therefore that the non-interchange masses in equation (13) should be regarded as the standard masses of the proton and electron. It appears from the theory of interchange that the mass of the internal particle of a hydrogen atom (and therefore approximately the mass of an electron) derived by applying the formulae of current quantum theory to observation is 137/136 times the standard mass (*P. and E.* § 15·8).

In many other parts of my theory the formulae are more easily derived and understood when we interpret the numerical coefficients as degrees of degeneracy. Besides the coefficients 10, 136, 137 the coefficients most frequently occurring are 4 expressing the degeneracy of an ordinary (not a *complete*) space vector, and 3 for the three-dimensional vectors in static problems. But other combinations can arise according to the circumstances of the problem under consideration.

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The masses of the neutron and mesotron

By Sir Arthur Eddington, F.R.S.

THE NEUTRON

1. The development of relativistic wave mechanics in the preceding paper makes it possible to calculate the mass of the neutron. References, unless otherwise stated, are to the sections of that paper. We have only to express the fact that a proton and electron by emitting a neutrino yield a neutron.

We consider the proton and electron initially without interaction, since the interaction energy would in any case have to be recalculated after the emission of the neutrino. They are equivalent to an external and an internal particle of masses $M = m_1 + m_2$, $\mu = m_1 m_2/(m_1 + m_2)$. These, like the electron and proton, are specified by complete momentum vectors having the full degree of degeneracy 10. I take it that the emission of a neutrino is a way of saying that one of the particles loses its spin. Accordingly its complete

momentum vector, which (leaving aside the six dormant electrical components) consists of an ordinary momentum 4-vector and a spin 6-vector (§ 17), is reduced to a momentum 4-vector, and its degree of degeneracy is changed from 10 to 4. The particle which undergoes the modification is clearly the internal particle, for the neutron in its external relations resembles a hydrogen atom; indeed a spinless external momentum vector would represent a particle capable of existing only in one Lorentz frame and therefore immobile, since the spin 6-vector is necessary for a Lorentz transformation as Dirac's pioneer investigation demonstrated.

The reduction of the degeneracy factor from 10 to 4 increases the energy of the internal particle in the ratio 10 to 4, i.e. from μ to $2\cdot 5\mu$. This follows from § 16, equation (21), bearing in mind that it is the generalized form of equation (20) whose significance is followed up later in the section. The last term $\mu'^2/\mu\nu$ is the energy of the internal particle, which is increased in the ratio $2\cdot 5$ by the change of the degeneracy factor ν from 10 to 4.

To satisfy ourselves that the change is effective—that it adds $1\cdot 5\mu$ to the whole mass and is not merely a formal change of reckoning—we must recall that (21) is not imposed as a condition to be satisfied unless we intend the system referred to on the right-hand side to be merely an alternative description of the dissociated proton and electron referred to on the left; in other cases the discrepancy of the two sides indicates the excess of energy of the system represented by the internal and external wave functions over the proton-electron system. By changing ν from 10 to 4 this discrepancy is increased by $1\cdot 5\mu$; that accordingly is the increase of mass necessitated by the emission of a neutrino. The point to notice is that although we start with a discrepancy μ between the two sides of the equation, attributable to a formal change of reckoning in the transition from an absolute to a relative description, variations of the discrepancy are not formal changes.

Accordingly the mass of a neutron exceeds the mass of a hydrogen atom by $1\cdot 5\mu$, or very nearly $1\cdot 5$ electron masses.

This result has already been obtained in an investigation by H. O. W. Richardson* based on the theory as given in *Relativity theory of protons and electrons*. Richardson's work has been known to me for a year; and, although it did not profess to give a complete proof (some details being left obscure), I could feel no doubt as to its substantial truth. By indicating a new direction in which advance was undoubtedly possible if certain parts of my theory were brought into better order, it has greatly helped me in these developments.

* Owing to incompleteness, Mr Richardson has withheld publication of the investigation.

2. There can be no internal angular momentum in the neutron. Although spin momentum is normally only part of the whole angular momentum, relativistic conditions make it impossible to attribute angular momentum to a spinless particle (*P. and E.* § 8·3). The neutron therefore does not possess an internal structure with negative energy-levels analogous to those of a hydrogen atom. I think that the only possible form of internal motion open to it would be pulsation, but no solutions of the wave equation are of that form. It seems clear therefore that the neutron has only one state, namely, that in which the momentum vector of the internal particle is directed along the τ-axis. Thus there will be no correction for internal energy, and the result found above is the true mass of the neutron.

The magnetic moment of a free neutron is easily determined. Since there is no internal angular momentum, the magnetic moment is that of the external wave function only, which is identical with the external wave function of a hydrogen atom. This moment is

$$2·5eh/4\pi c\textsc{m}, \tag{1}$$

that is to say, it is the moment erroneously attributed to the proton in current quantum theory. A derivation of (1) is given in *P. and E.* § 12·8, but this could now be considerably improved. It may be noted that (1) applies to the neutron as it stands, *not* with the mass of the neutron substituted for $\textsc{m}$.

THE MESOTRON

3. The mesotron is more difficult to treat, and the following determination of its mass may not be so definitive as the corresponding investigation for the neutron. We have to express the fact that a mesotron by emitting a neutrino becomes an electron, or, more conveniently, that an electron by combining with a neutrino becomes a mesotron.

The complete momentum vector representing an electron cannot *absorb* a neutrino in the sense in which it can emit one; it can lose the spin 6-vector incorporated in it, but it cannot be extended to accommodate an additional 6-vector. Thus we can only represent the association of an electron and neutrino as a *combination*, their respective vectors being combined into a double vector occupied by a bi-particle. The 10-fold degeneracy of the electron vector coupled with the 6-fold degeneracy of the neutrino spin-vector gives a double vector with degeneracy 60. Since the pure spin-vector contains no electrical components to combine with the electrical components of the electron vector, there are no additional dimensions of degeneracy.

I conclude that a mesotron is a bi-particle with degree of degeneracy 60.

One of the very few possibilities that present themselves for consideration is that the mesotron bi-particle includes what will ultimately be the comparison particle; so that the actual transformation is

$$\text{mesotron} - \text{neutrino} \rightarrow \text{electron} + \text{comparison particle.}$$

This seems probable when we consider the origin of the mesotron. The nucleus appears to be a region of intense angular momentum. The ordinary macroscopic theory of Riemannian space, which assumes that microscopic vorticity cancels out on the average, ceases to apply even approximately; so that both the geometrical frame and the physical comparison fluid which materializes it require the fuller specification explained in *P. and E.* § 11·4. We noticed (§ 8) that the ordinary comparison particles do not provide a standard of non-rotation, and that special spin-comparison particles might be required in certain types of problem. It seems reasonable to suppose that in the nucleus we have a certain number of ordinary comparison particles furnishing a standard of rest for the measurement of momentum, and a certain number of spin-comparison particles furnishing a standard of non-rotation for the measurement of angular momentum. The number and nature of the comparison particles is, of course, determined by the nature of the ideal measurements which we must suppose ourselves to have made in order to be in possession of the knowledge which we should regard as a full description of the system.

When an electron emerges from the nucleus it may bring with it an ordinary comparison particle or a spin-comparison particle, depending on the nature of the measurements required to obtain full knowledge of the transformed nucleus. In the first case the comparison particle merges at once into the comparison fluid, and we observed only an ordinary electron in undisturbed environment. In the second case the combination remains for some time a distinct bi-particle—an electron existing in a vortex. The vorticity is gradually dissipated; that is to say there is a probability of transition into an ordinary electron and comparison particle. The transition is described as emission of a neutrino, but it is an altogether different process from the emission of a neutrino in forming a neutron. I suppose that a physical picture of what happens is that, when released from the constraint of the nucleus, the bi-particle has new degrees of freedom which, by non-commutation with the neutrino spin, cause a precession which ultimately makes the original plane of spin untraceable. When it is necessary to distinguish the two ways of losing spin, I shall describe the mesotron process as emission of an *anti-neutrino*. The *anti*-ness lies in the fact that the spin is in the comparison part of the bi-particle instead of in the object part.

We have therefore to consider the conversion of a pseudo-discrete double vector of degeneracy 60 into the corresponding pseudo-discrete double vector of full degeneracy 136 which is immediately replaceable by an electron and comparison particle. In the case of the neutron we found that the change of degeneracy alters the mass in inverse ratio, and the same rule applies here; but the argument requires restating since we are now considering a bi-particle.

As in the case of the neutron, the key-condition is that energy must be gained, or lost in the transition in such a way as to preserve a one-to-one correspondence of the particles. This requires that the mesotron wave function shall have the same normalization volume V as the electron and comparison particle.

Each double vector consists of a mass-product multiplied by a unitary double matrix. The transformation of the double matrix of a mesotron into the double matrix of an electron + comparison particle is merely a relativity rotation of the double frame $E_\mu F_\nu$; thus apart from degeneracy, a mesotron in one frame would actually be an electron + comparison particle in another equivalent frame. The effect of degeneracy is contained in the formula relating the density to the mass-product (§ 12, formula (8)); the relation $\rho = 136 cm_0 m_e$ must be replaced in the mesotron by $\rho' = 60 cm_0' m_e'$. Treating this as a change of scale of energy, we must change ρ in the same ratio as m_0 and m_e, since the normalization volume has to be kept constant. Setting

$$m_0' = am_0, \quad m_e' = am_e, \quad \rho' = a\rho,$$

the conditions $\rho = 136 cm_0 m_e$, $\rho' = 60 cm_0' m_e'$ give

$$a = 136/60.$$

Thus if we build an electron + comparison particle system on 136/60 times the ordinary scale of energy, the result is equivalent to a mesotron; and conversely the transformation of a mesotron into an electron + comparison particle is a deflation of this system to the normal scale by emission of the surplus energy.

It may be noticed that the presence of the additional mass of the mesotron in the volume V must alter the curvature of space. This is represented in our formulae by the increase of the mass m_0 of the physical reference frame. The fact that *both* m_0 and m_e have to be altered follows from the relativistic condition that we cannot alter what is put into the frame without altering the frame itself (§ 9). The ordinary approximation of quantum theory which neglects the alteration of the frame would be hopelessly in error in this type of problem.

The mass $\frac{136}{60}(m_0 + m_e)$ of the mesotron is measured from a zero-level $-m_0$; for, after the emission of the excess energy, we are left with the mass $(m_0 + m_e)$ of which only m_e counts as object-mass. The actual object-mass of the mesotron is therefore

$$M = \tfrac{136}{60}(m_0 + m_e) - m_0. \tag{2}$$

Since $m_0 = 135 \cdot 93 m_e$, this gives

$$M = 174 \cdot 44 \text{ standard electron masses}$$

$$= 173 \cdot 17 \text{ current electron masses,}$$

the mass currently assigned to the electron being 137/136 times the standard mass.

Presumably there can exist also "heavy mesotrons" which change into protons (or negatrons). Their mass, obtained by substituting m_p for m_e in (2), is 2·36 proton masses.

4. A very rough idea of the lifetime of a mesotron is obtained in the following way. We have first to find a basal time t, or equivalently a length $l = ct$, which is naturally connected with the mass-product $m_1 m_2$ which characterizes the mesotron or the system into which it is transformed. One of the two mass factors is usually replaced by the reciprocal of the natural normalization volume V; and the most relevant association of length and mass is obtained by identifying V^{-1} with the mass of one of the simple particles which can occupy it. Then V^{-2} is identified with the mass-product of two such masses. Accordingly for the most simple double vector the characteristic length l is $V^{\frac{1}{3}}$. Since we have left vague the particular mass $(m_e, m_0$ or $m_p)$ associated with V^{-1}, we cannot define more closely the l corresponding to the mass-product $m_e m_0$ of the mesotron; but since l varies only as the sixth root of the mass-product the indefiniteness is comparatively small.

By cosmological theory the volume V is found to be 250 cm.[3] (*P. and E.* § 14·9). Hence $l = 6$ cm., and $t = 2 \times 10^{-10}$ sec. We cannot expect this basal time to be a close guide to the lifetime of the mesotron, because rather large factors, such as 60 and 136, will appear in the precise calculation of the transition probability. It seems clear that the basal time would apply most directly to a structure which offers no resistance to collapse. The fact that a neutrino is emitted would seem to place the mesotron transformation in the category of forbidden transitions. It is therefore not surprising that metastability prolongs the lifetime of a mesotron to 10^4 times the basal time.

NUCLEAR STRUCTURE

5. One or two points preliminary to an understanding of nuclear structure appear to be settled by the foregoing results, coupled with an earlier investigation of the non-Coulombian force between protons (Eddington 1937). Perhaps the most interesting conclusion is that mesotrons have no connexion with the so-called Yukawa particles.

The elementary material to be combined in the nucleus consists of protons (p), electrons (e), ordinary comparison particles (c) and spin-comparison particles (s). The comparison particles are virtual particles; they are not in the nucleus until we make the measurements necessary to obtain the knowledge which forms the quantum description of the nucleus. This material yields four possible kinds of bi-particles p_c, e_c, p_s, e_s. Owing to the large rest-energy $(2 \cdot 36 m_p)$ of the heavy mesotron p_s, the tendency will be to form the combinations p_c, e_s rather than the opposite combinations p_s, e_c. Provisionally we may suppose that an electron can only be inside the nucleus if it is attached to a spin-comparison particle; then if the number of electrons exceeds the number of spin-comparison particles, the electrons with ordinary comparison particles will remain outside as satellite electrons. This reduces the nuclear constituents to protons with ordinary comparison particles p_c and mesotrons e_s, the former being in excess.

We can advance one step in investigating the fusion of this material by complex interaction. Consider a combination of one p_c with one e_s. In a combination of p_c with e_c, p and e combine into the internal and external particles of a hydrogen atom and the two comparison particles reduce to one. To obtain the same combination of p_c and e_s, we must make e_s convert itself into e_c by emitting an anti-neutrino; this (presumably) will be cancelled if at the same time we make the internal particle of the hydrogen atom emit a neutrino, thereby converting the hydrogen atom into a neutron. Thus the result of the combination of p_c and e_s is a neutron with an ordinary comparison particle n_c.

We cannot suppose that the mesotrons are selectively combined each with a particular p_c; but it is not unlikely that the condition can be represented with sufficient approximation as combination with a p_c whose identity is being continually varied by interchange. But leaving aside the question whether it is the best starting-point for detailed investigation, we can see why the analysis of the nucleus into protons and neutrons (p_c, n_c) has seemed so much simpler than the analysis into protons and electrons (p_c, e_s). In the proton-neutron analysis all the comparison particles are ordinary, so that the protons and neutrons are represented in a physical frame corresponding

to ordinary Riemannian space; the proton-electron analysis requires spin-comparison particles and cannot be represented in Riemannian space.

The ordinary comparison particle was chosen so as to provide an exact reference frame for momentum; individually it provides no reference origin for position and no standard of non-rotation for the determination of angular momentum. But when we pass from horizontal to vertical section (§ 9), so that the volume V contains a mean of a large number of the original comparison particles, we obtain a fairly good standard of position and of non-rotation provided that the conditions are such that the law of chance is applicable. For example, different parts of the comparison fluid will (by averaging) furnish nearly the same standard of non-rotation unless we deliberately select a permanently disturbed region such as a vortex. The nucleus occupies such an exceptional region. It is well known that this occasional failure of averaging depends on the existence of integrals of the dynamical equations. There appears to be no integral which could cause a failure of positional averaging; and in any case the nucleus is not an exceptional region in this respect. The uncertainty of the origin provided by the comparison fluid (or by a single comparison particle in vertical section) is found to be e^{-r^2/k^2}, where (in terms of cosmical constants)

$$k = \sqrt{(R^2/2N)} = 1\cdot56 \times 10^{-13} \text{ cm.}$$

(Eddington 1937, equation (7·2))*. By this uncertainty a point in the physical reference frame corresponds to a Gaussian distribution in the geometrical reference frame and *vice versa*. It is this Gaussian spread which provides the "range of nuclear forces" between protons.

It is therefore the comparison particle attached to the proton or neutron which (by the uncertainty of its position relative to a Galilean frame) determines the range of nuclear forces. The "Yukawa particles" introduced by nuclear physicists to account for this range are therefore identical with the comparison particles already familiar in extra-nuclear theory. They are neutral spinless particles of mass $m_0 = 136m_e$.

When the mesotron was discovered experimentally it was assumed to be identical with the Yukawa particle. I found this very puzzling because the comparison particle is essentially a virtual particle; and it was difficult to see how it could be turned into an object particle and lose its electrically neutral character. We see now that the mesotron is an altogether different particle. If we adopt the analysis p_c, e_s of the nucleus, the Yukawa or com-

* This is one of the investigations which can be simplified by the degeneracy method of treatment. A note by H. M. Thaxton and the author (not yet published) suggests that there will probably be a small change of k in the revised investigation.

parison particles c and the mesotrons e_s are exhibited as distinct constituents. Since the mesotrons have an altogether different type of wave function from the comparison particles, the present tendency to confuse them is likely to hamper the progress of nuclear physics.

Our conclusions may be summarized as follows: In its normal state the nucleus may be taken to consist of protons and neutrons each with a comparison (Yukawa) particle (p_c, n_c). When a transition is about to occur the neutron is to be regarded as a combination of p_c and e_s. If the transition is spontaneous, the available energy is insufficient to provide for the emission of a mesotron e_s. Accordingly e_s is transformed into e_c before emission, by the process described as emission of an anti-neutrino, and we have the ordinary β-ray disintegration. If the transition is a consequence of the entry into the nucleus of a large amount of energy sufficient to provide the rest-mass $173m_e$ of the mesotron, direct emission of e_s becomes possible and the second transition $e_s \rightarrow e_c$ is deferred until the end of the natural lifetime of the mesotron in free space.

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