

XXIII. *On the Determination of the Number of Electrostatic Units in the Electromagnetic Unit of Electricity.*

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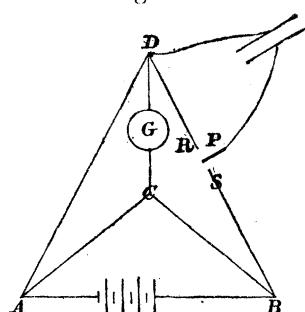
Communicated by Lord RAYLEIGH, F.R.S.

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THE values which various physicists have found for “*v*,” the number of electrostatic units in the electromagnetic unit of electricity, differ so widely from each other that it seems important that additional experiments should be made in order to help to determine the value of this important constant. Six determinations of “*v*” have been published. The first determination was made by WEBER, who measured the capacity of a condenser, both electrostatically and electromagnetically. HOCKIN and Professors AYRTON and PERRY have also determined “*v*” in this way. MAXWELL determined it by balancing the electrostatic attraction between two discs maintained at different potentials against the repulsion between electric currents circulating at the back of the discs, the currents being derived from the battery which maintained the discs at different potentials. Sir WILLIAM THOMSON and Mr. SHIDA have determined it by measuring an electromotive force both electrostatically and electromagnetically.

The following method was employed in this investigation: it is a very slight modification of the method described in § 776 of MAXWELL’s ‘Electricity and

Fig. 1.



Magnetism.’ In a WHEATSTONE’s bridge, A B C D, with the galvanometer at G, and the battery between A and B, the circuit B D is not closed, but the points B and D are connected with two poles, R and S, of a commutator, between which a travelling piece, P, moves backwards and forwards; P is connected with one plate of a condenser, the other plate of which is connected with D. Thus when P is in contact

with S, the condenser will be charged, and until it is fully charged, electricity will flow into it from the battery; this will produce a momentary current through the various arms of the bridge. When the moving piece P is in contact with R, the two plates of the condenser are connected, and the condenser will discharge itself through D R, and as the resistance of D R is infinitesimal in comparison with the resistance of any other circuit, the discharge of the condenser will not send an appreciable amount of electricity through the galvanometer. Thus, if we make the moving piece P oscillate quickly from R to S, there will, owing to the flow of electricity to the condenser, be a succession of momentary currents through the galvanometer. The resistances are so adjusted that the deflection of the galvanometer produced by these momentary currents is balanced by the deflection due to the steady current through the galvanometer, and the resultant deflection is zero. When this is the case there is a relation between the capacity of the condenser, the number of times the condenser is charged and discharged per second, and the resistances in the various arms of the bridge.

As the investigation of this relation given in MAXWELL's 'Treatise on Electricity and Magnetism' is only an approximation, it may be worth while to give here an exact investigation of the relation between the capacity of the condenser and the resistances in the arms of the bridge; though we shall find that when the resistances have the values which they had in the present investigation, MAXWELL's formula is very nearly correct. This relation will enable us to calculate the electromagnetic measure of the capacity of the condenser.

Let \dot{x} be the current in A B

$$\begin{array}{lll} \dot{y} & , & , \quad \text{B S} \\ \dot{z} & , & , \quad \text{D C} \end{array}$$

then the currents in

$$\text{A C} = \dot{x} - (\dot{y} + \dot{z})$$

$$\text{A D} = \dot{y} + \dot{z}$$

$$\text{C B} = \dot{x} - \dot{y}$$

Let b be the resistance of A B

$$\begin{array}{lll} a & , & , \quad \text{A C} \\ c & , & , \quad \text{A D} \\ g & , & , \quad \text{D C} \\ d & , & , \quad \text{B C} \end{array}$$

The resistances of D R, S B are so small in comparison with the other resistances that they may be neglected.

The Dissipation Function

$$= \frac{1}{2} \{ b\dot{x}^2 + a(\dot{x} - (\dot{y} + \dot{z}))^2 + c(\dot{y} + \dot{z})^2 + g\dot{z}^2 + d(\dot{x} - \dot{y})^2 \}$$

The Potential Energy

$$= \frac{1}{2} \frac{y^2}{C}$$

where C is the capacity of the condenser.

Thus if E be the electromotive force of the battery, we have, neglecting the self-induction of the resistance coils in the circuit,

$$\begin{aligned} b\dot{x} + a(\dot{x} - (\dot{y} + \dot{z})) + d(\dot{x} - \dot{y}) &= E \\ -a\{\dot{x} - (\dot{y} + \dot{z})\} + c(\dot{y} + \dot{z}) - d(\dot{x} - \dot{y}) + \frac{y}{C} &= 0 \\ -a(\dot{x} - (\dot{y} + \dot{z})) + c(\dot{y} + \dot{z}) + g\dot{z} &= 0 \end{aligned}$$

or

$$\begin{aligned} (a + b + d)\dot{x} - (a + d)\dot{y} - a\dot{z} &= E \\ -(a + d)\dot{x} + (a + c + d)\dot{y} + (a + c)\dot{z} + \frac{y}{C} &= 0 \\ -a\dot{x} + (a + c)\dot{y} + (a + c + g)\dot{z} &= 0 \end{aligned}$$

To solve these equations, assume

$$\begin{aligned} \dot{x} &= u + p\epsilon^{-\lambda t} \\ \dot{y} &= q\epsilon^{-\lambda t} & y &= \frac{q}{\lambda} (1 - \epsilon^{-\lambda t}) \\ \dot{z} &= w + r\epsilon^{-\lambda t} \end{aligned}$$

where t is measured from the instant when the moving piece P first touches S .

Substituting we get

$$\begin{aligned} w &= \frac{aE}{(a+c+g)(a+b+d) - a^2} \\ u &= \frac{(a+c+g)E}{(a+c+g)(a+b+d) - a^2} \\ \frac{q}{\lambda C} &= \frac{\{(a+d)(a+c+g) - (a+c)a\}E}{(a+c+g)(a+b+d) - a^2} \\ r &= \frac{q\{(a+b+d)(a+c) - a(a+d)\}}{a^2 - (a+b+d)(a+c+g)} \end{aligned}$$

therefore

$$\frac{r}{\lambda} = - \frac{CE\{(a+b+d)(a+c) - a(a+d)\}\{(a+d)(a+c+g) - a(a+c)\}}{\{(a+c+g)(a+b+d) - a^2\}^2}$$

But r/λ is the quantity of electricity that flows through the galvanometer whilst the condenser is being charged. If the condenser is charged and discharged n times in a second, the quantity of electricity which flows through the galvanometer in one second is nr/λ , and if this is to balance the steady current, we must have

$$n\frac{r}{\lambda} + w = 0$$

or

$$nC = \frac{\{(a+c+g)(a+b+d)-a^2\}a}{\{(a+b+d)(a+c)-a(a+d)\}\{(a+d)(a+c+g)-a(a+c)\}}$$

or

$$nC = \frac{a \left\{ 1 - \frac{a^2}{(a+c+g)(a+b+d)} \right\}}{cd \left\{ 1 + \frac{ab}{c(a+b+d)} \right\} \left\{ 1 + \frac{ag}{d(a+c+g)} \right\}}$$

Now in the actual experiment the resistances a, b, c, d, g had about the following values :—

$$a = 1,200 \text{ B.A. units.}$$

$$b = 2,500 \text{ , ,}$$

$$c = 100,100 \text{ , ,}$$

$$d = 900,000 \text{ , ,}$$

$$g = 11,000 \text{ , ,}$$

So that in this case the formula $nC = a/cd$ is correct to within 0.1 per cent., and it is the one we shall use to calculate the electromagnetic measure of the capacity of the condenser.

With these values of the resistances we find that λ is greater than 5000, thus the time constant of the system is very small compared with the time during which the plates of the condenser are connected together, so that the condenser is completely discharged each time.

The electrostatic measure of the capacity must be calculated from the geometrical constants of the condenser. It was necessary to use a guard ring in order to simplify the calculation, and to avoid the influence of the irregular distribution of electricity near the edges of the condenser, but as a condenser with a guard ring could not be worked by the commutator, the capacity of the guard ring condenser had to be compared experimentally with that of a condenser without a guard ring which could be worked by the commutator.

The investigation thus divides itself naturally into three parts :—

First, the theoretical calculation of the electrostatic capacity of the guard ring condenser. For this purpose it was necessary to determine the geometrical constants of the guard ring condenser.

Secondly, the comparison of the capacity of this guard ring condenser with that of a condenser without a guard ring.

Thirdly, the determination by the method just described of the electromagnetic measure of the capacity of the condenser without a guard ring.

I shall describe these parts separately.

PART I.

The calculation of the electrostatic capacity of the guard ring condenser.

The condenser, which was designed by Lord RAYLEIGH, is represented in section in fig. 2, and in plan in fig. 3.

Fig. 2.

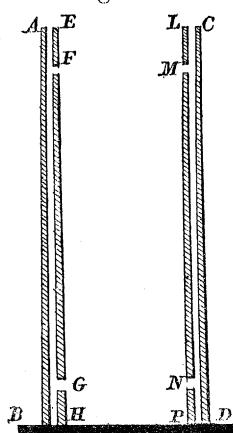
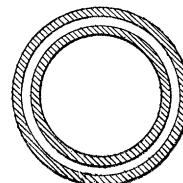


Fig. 3.



B H P D is a thick ebonite board placed in an approximately horizontal position, in this board two concentric circular grooves are cut. A cylindrical brass ring, H P, whose external diameter is about 23 centims., and whose height is about 10 centims., fits into the smaller of these grooves. Three pieces of ebonite carefully ground down to the same thickness (about 3 centims.), are placed at equal intervals on the top of this ring. On these the brass cylinder F G M N is placed ; this cylinder is of exactly the same diameter as the cylindrical ring H P, and is about 60 centims. long. The cylinders, G F M N and H P, are placed so that their axes are coincident ; this is tested by placing a straight-edge against the sides of the cylinder. On the top of this cylinder three pieces of ebonite are placed, and upon the top of these a cylindrical ring E L, similar to the one at the bottom ; another brass cylinder, A B D C, made in three pieces, two rings similar in dimensions to the rings H P E L, and a long middle piece of the same length as the cylinder F G M N is then fitted over the other cylinders, the bottom ring fitting into the outer groove in the ebonite board ; the internal diameter of this cylinder is about 25 centims. The distance between the cylinders at the top is tested by observing how far a wedge, whose vertical angle is very small, sinks down between the cylinders. When the system is properly adjusted, the variation in the distance is only a small percentage of its mean value.

The dimensions of this condenser were ascertained in the following way:—The length of the cylinder was measured by beam compasses, and the diameters of the inner and outer cylinders by callipers; the difference of these readings was not, however, taken as the distance between the cylinders, for though the error made in determining the diameter of either cylinder may be a small fraction of either diameter, yet since the diameters are nearly equal, it may not be a small fraction of their difference. The distance between the cylinders was determined by fastening the middle pieces of the two cylinders down to a flat board by a thin layer of shellac, and then filling the space between them with water which had been boiled a few hours before the experiment so as to be in a condition to absorb any air-bubbles that might be formed. The quantity of water required to fill this space was carefully weighed. This gives the volume of the water, and knowing the length of the cylinder and the diameter of one of them, the difference of the diameters can be calculated.

The results of these measurements are:—

LENGTH of cylinder, measured by beam compasses.

60·97

60·965

60·97

—

Mean 60·968 centims.

INTERNAL diameter of outer cylinder, measured by callipers.

9·986

9·989

9·992

—

Mean 9·989 inches, or 25·372 centims.

EXTERNAL diameter of inner cylinder, measured by callipers.

9·254

9·255

9·250

—

Mean 9·253 inches, or 23·50 centims.

WEIGHT of water required to fill the space between the cylinders.

4406·8 grammes at 17·5° C.

4404·6 „ 13·5°

4401 „ 12·2

4403 „ 11·5

—

Mean 4405·1 grammes.

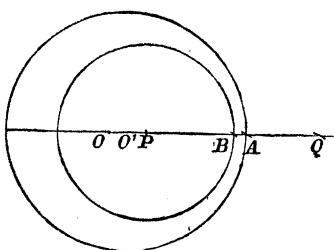
Since the greatest difference in temperature does not affect the result by one part in a thousand, the correction for temperature is neglected.

We find from these numbers that the distance between the cylinders is '941 centim.

When the distance between the cylinders was measured by hair dividers, the least distance was found to be '826 centim., the greatest '984 centim., giving '79 centim. as the distance between the axes of the cylinders.

Since the axes of the cylinders are not quite coincident, we cannot calculate the capacity by the ordinary formula. We proceed to investigate a formula which will hold in this case.

Fig. 4.



Let the figure represent a section of the cylinders by a plane perpendicular to their axes. Let O be the centre of the section of the cylinder O A, O' the centre of the section of O' B. Let O A = a , O' B = b .

Let P and Q be inverse points with respect to both circles, so that

$$OP \cdot OQ = a^2$$

$$O'P \cdot O'Q = b^2$$

Then if $\phi = A - B \log r_1/r_2$, when r_1, r_2 are the distances of a point T from P, Q respectively, ϕ will satisfy LAPLACE's equation and will be constant over both cylinders. Thus ϕ will be the potential of the electrical distribution, and by comparison with the ordinary form for the potential of an electrified cylinder we see that $\frac{1}{2} B$ will be the quantity of electricity per unit length upon either cylinder. Let the outer cylinder be connected with the earth so that its potential is zero, and let the potential of the inner cylinder be V.

Then we have

$$0 = A - B \log \frac{PA}{QA}$$

$$V = A - B \log \frac{PB}{QB}$$

therefore
$$V = B \left\{ \log \frac{PA}{QA} - \log \frac{PB}{QB} \right\} = B \log \frac{PA \cdot QB}{QA \cdot PB}$$

$$B = \frac{V}{\log \frac{PA \cdot QB}{QA \cdot PB}}$$

but $\frac{1}{2} B$ is the quantity of electricity per unit length upon either cylinder, and since the capacity is the quantity of electricity divided by the difference of potential, the capacity of the two cylinders

$$= \frac{1}{2} \frac{l}{\log \frac{PA \cdot QB}{QA \cdot PB}}$$

where l is the length of either cylinder.

Let

$$OP = x \quad OO' = c$$

then

$$OQ = \frac{a^2}{x} \quad PA = a - x \quad QA = \frac{a^2}{x} - a$$

therefore

$$\frac{PA}{QA} = \frac{x}{a}$$

similarly

$$\frac{QB}{PB} = \frac{b}{x-c}$$

Since $O'P \cdot O'Q = b^2$ we have

$$(x-c) \left(\frac{a^2}{x} - c \right) = b^2$$

therefore

$$(a^2 + c^2 - b^2) - c \frac{a^2}{x} - cx = 0$$

or

$$x^2 + x \frac{b^2 - (a^2 + c^2)}{c} + a^2 = 0$$

Solving we find that

$$x = \frac{ca^2}{a^2 - b^2} \left\{ 1 + \frac{b^2 c^2}{(a^2 - b^2)^2} \right\}$$

approximately, supposing that as in our condenser $\frac{c^2}{a^2 - b^2}$ is small.

therefore

$$x - c = \frac{cb^2}{a^2 - b^2} \left\{ 1 + \frac{a^2 c^2}{(a^2 - b^2)^2} \right\}$$

$$\log \frac{PA \cdot QB}{QA \cdot PB} = \log \frac{x \cdot b}{a(x-c)} = \log \frac{a \left(1 + \frac{b^2 c^2}{(a^2 - b^2)^2} \right)}{b \left(1 + \frac{a^2 c^2}{(a^2 - b^2)^2} \right)}$$

so that the capacity of the condenser

$$= \frac{1}{2} \frac{l}{\log \left\{ \frac{a \left(1 + \frac{b^2 c^2}{(a^2 - b^2)^2} \right)}{b \left(1 + \frac{a^2 c^2}{(a^2 - b^2)^2} \right)} \right\}}$$

$$= \frac{1}{2} \cdot \frac{l}{\log \frac{a}{b} - \frac{c^2}{a^2 - b^2}} \text{ approximately.}$$

Substituting the values of $a-b$ and b given above we find

$$\log \frac{a}{b} = 0.07705$$

$$\frac{c^2}{a^2-b^2} = 0.00027$$

and the electrostatic measure of the capacity of the condenser is consequently 396.8.

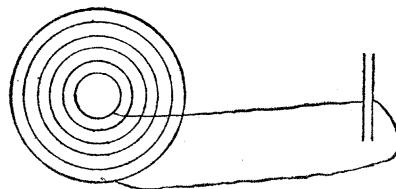
PART II.

The comparison of the capacity of this condenser with that of one without a guard ring.

As this condenser could not be worked by a commutator on account of the guard ring, it was necessary to compare its capacity with the capacity of a condenser without a guard ring. At first it was intended to compare the guard ring condenser with one of considerably greater capacity. Such a condenser was constructed, having a number of brass discs separated by thin pieces of shellac, the alternate discs being electrically connected, a weight was placed upon the disc at the top to keep the system steady; and the system was placed in a vessel formed by putting a bell-jar on a surface plate. There were two openings into this vessel, one of these was connected with a water pump; the other with the air outside the jar by a series of tubes filled with cotton wool and chloride of calcium, to free the air passing through them from dust and moisture; air was then pumped through the vessel for about 24 hours, and both openings were then closed. The capacity of this condenser was compared with that of the guard ring condenser, by connecting one plate of each condenser to earth, and the other with two points, P and Q, of a battery circuit; resistance boxes being placed between P and Q. A point O of the circuit between P and Q was then put to earth, and the resistance in the parts O P, O Q, so adjusted that when the charges of the two condensers were sent simultaneously into an electrometer there was no deflection of the needle, showing that the charges in the two condensers were then equal and opposite. In this case, the capacities of the condensers, whose plates were connected with P and Q respectively, would bear the same ratio to each other as the resistance in O Q bears to the resistance in O P. With the battery-power obtainable, this method however was found not to be sufficiently sensitive, as the resistance in either of the arms O P, O Q, could be altered by about .75 per cent. without appreciably disturbing the equilibrium of the needle of the electrometer when the charges of the condenser were sent into it. It was therefore decided to make a condenser without a guard ring equal in capacity to the guard ring condenser, and employ the method given in § 229 of MAXWELL's 'Electricity and Magnetism,' to determine when the two condensers were of equal capacity; this method can be made

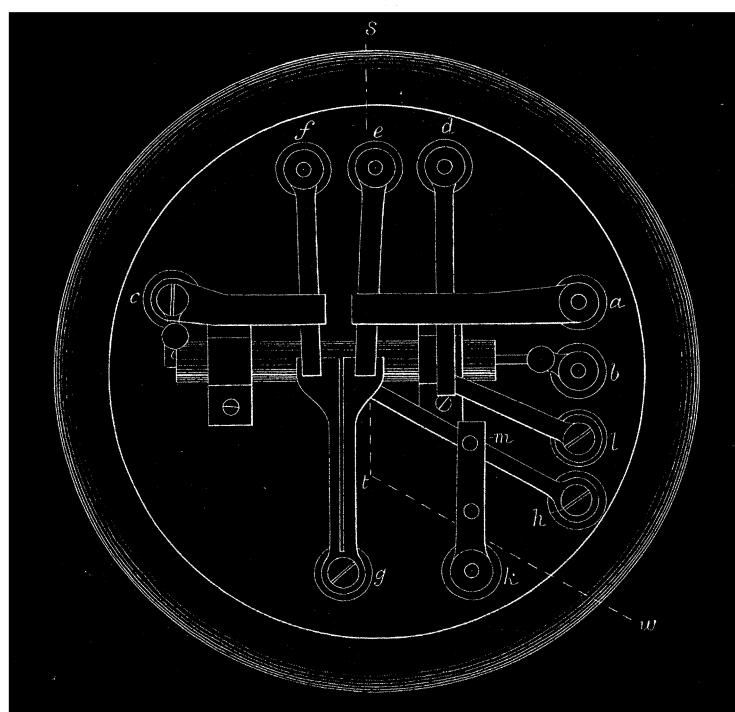
much more sensitive than the one just described, as the Leyden-jar used in MAXWELL's method can easily be raised by an electrophorus to a very high potential. The new condenser consisted of several co-axial tubes represented in section in fig. 5, The alternate tubes were connected together, and the two series connected with opposite plates of a very fine plate condenser, which was very kindly lent to me by the Rev. COUTTS TROTTER, Fellow of Trinity College.

Fig. 5.

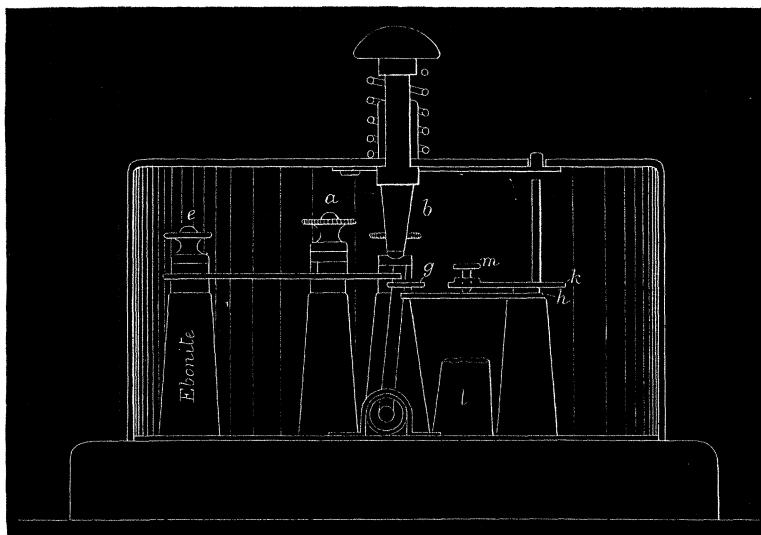


A rough adjustment could be made by altering the number of tubes connected together, while the fine adjustment was effected by altering the distance between the plates of the plate condenser by means of a finely cut screw. The equality of this condenser and the guard ring condenser was tested by the method given in § 229 of MAXWELL's 'Electricity and Magnetism,' using a key which was very kindly lent to me by Dr. JOHN HOPKINSON, F.R.S., and which had been used by him for a similar purpose. I quote the description given of it by him in his paper on the "Electrostatic Capacity of Glass," Part II., p. 360, Phil. Trans., Part II., 1881:—

"Fig. 3.



“Fig. 4.

“Section on line *s, t, w.*

“*a, c* are stiff insulated horizontal contact bars connected to the two poles of the battery. *d, e, f* are insulated springs normally touching *a* and *c* on the under side. *d* is connected by a wire to the guard ring, *e* to the plate of guard ring condenser, *f* to the sliding condenser. *b* is an insulated binding screw connected with *c* for the purpose of more conveniently introducing the battery wire. *l* is a spring connected to earth. *k* is a stiff insulated piece carrying an adjustable point *m*, normally in contact with the upper side of the insulated spring *h*. From *k* a wire leads to the quadrant of the electrometer. *k* can at any moment be put to earth by a spring key. The insulated spring *g* has its end between *e, f*, and *h*, and is normally in contact with neither. The springs *d, e, f* can be simultaneously bent downwards by an insulated plunger. When this plunger is struck downwards we have the following operations effected in a fraction of a second—

- 1°. $\begin{cases} d \text{ and } e \text{ are in contact with } a. \\ f \text{ in contact with } c. \end{cases}$
- 2°. *d, e, and f* insulated.
- 3°. $\begin{cases} d \text{ connected to } l. \\ e, f, \text{ and } g \text{ connected together.} \end{cases}$
- 4°. *e, f, g, h, k* connected together.
- 5°. Connexion of *k* and *h* broken.

* * * * *

“The whole switch, binding screws and all, is covered with a brass cover connected to earth and provided with apertures for the connecting wires. The ebonite legs which carry the pieces *a, b, c, d, e, f, g, k* are attached to a brass base plate, so that if any leakage occur from *a, b, c, d, e, or f*, it shall be to earth and not to the electrometer.”

The connexions are made in the following way. Let A denote the outer cylinder of the guard ring condenser, B the guard ring pieces, and C the inner cylinder; let A' and C' denote the plates of the other condenser; T the armature of the Leyden-jar, which is not connected with the earth. Then A is connected with the earth; B to A' and to *d* of the key, C to *e* of the key, C' to *f* and T to *a*; *b* and *l* are connected with the earth, and *h* is connected with the electrometer.

Before the plunger is pushed down A is put to earth; B and C to T; A' to T; C' to earth.

When the plunger is pushed down, before it reaches *e* and *g*, A is to earth. B and C are charged and disconnected. A' and C' have equal and opposite charges.

When the plunger is pushed down a little further, so that *d* comes into contact with *l*, B and A' are put to earth, so that the charges on C and C' are free to flow into the electrometer when the plunger goes a little further and strikes *h*.

If the capacities of the two condensers are equal, the charges on C and C' are equal, and of opposite signs, and when they flow together into the electrometer, their combined effect will be zero. The distance between the plates of the plate-condenser was altered, until the needle of the electrometer was not deflected when the plunger of the key was pushed down. This method was found to be very sensitive; if after a balance had been obtained, the capacity of one condenser was altered by 1 per cent., the quantity of electricity sent to the electrometer was sufficient to drive the spot of light off the scale.

The insulation of the two condensers and the key was tested several times, both electrostatically and by attempting to pass a current through them. If either condenser was charged, and the key put in electrical connexion with it, the loss of charge in five minutes was not quite 2 per cent., so that the loss in the small time required to push the plunger down is quite negligible. When the condensers and the key were put in circuit with a battery of 150 DANIELL's elements, no current could be detected with a galvanometer whose resistance was 11,000 ohms.

PART III.

The determination of the electromagnetic measure of the capacity of the condenser without the guard ring.

This was effected by the method described at the commencement of this paper.

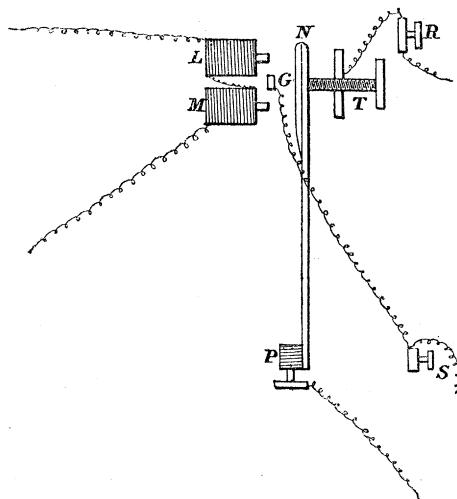
The galvanometer used had a resistance of about 11,000 ohms. It was insulated by placing it upon pieces of glass coated with paraffin.

The battery consisted of 150 DANIELL's cells, coupled up with 25 LECLANCHÉ. The DANIELL's cells were put into trays, containing 10 each. The resistance of the battery was about 2500 ohms. The insulation of the battery gave a considerable amount of trouble, but the following plan was found successful. The case containing the trays

was placed on glass supports about 2 inches thick covered with a thin layer of paraffin, while each tray was insulated from the case by pieces of ebonite.

The commutator was one which had been previously used by Lord RAYLEIGH, and had been designed by him.

Fig. 6.



The current from two GROVES' cells passes first through a tuning-fork interruptor, and then through the coils L M of an electromagnet. P N is a strip of brass with a piece of iron attached to it. When there is no current passing through the electromagnet, the elasticity of the rod P N makes it press against a screw T, which is electrically connected with a binding screw R: when the current passes through the electromagnet the magnet attracts the iron attached to the rod P N and brings it into contact with the stop G, which is electrically connected with the binding screw S. The letters P, R, S indicate the same points in this figure as in fig. 1. All the places where contact is made by the vibrating piece P N are covered with platinum, and the whole arrangement is fastened down to an ebonite board. As the current passes intermittently through the coils L N of the electromagnet, the vibrating piece P N strikes alternately against the parts G and T; when it strikes against G the opposite plates of the condenser are connected with the two poles of the battery; when it strikes against T the condenser is discharged (see fig. 1).

This commutator was found to work extremely well. When it was in good order the spot of light reflected from the mirror of the galvanometer through which the intermittent current passed never moved off one division of the scale, and the only thing by which the deflection could be distinguished from one due to a steady current was a slight indistinctness in the edge of the image of the spot of light.

The speed of the tuning-fork interruptor was found by comparing it with that of the standard fork used by Lord RAYLEIGH in his determination of the ohm in absolute measure. The standard fork vibrates about 128 times per second, while the tuning-fork used in this investigation vibrates about 32 times per second. This fork was

used to drive another of about four times its frequency, and the number of beats per second between this driven fork and Lord RAYLEIGH's standard fork was counted. At the temperature of 15° C. there were 12 beats in 20 seconds between the two forks, and the standard fork vibrated more slowly than the other. The standard fork makes 128.15 vibrations per second, so that if n be the number of vibrations per second of the fork used to drive the commutator, we have

$$4n \times 20 - 12 = 128.15 \times 20$$

$$n = 32.1875.$$

The observations.

The observations consisted of two parts. The capacity of the movable condenser had to be adjusted until it was equal to the capacity of the guard ring condenser. This was ascertained by the method described in Part II.; and then this adjustable condenser was put in the WHEATSTONE's bridge as in fig. 1, and the resistances of the arms of the bridge adjusted so that the deflection of the galvanometer due to the steady current was just balanced by the deflection due to the intermittent current arising from the flow of electricity to the condenser when the movable piece P was in contact with S. The resistances in the arms A D, B C (fig. 1) were kept constant, and the adjustment was effected by altering the resistance in A C.

The steady current, when it was not balanced by the current arising from the charging of the condenser, produced a deflection of the dot of light reflected from the mirror of the galvanometer of about 120 scale divisions, and as a fine wire was placed before the lamp of the galvanometer and focussed on the scale, readings could easily be made to quarter of a division.

The following are the results of the observations, and it may be worthy of remark that, as many of the pieces of apparatus used were required for the ordinary work of the laboratory, the whole arrangement had to be taken down and put together again between each determination. This must have had the effect of getting rid of a good many accidental errors, and taking it into consideration the following numbers seem as near together as could be expected for such complex observations. The resistances are given in B.A. units.

RESISTANCES in the various arms of the WHEATSTONE's bridge, when there was no deflection of the galvanometer.

	Resistance in the arm A. D.	Resistance in B. C.	Resistance in A. C.
1.	899,666	99,920	1294
2.	899,666	99,920	1285
3.	899,930	99,950	1297
4.	899,700	99,925	1287
5.	899,700	99,925	1297

The mean of these correct to 1/10 per cent. is

$$899,700 \quad 99,925 \quad 1292$$

According to Lord RAYLEIGH's determination of the ohm the B.A.

$$= \cdot 987 \times 10^9$$

so that from the formula $nC = a/cd$ we find that the electromagnetic measure of the capacity of the condenser $= \cdot 4517 \times 10^{-19}$.

The electrostatic measure of the capacity of the same condenser is 396.8.

So if v be the ratio of the electrostatic unity of electricity to the electromagnetic

$$v^2 = \frac{396.8 \times 10^{19}}{\cdot 4517}$$

$$v = 2.963 \times 10^{10} \text{ in C.G.S. units.}$$

Some experiments were made with a tuning-fork vibrating 44 times a second; the results of those were found to agree very closely with those obtained when the tuning-fork vibrated 32.18 per second. The above experiments were made in the Cavendish Laboratory, Cambridge, and I have much pleasure in thanking Lord RAYLEIGH for the very valuable advice which he gave to me throughout the investigation, as well as for his kindness in designing several of the more important pieces of apparatus.