Comments

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Comment on "Quantum Zeno effect"

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The quantum Zeno effect is not a general characteristic of continuous measurements. In a recently reported experiment [Itano *et al.*, Phys. Rev. A **41**, 2295 (1990)], the inhibition of atomic excitation and deexcitation is not due to any "collapse of the wave function," but instead is caused by a very strong perturbation due to the optical pulses and the coupling to the radiation field. The experiment should not be cited as providing empirical evidence in favor of the notion of "wave-function collapse."

The quantum Zeno effect, or "watched-pot" paradox, is a theoretical argument to the effect that continuous observation should inhibit the change of a quantum state. Itano *et al.*[1] have recently performed an interesting experiment related to that paradox. Unfortunately in the discussion and interpretation of their results they make statements that may be misleading.

Measurement is the determination of the magnitude or quantity of something (to paraphase the *Oxford English Dictionary*), and it involves an interaction between the object of interest and an apparatus. The assertion that "measurement causes a collapse (reduction) of the wave function," although common in the older literature of quantum mechanics, is, strictly speaking, not true. The effect of measurement is rather to establish a *correlation* between the measured object and the measuring apparatus.[2] If the apparatus is designed to measure the object's dynamical variable R, with eigenvectors $\{|r\rangle\}$, then the evolution of the state of the whole system (object plus apparatus) will carry an initial state of the form $|r\rangle\otimes|\alpha_0\rangle$ into a final state of the form $|r\rangle\otimes|\alpha_r\rangle$, where the first factor is an object state vector and the second is an apparatus state vector. (It is assumed for simplicity that this is a nondisturbing measurement, which does not change the value of the dynamical variable being measured.) If the initial state of the object is a coherent superposition of R eigenvectors, then the uncorrelated initial state $|\Psi(t_i)\rangle=\sum_r c_r|r\rangle\otimes|\alpha_0\rangle$ will evolve into the final state

$$|\Psi(t_f)\rangle=\sum_r c_r|r\rangle\otimes|\alpha_r\rangle \ , \tag{1}$$

which exhibits a correlation between the object and the apparatus. The state operator for the whole system, $\rho(t_r)=|\Psi(t_r)\rangle\langle\Psi(t_r)|$, describes a pure state. But the partial (or reduced) state operator of the object (o), $\rho^{(o)}(t_f)=\mathrm{Tr}^{(a)}\rho(t_f)$, obtaining by tracing over the variables of the apparatus (a), need not describe a pure state. If the various apparatus states in Eq. (1) are mutually orthogonal, then it will describe an incoherent mixture,

$$\rho^{(o)}(t_f)=\sum_r |c_r|^2|r\rangle\langle r| \ . \tag{2}$$

It is only the partial state for a component of the system that may be "reduced"; the state of the whole system remains pure and retains its coherence. This coherence can, in principle, be recovered by the object state, as is shown by the so-called "haunted measurement."[3] Thus the statement that measurement causes a reduction of the wave function is objectionable. If it refers to the state of the whole system it is false, and even if it is restricted to the partial state of the object it need not be true. The notion of "collapse of the wave function" is not essential to quantum mechanics. The entire theory[4] can be developed without it. There are some physicists who regard questions of interpretation in quantum mechanics as unimportant unless they lead to different experimental predictions. It should, therefore, be pointed out that plausible uses of the "collapse" postulate can lead to incorrect results in repeated measurements[5] (although this did not happen in Ref. 1).

The quantum Zeno paradox arises from the assumption that every measurement or observation brings about a "reduction of the wave function," and in particular, that the observation of an unstable object not yet decayed must be accompanied by a "reduction" of the state to the initial undecayed form. In the limit of continuous observation this argument leads to the paradoxical conclusion that the state can not change. This unphysical prediction is due to the unwarranted assumption of "state reduction" in such measurements.[4,5] Peres[6] has studied an explicit model of sequential measurements on a time dependent quantum system. He shows that there is an optimum strength of coupling between the object and the apparatus: too weak and no information is transferred to the apparatus; too strong and the apparatus overwhelms the dynamics of the object. In the physically unrealizable limit of infinitely strong coupling, the state of the object never changes, and one obtains the quantum Zeno effect. The resolving power of the measurement is proportional to the product of the coupling strength and the duration of the measurement, so the need to limit the coupling strength is equivalent to a need to limit the resolution. A reasonable model of continuous measurement, without any "Zeno" paradox, can be obtained by taking the limit in which the duration τ of a measurement goes to zero while its resolution varies as τ^{-1}. (This has also been pointed out by Caves and Milburn.[7]) Thus the quantum Zeno effect is not really a consequence of continuous observation, but rather of an excessively strong perturbation by an inappropriate apparatus.

In the light of these general theoretical results, let us now consider the interpretation of the experiment of Ref. 1. We shall treat only a highly simplified model of it, consisting of a three-level atom and a radiation field mode. The atom has three states: the ground state $|\phi_1\rangle$, a metastable state $|\phi_2\rangle$, and an unstable state $|\phi_3\rangle$ that couples strongly to the ground state via spontaneous emission. Spontaneous radiation between any other pair of atomic states is neglected, and the driving fields at rf frequency $\omega_{12}=(E_2-E_1)/\hbar$ and at optical frequency $\omega_{13}=(E_3-E_1)/\hbar$ are treated as classical. In the experiment an rf field of frequency ω_{12} and Rabi frequency Ω (proportional to the field strength) is applied continuously, and intense short optical pulses of frequency ω_{13} are applied at time intervals of length τ.

Initially the atom and field mode are in their ground state, and so the system state vector is

$$|\Psi(0)\rangle=|\phi_1\rangle\otimes|0\rangle \ , \tag{3}$$

where $|0\rangle$ denotes the vacuum state of the ω_{13} radiation field mode. During the interval $0<t<\tau$ the rf field is applied, and the state vector becomes

$$|\Psi(t)\rangle=[\cos(\Omega t/2)|\phi_1\rangle+\sin(\Omega t/2)|\phi_2\rangle]\otimes|0\rangle \ . \tag{4}$$

At time $t=\tau$ a short optical pulse, which couples only atomic levels 1 and 3, is applied. For simplicity we assume that it is a π pulse (although in the actual experiment it was probably much longer), and that its temporal duration is negligible compared to Ω^{-1}. Then just after this pulse the state will be[8]

$$|\Psi(\tau)\rangle=\{\cos(\Omega\tau/2)|\phi_3\rangle+\sin(\Omega\tau/2)|\phi_2\rangle\}\otimes|0\rangle \ . \tag{5}$$

The atomic $3\rightarrow1$ transition couples strongly to the radiation field, and we assume that spontaneous radiation occurs promptly within a time negligible compared to Ω^{-1}. Then after a very short time the state vector will become

$$|\Psi(\tau^+)\rangle=\cos(\Omega\tau/2)|\phi_1\rangle\otimes|1\rangle+\sin(\Omega\tau/2)|\phi_2\rangle\otimes|0\rangle \ , \tag{6}$$

where $|1\rangle$ denotes one photon in the radiation field. The net effect of the optical pulse and the spontaneous radiation is analogous to a measurement, in that it brings about a correlation[9] between the states of the atom and the field [compare (6) with (1)].

It is easy to continue this analysis. The effect of the rf field during the interval $\tau<t<2\tau$ is

$$|\Psi(2\tau)\rangle=\cos(\Omega\tau/2)\{\cos(\Omega\tau/2)|\phi_1\rangle+\sin(\Omega\tau/2)|\phi_2\rangle\}\otimes|1\rangle+\sin(\Omega\tau/2)\{-\sin(\Omega\tau/2)|\phi_1\rangle+\cos(\Omega\tau/2)|\phi_2\rangle\}\otimes|1\rangle \ . \tag{7}$$

After the second optical pulse and subsequent spontaneous radiation, the state vector will become

$$\begin{aligned}|\Psi(2\tau^+)\rangle= &[\cos(\Omega\tau/2)]^2|\phi_1\rangle\otimes|2\rangle+\cos(\Omega\tau/2)\sin(\Omega\tau/2)|\phi_2\rangle\otimes|1\rangle\\ &-[\sin(\Omega\tau/2)]^2|\phi_1\rangle\otimes|1\rangle+\sin(\Omega\tau/2)\cos(\Omega\tau/2)|\phi_2\rangle\otimes|0\rangle \ .\end{aligned} \tag{8}$$

From this result we can see that the probability of photons being emitted following both optical pulses and the atom being left in its ground state is $[\cos(\Omega\tau/2)]^4$. On the other hand, if no optical pulses had occurred the expression (4) would have applied at $t=2\tau$, and the probability of the atom remaining in the ground state would have been $[\cos(\Omega\tau)]^2$.

Extending the analysis to n optical pulses at times $t=\tau,2\tau,\ldots,n\tau$, and putting $\Omega\tau=\pi/N$, we obtain the probability that photons are emitted after each of the n pulses and the atom is left in its ground state[10] to be $[\cos(\pi/2N)]^{2n}$. In the limit $n=N\rightarrow\infty$ this probability approaches 1. (In this limit the duration of a pulse must go to zero and its intensity must approach infinity.) The strong perturbation keeps the atom in its ground state, whereas if no optical pulses had been applied the probability of the atom remaining in its ground state would have been $[\cos(n\pi/2N)]^2$, which vanishes for $n=N$. Thus the quantum Zeno effect would occur for this system in the limit $n\rightarrow\infty$. But it is misleading to explain it as being due to a "collapse of the wave function" caused by measurement. No "collapse" actually occurs, rather the excitation of the atom is impeded by the strong perturbation by the optical pulses, and the coupling to the radiation field. Moreover (as was also pointed out in Ref. 1) the effect occurs regardless of whether or not any measurement (detection of the photons) is actually performed.

If we begin with the atom in the metastable state, and so instead of (3) we take $|\Psi(0)\rangle=|\phi_2\rangle\otimes|0\rangle$, the analysis proceeds in a very similar fashion. In this case after n optical pulses the probability that no photons are emitted and the atom is left in the metastable state is equal to $[\cos(\pi/2N)]^{2n}$. This also shows the quantum Zeno effect. But it is particularly misleading to assert that the effect is caused by "wave-function collapse due to a null measurement," as that expression suggests that the decay of the metastable state was halted by a measurement that does not strongly perturb the atom. That is not true. Just in the previous case, the Zeno effect is caused by the very strong perturbation by the optical pulses and the coupling to the radiation field.

[1]W. M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. A **41**, 2295 (1990).

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[3]D. M. Greenberger and A. Ya'sin, Ann. NY Acad. Sci. **480**, 449 (1986).

[4]L. E. Ballentine, *Quantum Mechanics* (Prentice Hall, Englewoods Cliffs, NJ, 1990).

[5]L. E. Ballentine, Found. Phys. **20**, 1329 (1990).

[6]A. Peres, Phys. Rev. D **39**, 2943 (1989).

[7]C. M. Caves and G. J. Milburn, Phys. Rev. A **36**, 5543 (1987).

[8]Here we disagree with the interpretation given in Ref. 1, that the optical pulse "causes a collapse of the wave function." In fact, it produces a coherent superposition, as does the rf pulse. We have assumed for simplicity that the duration of the optical pulse is shorter than the inverse of the spontaneous radiation rate. If this is not so the intermediate result (5) must be modified, but (6) will still be obtained.

[9]In view of the fact that net result of the optical pulse is to produce a *correlation* between the states of the atom and the field, and not a "reduction" of the state, it is unclear what the authors of Ref. 1 meant when they said, "The pulses were long enough to collapse each ion's wave function . . . ."

[10]In Ref. 1 the optical pulses were referred to as "measurement pulses," from which one might reasonably infer that each of them measures the state of the atom by means of the emission or nonemission of a photon. Thus one can, in principle, determine the joint probability for emission of a photon after each of the n pulses. However in the actual experiment of Ref. 1, it seems that the so-called "measurement" pulses did not actually yield information about the atomic state (and so did not really measure anything); rather a subsequent measurement was performed to determine the relative numbers of atoms in states 1 and 2. This determines the probability that the atom is finally left in its ground state, regardless of the number of photons emitted. This probability can be obtained by summing the probabilities that the atom is left in its ground state with the emission of $1,2,,\ldots,n$ photons, or more directly by the density matrix method used in Ref. 1, and is equal to $\frac{1}{2}\{1+[\cos(\pi/n)]^n\}$ for $n=N$. This probability and the one calculated in the text both approach 1 in the limit $n\rightarrow\infty$, illustrating the Zeno effect.