PROCEEDINGS

OF

THE ROYAL SOCIETY.

May 2, 1889.

Professor G. G. STOKES, D.C.L., President, in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

In pursuance of the Statutes the names of the Candidates recommended for election into the Society were read from the Chair as follows:—

Aitken, John.
Ballard, Edward, M.D.
Basset, Alfred Barnard, M.A.
Brown, Horace T., F.C.S.
Clark, Latimer, C.E.
Cunningham, Professor David Douglas, M.B.
Fletcher, Lazarus, M.A.
Hemsley, William Botting, A.L.S.
Hudson, Charles Thomas, M.A.
Hughes, Professor Thomas McKenny, M.A.
Poulton, Edward B., M.A.
Sollas, Professor William Johnson, D.Sc.
Todd, Charles, M.A.
Tomlinson, Herbert, B.A.
Yeo, Professor Gerald F., M.D.

Professor Georg Friedrich Julius Arthur Auwers, Foreign Member (elected 1879), signed the obligation in the Charter Book and was admitted into the Society.

The following Papers were read:—

I. "Note on the Effect produced by Conductors in the Neighbourhood of a Wire on the Rate of Propagation of Electrical Disturbances along it, with a Determination of this Rate." By J. J. THOMSON, M.A., F.R.S., Cavendish Professor of Experimental Physics, Cambridge. Received April 1, 1889.

In a paper on "The Resistance of Electrolytes to the Passage of very rapidly Alternating Currents" ('Roy. Soc. Proc.,' vol. 45,

p. 270), I have shown that if Maxwell's theory that electricity moves like a perfectly incompressible fluid is not true, the rate of propagation of very rapidly alternating currents along a wire placed at an infinite distance from other conductors cannot be the same as the rate of propagation of the electrodynamic action through the surrounding dielectric. As Hertz, in his experiments on the rate of propagation of electrical waves along a metal wire, found that these rates were not the same, it might appear that this proved unmistakably that Maxwell's theory is untenable. I wish in this note to show that, assuming Maxwell's theory, we can explain the smaller velocity of propagation along wires found by Hertz, by taking into account the capacity of the wire, if the wire is not at a very great distance from other conductors; in fact that the capacity of the wire produces much the same effect as the "compressibility" of the electricity which is supposed to exist in all theories other than Maxwell's. In the case of a "free" wire in the laboratory, the electrical effects produced by the walls and floors are indefinite. I shall, therefore, consider on the most general theory the case of a wire surrounded by a coaxial metal cylinder, a case where the electrical conditions are perfectly definite if the electrical oscillations are very rapid.

I shall consider the case of a cylindrical wire surrounded by a cylindrical sheath of dielectric, which in its turn is surrounded by a third substance, either a conductor or another dielectric. The axis of the wire is taken as the axis of z, and all the variable quantities are supposed to vary as $e^{\iota(mz+pt)}$: the notation is as nearly as possible the same as the former paper. Let a be the radius of the wire, σ its specific resistance, b the outer radius of the sheath of dielectric, and K its specific inductive capacity; let us first suppose that this is surrounded by a substance whose specific resistance is σ'.

Let ϕ denote the electrostatic potential, and let

$$\phi = e^{\iota(mz+pt)}\mathrm{A}\mathrm{J}_0(\iota qr), \text{ in the wire,}$$

$$= e^{\iota(mz+pt)}(\mathrm{B}\mathrm{J}_0(\iota q'r)+\mathrm{C}\mathrm{I}_0(\iota q'r)), \text{ in the dielectric,}$$

$$= e^{\iota(mz+pt)}\mathrm{D}\mathrm{I}_0(\iota q''r), \text{ in the outer conductor;}$$

where $$q^2 = m^2-\frac{p^2}{\omega^2}, \qquad q'^2 = m^2-\frac{p^2}{\omega'^2}, \qquad q''^2 = m^2-\frac{p^2}{\omega''^2},$$

where ω, ω', ω'' are the velocities of propagation of the electrostatic potential in the wire, sheath, and outer conductor respectively; in Maxwell's theory these are all infinite, and $q = q' = q'' = m$.

If F, G, H are the components of the vector potential, we may put

$$H = e^{\iota(mz+pt)}EJ_0(\iota nr) + \frac{\nu}{\iota p}\frac{d\phi}{dz}, \text{ in the wire,}$$

$$= e^{\iota(mz+pt)}(FJ_0(\iota\kappa r) + GI_0(\iota\kappa r)) + \frac{\nu'}{\iota p}\frac{d\phi}{dz}, \text{ in the dielectric sheath,}$$

$$= e^{\iota(mz+pt)}LI_0(\iota n'r) + \frac{\nu''}{\iota p}\frac{d\phi}{dz}, \text{ in the outer conductor;}$$

where

$$n^2 = m^2 + \frac{4\pi\iota p\mu}{\sigma}, \qquad \kappa^2 = m^2 - \frac{p^2}{v^2}, \qquad n'^2 = m^2 + \frac{4\pi\iota p\mu'}{\sigma'},$$

v being the velocity of propagation of electrodynamic action through the sheath, μ and μ' the magnetic permeabilities of the wire and outer conductor respectively, and ν and ν' constants.

$$\left.\begin{aligned} F &= \frac{d\chi}{dx} + \frac{\nu}{\iota p}\frac{d\phi}{dx} \\ G &= \frac{d\chi}{dy} + \frac{\nu}{\iota p}\frac{d\phi}{dy} \end{aligned}\right\} \text{ in the wire,}$$

where

$$\chi = \frac{\iota m}{n^2}e^{\iota(mz+pt)}EJ_0(\iota nr);$$

$$\left.\begin{aligned} F &= \frac{d\chi'}{dx} + \frac{\nu'}{\iota p}\frac{d\phi}{dx} \\ G &= \frac{d\chi'}{dy} + \frac{\nu'}{\iota p}\frac{d\phi}{dy} \end{aligned}\right\} \text{ in the dielectric,}$$

where

$$\chi' = -\frac{\iota m}{\kappa^2}e^{\iota(mz+pt)}(FJ_0(\iota\kappa r) + GI_0(\iota\kappa r));$$

and

$$\left.\begin{aligned} F &= \frac{d\chi''}{dx} + \frac{\nu''}{\iota p}\frac{d\phi}{dx} \\ G &= \frac{d\chi''}{dx} + \frac{\nu''}{\iota p}\frac{d\phi}{dy} \end{aligned}\right\} \text{ in the outer conductor,}$$

where

$$\chi'' = \frac{\iota m}{n'^2}LI_0(\iota n'r).$$

From the continuity of ϕ at the surfaces $r = a$, $r = b$, we have

$$\left.\begin{aligned} AJ_0(\iota qa) &= BJ_0(\iota q'a) + CI_0(\iota q'a) \\ DI_0(\iota q''b) &= BJ_0(\iota q'b) + CI_0(\iota q'b) \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (1).$$

From the continuity of H we get

$$\left.\begin{aligned} \mathrm{EJ}_0(\iota na) &= \mathrm{FJ}_0(\iota\kappa a) + \mathrm{GI}_0(\iota\kappa a) + \frac{\nu' - \nu}{\iota p}\,\iota m \mathrm{AJ}_0(\iota q a) \\ \mathrm{LI}_0(\iota n' b) &= \mathrm{FJ}_0(\iota\kappa b) + \mathrm{GI}_0(\iota\kappa b) + \frac{\nu' - \nu''}{\iota p}\,\iota m \mathrm{DI}_0(\iota q'' b) \end{aligned}\right\} \ldots (2).$$

From the continuity of F and G we get

$$\left.\begin{aligned} \frac{m}{n}\mathrm{EJ}_0'(\iota na) &= \frac{m}{\kappa}(\mathrm{FJ}_0'(\iota\kappa a) + \mathrm{GI}_0'(\iota\kappa a)) + \frac{\nu'\iota q'}{\iota p}(\mathrm{BJ}_0'(\iota q' a) \\ &\qquad + \mathrm{CJ}_0'(\iota q' a)) - \frac{\nu \iota q}{\iota p}\mathrm{AJ}_0'(\iota q a), \\ \frac{m}{n'}\mathrm{LI}_0'(\iota n' b) &= \frac{m}{\kappa}(\mathrm{FJ}_0'(\iota\kappa b) + \mathrm{GI}_0'(\iota\kappa b)) + \frac{\nu'\iota q'}{\iota p}(\mathrm{BJ}_0'(\iota q' b) \\ &\qquad + \mathrm{CJ}_0'(\iota q' b)) - \frac{\nu''\iota q''}{\iota p}\mathrm{DJ}_0'(\iota q'' b). \end{aligned}\right\} \ldots\ldots (3).$$

The magnetic force parallel to the surface of the wire is

$$\frac{1}{\mu}\left(\frac{d}{dz}\frac{d\chi}{dr} - \frac{d\mathrm{H}}{dr}\right),$$

and, since this is continuous, we have

$$\left.\begin{aligned} \frac{1}{\mu}\frac{m^2 - n^2}{n}\mathrm{EJ}_0'(\iota na) &= \frac{m^2 - \kappa^2}{\kappa}(\mathrm{FJ}_0'(\iota\kappa a) + \mathrm{GI}_0'(\iota\kappa a)), \\ \frac{1}{\mu'}\frac{m^2 - n'^2}{n'}\mathrm{LI}_0'(\iota n' b) &= \frac{m^2 - \kappa^2}{\kappa}(\mathrm{FJ}_0'(\iota\kappa b) + \mathrm{GI}_0'(\iota\kappa b)). \end{aligned}\right\} \ldots (4).$$

Eliminating E and L from equations (2) and (4), we get

$$\begin{aligned} &\mathrm{F}\left(\frac{m^2 - n^2}{\mu n}\mathrm{J}_0(\iota\kappa a)\mathrm{J}_0'(\iota na) - \frac{m^2 - \kappa^2}{\kappa}\mathrm{J}_0'(\iota\kappa a)\mathrm{J}_0(\iota na)\right) \\ &+ \mathrm{G}\left(\frac{m^2 - n^2}{\mu n}\mathrm{J}_0'(\iota na)\mathrm{I}_0(\iota\kappa a) - \frac{m^2 - \kappa^2}{\kappa}\mathrm{J}_0(\iota na)\mathrm{I}_0'(\iota\kappa a)\right) \\ &= \frac{\nu - \nu'}{\iota p}\,\iota m\,\frac{m^2 - n^2}{\mu n}\mathrm{AJ}_0(\iota pa)\mathrm{J}_0'(\iota na) \quad \ldots\ldots\ldots\ldots\ldots\ldots (5), \end{aligned}$$

$$\begin{aligned} &\mathrm{F}\left(\frac{m^2 - n^2}{\mu' n'}\mathrm{J}_0(\iota\kappa b)\mathrm{I}_0'(n\ \ b) - \frac{m^2 - \kappa^2}{\kappa}\mathrm{J}_0'(\iota\kappa b)\mathrm{I}_0(\iota nb)\right) \\ &+ \mathrm{G}\left(\frac{m^2 - n'^2}{\mu' n'}\mathrm{I}_0(\iota\kappa b)\mathrm{I}_0'(\iota n' b) - \frac{m^2 - \kappa^2}{\kappa}\mathrm{I}_0'(\iota\kappa b)\mathrm{I}_0(\iota nb)\right) \\ &= \frac{\nu'' - \nu'}{\iota p}\,\iota m\,\frac{(m^2 - n'^2)}{\mu' n'}\mathrm{DI}_0(\iota pb)\mathrm{I}_0'(\iota n' b) \quad \ldots\ldots\ldots\ldots\ldots\ldots (6). \end{aligned}$$

Eliminating E and F from equations (3) and (4), we get

$$\{\mathrm{F}\mathrm{J}_0'(\iota\kappa a)+\mathrm{G}\mathrm{I}_0'(\iota\kappa a)\}\,\frac{1}{\kappa}\left\{1-\frac{m^2-\kappa^2}{m^2-n^2}\mu\right\}$$

$$=\frac{\nu q}{pm}\,\mathrm{A}\mathrm{J}_0'(\iota q a)-\frac{\nu' q'}{pm}\,(\mathrm{B}\mathrm{J}_0'(\iota q' a)+\mathrm{C}\mathrm{I}_0'(\iota q' a))\;\ldots\ldots\;(7),$$

$$\{\mathrm{F}\mathrm{J}_0'(\iota\kappa b)+\mathrm{G}\mathrm{I}_0'(\iota\kappa b)\}\,\frac{1}{\kappa}\left\{1-\frac{m^2-\kappa^2}{m^2-n^2}\mu'\right\}$$

$$=\frac{\nu'' q''}{pm}\,\mathrm{D}\mathrm{I}_0'(\iota q'' b)-\frac{\nu' q'}{pm}\,(\mathrm{B}\mathrm{J}_0'(\iota q' b)+\mathrm{C}\mathrm{I}_0'(\iota q' b))\ldots\ldots\;(8).$$

We can substitute for B and C from equations (1) and get four equations from which we can eliminate F, G, A, and D; as, however, the result is lengthy, we shall only solve it for the particular case with which we are concerned, when qa, $q'a$, $q'b$, and $q''b$, κa and κb, are all small. We get, substituting in the terms multiplied by $\nu'-\nu$, the approximate values of the Bessel's functions for small quantities of the variable

$$\left\{\frac{(m^2-n^2)}{\mu n}\,\mathrm{J}_0(\iota\kappa a)\mathrm{J}_0'(\iota n a)-\frac{(m^2-\kappa^2)}{\kappa}\,\mathrm{J}_0'(\iota\kappa a)\mathrm{J}_0(\iota n a)\right\}$$

$$\left\{\frac{(m^2-n'^2)}{\mu' n'}\,\mathrm{I}_0(\iota\kappa b)\mathrm{I}_0'(\iota n' b)-\frac{(m^2-\kappa^2)}{\kappa}\,\mathrm{I}_0'(\iota\kappa a)\mathrm{I}_0(\iota n b)\right\}$$

$$=\left\{\frac{m^2-n^2}{\mu n}\,\mathrm{I}_0(\iota\kappa a)\mathrm{J}_0'(\iota n a)-\mathrm{I}_0'(\iota\kappa a)\left[\frac{m^2-\kappa^2}{\kappa}\,\mathrm{J}_0(\iota n a)\right.\right.$$

$$\left.\left.-\iota\,\frac{(\nu'-\nu)}{\nu'}\,\frac{m^2}{\kappa}\,\frac{m^2-n^2}{\mu n}\,a\mathrm{J}_0'(\iota n a)\log\frac{b}{a}\right]\right\}$$

$$\times\left\{\frac{m^2-n'^2}{\mu n'}\,\mathrm{J}_0(\iota\kappa b)\mathrm{I}_0'(\iota n' b)-\frac{m^2-\kappa^2}{\kappa}\,\mathrm{J}_0'(\iota\kappa b)\mathrm{I}_0(\iota n b)\right\}.$$

In this equation the approximate values of the Bessel's functions have been used only in the term multiplied by $\nu'-\nu$.

This equation simplifies very much, since κa and κb are very small, and therefore approximately

$$\mathrm{J}_0(\iota k a)=1,\qquad \mathrm{J}_0'(\iota k a)=-\tfrac{1}{2}\iota k a,\qquad \mathrm{I}_0(\iota k a)=\log\gamma i k a,$$
$$\text{and }\mathrm{I}_0'(\iota k a)=1/\iota k a.$$

Making these substitutions, the above equation reduces to

$$\kappa^2=\iota\,\frac{p^2}{v^2}\left\{\frac{\mu n}{m^2-n^2}\,\frac{1}{a}\,\frac{\mathrm{J}_0(\iota n a)}{\mathrm{J}_0'(\iota n a)}-\frac{\mu' n'}{m^2-n'^2}\,\frac{1}{b}\,\frac{\mathrm{I}_0(\iota n b)}{\mathrm{I}_0'(\iota n' b)}\right\}\frac{1}{\log(b/a)}+\frac{\nu'-\nu}{\nu'}\,m^2,$$

(see also "Electrical Oscillations on Cylindrical Conductors," 'London Math. Soc. Proc.,' vol. 17, p. 320),

or,

$$m^2\frac{\nu}{\nu'} = \frac{p^2}{v^2}\left\{1+\iota\left(\frac{\mu n}{m^2-n^2}\frac{1}{a}\frac{J_0(\iota na)}{J_0'(\iota na)}-\frac{\mu' n'}{m^2-n'^2}\frac{1}{b}\frac{I_0(\iota n'b)}{I_0'(\iota n'b)}\right)\frac{1}{\log(b/a)}\right\} \quad (9).$$

The nature of the solution of this equation will depend upon the magnitudes of na and $n'b$.

Case I. na and $n'b$ both small. In this case

$$J_0(\iota na)/J_0'(\iota na) = -2/\iota na \quad \text{and} \quad I_0(\iota n'b)/I_0'(\iota n'b) = \iota n'b \log \gamma\iota n'b.$$

Making these substitutions, equation (9) becomes

$$m^2\frac{\nu}{\nu'} = \frac{p^2}{v^2}\left\{1-\frac{\log\gamma\,\iota n'b}{\log(b/a)}+\iota\frac{\sigma}{2\pi a^2 p}\frac{1}{\log(b/a)}\right\} \quad \ldots\ldots (10).$$

Since na is by hypothesis small, $\sigma/2\pi a^2 p$ is large compared with unity, and unless $\log(b/a)$ is very great, it will be much the largest term inside the bracket, so that (10) may be written

$$m^2\frac{\nu}{\nu'} = \frac{p^2}{v^2}\frac{\iota\sigma}{2\pi a^2 p}\frac{1}{\log(b/a)},$$

$$m = \frac{p}{v}\sqrt{\left\{\frac{\nu'\sigma}{\nu 4\pi a^2 p\log(b/a)}\right\}}\cdot(1+\iota).$$

This represents a disturbance propagated with the velocity

$$v\left\{\left(\frac{4\ \nu a^2 p}{\sigma\nu'}\right)\log(b/a)\right\}^{\frac{1}{2}},$$

and dying away to $1/e$ of its original value after traversing a distance

$$\left\{\left(\frac{4\pi\nu a^2}{\nu'\sigma p}\right)\log(b/a)\right\}^{\frac{1}{2}}.$$

This case, which is that of slowly alternating currents, was solved many years ago by Sir William Thomson.

Case II. na large, $n'b$ small. This is the case of rapidly alternating currents travelling along a wire which is surrounded by a substance whose conductivity is so small that $4\pi\mu' pb^2/\sigma'$ is a small quantity.

In this case, since $J_0'(\iota na) = \iota J_0(\iota na)$, equation (9) reduces to

$$m^2\frac{\nu}{\nu'} = \frac{p^2}{v^2}\left\{1-\frac{\mu}{na}\frac{1}{\log(b/a)}-\frac{\log\gamma\,\iota n'b}{\log(b/a)}\right\} \quad \ldots\ldots (11).$$

Since na is large, the second term in the bracket will be small for wires made of non-magnetic metals; so that for this case (11) reduces to

$$m^2\frac{\nu}{\nu'} = \frac{p^2}{v^2}\left\{1+\frac{\log \gamma\, \iota n' b}{\log(b/a)}\right\},$$

or, substituting for n' the approximate value $\sqrt{\left(\frac{4\pi\mu'\iota p}{\sigma'}\right)}$,

$$m^2\frac{\nu}{\nu'} = \frac{p^2}{v^2}\left\{1+\tfrac{1}{2}\,\frac{\log(\sigma_1/4\pi\mu' p b^2)}{\log(b/a)}+\frac{\iota(\pi/4)}{\log(b/a)}\right\} \text{ approximately,}$$

or

$$m = \frac{p}{v}\left\{\frac{\nu'}{\nu}\right\}^{\frac{1}{2}}\left\{\frac{\log(\sigma'/4\pi\mu' p a^2)}{\log(b^2/a^2)}\right\}^{\frac{1}{2}}\left\{1+\iota\frac{\pi}{4}\frac{1}{\log(\sigma'/4\pi\mu' p a^2)}\right\}.$$

This represents a disturbance propagated with the velocity

$$v\sqrt{\frac{\nu}{\nu'}}\cdot\frac{1}{\sqrt{\left(1+\frac{\log(\sigma'/4\pi\mu' p b^2)}{\log(b^2/a^2)}\right)}},$$

and fading away to $1/e$ of its original value, after traversing a distance

$$\frac{4}{\pi}\frac{v}{p}\sqrt{\frac{\nu}{\nu'}}\sqrt{\left(\log\frac{b^2}{a^2}\log\frac{\sigma'}{4\pi\mu' p a^2}\right)},$$

or if λ is the wave-length of the electrical vibration, the distance a disturbance travels before falling to $1/e$ of its original value is

$$\frac{2\lambda}{\pi^2}\log\frac{\sigma'}{4\pi\mu' p a^2}.$$

Thus in this case, even if ν' equals ν, that is, if Maxwell's theory is correct, the rate of propagation of the disturbance along the wire will not be the same as that of electrodynamic action through air; and yet the conditions may be such as to allow a disturbance to pass over several wave-lengths before falling to $1/e$ of its original value. It will be noticed that the velocity of propagation does not depend on the specific resistance of the wire, and that it increases with the rapidity of the reversal, and that the rate at which the vibrations die away is independent of the resistance of the wire, and only varies slowly with the resistance of the outer conductor, since σ' only enters in the form $\log \sigma'$.

We can see the reason of this if we consider the amount of heat produced in the outer conductor.

If i is the current parallel to the axis of z passing through a section of the wire, then, assuming in the investigation that $\nu' = \nu$,

$$i = \frac{\iota p}{\sigma} \mathrm{E} e^{\iota(mz+pt)} \int_0^a 2\pi r \mathrm{J}_0(\iota n r) dr,$$

$$= \frac{\iota p}{\sigma} e^{\iota(mz+pt)} \mathrm{E} \frac{2\pi}{n^2} \iota n a \mathrm{J}_0'(\iota n a).$$

The rate of production of heat in the wire is

$$\frac{p^2}{\sigma} \mathrm{E}^2 e^{2\iota(mz+pt)} \int_0^a 2\pi r (\mathrm{J}_0(\iota n r))^2 dr$$

$$= \frac{p^2}{\sigma} \mathrm{E}^2 \pi a^2 \{\mathrm{J}_0'^2(\iota n a) + \mathrm{J}_0^2(\iota n a)\} e^{2\iota(mz+pt)}.$$

Hence the ratio of the heat generated in the wire to $\sigma i^2/\pi a^2$, the heat which would be generated if the current were uniformly distributed,

$$= \tfrac{1}{4} n^2 a^2 \frac{\{\mathrm{J}_0'^2(\iota n a) + \mathrm{J}_0^2(\iota n a)\}}{\mathrm{J}_0'^2(\iota n a)}.$$

Since when na is large $\mathrm{J}_0(\iota n a) = (e^{na}/\sqrt{2\pi n a})\left(1 + \frac{1}{8na}\right)$, this ratio

$$= \tfrac{1}{4} na.$$

The rate at which heat is generated in the outer conductor is

$$\frac{p^2}{\sigma'} e^{2\iota(mz+pt)} \mathrm{L}^2 \int_b^\infty 2\pi r \mathrm{I}_0^2(\iota n' r) dr,$$

$$\frac{p^2}{\sigma'} \mathrm{L}^2 \pi b^2 \{\mathrm{I}_0'^2(\iota n' b) + \mathrm{I}_0^2(\iota n' b)\} e^{2\iota(mz+pt)}.$$

By equations (7) and (8) we have

$$\frac{\mathrm{L}\mathrm{I}_0(\iota n' b)}{\mathrm{E}\mathrm{J}_0(\iota n a)} = \frac{a}{b} \frac{m^2 - n^2}{\mu n} \frac{\mu' n'}{m^2 - n'^2} \frac{\mathrm{J}_0'(\iota n a)}{\mathrm{J}_0(\iota n a)} \frac{\mathrm{I}_0(\iota n' b)}{\mathrm{I}_0'(\iota n' b)},$$

so that

$$\pi \mathrm{L} b \mathrm{I}_0'(\iota n' b) e^{\iota(mz+pt)} = \tfrac{1}{2} \frac{m^2 - n^2}{\mu} \frac{\mu' n'}{m^2 - n'^2} \cdot \frac{\sigma i}{p}$$

$$= \frac{2\pi\mu'}{n'} \cdot i, \text{ approximately.}$$

Thus the heat generated in the outer conductor

$$= 4 \frac{\pi p^2}{\sigma'} i^2 \cdot \frac{\mu'^2}{n_1^2} \left\{ 1 + \frac{\mathrm{I}_0^2(\iota n' b)}{\mathrm{I}_0'^2(\iota n' b)} \right\},$$

since $n'b$ is small, $\mathrm{I}_0'(\iota n' b)$ is large compared with $\mathrm{I}_0(\iota n' b)$, and n'^2 is approximately

$$4\pi\mu' \iota p/\sigma'.$$

Thus the rate at which heat is generated in the outer conductor is approximately

$$\mu' p i^2,$$

and is therefore approximately independent of the resistances of the wire and of the outer conductor, and large compared with the heat developed in the wire.

The case of an iron wire would differ from that investigated in the case when though na is large, μ/na is also large; in this case equation (9) becomes approximately

$$m^2 \frac{\nu}{\nu'} = \frac{p^2}{v^2}\left\{1 - \frac{\mu}{na}\right\},$$

which represents a vibration travelling with a smaller velocity than that of the electrodynamic action through the dielectric, and dying away to $1/e$ of its original value after traversing a space comparable with a wave-length. When the rate of alternation of the currents gets sufficiently rapid, $n'b$ gets large, and μ/na small, and we get

Case III. na and $n'b$ both large.

In this case, since $J_0'(\iota na) = \iota J_0(\iota na)$ and $I_0'(\iota n'b) = -\iota I_0(\iota n'b)$ and equation (9) reduces to

$$m^2 \frac{\nu}{\nu'} = \frac{p^2}{v^2}\left\{1 - \left(\frac{\mu}{na} + \frac{\mu'}{n'b}\right)\frac{1}{\log(b/a)}\right\}$$

$$= \frac{p^2}{v^2}\left\{1 + \frac{\iota}{\sqrt{(8\pi p)}}\left\{\sqrt{\left(\frac{\sigma\mu}{\mu a^2}\right)} + \sqrt{\left(\frac{\sigma'\mu'}{b^2}\right)}\right\}\frac{1}{\log(b/a)}\right\}.$$

$$m = \frac{p}{v}\sqrt{\frac{\nu}{\nu'}}\left\{1 + \frac{\iota}{2\sqrt{(8\pi p)}}\left\{\sqrt{\left(\frac{\sigma\mu}{a^2}\right)} + \sqrt{\left(\frac{\sigma'\mu'}{b^2}\right)}\right\}\frac{1}{\log(b/a)}\right\}.$$

This represents a vibration travelling with the velocity $v\sqrt{(\nu/\nu')}$, and dying away to $1/e$ of its original value after traversing a distance

$$4v\sqrt{\frac{\nu}{\nu'}}\sqrt{\frac{2\pi}{p}}\left\{\sqrt{\frac{\sigma\mu}{a^2}} + \sqrt{\frac{\sigma'\mu'}{b^2}}\right\}^{-1}\log(b/a).$$

From this equation we see that if σ/a^2 is very much greater than σ'/b^2, the decay of the disturbance will be due chiefly to the resistance of the wire, but if, on the other hand, σ'/b^2 is very much greater than σ/a^2, the decay will be due chiefly to the resistance of the outer conductor. This case includes that of a wire surrounded by a metal tube, the space between the tube and the wire being occupied by any dielectric, in this case the electrical conditions are perfectly definite, and we see that the velocity of propagation along the wire will be $v\sqrt{(\nu/\nu')}$, where v is the velocity of propagation of the electrodynamic action through the dielectric. Thus if $\nu' = \nu$, as in Maxwell's theory, the velocity along the wire will be the same as that through the dielectric, but it will not be so unless this condition is fulfilled. Thus this case would afford a definite means of testing whether or not Maxwell's theory is true. The thickness of the outer tube would be immaterial, as with these very rapid vibrations the currents are entirely confined to the inner skin of the tube.

By comparing the results for this case with those of Case II, we see that if the rate at which the electrical disturbances die away depends on the conductivity of the wire, the velocity of propagation through the wire must be the same as that through the dielectric if Maxwell's theory is true.

The preceding equations can be modified so as to include the case when the outer conductor is replaced by another dielectric; all that we have to do is in equation (9) to replace n' by κ', where

$$\kappa'^2 = m^2 - (p^2/v_1^2),$$

v_1 being the velocity of propagation through the outer dielectric.

In this case equation (9) becomes

$$\kappa^2 = \frac{\iota p^2}{v^2}\left\{\frac{\mu n}{m^2-n^2}\frac{J_0(\iota n a)}{J_0'(\iota n a)} - \frac{\kappa'}{m^2-\kappa_1^2}\frac{I_0(\iota\kappa' b)}{I_0'(\iota\kappa' b)}\right\}\frac{1}{\log(b/a)} + \frac{\nu'-\nu}{\nu'}m^2.$$

If both $\kappa' b$ and na are large, the velocity is the same as before, viz., $v\sqrt{(\nu/\nu')}$. If na is large and $\kappa' b$ small, the equation becomes approximately

$$\frac{\nu}{\nu'}m^2 - \frac{p^2}{v^2} = \kappa_1^2 \frac{I_0(\iota\kappa' b)}{\log(b/a)}\frac{v_1^2}{v^2},$$

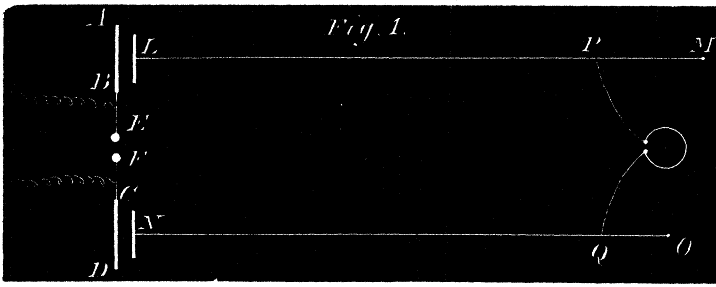
or substituting for $I_0(\iota\kappa' b)$ we get

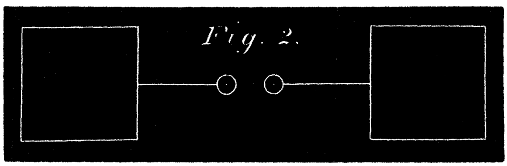
$$m^2\left\{\frac{\nu}{\nu'} + \frac{\log(1/\gamma\kappa' b)}{\log(b/a)}\frac{v_1^2}{v^2}\right\} = \frac{p^2}{v^2}\left\{1 + \frac{\log(1/\gamma\kappa' b)}{\log(b/a)}\right\}.$$

Thus the velocity of propagation is

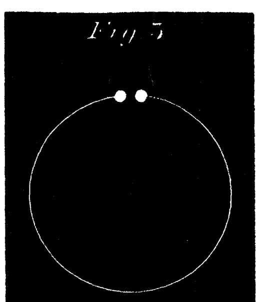
$$\frac{\left\{v^2\frac{\nu}{\nu'} + \frac{\log(1/\gamma\kappa' b)}{\log(b/a)}v_1^2\right\}^{\frac{1}{2}}}{\left\{1 + \frac{\log(1/\gamma\kappa' b)}{\log(b/a)}\right\}^{\frac{1}{2}}}.$$

As it has been shown above that if the rate at which the vibrations decay depends upon the nature of the wire, the rate of propagation of the disturbance along the wire will be $v\sqrt{(\nu/\nu')}$, I thought it would be of interest to determine the rate of propagation in this case, in order to see whether the velocity would still differ as much as in Hertz's experiments from that of the propagation of the electrodynamic action through air.





The method used is shown in fig. 1. AB, CD is the action of a vibrator (shown in elevation in fig. 2) of the same shape and size as the one used by Hertz in the experiments described in Wiedemann's 'Annalen,' vol. 34, p. 553, AB, CD being squares of tin-plate, 40 cm. square. BE, EF wires, each 30 cm. long, terminating in the brightly polished balls E and F, these balls being separated by an air space of 3 or 4 mm. The terminals of the induction coil are fastened to BE, CF respectively. L, N are pieces of tin-plate placed in front of AB and CD, having insulated wires about 25 metres long fastened to them, the ends M and O being covered with sealing-wax.



The resonator (fig. 3) is, as in Hertz's experiments, a ring of wire about 70 cm. in diameter, terminating in two balls, the distance between which can be accurately adjusted by means of a screw. The way in which the resonator was used was different from Hertz's method. Two wires of equal length covered with gutta-percha and surrounded by tin-foil, connected at both ends with the earth, were fastened close to the balls of the resonator; the other extremities of these wires could move along the wires LM, NO respectively. When the coil was working, sparks passed between the balls of the resonator, and it was found that the intensity of these sparks depended on the position of the points P and Q, to which the extremities of the wires of the resonator were attached. The experiments made to determine the velocity through the wire were as follows: the end Q of one of the wires of the resonator was placed at O, the end of the wire NO and the extremity P of the other wire moved along LM until the sparks in the resonator were as faint as possible; the distance P_1M, when this was the case, was about 5 metres. We may conclude that in this position the points P_1 and O are nearly at the same potential. The end of the other wire was then moved along NO until the sparks were again as faint as possible; the position Q_1, when this was the case, was such that Q_1O was between 10 and 10·25 metres. Since the sparks are again a minimum, we may conclude that P_1 and Q_1 are again at nearly the same potential, hence the potentials at Q_1 and O must be very nearly equal, but when this is the case, Q_1O must be very nearly a wave-length; the wave-length in the wire LM was found in a similar way to be also about 10 metres. Hence the wave-length of the electrical vibration in the wire must in this case be about 10 metres, but Hertz has shown by the interference of the direct electrical waves, and those reflected from a large metal reflector, that the wave-length of the action propagated through the air from this vibrator is also about 10 metres, and the length of the wave must be approximately the same in our experiments as the resonator which responded to the vibrations was of the same dimensions. So that in this case the velocity of propagation through the wire is the same as that through the air.

Since the sparks between the balls of the resonator never actually vanish, the determination of the places where they are as faint as possible is a matter of judgment, and thus the method is not capable of any very great accuracy. I found, however, on comparing my results with those of another observer, Mr. E. Everett, that the two sets agreed within about 2 feet in 10 metres.

The rate at which the disturbances die away was determined by a preliminary experiment. In this only one wire was used, and this was carried over the laboratory until when the resonator was used in the way described by Hertz no sparks passed between the balls; the length of wire necessary for this was more than twice as great for copper as for German silver wire of the same diameter. Thus the rate of decay depends on the material of which the wire is made, and, therefore, by the above investigation the velocity through the wire is $v\sqrt{(\nu/\nu')}$.

[On repeating the experiments after the Easter Vacation, an effect was observed which may explain the difference between the value of the wave-length along the wire found in the above experiments and those of Hertz. It happened that the plates after the vacation were placed further from the wall than they had been before, and it was found that the wave-length was much less, being now between seven and eight metres; on moving the plates nearer the wall the wave-length increased, the increase being evidently due to the increase in the capacity of the plate produced by the proximity of the wall. Thus if the distance of the plates from the walls was different in the determination of the wave-length along the wire from what it was in the determination of the wave-length through air, the wave-lengths would not be equal even if the velocity of propagation were the same. I endeavoured to determine the wave-length in air by measuring the distance between the nodes after reflection from a large metal screen, but could not succeed in fixing the position of the nodes with sufficient definiteness to determine the wave-length with any accuracy. The fact, however, that I got a wave-length in the wire the same as that obtained by Hertz through air, is sufficient to show that it is not necessary to suppose that the velocities through the wire and air are different, but that the difference in Hertz's results may have been due to a change in the position of the vibrator relatively to the walls of the room.—May 15.]

II. "Researches in the Chemistry of Selenic Acid and other Selenium Compounds" By Sir CHARLES A. CAMERON, M.D., F.R.C.S.I., V.P.I.C., Professor of Chemistry and Hygiene, R.C.S.I., and JOHN MACALLAN, F.I.C., Demonstrator of Chemistry, R.C.S.I. Communicated by Sir HENRY ROSCOE, F.R.S. Received April 6, 1889.

Although selenic acid was prepared by Mitscherlich so far back as the year 1827, few chemists appear to have studied its properties. This want of interest in selenic acid is rather surprising, seeing that it possesses so close a relationship to sulphuric acid, which is so important a compound. Finding the chemistry of selenic acid so meagre, we resolved to make an investigation of this body, with the view of bringing, so far as we could, its chemistry abreast with that of sulphuric acid, and also in the hope that its study would yield results which might throw additional light on the relations of the latter acid. The following pages contain the results at which we have arrived.

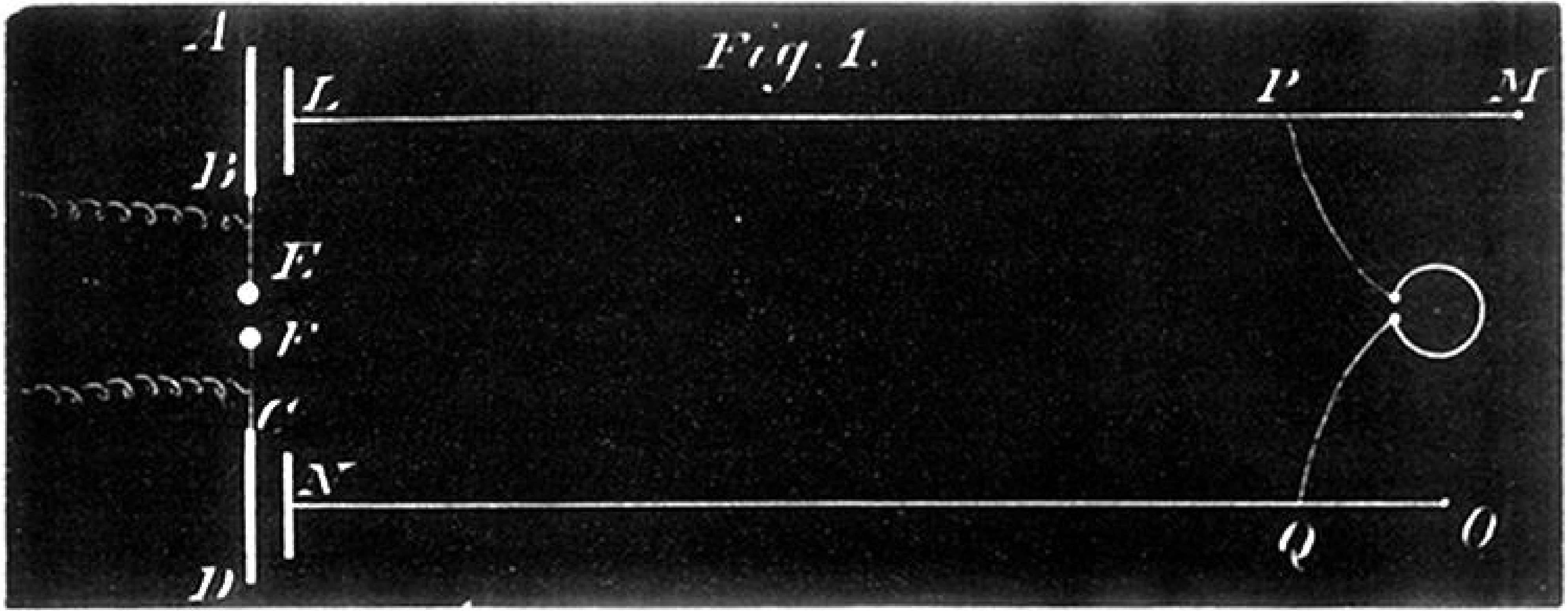


Fig. 1.

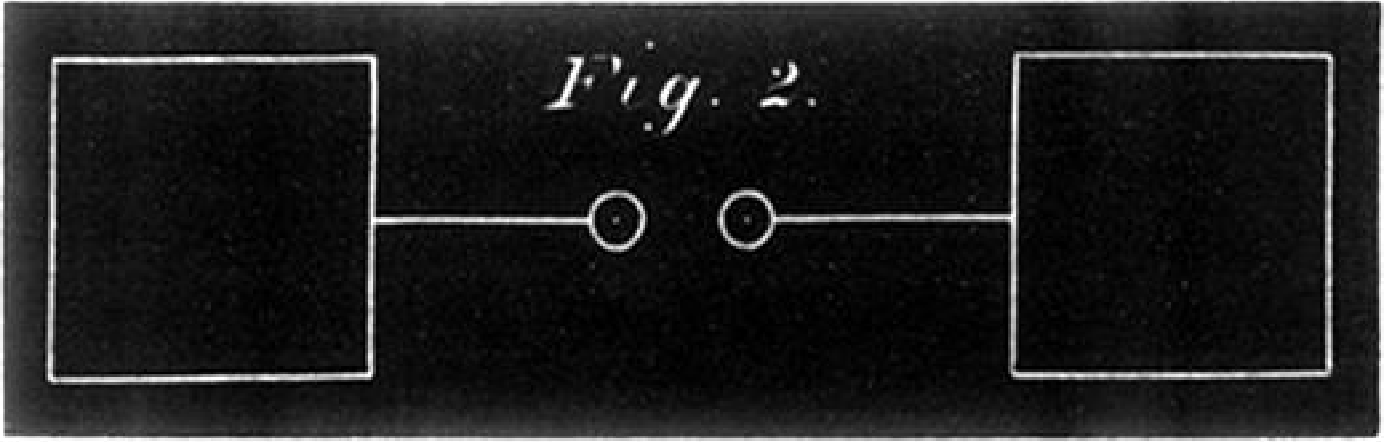


Fig. 2.

Fig. 5.