

Circumstances prevented the realisation of this scheme till quite recently, when I put into Mr. Hilger's hands a grating presented to me by Mr. Rutherford.

The result is excellent. It is possible to observe C and F, for instance, together quite conveniently, with either a normal or a tangential slit. The only precautions necessary are to see that half of the light passing through the object-glass falls on the half grating, and that the rays which come to a focus on the slit plate are those the wave-lengths of which are half way between the wave-lengths of the two lines compared.

V. "On the Formation of Vortex Rings by Drops falling into Liquids, and some allied Phenomena." By J. J. THOMSON, M.A., F.R.S., Fellow of Trinity College, Cavendish Professor of Experimental Physics, Cambridge, and H. F. NEWALL, M.A., Trinity College, Cambridge. Received November 28, 1885.

When a drop of ink falls into water from not too great a height, it descends through the water as a ring, in which there is evidently considerable rotation about the circular axis passing through the centres of its cross sections; as the ring travels down through the water inequalities make their appearance: more ink seems to collect in some parts of it than in others, and as these parts of the ring descend more rapidly than the rest, it assumes some such appearance as that shown in fig. 1. These aggregations as they descend develope

FIG. 1.

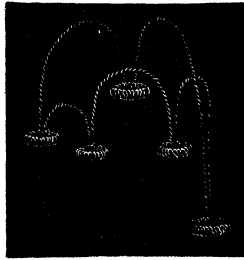


fresh rings in the same way as the original ring was developed from the drop. The ring is thus split up into several rings, each of which is connected with its neighbours by threads of ink affording a very pretty illustration of the continuity of motion in a liquid (see fig. 2).

As the secondary rings descend they develope other rings just as they were developed from the original one, and the process of reproduction seems to go on indefinitely.

It is not every liquid, however, which, when dropped into water, gives rise to rings, for if we drop into water any liquid which does not mix with it, such as chloroform, the drop in consequence of the

FIG. 2.



surface tension remains spherical as it descends. In fact, we may say that, with some few exceptions to be noticed later, rings are formed only when a liquid is dropped into one with which it can mix. This is important, because surface tension has been supposed to play an important part in the formation of these rings; it is difficult, however, to see how any appreciable surface tension can exist between liquids that can mix, and as far as our experiments go they tend to show that it is only the absence of surface tension which is necessary for their production. There are, as we shall show later, many cases where rings are formed under circumstances in which there is no possibility of capillary action, such as when the liquid into which the drop falls is the same as the drop itself. As capillarity was found not to be involved in the production of the rings, it seemed interesting to investigate the subject further, and the following investigation was undertaken with this object. As the result of our experiments we have been led to a theory of the production of these rings of which we shall now give a brief sketch in order to render the sequence of our experiments more intelligible.

Let us suppose that a spherical drop falls into a liquid; the motion of the liquid surrounding the drop will at first be much the same as if a solid sphere of the same size were to fall into the liquid. Now, when a sphere moves through a liquid the tangential velocity of the liquid is different from the tangential velocity of the sphere, so that the liquid flows past the sphere. If the sphere be fluid as well as the medium in which it moves, there will not be an absolute discontinuity in the motion; but only a very rapid change, so that there is a finite alteration in an exceedingly small distance. This alteration is equivalent to a vortex film covering the sphere, the lines of vortex motion being horizontal circles, and if the liquid be viscous the vorticity in the film will diffuse inwards and outwards. As the drop falls the resistance makes it get flatter and flatter until it becomes disk-shaped; by this time, however, it is full of vortex motion, and as the disk-shape is an unstable arrangement of vorticity, the disk must break up into the stable arrangement—that of an anchor ring. This is a

rough outline of the theory we adopt. It will be seen that the most important property of the liquid concerned is its viscosity—the viscosity must be such that when the drop has become disk-shaped there should be enough vortex motion in it to cause it to break up; if the viscosity is too small the vortex motion in it will not have had time to spread far by the time the drop has become disk-shaped, and so the drop will continue to flatten and get into thin sheets with streaks of vortex motion in it instead of breaking up into a ring, whilst if the viscosity is too great the vortex motion will be dissipated before the drop becomes disk-shaped.

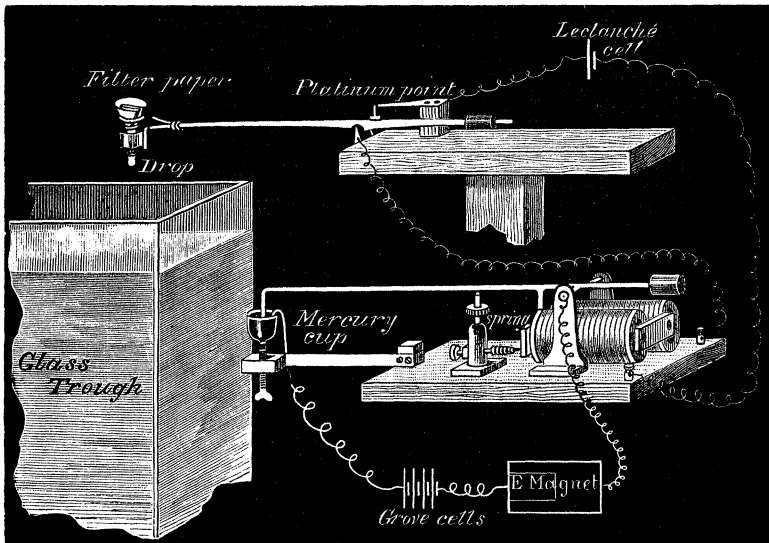
We shall now give the experiments which led us to form the conclusions. We began by investigating the change in the shape of the drop before it became disk-shaped.

*Shape of the Drop before it becomes a Ring.*

To study the change of shape of the drop as it falls through the liquid we have had recourse to instantaneous illumination, and have used for this the bright spark formed at breaking in a mercury cup a circuit containing an electromagnet. It was necessary that we should be able to illuminate the drop at any point of its fall, and it was obviously convenient to make the actual fall of each drop start a set of processes which should result in the spark. The figure shows diagrammatically the arrangement we have used.

The drop is formed at the end of a small piece of glass tubing

FIG. 3.



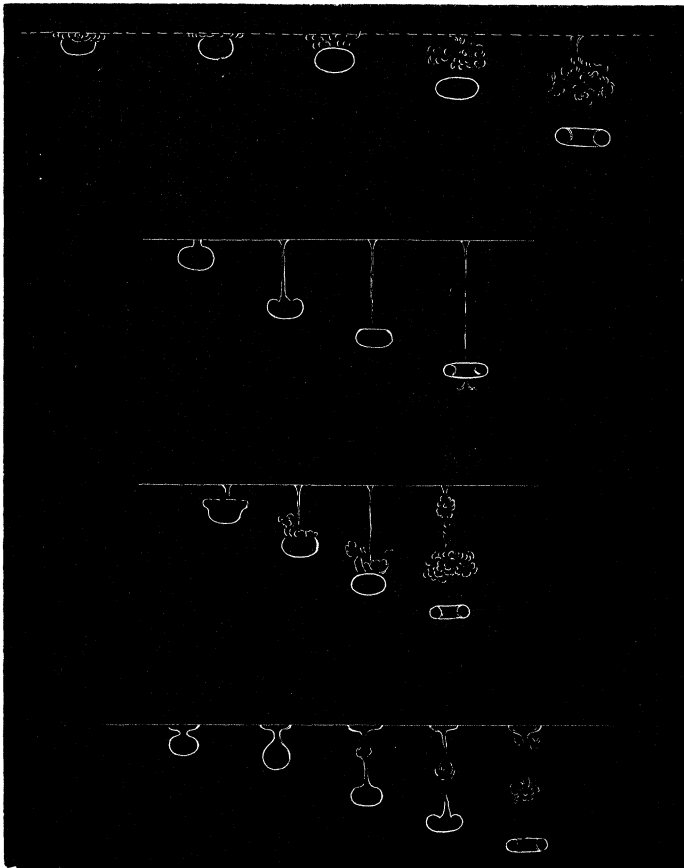
$\frac{3}{16}$  inch in diameter, the liquid filtering slowly through a small filter paper held by a wire ring in the upper end of the glass tube. The tube and filter are fixed at one end of a light lever, which is counterpoised at the other end and balanced carefully on a knife edge. The lever was made of a narrow strip of brass foil, folded for the sake of increased rigidity into a  $\Lambda$ -shape. On the lever between the knife edge and the adjustable counterpoise a small piece of platinum is fixed, and when the balance is set this is pressed lightly upwards against a platinum point which is held in position by a brass arm. The knife edge is connected with one terminal, the brass arm with the other terminal of a circuit containing one or two Leclanché cells and a Morse sounder, which we have adapted so as to serve as a contact breaker in the spark circuit. The moving brass lever of the Morse is prolonged by means of a light wooden arm, and the arm carries a wire tipped with platinum and bent so as to dip into a mercury cup. A current from three or four Grove cells is passed along this wire, and the circuit is completed through a strong electromagnet. The mercury cup is placed conveniently so as to illuminate the drop as it falls.

In using the apparatus the following is the procedure:—Several drops are passed into the filter paper, and the counterpoise is adjusted so that contact is just made at the platinum points. Thus, the relay circuit is complete, and as a consequence the lever of the Morse is held down, and so the electromagnet circuit is made by the dipping of the platinum tip into the mercury cup. The spring and counterpoise of the relay and the mercury cup have been “set” so as to give the spark at break at about the moment required. When the drop detaches itself from the tube the equilibrium of the balance is upset, the counterpoise sinks, and so breaks the relay circuit, which consequently breaks the electromagnet circuit at the mercury surface, and the spark so produced illuminates the drop at some point in its descent. This point may be varied by altering the “setting” of the relay and mercury cup.

Various liquids were used for dropping: fluorescein dissolved in weak ammonia gave very good results for two or three drops when dropped into water, but it soon diffused through the column of water, and the drops were then no longer clear, as the efficient rays had been absorbed at the surface of the column. Milk gave excellent results; but even when considerably diluted it is disagreeable to work with on account of the greasiness that accumulates in the dropping tube, and of the cloudiness produced in the column. A weak solution of nitrate of silver dropped into a column of weak sodic chloride solution was found to do best, as very good rings are formed, and the precipitate may be discharged by the addition of a few drops of ammonia. The results figured below are those obtained in this way.

We have observed a great number of drops at many different points of their fall through the column into which they have been dropped, and at any given instant have found that a few definite forms repeat themselves. Thus we are led to believe that the conditions of fall, though seemingly not altered, vary in a few definite ways, so that successive drops do not always pass through the same series of phases. Indeed, simple observation in continuous light shows clearly differences in the perfection of formation of the rings under apparently similar conditions. We have found a set of forms for a particular instant, and from the various sets taken for successive instants, separated by small intervals, we have picked out series of forms which strike us as continuously derivable from one another.

Fig. 4.



It will be seen at a glance from the figures that the variations are only in the unimportant parts, whilst the essential parts are recognisably the same. At the early stages the drop is more or less spherical; but as it descends, it gets flatter and flatter, as we might expect since its motion is resisted, and at one of the stages becomes almost disk-shaped, and passes very quickly from this into the ring shape.

This change of shape, we imagine, is due to the presence of the vortex motion, the distribution of rotation in the ring being a stable arrangement of vortex motion, whilst that in the disk is not. The vortex motion in the drop has travelled from the boundary, diffusing according to the same laws as those which govern the conduction of heat, the quantity corresponding to the diffusivity in the conduction of heat being in this case  $\mu/\rho$ , where  $\mu$  is the coefficient of viscosity and  $\rho$  the density of the liquid. If  $\mu/\rho$  be very small, the vortex motion will not travel far into the drop, whilst if it be large it will all have diffused before the ring has become disc-shaped. In neither of these cases should we expect the disk to change into a ring, and it will be seen later on that as a matter of fact it does not.

*Effect of Dropping one Liquid into another.*

A drop does not always make a ring when it falls into a vessel containing liquid. We have tried the effect of letting a drop of one liquid fall into a vessel containing another liquid for a good many substances. The results are given in the following table:—

Column of Water:—

Rings are formed when drops of the following liquids are let fall into a column of water: milk, alcohol, blood, aqueous solutions of sugar, of gum arabic, of potash, of permanganate of potash, of carbonate of potash, of sodic chloride, of copper sulphate, of nitrate of potash, of oleate of soda, of nitrate of silver, of cobalt chloride, of carbonate of soda, of ammoniac chloride, hydrochloric acid, acetic acid, nitric acid, Plateau's soap solution, essence of carraway, solution of iodine in ammonia, solution of fluorescein in ammonia, glycerine and water, sulphuric acid.

Globules are formed when drops of the following liquids are let fall into a column of water: carbon bisulphide, chloroform, ether, olive oil, paraffin oil, turpentine.

Column of Paraffin Oil:—

Rings: carbon bisulphide, chloroform, ether, olive oil, turpentine, butylic alcohol.

Globules: water, alcohol, essence of carraway, glycerine and water, dilute sulphuric acid.

Column of Alcohol :—

Rings : carbon bisulphide, chloroform, water, turpentine, butylic alcohol, sulphuric acid.

Globules : olive oil, paraffin oil.

It will be seen from the tables that a drop of one liquid only makes a ring when let fall into another liquid, when the two liquids can mix, and, therefore, when the surface tension is very small.

The following experiment shows that an exceedingly small amount of surface tension is sufficient to prevent the formation of the rings. Absolute alcohol dropped into benzene gives rings; water gives globules : to about 10 c.c. of absolute alcohol water was added drop by drop, the mixture was stirred, and a drop was let fall into benzene. Until after the third drop of water had been added, little change in the appearance was noticed; a ring was formed, and this subdivided into secondary rings, and so on. After the fourth drop was added, very small globules began to appear after a good many subdivisions. After the fifth drop was added, the ring first formed subdivided not into rings but into flattened globules. After the sixth or seventh drop was added, the appearance of the primary ring changed; there seemed to be a more definite surface to it; in fact a small surface tension had sprung up. The globules formed on subdivision of the ring were quite disconnected from one another, whereas before there had been trails or festoons following each. After the seventh or eighth drop of water was added, the formation of the primary ring seemed uncertain; the flattened globule, if large, broke up irregularly into smaller globules without the intervention of the ring shape. This experiment shows that if a liquid A forms spheres when let fall into another B, then A may be diluted with more than 1000 times its volume of some liquid, which has no surface tension with B, before it loses the property of making spheres.

The most striking proof, however, that the formation of the rings does not depend on surface tension, is the fact that the rings are formed when the liquid of which the drop is made is the same in all respects as that into which it falls. If we take a vessel full of water, and raise from it by means of glass tubing enough water to make a drop, then, when this drop falls back again into the vessel from the proper height, it forms a ring. After a little practice, it is easy to distinguish the ring from the rest of the liquid, and this may be done still more easily if we mix some insoluble powder with the water.

#### *Experiments with one Liquid.*

We found on trying different liquids that they behaved very differently when treated in the way we have just described. In some of them a distinct ring was formed by the drop, whilst in others the

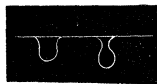
drop after falling into the liquid seemed to spread through it without assuming any definite shape. We found when we used various liquids that if the drops were let fall under similar circumstances, the results depended only on the value of  $\mu/\rho$  (the kinematic coefficient of viscosity for the liquid). This is shown in the following table, where the behaviour of the drop and the value of  $\mu/\rho$  are given for several liquids, which we have divided into classes. The viscosity coefficient  $\mu$  has been determined by Poiseuille's transpiration method; that is, from the time in which a constant quantity of liquid flows under constant pressure through a fine capillary tube. To test the character of the rings formed in the case of any particular liquid, drops of it are let fall into a column of it from three different heights: 1st, such that the drop just touches the surface of the liquid column at the moment it detaches itself from the tube; this height of fall is denoted in the table by "fall = 0"; 2nd, such that the tube is held  $\frac{1}{2}$  inch above the surface of the column; 3rd, such that the tube is held  $\frac{3}{4}$  inch above the surface. In describing the character of the rings we have used terms which it will be well to define. "*Splash*" denotes the irregular spread of the drop through the liquid column; it takes place with whirls and eddies irregularly, and is difficult to represent, but the figure (5) would probably recall the appearance to

FIG. 5.



one who had seen the reality. "*Uncertain*" expresses that a ring or a splash is formed, one as often as the other. "*Blob*" denotes the appearance of a drop that does not break and spread through the column, but remains within its boundary; the figure (6) represents

FIG. 6.



the case. "*Doubtful*" denotes that the drop tends to become a ring, but that it is a question whether it ever leaves the state of a blob.



	$\rho$ . Spec. gravity at 17° C.	$\mu$ . Transpiration time. Water = 1.	$\mu/\rho$ . Kin. visc. coefficient.	Character of ring formed.		
				Fall = 0.	$\frac{1}{2}$ inch.	$\frac{3}{4}$ inch.
Class I, or Ether Class.	Carbon bisulphide . . . . .	0.467	0.418	Uncertain.	Splash.	Splash.
	Chloroform . . . . .	0.6951	0.589	Uncertain.	Uncertain.	Splash.
	Ether . . . . .	0.4428	0.654	Fair ring.	Uncertain.	Splash.
	Benzene . . . . .	0.494	0.700	Fair ring.	Generally ring.	Splash.
Class II, or Water Class.	Water . . . . .	1.0	1.0	Good ring.	Good ring.	Very good ring.
	Oleate of soda (i) . . . . .	1.003	1.226	Slow ring.	Fair ring.	Fair ring.
	Absolute alcohol . . . . .	0.7963	1.514	Slow ring.	Good ring.	Fair ring.
	Turpentine . . . . .	0.879	1.753	Doubtful ring.	Fair ring.	Good ring.
	Sulphuric acid III (ii) . . . . .	1.419	3.804	Fair ring.	Fair ring.	Good ring.
	Paraffin oil I (iii) . . . . .	0.803	2.176	Doubtful ring.	Fair ring.	Fair ring.
	Glycerine II (iv) . . . . .	1.085	2.951	Blob.	Slow ring.	Fair ring.
Class III, or Sugar Class.	Sugar (v) . . . . .	1.157	3.90	Blob.	Blob.	Slow ring.
	Paraffin oil II (vi) . . . . .	0.820	4.050	Very slow ring.	Very slow ring, quickly stopped.	Slow ring, quickly stopped.
Class IV, Butyl Class.	Butylic alcohol . . . . .	0.8051	4.31	Very slow ring.	Fair ring.	Fair ring.
	Sulphuric acid II (vii) . . . . .	1.589	5.053	Very slow ring.	Doubtful ring, not quite breaking.	Fair ring.
Class V.	Sulphuric acid I (viii) . . . . .	1.827	14.79	Blob.	Blob.	Blob.
	Glycerine I (ix) . . . . .	1.179	18.28	Blob.	Blob.	Blob.

*Notes.*—(i.) Oleate of soda solution in water, made of that strength which best gave certain results when a drop was let fall into a special sample of paraffin oil (see below).

(ii.) Sulphuric acid III, a mixture of acid and water, made by adding water to strong acid until drops taken from the mixture and let fall into it gave rings as good as many of the liquids in Class II.

(iii.) Paraffin oil such as is ordinarily used in lamps.

(iv.) Glycerine II, a mixture of glycerine and water, dilute enough to bring it only just into Class II.

(v.) Sugar solution in water, so strong as just to give rings when dropped into itself.

(vi.) Paraffin oil II, a special sample of oil that we have not been able to match, and which will be referred to below (p. 432).

(vii.) Sulphuric acid II, made of such strength that it should belong by the character of its ring to Class III.

(viii.) Sulphuric acid I, strong commercial acid.

(ix.) Glycerine I, diluted with water, but still of such strength that rings were never formed by drops of it falling into a column of it.

These experiments indicate that the nature of the motion, after the drop has fallen into the liquid, depends upon the value of  $\mu/\rho$ . Now  $\mu/\rho$  is a quantity of the dimensions of the product of a length and a velocity, so that nothing can depend upon the absolute value of this quantity, but only on the ratio of it to the product of some length and velocity in the system. The most obvious length in the system with which  $\mu/\rho$  can be connected is the size of the drop or ring. If this is so, diminishing the size of the drop when  $\mu/\rho$  is kept constant ought to produce the same effect as increasing  $\mu/\rho$  when the size of the drop is kept constant. Now if we take a liquid from Class I, the effect of increasing  $\mu/\rho$  sufficiently would be to make it behave like one in Class II, that is, give very much better rings, so that for a liquid of this kind we should expect that small drops would make better rings than large ones. To test this, we repeated the experiments with the liquids of Class I, using much smaller drops than we used before, and we found that now they made rings much more readily and certainly, in fact, with this sized drop, they deserved to be put into Class II. If we try the same thing with liquids in Class III, we find that the rings are worse with small drops than with large. On the other hand, increasing the size of the drop when  $\mu/\rho$  is kept constant, should have the same effect as diminishing  $\mu/\rho$  when the size of the drop remains constant, so that if we repeat the experiments with liquids from Class IV, using larger drops than before, we should expect to get better rings. Experiment showed this to be the case. Large drops were obtained by dipping a piece of glass tubing into the liquid (*e.g.*, strong sulphuric acid), and then raising the tube after closing the top with the finger; on removing the finger, a considerable quantity of liquid flows through the tube, making a large ring. The velocity with which  $\mu/\rho$  would be most

naturally connected is the velocity of the drop; it is not possible, however, to alter this very much without introducing great disturbance into the liquid.

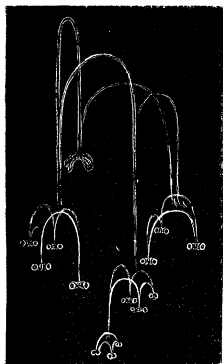
These results admit of very easy explanation, if our theory of the formation of the rings is correct. According to this, the reason that the liquids in Class I do not make rings, is because the vortex motion has not penetrated sufficiently into the drop to make it break up, when it becomes disk-shaped. If, however, the drops be made smaller, the vortex motion has a better chance of filling it before it becomes disk-shaped, and so causing it to break up into a ring. When  $\mu/\rho$  is very large, as is the case for liquids of Class IV, the vortex motion is dissipated so quickly, that though the drop may have been filled with vortex motion, this has all diffused away before the drop reaches the disk-stage. If we make the drop larger, it will take longer to get full of vortex motion, and there may be enough left in, by the time it gets disk-shaped, to cause it to break up into a ring. There is another way in which the formation of rings by liquids of Class IV may be promoted: suppose that, instead of letting a drop fall into a column of the same liquid, we let it fall into another liquid for which  $\mu/\rho$  is smaller; then since this liquid is a worse conductor of vortex motion than the drop, the vortex motion will not diffuse so readily into the surrounding liquid. Thus we should expect the drop to form a ring more readily than before; and we have found this to be the case. Thus, for example, drops of sulphuric acid I from Class IV, let fall into either sulphuric acid II of Class III or sulphuric acid III of Class II, give rings. Similarly with solutions of sugar, of caustic potash, and of glycerine.

These effects are sometimes masked by the effects produced by difference of density; for if the drop is much heavier than the liquid into which it falls, it will fall faster, and this will promote the formation of the ring. If we guard against this source of error, we may see that if a drop does not make rings when it falls into a column of the same liquid, it will not make rings when it falls into a column of another liquid of the same density, but for which  $\mu/\rho$  is greater; but it may make rings if  $\mu/\rho$  be less than for the drop.

#### *On the Splitting up of the Rings.*

When a ring has travelled some little distance through the liquid, its outline generally becomes irregular, and after a time takes the corrugated appearance shown in fig. 1. The corrugations become more and more marked as the ring falls, until the appearance is that represented in fig. 8,  $\gamma$ : the drops at the bottom of the bends develop rings in the same way as the ring itself was originally developed. This process of subdivision is repeated several times, until the ring assumes the appearance shown in fig. 7.

Fig. 7.



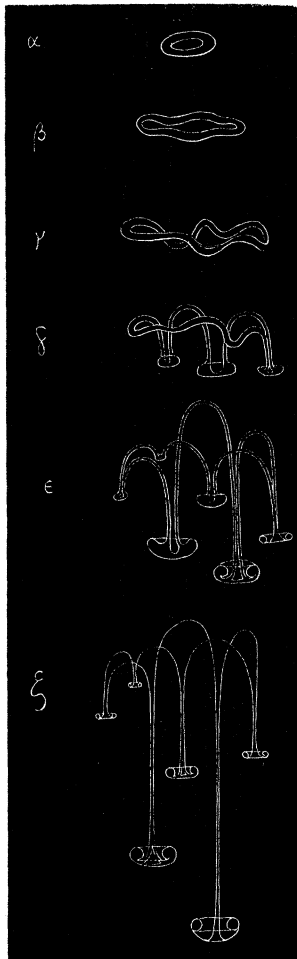
The separate rings always remain connected with each other by threads of liquid; this shows that there is no surface tension; for if there were, these threads would break up into drops. The only stage at which the motion is discontinuous is when the drops are changed into rings. The change from the ring shape to stage 2 (fig. 1) occurs quite gradually, and there is nothing analogous to the separation which takes place when a cylinder of liquid splits up into drops. We have made a great many experiments on the subject, and have arrived at the conclusion that two conditions are necessary for the splitting up of the rings.

1. The liquid forming the ring must be of density different from that of the liquid into which it falls.

2. There must be motion in the liquid into which the drop falls.

We suppose that the splitting up takes place in the following way. In consequence of the motion in the column of liquid the ring gets a little uneven, more of the liquid collecting in one part of the ring than another. Now, if there is plenty of vortex motion in the ring, this irregularity will not be permanent, as the anchor ring with uniform cross section is the stable form for the motion, so that unless the disturbing force is too great, the ring will oscillate about the anchor-ring shape, and the irregularity will not increase. If, however, the vortex motion in the ring be small, it may not be able to balance the disturbing forces and the irregularities will increase. The disturbance is due to the resistance of the liquid in the column, the places where the liquid in the ring has collected falling faster than the remaining portions of the ring; the ring consequently takes in time some such appearance as that shown in fig. 8 ( $\gamma$  and  $\delta$ ). The thicker portions will behave now as the drop did when it fell into the column, and they will develop rings in just the same way.

FIG. 8.



The observations on which we base these conclusions are the following:—

We have never seen a ring break up when the liquid forming the ring was the same in all respects as that surrounding it. If the temperature of the drop be different from that of the column, or if so much powder has been added to the drop as to make the difference of the density appreciable, there is breaking up.

The number and size of rings into which the original ring breaks, and their manner of distribution round its circumference, are quite irregular.

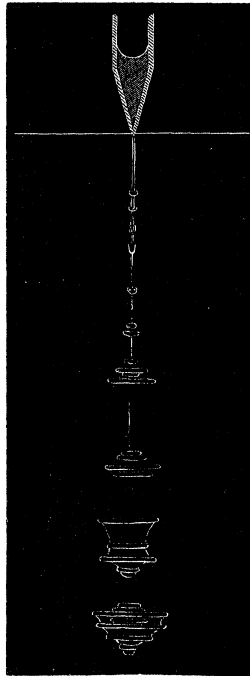
When the liquid into which the drop falls has been allowed to rest for some hours, a ring will go much further and will last much longer without splitting up, than when it follows on a succession of rings. Some rings of very dilute permanganate have thus been observed to last for as long as ten minutes.

When there is little difference in the density of the drop and the liquid into which it falls, the ring does not break up until there is no vortex motion in it.

When, however, the difference in density is large, the ring may break up while it is still rotating.

If a tube be drawn out into a fine capillary and be filled with sulphuric acid, and held so that its capillary end is just beneath the surface of a column of water, a fine stream of acid flows down; and on it marked beadings appear. Each bead gives rise to a vortex ring, and the rings so formed behave in characteristic manner (fig. 9.)

FIG. 9.



Here there seems strong evidence of a tension between the acid and the water, but the appearances are to be explained by differences of velocity in the stream, brought about by motion in the column of

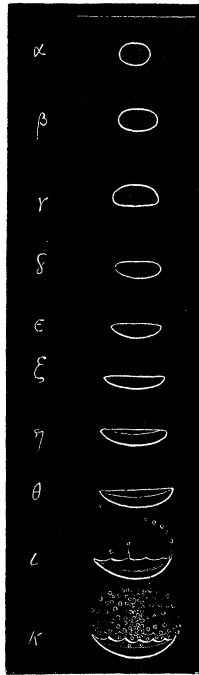
water, or by vibrations communicated to the capillary tube. If the experiment be made with all care to avoid vibration, the stream falls unbroken through a column of 8 inches of water: whilst if a tap be given to the acid tube a break occurs in the stream, in consequence of a momentary stop in the flow of acid, a small bead is formed, and from it a ring. If no care is taken to avoid vibration the beads will follow one another very rapidly. It may be objected that if there existed a surface tension, it would only be when disturbances were communicated that beadings would appear. But in such a case, the resolution into drops would be complete, and small spherules would be formed between the larger drops. In fact, however, the connexions between the beadings are fine filaments of acid, so that the beadings are never really separated from one another. We have, moreover, convinced ourselves of the correctness of this explanation, by allowing a stream of cold water with lycopodium powder to flow from a fine tube into a column of slightly warm water; similar cessations in flow and formations of beadings may be observed; the rings are not well formed, but this is to be expected, for the conditions are not nearly so favourable.

*Experiments with two Liquids between which there is Tension.*

When there is a very small tension between the liquid of the drop and that in the column, some very interesting results are obtained. In a few cases a ring is formed for a moment, but is almost immediately broken up into drops and spherules. As instances of such cases we may mention strong caustic potash solution dropped into paraffin oil: strong sulphuric dropped into turpentine or into paraffin oil: a certain mixture of turpentine and alcohol dropped into paraffin oil: a mixture of alcohol and chloroform dropped into water: and the mixture of alcohol and a few drops of water dropped into benzene (see above, p. 423). In most cases the drop falls through the liquid shaped like a disk, and in the first part of its course changes its shape very considerably. These changes may be well observed in the case of a drop of sodium sulphate solution falling through paraffin oil; they are shown in fig. 10.

At first the drop is nearly spherical ( $\alpha$ ); then it becomes flattened ( $\beta$ ); next it becomes quite flat underneath or sometimes even hollow ( $\gamma$ ), the top remaining curved. During all this time the velocity of the drop has been changing. The top now gets flatter and flatter ( $\delta$ ,  $\epsilon$ ,  $\zeta$ ), while it begins to bulge out at the bottom till the drop is saucer-shaped ( $\eta$ ,  $\theta$ ), hollow at the top, and rounded at the bottom, and we perceive that it is followed close on by a vortex ring in the liquid of the column. It now moves with constant velocity through the liquid as though it met with no resistance, differing in this respect from the case where the drop falls as a ring, when its velocity—unless th

FIG. 10.



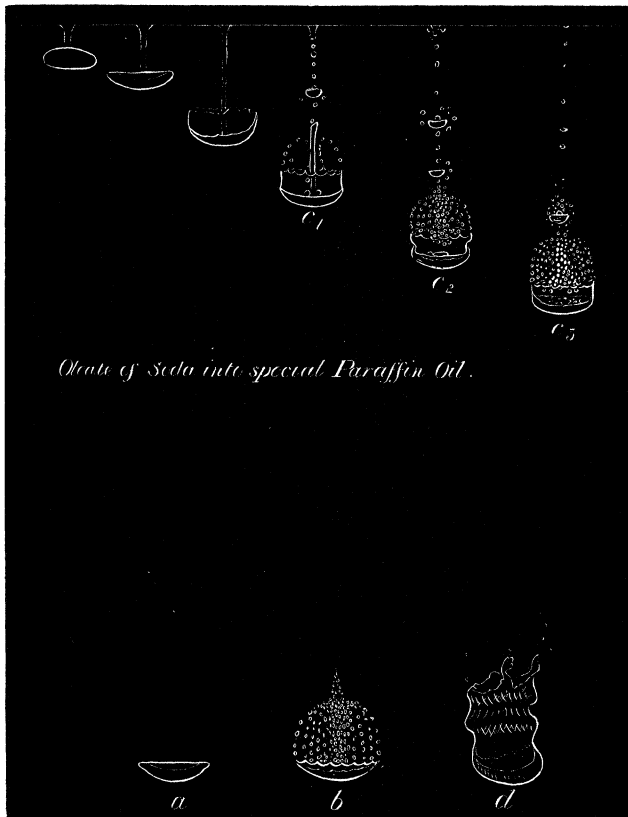
difference of density is very great—continually diminishes. We see the reason for this to be that the vortex at the back of the disk since it tends to make the liquid move along its stream lines, will increase the pressure in the rear and thus diminish the resistance. In fact we see that what the vortex does is to make the conditions very much the same as if the saucer-shaped drop were moving in a stream flowing with the same velocity. The existence of the following vortex is shown in a very beautiful manner in the next stages represented at  $\iota$  and  $\kappa$  in the figure: the edges of the saucer-shaped disk become thinned out to such an extent by the action of the radial streams above and below, that they are drawn out into fine filaments which immediately break up into small spherules, and these are carried round in the vortex behind the drop.

These latter stages are best shown in the case of a certain solution of oleate of soda let fall into a special sample of paraffin referred to above (p. 426), and the changes are represented in fig. 11.

If only a small drop of oleate is let fall, the steady state is reached before the spherules are detached, and the drop falls in the shape shown at  $a$ . If a slightly larger drop falls, then the form  $b$  is reached,



FIG. 11.



and persists until so much of the drop has been whirled off into the following vortex that the form *a* is reached again, and the velocity of fall never attains such a value as to bring about the thinning out of the edges. If a still larger drop is let fall, the forms  $C_1$ ,  $C_2$ , and  $C_3$  are assumed, in which the thin cylindrical part at the edge is at first vibrating up and down, until the steady phase,  $C_3$ , is reached: this will by degrees be reduced to the form *b*, but the column of paraffin was not long enough for us to observe the final form *a*. If a still larger drop of oleate is used, the vibrations of the cylindrical part, *d*, are so great that the drop is torn asunder and such disturbance is produced in the column that a steady fall is not attained. These results may be obtained by increasing the strength of the oleate solution and keeping the size of the drop constant.

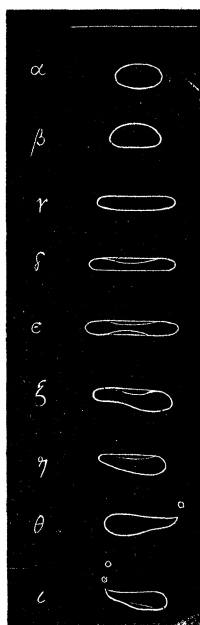
One more case of interest may be described, namely, that of drops

of carbonate of soda dropped into paraffin oil. The phases are represented in fig. 12. The earlier ones are somewhat similar to those above described. But the phase shown at *e* is different. Here it seems quite a chance, as it were, that the disk does not break into a ring. The instability is immediately shown by the oscillations, which begin at this point and continue in a regular manner through a fall of more than 3 feet. An attempt is made to show this in the phases  $\zeta$ ,  $\eta$ ,  $\theta$ . A thickening appears at one side, and the opposite edge thins off considerably, and these irregularities travel regularly round the drop as it falls through the column.

*Effect of the Height of Fall on the Formation of the Ring.*

We have found that alteration of the height from which a drop falls before reaching the surface of the column modifies the formation of

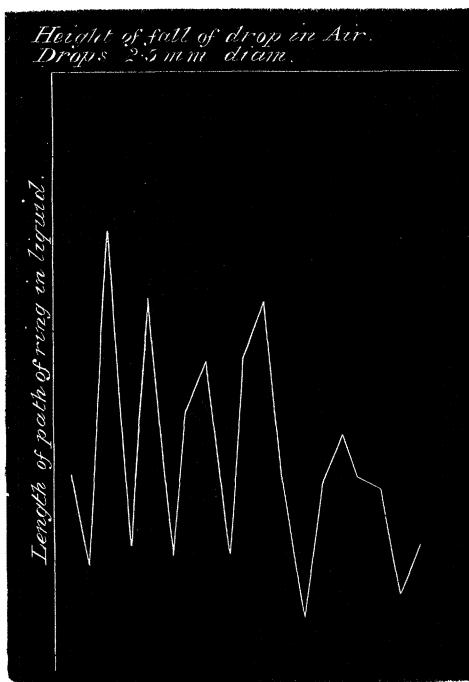
FIG. 12.



the rings considerably. Good rings are formed only within small limits of height—from about 1 to 3 inches, according to the size of the drop. Above 3 inches it is only when the drop falls back into the column after rebound that a ring is formed, and that very irregularly. But within the limits for any particular size of drop rings are much

better formed from some heights than from others. We have observed many cases, varying the size of the drop, the liquid of which it is composed, and the tension of the surface, and have found that the variations in goodness of the rings—where goodness is judged by the length of path in the liquid—depend on the variations in the shape of the drop at the moment it touches the surface. Curves were plotted out, abscissæ being taken equal to the height of fall, and ordinates equal to the depth to which the ring goes without breaking up. The curves show two or more maximum points: the abscissa values for these differ from one another by lengths which may be reduced to time intervals. We find that there is very fair agreement between the intervals so calculated and the periods of vibration of the drop about the spherical form. With small drops of water there are several maxima close together, but more widely separated as the fall gets longer (fig. 13). The equivalent intervals are about  $\frac{1}{85}$  of a second,

FIG. 13.



whilst the period of vibration of a drop of the size used is about  $\frac{1}{90}$  of a second. With as large drops as can be formed from a tube there are only two maxima separated by an interval equivalent to  $\frac{1}{2}$

of a second. The period of vibration of such a drop was calculated to be  $\frac{1}{21}$  of a second.

The effect of diminishing the tension would be to increase the period of vibration. In experiment we have observed that the separation of the maxima is much greater in the cases of paraffin and of turpentine and of alcohol. But difficulties were experienced in the fact that drops of these liquids when let fall from small heights up to about half an inch, generally assumed the spheroidal state. Equal drops of water and of weak oleate of soda solution were let fall into water, and it was found that the separation of maxima was nearly twice as great for the oleate as for the pure water.

In the case of large drops, drops let fall from a point midway between the points to which maxima correspond are nearly always broken up without the formation of a ring. In the case of small drops rings are nearly always formed, but the depth of their paths in the liquid varies from about half an inch to 5 inches.

- VI. "A Preliminary Account of a Research into the Nature of the Venom of the Indian Cobra (*Naja tripudians*). By R. NORRIS WOLFENDEN, M.D. Cantab. (from the Physiological Laboratory, University College, London). Communicated by E. A. SCHÄFER, F.R.S. Received November 17, 1885.

The Society adjourned over the Christmas Recess to Thursday, January 7th, 1886.

*Presents, December 10, 1885.*

Transactions.

- Adelaide :—Royal Society of South Australia. Report. Vol. VII. 8vo. *Adelaide* 1885. The Society.
- Baltimore :—Johns Hopkins University. Studies from the Biological Laboratory. Vol. III. Nos. 3-4. 8vo. *Baltimore* 1885. Studies (Third Series), V-X. 8vo. *Baltimore* 1885. The University.
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FIG. 1.

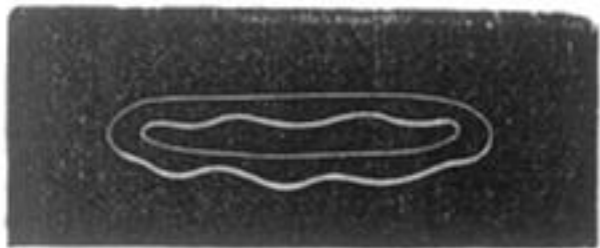


FIG. 2.

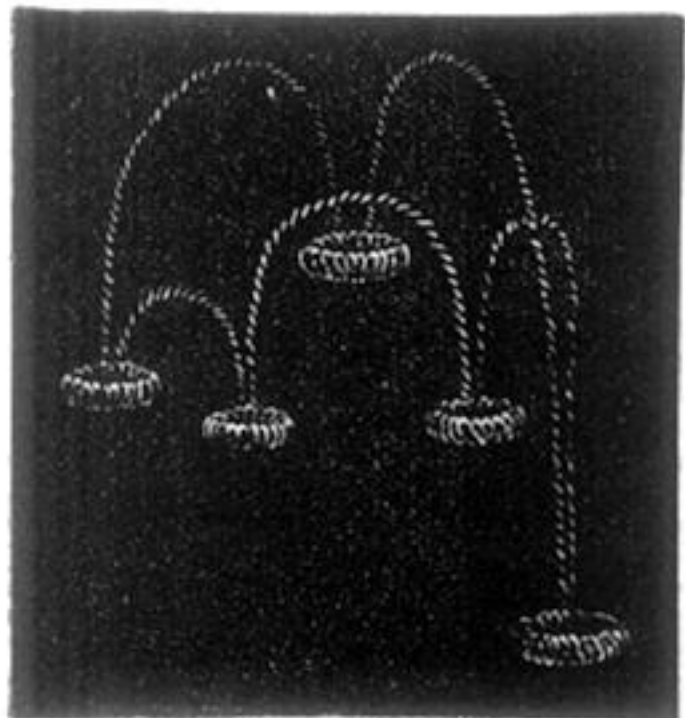


FIG. 3.

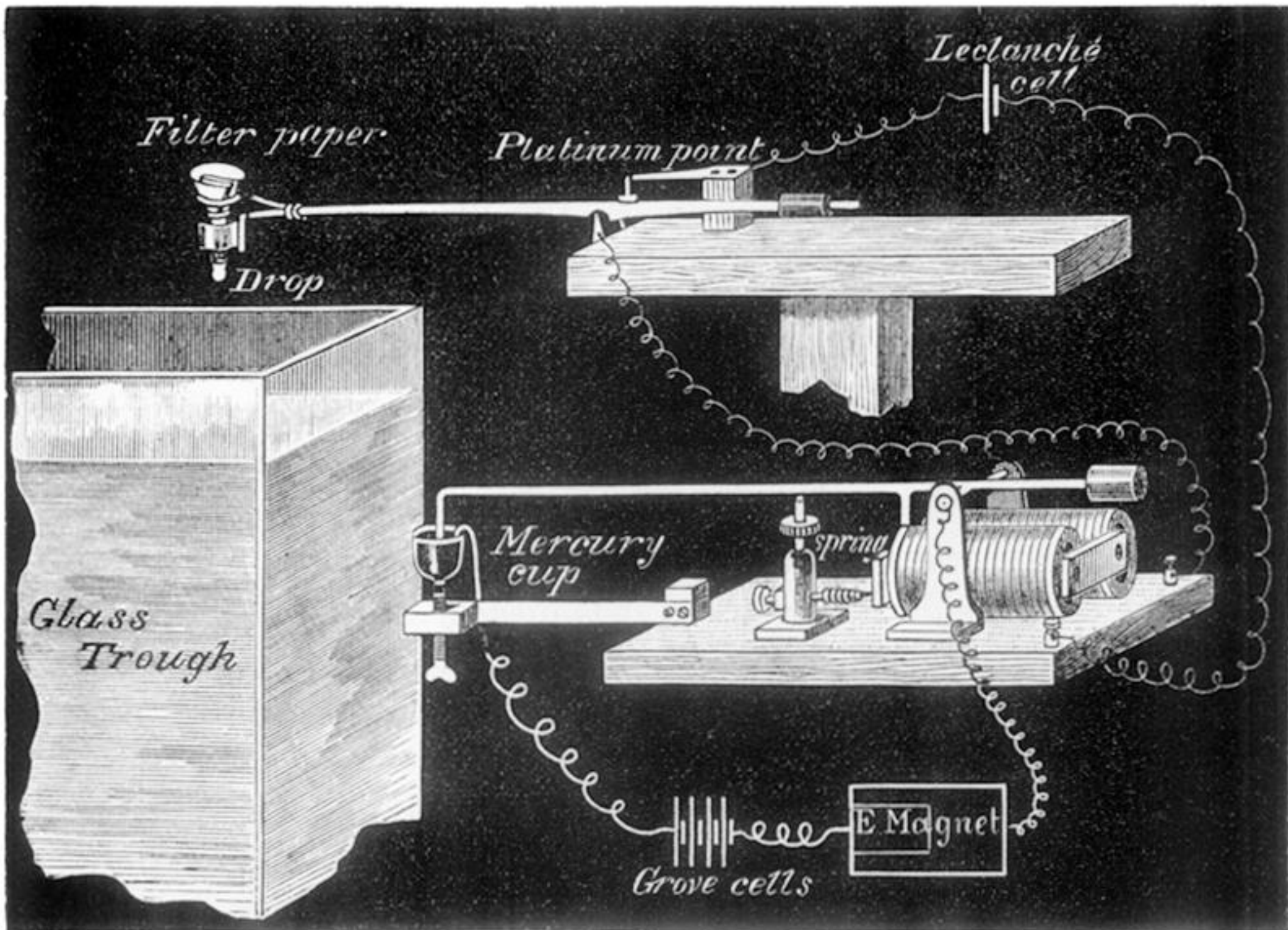


Fig. 4.

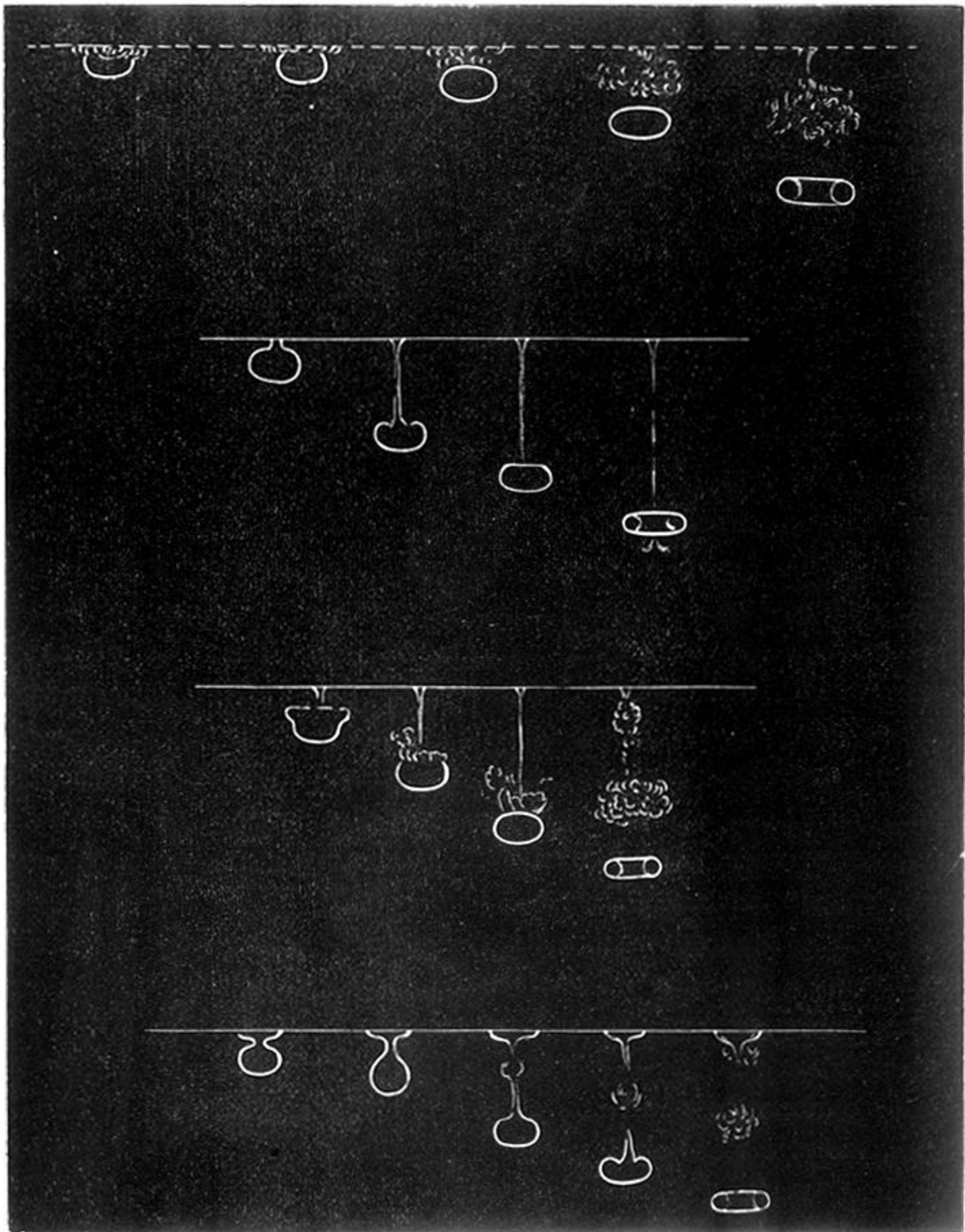




FIG. 5.



FIG. 6.

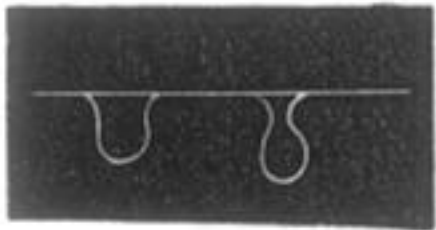


Fig. 7.

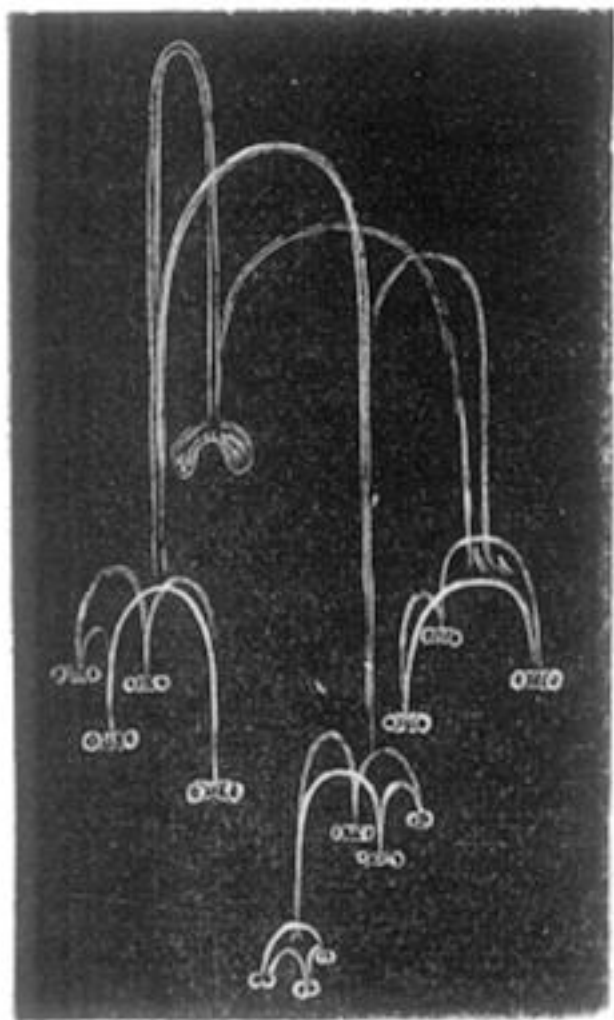


FIG. 8.

$\alpha$



$\beta$



$\gamma$



$\delta$



$\epsilon$



$\xi$

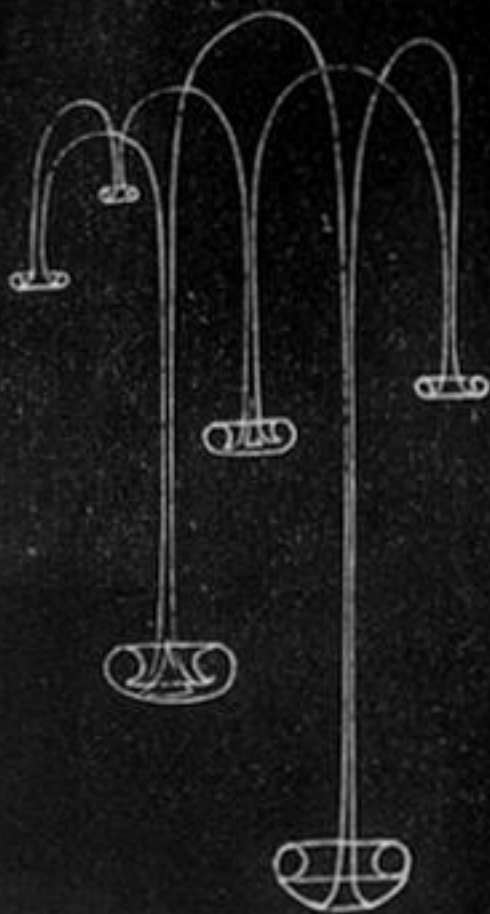


FIG. 9.

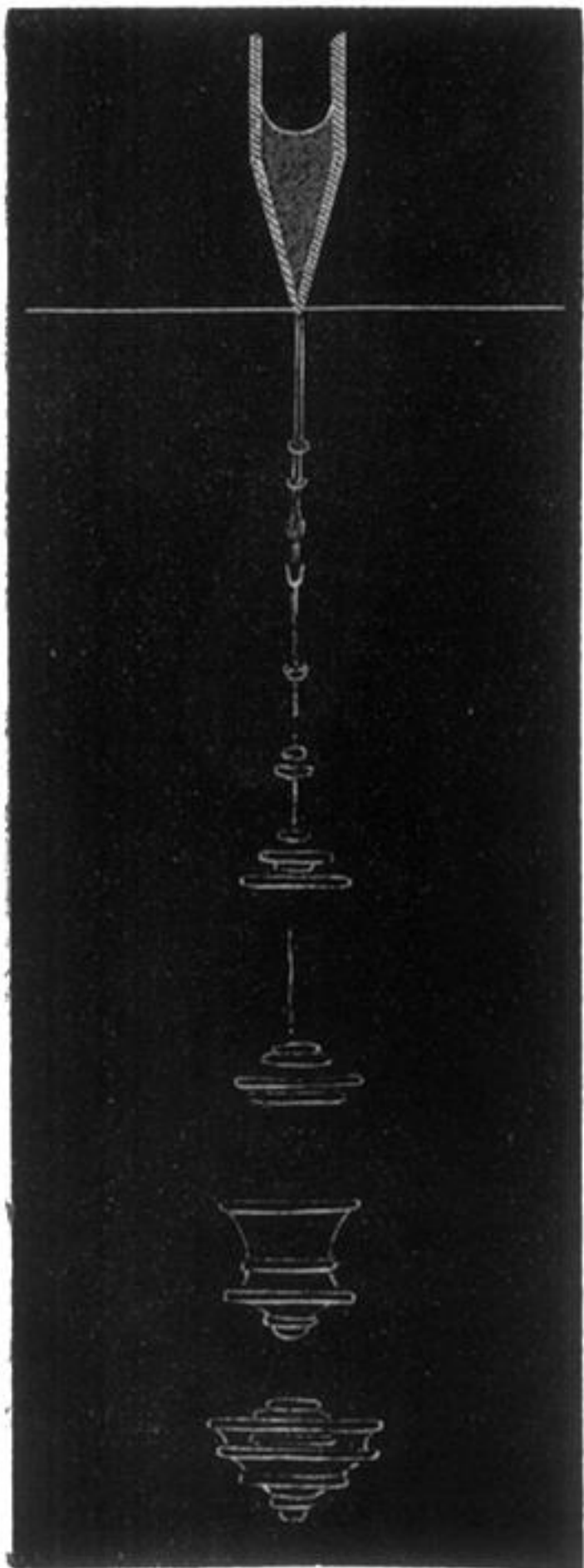


FIG. 10.

$\alpha$



$\beta$



$\gamma$



$\delta$



$\epsilon$



$\xi$



$\eta$



$\theta$



$\iota$



$\kappa$

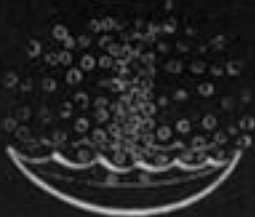
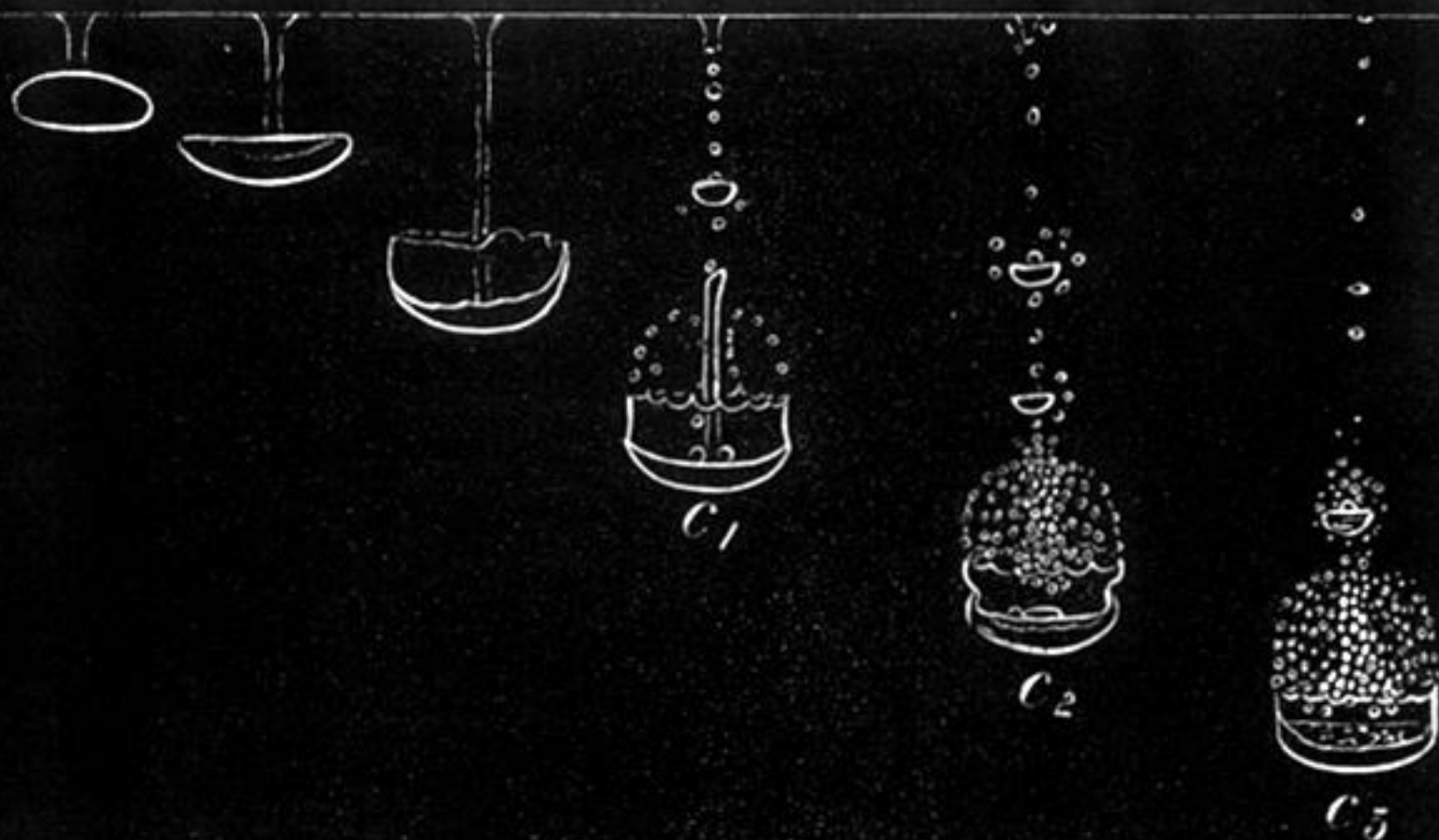


FIG. 11.



*Oleate of Soda into special Paraffin Oil.*



FIG. 12.

$\alpha$



$\beta$



$\gamma$



$\delta$



$\epsilon$



$\xi$



$\eta$



$\theta$



$\iota$





FIG. 13.

*Height of fall of drop in Air.  
Drops 2.5 mm diam.*

*Length of path of ring in liquid.*

