of accuracy. Hence the variation of intrinsic energy in a mass of air is, according to those laws, simply the difference of temperatures multiplied by a constant, irrespectively of any expansion or condensation that may have been experienced. Hence, if N denote the capacity for heat of a pound of air in constant volume, and J the mechanical value of the thermal unit, we have

$$E - e = JN (T - t).$$

Thus the preceding equation of mechanical effect becomes

$$\frac{q^2}{2g} = PV \left(1 - \frac{t}{T}\right) + JN (T - t).$$

Now (see "Notes on the Air-Engine," Phil. Trans. March 1852, p. 81, or "Thermal Effects of Fluids in Motion," Part 2, Phil. Trans. June 1854, p. 361) we have

$$JN = \frac{1}{k-1} \frac{H}{t_0} = \frac{1}{k-1} \frac{PV}{T},$$

where k denotes the ratio of the specific heat of air under constant pressure to the specific heat of air in constant volume; H, the product of the pressure into the volume of a pound, or the "height of the homogeneous atmosphere" for air at the freezing-point (26,215 feet, according to Regnault's observations on the density of air), and t_0 the absolute temperature of freezing (about 274° Cent.). Hence we have

$$\frac{q^2}{2g} = \mathrm{PV}\left(1 + \frac{1}{k-1}\right)\left(1 - \frac{t}{\mathrm{T}}\right) = \frac{k\,\mathrm{PV}}{k-1}\left(1 - \frac{t}{\mathrm{T}}\right).$$

Now the velocity of sound in air at any temperature is equal to the product of $\sqrt{k}$ into the velocity a body would acquire in falling under the action of a constant force of gravity through half the height of the homogeneous atmosphere; and therefore if we denote by α the velocity of sound in air at the temperature T, we have

$$\alpha^2 = kg\mathrm{PV}.$$

Hence we derive from the preceding equation,

$$\frac{\mathrm{T}-t}{\mathrm{T}} = \frac{k-1}{2}\left(\frac{q}{\alpha}\right)^2,$$

which expresses the lowering of temperature, in any part of the narrow channel, in terms of the ratio of the actual velocity of the air in that place to the velocity of sound in air at the temperature of the stream where it moves slowly up towards the rapids. It is to be observed, that the only hypothesis which has been made is, that in all the states of temperature and pressure through which it passes the air fulfils the three gaseous laws mentioned above; and that whatever frictional resistance, or irregular action from irregularities in the channel, the air may have experienced before coming to the part considered, provided only it has not been allowed either to give out heat or to take in heat from the matter round it, nor to lose any mechanical energy in sound, or in other motions not among its own particles, the preceding formulæ will give the lowering of temperature it experiences in acquiring the velocity q. It is to be observed that this is not the velocity the air would have in issuing in the same quantity at the density which it has in the slow stream approaching the narrow passage. Were no fluid friction operative in the circumstances, the density and pressure would be the same in the slow stream flowing away from, and in the slow stream approaching towards the narrow passage; and each would be got by considering the lowering of temperature from T to t as simply due to expansion, so that we should have

$$\frac{t}{\mathrm{T}} = \left(\frac{\mathrm{V}}{v}\right)^{k-1}$$

by Poisson's formula. Hence if Q denote what we may call the "reduced velocity" in any part of the narrow channel, as distinguished from q, the actual or true velocity in the same locality, we have

$$\mathrm{Q} = \frac{\mathrm{V}}{v} q = \left(\frac{t}{\mathrm{T}}\right)^{\frac{1}{k-1}} q,$$

and the rate of flow of the air will be, in pounds per second, $w\mathrm{QA}$, if w denote the weight of the unit of volume, under pressure P, and A the area of the section in the part of the channel considered. The preceding equation, expressed in terms of the "reduced velocity," then becomes

$$1 - \frac{t}{\mathrm{T}} = \frac{k-1}{2} \left(\frac{\mathrm{T}}{t}\right)^{\frac{2}{k-1}} \left(\alpha\right)^{2},$$

and therefore we have

$$\frac{\mathrm{Q}}{\alpha} = \sqrt{\left\{\frac{2}{k-1} \left(\frac{t}{\mathrm{T}}\right)^{\frac{2}{k-1}} \left(1 - \frac{t}{\mathrm{T}}\right)\right\}}.$$

The second member, which vanishes when $t = 0$, and when $t = \mathrm{T}$, attains a maximum when

$$t = \cdot 83\,\mathrm{T},$$

the maximum value being

$$\frac{\mathrm{Q}}{\alpha} = \cdot 578.$$

Hence, if there were no fluid friction, the "reduced velocity" could never, in any part of a narrow channel, exceed ·578 of the velocity of sound, in air of the temperature which the air has in the wide parts of the channel, where it is moving slowly. If this temperature be 13° Cent. above the freezing-point, or 287° absolute temperature (being 55° Fahr., an ordinary atmospheric condition), the velocity of sound would be 1115 feet per second, and the maximum reduced velocity of the stream would be 644 feet per second. The cooling effect that air must, in such circumstances, experience in acquiring such a velocity would be from 287° to 268° absolute temperature, or 19° Cent.

The effects of fluid friction in different parts of the stream would require to be known in order to estimate the reduced velocity in any narrow part, according to either the density on the high-pressure side or the density on the low-pressure side. We have not as yet made any sufficient investigation to allow us to give even a conjectural estimate of what these effects may be in any case. But it appears improbable that the "reduced velocity," according to the density on the high-pressure side, could ever with friction exceed the greatest amount it could possibly have without friction. It therefore seems improbable that the "reduced velocity" in terms of the density on the high-pressure side can ever, in the narrowest part of the channel, exceed 644 feet per second, if the temperature of the high-pressure air moving slowly be about the atmospheric temperature of 13° Cent. used in the preceding estimate.

Experiments in which we have forced air through apertures of $\frac{29}{1000}$, $\frac{53}{1000}$, and $\frac{84}{1000}$ths of an inch in diameter drilled in thin plates of copper, have given us a maximum velocity reduced to the density of the high-pressure side equal to 550 feet per second. But there can be little doubt that the stream of air, after issuing from an orifice in a thin plate, contracts as that of water does under similar circumstances. If the velocity were calculated from the area of this contracted part of the stream, it is highly probable that the maximum velocity reduced to the density on the high-pressure side would be found as near 644 feet as the degree of accuracy of the experiments warrants us to expect.

As an example of the results we have obtained on examining the temperature of the rushing stream by a thermo-electric junction placed $\frac{1}{8}$th of an inch above the orifice, we cite an experiment, in which the total pressure of the air in the receiver being 98 inches of mercury, we found the velocity in the orifice equal to 535 and 1780 feet respectively as reduced to the density on the high-pressure and that on the atmospheric side. The actual velocity in the small aperture must have been greater than either of these, perhaps not much greater than 1780, the velocity reduced to atmospheric density. If it had been only this, the cooling effect would have been exactly $T\frac{k-1}{2}\left(\frac{1780}{1115}\right)^2$, that is, a lowering of temperature amounting to 150° Cent. But the amount of cooling effect observed in the experiment was only 13° Cent.; nor have we ever succeeded in observing (whether with thermometers held in various positions in the stream, or with a thermo-electric arrangement constituted by a narrow tube through which the air flows, or by a straight wire of two different metals in the axis of the stream, with the junction in the place of most rapid motion, and in other positions on each side of it,) a greater cooling effect than 20° Cent; we therefore infer *that a body round which air is flowing rapidly acquires a higher temperature than the average temperature of the air close to it all round.* The explanation of this conclusion probably is, that the surface of contact between the air and the solid is the locality of the most intense frictional generation of heat that takes place, and that consequently a stratum of air round the body has a higher average temperature than the air further off; but whatever the explanation may be, it appears certainly demonstrated that the air does not give its own temperature even to a tube through which it flows, or to a wire or thermometer-bulb completely surrounded by it.

Having been convinced of this conclusion by experiments on rapid motion of air through small passages, we inferred of course that the same phenomenon must take place universally whenever air flows against a solid or a solid is carried through air. If a velocity of 1780 feet per second in the foregoing experiment gave 137° Cent. difference of temperature between the air and the solid, how probable is it that meteors moving at from six to thirty miles per second even through a rarefied atmosphere, really acquire, in accordance with the same law, all the heat which they manifest! On the other hand, it seemed worth while to look for the same kind of effect on a much smaller scale in bodies moving at moderate velocities through the ordinary atmosphere. Accordingly, although it has been a practice in general undoubtingly followed, to whirl a thermometer through the air for the purpose of finding the atmospheric temperature, we have tried and found, with thermometers of different sizes and variously shaped bulbs, whirled through the air at the end of a string, with velocities of from 80 to 120 feet per second, temperatures always higher than when the same thermometers are whirled in exactly the same circumstances at smaller velocities. By alternately whirling the same thermometers for half a minute or so fast, and then for a similar time slow, we have found differences of temperature sometimes little if at all short of a Fahrenheit degree. By whirling a thermo-electric junction alternately fast and slow, the same phenomenon is most satisfactorily and strikingly exhibited by a galvanometer. This last experiment we have performed at night, under a cloudy sky, with the galvanometer within doors, and the testing thermo-electric apparatus whirled in the middle of a field; and thus, with as little as can be conceived of disturbing circumstances, we confirmed the result we had previously found by whirling thermometers.

Velocity of Air escaping through narrow Apertures.*

In the foregoing part of this communication, referring to the circumstances of certain experiments, we have stated our opinion that the velocity of atmospheric air impelled through narrow orifices was, in the narrowest part of the stream, greater than the reduced velocity corresponding to the atmospheric pressure; in other words, that the density of the air, kept at a constant temperature, was, in the narrowest part, less than the atmospheric density. In order to avoid misconception, we now add, that this holds true only when the difference of pressures on the two sides is small, and friction plays but a small part in bringing down the velocity of the exit stream. If there is a great difference between the pressures on the two sides, the reduced velocity will, on the contrary, be *less* than that corresponding with the atmospheric pressure; and even if the pressure in the most rapid part falls short of the atmospheric pressure, the density may, on account of the cooling experienced, exceed the atmospheric density.

We stated that, at 57° Fahr., the greatest velocity of air passing through a small orifice is 550 feet per second, if reduced to the density on the high-pressure side. The experiments from which we obtained this result enable us also to say that this maximum occurs, with the above temperature and a barometric pressure of 30·14 inches, when the pressure of the air is equal to about 50 inches of mercury above the atmospheric pressure. At a higher or lower pressure, a smaller volume of the compressed air escapes in a given time.

* Received June 19, 1856.

Surface Condenser.—A three-horse power high-pressure steam-engine was procured for our experiments. Wishing to give it equal power with a lower pressure, we caused the steam from the eduction port to pass downwards through a perpendicular iron gas-pipe, ten feet long and an inch and a half in diameter, placed within a larger pipe through which water was made to ascend. The lower end of the gas-pipe was connected with the feed-pump of the boiler, a small orifice being contrived in the pump cover in order to allow the escape of air before it could pass, along with the condensed water, into the boiler. This simple arrangement constituted a "surface condenser" of a very efficient kind, giving a vacuum of 23 inches, although considerable leakage of air took place, and the apparatus generally was not so perfect as subsequent experience would have enabled us to make it. Besides the ordinary well-known advantages of the "surface condenser," such as the prevention of incrustation of the boiler, there is one which may be especially remarked as appertaining to the system we have adopted, of causing the current of steam to move in an opposite direction to that of the water employed to condense it. The refrigerating water may thus be made to pass out of the condenser at a high temperature, while the vacuum is that due to a low temperature; and hence the quantity of water used for the purpose of condensation may be materially reduced. We find that our system does not require an amount of surface so great as to involve a cumbrousness or cost which would prevent its general adoption, and have no doubt that it will shortly supersede that at the present time almost universally used.

IV. "On the Stability of Loose Earth." By W. J. Macquorn Rankine, Esq., C.E., F.R.SS. L & E., Regius Professor of Civil Engineering and Mechanics in the University of Glasgow.

(Abstract.)

The object of this paper is to deduce the mathematical theory of that kind of stability which depends on the mutual friction of the parts of a granular mass devoid of tenacity, from the known laws of friction, unaided by any hypothesis.