

A 500 c.c. flask was filled with water and weighed, its contents were then transferred to the cylinders, and it was then weighed again. The difference in weight gives the weight of water transferred to the cylinder. This process was repeated until the space between the cylinders was full.

The weights of the full and empty flasks were determined to 1 centigram.

The 500 gm. weight used to balance the water was compared with the standard 500 gm. weight of the Laboratory and found too heavy by .055 gm. This has been allowed for. The equality of the arms of the balance was also tested.

The two cylinders were fastened down to a flat metal plate with a thin layer of cement so as to be quite water-tight. To get any accurate estimate of the volume of water required to fill the space it was necessary to provide some means to ascertain when the space was exactly full. The effects of capillarity, grease, &c., preclude any very accurate result being obtained when there is no top or cover fitted to the top of the cylinder, and as it was necessary to see whether any air bubbles were left inside a glass top had to be used. Two holes were bored through the glass, and tubes were fixed into these. The water was introduced through one tube, and the air escaped through the other. Although no difficulty was experienced in making the joint at the bottom quite water-tight with any of the cements employed, it took us several days to make a satisfactory joint at the top. The ease with which tightness at the bottom was secured was probably owing to the great weight of the cylinders.

The top gave us all the more trouble, because we could not tell whether it was watertight or not until we had almost completed the filling in of the water. If, then, the joint proved bad the whole of the time spent in weighing the water poured in was wasted.

We tried Prout's elastic glue, then gutta-percha dissolved in benzene, but both of these failed. The water seemed to loosen the hold of the glue upon the glass, so that although the system seemed air-tight it would not remain water-tight for more than a minute or two.

We finally tried some red wax which had been sent to the Laboratory by Professor THRELFALL, who obtained it in Germany, and this answered very well. It looks somewhat like a mixture of bees'-wax and sealing wax, and as it never gets quite hard it never cracks. It possessed another property which was also useful for our purpose, viz., that of melting at a comparatively low temperature. To apply the other cements in a satisfactory manner the glass had to be heated to a somewhat high temperature, and this frequently cracked it.

The wax when melted became very fluid, so that only an extremely thin layer was included between either the top or bottom plate and the cylinders.

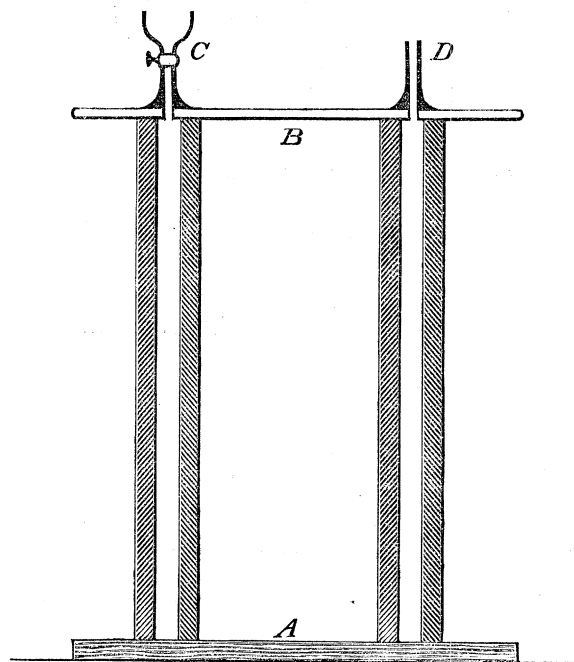
At first the water was poured in through a funnel inserted into one of the tubes, but with this arrangement it was found impossible to get rid of the last air bubble, since fresh quantities of air were continually carried down the tube. After some experiments with different arrangements for filling, we finally adopted as the means

of introducing the water a tube with a funnel top provided with a tap. The funnel could always be kept partially full of water by means of the tap, so that no air was introduced. The cylinders were slightly tilted so that the exit tube was at the highest place. As the last few grammes were poured in the air was gradually swept along by the advancing water and driven up the exit tube. The small bubble which sometimes remained under the exit tube was easily removed by agitating it by a fine wire introduced through the exit tube.

The amount of water remaining in the tap and the amount which rose up the exit tube were ascertained afterwards by detaching the tap and tube, and finding the weight of water required to fill them to the same extent as when the cylinders were filled.

While the water was still about 3 cm. from the glass the air was exhausted from the cylinders by a water pump; the pressure of the air was reduced to about 50 mm. of mercury. This had the effect of removing a good many bubbles, and we may hope that the experiment was free from error in this respect. The wax employed for fixing the top on to the cylinders stood this difference of pressure perfectly. A drying tube was placed between the cylinders and the tube leading to the pump in order to catch any water which might be carried off in the pumping. The increase in weight of the drying tube was found to be not more than  $\cdot 02$  or  $\cdot 03$  gramme, and this has been allowed for. The weighings have also been corrected to a vacuum.

The annexed sketch shows a section of the cylinders by a plane through their axis, and through the tap and exit tube.



A is the flat metal plate.  
B the glass plate.

C the funnel and tap.  
D the exit tube.

The following are the results of the weighings on two separate days :—

	[1.] Temp. 17.	[2.] Temp. 15·3.
Weight of water put into funnel . . . . .	4403·15	4403·53
Weight of water left in tap and tube . . . . .	1·33	1·35
Volume of a piece of wax underneath the glass top . . . . .	·70	0
Weight of water in space between the cylinders is therefore	4402·52	4402·18
Correction to vacuum . . . . .	4·66	4·66
Correction for temperature . . . . .	5·10	3·91
Correction for error in 500-grm. weight . . . . .	·50	·50
Correction for inequality of arms . . . . .	·09	·04
Volume between the cylinders is . . . . .	4412·87	4411·29

The mean of these is

$$4412\cdot08 \text{ c.c.}$$

which we take as the value of the volume between the cylinders.

If  $d$  is the mean distance between the cylinders,  $l$  the length of the inner cylinder,  $a$  and  $b$  the radii of the outer and inner cylinders respectively

$$d = \frac{4412\cdot08}{\pi(a+b)l};$$

so that

$$d = \cdot94128 \text{ cm.}$$

$$\frac{a}{b} = 1 + \frac{a-b}{b} = 1\cdot0800262;$$

$$2 \log \frac{a}{b} = \cdot15397063.$$

The thicknesses of the pieces of ebonite between the guard rings and the cylinder at the top and bottom were respectively  $\cdot2934$  and  $\cdot288$ . Correcting for the air space the effective length of the cylinder is

$$60\cdot9784 + \cdot2907 = 61\cdot2691 \text{ cm.}$$

Hence the electrostatic measure of the capacity

$$\begin{aligned} &= \frac{61\cdot2691}{\cdot15397063}; \\ &= \mathbf{397\cdot927} \text{ cm.} \end{aligned}$$

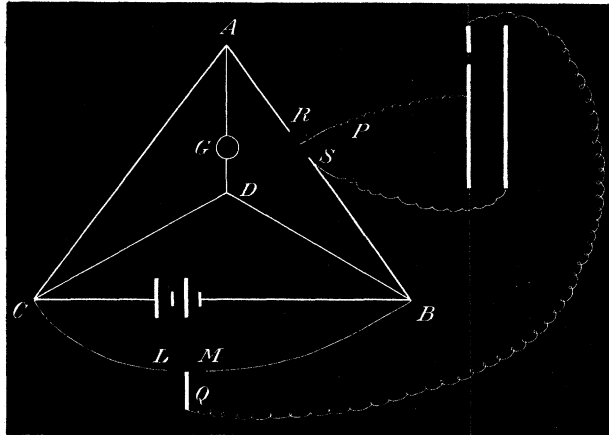
No correction is required for want of coincidence between the axes of the cylinders, for if  $c$  be the distance between the axes the correction is proportional to  $c^2/(a^2 - b^2)$ . In our experiments  $c$  was less than  $\cdot01$ , so that the correction only amounts to one part in more than 30,000.

The difference of potential between the guard ring and the cylinder depends upon the speed of the commutator, so that the correction on this account is made on the electromagnetic measure of the capacity.

*The Electromagnetic Measure of the Capacity.*

The arrangement employed in this measurement is represented in fig. 3.

Fig. 3.



ABCD is a Wheatstone's bridge with the galvanometer at G, and the battery between B and C. The arm AB is broken at R and S, which are two poles of a commutator, which alternately come into contact with a spring P, connected with the middle part of the inner cylinder of the condenser. The outer cylinder is connected to S. The points C and B are connected respectively with L and M, the two poles of a commutator, which alternately come into contact with a spring Q, attached to the guard ring of the condenser. The system is arranged so that when the commutators are working the order of events is as follows :—

- I. P on S. Condenser discharged.  
Q on M. Guard ring discharged.
- II. P on R. Condenser begins to charge.  
Q on M.
- III. P on R. Condenser completely charged to potential (A)--(B).  
Q on L. Guard ring charged to potential (C)--(B).
- IV. P on S. Condenser begins discharging.  
Q on L.
- V. P on S. Condenser discharged.  
Q on M. Guard ring discharged.

Thus, when the commutators are working, there will, owing to the flow of electricity to the condenser, be a succession of momentary currents through the galvanometer. The resistances are so adjusted that the effect of these momentary currents on the galvanometer just balances the effect due to the steady current, and there is no deflection of the galvanometer.

To investigate the relation between the resistances when this is the case, let us suppose that when the guard ring and condenser are charging

$$\begin{aligned} \dot{x} &= \text{current through BC.} \\ \dot{y} &= \text{current through AR.} \\ \dot{z} &= \text{current through AD.} \\ \dot{w} &= \text{current through CL.} \end{aligned}$$

Thus, if  $a, b, \alpha, \beta, \gamma$  are the resistances in the arms BC, AC, AD, BD, CD respectively,  $L$  the coefficient of self induction of the galvanometer, and  $E$  the electromotive force of the battery, we have

$$L\ddot{z} + (b + \gamma + \alpha)\dot{z} + (b + \gamma)\dot{y} + \gamma\dot{w} - \gamma\dot{x} = 0 \quad \dots \dots (1)$$

$$(a + \gamma + \beta)\dot{x} - (\gamma + \beta)\dot{y} - \gamma\dot{z} - (\gamma + \beta)\dot{w} - E = 0 \quad \dots \dots (2)$$

Now it is evident that the currents are expressed by equations of the following kind

$$\begin{aligned} \dot{x} &= \dot{x}_1 + \dot{x}_2, \\ \dot{z} &= \dot{z}_1 + \dot{z}_2, \end{aligned}$$

where  $\dot{x}_1$  and  $\dot{z}_1$  express the steady currents when no electricity is flowing into the condenser, and  $\dot{x}_2, \dot{z}_2$  are of the form  $A\epsilon^{-\lambda t}, B\epsilon^{-\lambda t}$ , and express the variable parts of the currents due to the charging of the condenser,  $\dot{y}$  and  $\dot{w}$  will be of the form  $C\epsilon^{-\lambda t}, D\epsilon^{-\lambda t}$ ;  $t$  in all these equations is the time which has elapsed since the condenser commenced to charge.

Equations (1) and (2) will thus contain constant terms, and terms multiplied by  $\epsilon^{-\lambda t}$ , the latter must separately vanish, hence we have

$$L\ddot{z}_2 + (b + \gamma + \alpha)\dot{z}_2 + (b + \gamma)\dot{y} + \gamma\dot{w} - \gamma\dot{x}_2 = 0 \quad \dots \dots (3)$$

$$(a + \gamma + \beta)\dot{x}_2 - (\gamma + \beta)\dot{y} - \gamma\dot{z}_2 - (\gamma + \beta)\dot{w} = 0 \quad \dots \dots (4)$$

Let  $Z, X$  be the quantities of electricity which have passed through the galvanometer and battery respectively, in consequence of the charging of the condenser, and

Y and W the charges in the condenser and guard ring. Then integrating equations (3) and (4), over a time extending from just before the condenser began to charge until it is fully charged, remembering that at each of these times  $\dot{z}_2 = 0$ , we get

$$(b + \gamma + \alpha) Z + (b + \gamma) Y + \gamma W - \gamma X = 0$$

$$(a + \gamma + \beta) X - (\gamma + \beta) Y - \gamma Z - (\gamma + \beta) W = 0,$$

hence eliminating X

$$Z \left( b + \gamma + \alpha - \frac{\gamma^2}{a + \gamma + \beta} \right) + Y \left( b + \gamma - \frac{\gamma(\gamma + \beta)}{a + \gamma + \beta} \right) + W \gamma \frac{a}{a + \gamma + \beta} = 0.$$

In our experiments, the battery resistance is very small, being less than 1 ohm, while  $\beta$  is 500,000 ohms,  $b$  200,000 ohms, and  $\gamma$  3000 ohms, thus the third term is less than  $\frac{1}{5,000,000}$ th part of the second, and may be neglected, and we get, neglecting the battery resistance

$$Z = - \frac{b}{b + \gamma + \alpha - \frac{\gamma^2}{\gamma + \beta}} Y.$$

If {A} {B} {D} denote the potentials of A, B, D when the condenser is fully charged, C the capacity of the condenser, then

$$Y = C[\{A\} - \{B\}].$$

But

$$\frac{\{A\} - \{B\}}{a + \beta \frac{(b + \alpha + \gamma)}{\gamma}} = \frac{\{A\} - \{D\}}{a}.$$

The right-hand side of this equation is  $\dot{z}_1$ , the steady current through the galvanometer, so that

$$Y = - C \dot{z}_1 \left( \alpha + \beta \frac{(b + \alpha + \gamma)}{\gamma} \right). \quad \dots \dots \dots (5)$$

$$Z = - \dot{z}_1 b C \frac{\left\{ \alpha + \beta \frac{(b + \alpha + \gamma)}{\gamma} \right\}}{b + \gamma + \alpha - \frac{\gamma^2}{\gamma + \beta}}. \quad \dots \dots \dots (6)$$

If the condenser is charged  $n$  times per second, the quantity of electricity which passes in consequence through the galvanometer per second is  $nZ$ . If the galvanometer

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 needle remains undeflected, the quantity of electricity which passes through the  
 galvanometer in unit time must be zero. But this quantity is  $nZ + \dot{z}_1$ , so that

$$nZ + \dot{z}_1 = 0.$$

Substituting this relation in equation (6), we get

$$C = \frac{1}{n} \frac{\gamma}{b\beta} \left\{ \frac{1 - \frac{\gamma^2}{(\gamma + \beta)(b + \gamma + \alpha)}}{1 + \frac{\gamma\alpha}{(b + \alpha + \gamma)\beta}} \right\}. \quad \dots \dots \dots (7)$$

From this equation, if we know the resistances and the speed, we can calculate the  
 capacity.

In order to apply the correction for the difference of potential between the guard  
 ring and the inner cylinder, we require  $\{A\} - \{C\}/\{A\} - \{B\}$ , now

$$\begin{aligned} \frac{\{A\} - \{C\}}{\{A\} - \{B\}} &= \frac{b}{\alpha} \cdot \frac{\{A\} - \{D\}}{\{A\} - \{B\}} \\ &= \frac{b}{\alpha + \beta} \frac{(b + \alpha + \gamma)}{\gamma}. \end{aligned}$$

Where  $n$  was equal to 64 the approximate values of the resistances were

- $b = 200,000$  ohms.
- $\alpha = 20,000$  „
- $\beta = 500,000$  „
- $\gamma = 3,000$  „

substituting the values

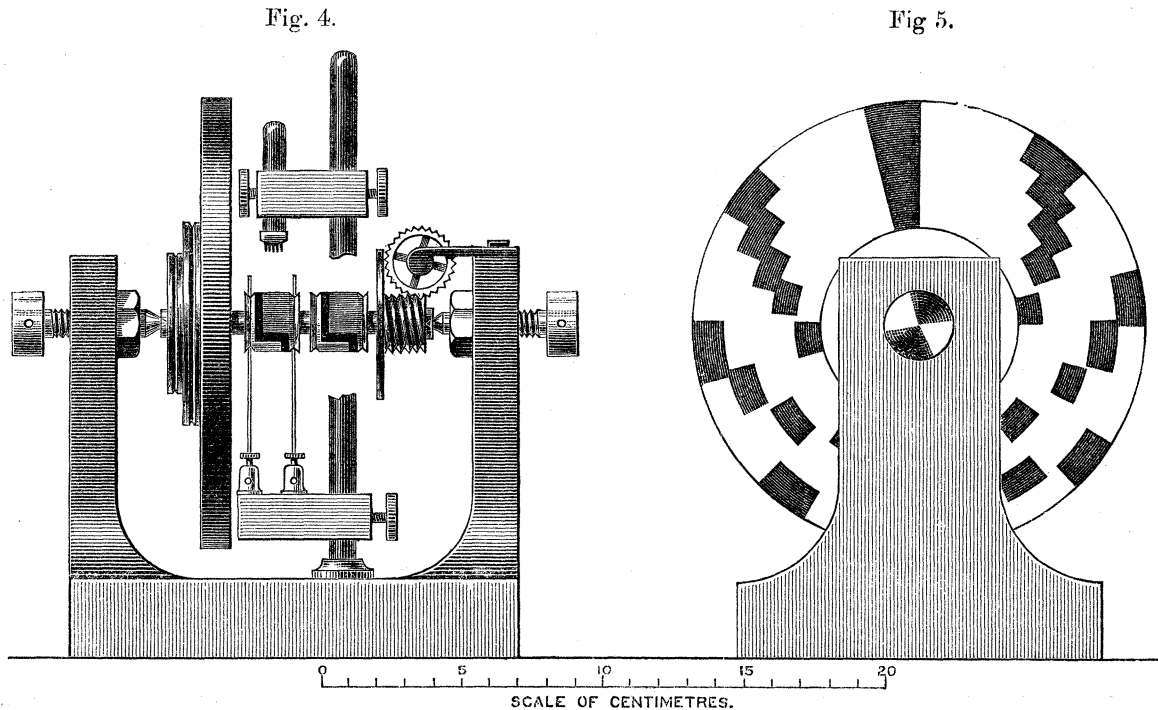
$$\frac{\{A\} - \{C\}}{\{A\} - \{B\}} = \frac{1}{1.83}.$$

We shall now go on to discuss the details of the method whose theory we have just  
 given.

*The Commutator.*

The general view of this is shown in figs. (4), (5), (6). The framework is  
 strongly made of cast iron, and somewhat resembles the headstock of a lathe, it is  
 provided with two hardened steel centres, capable of adjustment, between which runs  
 the axle of the commutator. This was made of tool steel but not hardened. The  
 centres were very good, and the apparatus ran with very little friction and wear.

We will now describe the different pieces fixed to the axle beginning from the left of fig. 4.



First there is a wooden disc with two grooves for the driving string, next comes the wooden disc on which the stroboscopic pattern is painted. The end view of this is shown in fig. 5.

Beyond this are the two commutators, which are exactly alike, each is made of two portions of brass tube fixed to an ebonite bush. Two grooves are turned in the continuous portion of the tube, in which rest two wires by means of which electrical connection is made with the parts of the commutator. Although the slits in the commutator were not more than about 2 mm. wide yet no trouble at all was experienced through want of insulation. The commutator could be easily cleaned by scraping the ebonite between the parts of the commutator with a sharp tool. There was also very good insulation between the commutator and the axle. The insulation was tested several times by means of the gold leaf electroscope, and was found practically perfect.

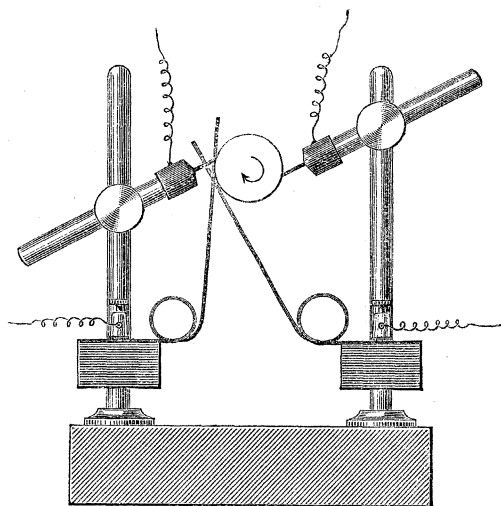
Following the two commutators is an ebonite disc to prevent oil from the centres getting to the commutators.

The last thing on the axle is an endless screw in which gears a worm-wheel of 30 teeth. This wheel is furnished with a pin which makes contact with a spring once in every revolution of the wheel, *i.e.*, every 30 revolutions of the commutator. The spring is insulated from the framework, and the contact of the pin with it completes the circuit required for exciting one of the electromagnets of the recording apparatus. The spring is not shown in the figure.



The supports for the contact wires of the commutator and for the brushes were two pillars screwing into the base of the framework. One of these is shown in the figure; it is shown broken, and the two parts separated in order to show both parts of the commutator. Fig. 6 shows the arrangement as it would look in an end view if the

Fig. 6.



stroboscopic disc were absent. The wires for making contact with the commutator are clamped under the two binding screws and are made with a curl  $Q$  so as to be very elastic and to ensure contact with the commutator. Before the curl was put into the wires it was found that the jarring of the apparatus gradually caused the wires (though made of hard drawn brass) to relax their pressure on the commutator and to make uncertain contacts. With the curled wires no trouble was found in this respect, and the pressure could be made much less, saving both friction and wear. The binding screws clamping the wires are fixed to a piece of ebonite which can slide on the pillar and be fixed in any position by a screw. The pillar also carries a support for the brush. The brush itself is made of very fine hard drawn brass wire, soldered into a brass piece of a suitable shape. The way in which the brush acted depended a good deal upon the regularity and straightness of its wires. We found it best not to have a thick bunch of wire, but a thin layer only a few wires thick. The brass piece into which the brush wires were soldered, fits on the end of an ebonite rod passing through the brush holder and capable of being fixed in its proper position by means of a screw.

The whole arrangement was clamped down to a thick iron slab, resting on a strong table. In this way the vibrations, which would otherwise have been set up, were avoided.

The tuning fork, by means of which the speed was observed and regulated, was placed on a separate table. At first the two were on the same table, but it was found that at the speed at which the commutator made one revolution for each

complete vibration of the fork, the vibrations set up by the fork were sufficient to make the contacts uncertain. As finally fixed, the apparatus worked extremely satisfactorily, and would run for several hours without either the brushes or the springs requiring any adjustment.

The commutator was driven from a water motor which was supplied from a tank at the top of the laboratory to secure a constant head of water. It was driven by a band of fine fishing line joined with a "long splice"; any rougher method of joining produced a joint, the effect of whose passage over the pulley of the commutator was plainly seen by the observer at the galvanometer. A second band went from a small pulley on the motor to a pulley fixed within easy reach of the observer stationed at the tuning fork. The regulation of the speed was done by letting the auxiliary band run through the fingers, and slightly pressing it. This was found to be a much better plan than regulating by the band driving the commutator. But, in spite of this, the speed of the commutator as judged by the steadiness of the pattern seen through the slits of the tuning fork was subject to incessant small agitations, and it required considerable vigilance on the part of the observer at the fork to keep the pattern quite at rest. A heavier disc on the commutator would no doubt have made this easier.

The supply of water was so adjusted that it was able to drive the commutator slightly faster than the speed required. The necessary fine adjustment was made by slightly pressing the regulating band.

#### *Determination of the Speed of the Commutator.*

To ascertain the speed at which the commutator was being driven its stroboscopic disc was observed through a pair of narrow slits fastened to the prongs of an electrically maintained fork. This fork made approximately 64 complete vibrations per second. The disc was provided with circles containing 4, 5, 6, 7, 8 spots at equal intervals, so that when a distinct pattern was observed through the slits on the fork the commutator made one of the following numbers of revolutions per second:—

16·0,	18·3,	21·35,	25·6,	32·0,	36·6	42·7,	48·0,
	51·2,	54·9,	64,	73·2,	76·9,	80·1,	85·4.

These numbers are respectively  $\frac{1}{4}$ ,  $\frac{2}{7}$ ,  $\frac{1}{3}$ ,  $\frac{2}{5}$ ,  $\frac{1}{2}$ ,  $\frac{4}{7}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$ ,  $\frac{4}{5}$ ,  $\frac{6}{7}$ , 1,  $\frac{8}{7}$ ,  $\frac{6}{5}$ ,  $\frac{5}{4}$ , and  $\frac{4}{3}$  of 64.

Any one of these speeds could be obtained by simply regulating the supply of water to the motor. Higher speeds could, of course, have been observed, but the motor would not drive the commutator much faster than 80 revolutions per second.

Experiments at most of these speeds will be found below.

It will be observed that this method gives a great choice of speeds, all of which can be determined with the same accuracy, and whose relation one to another is known with absolute accuracy.

The standard to which the speed was referred during the experiments was the MDCCCXC.—A.

standard fork used by Lord RAYLEIGH. This makes about 128 complete vibrations per second. The electrically driven fork maintained another fork whose natural period is about half its own. This gave beats with the standard, and by counting the beats the speed of the fork through which the stroboscopic disc was observed could be determined in terms of that of the standard fork.

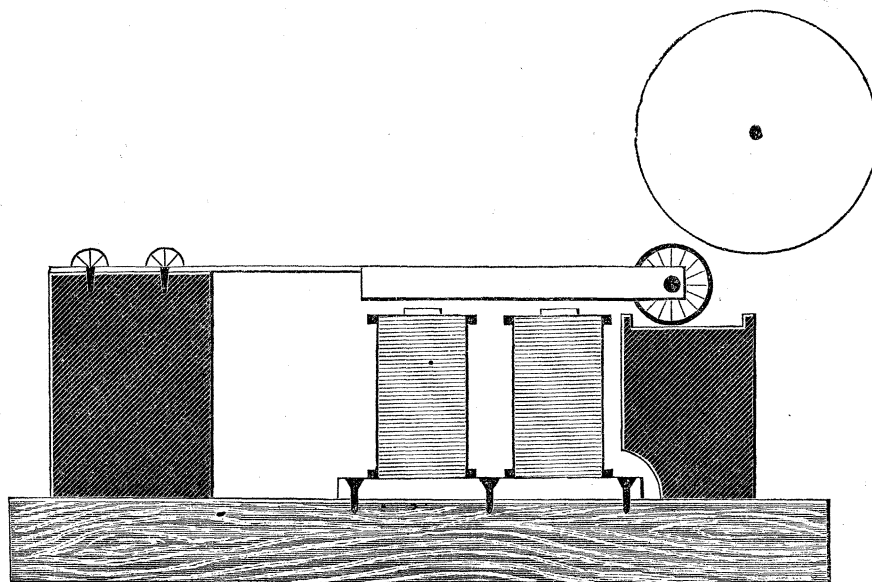
*Determination of the Speed of the Standard Fork.*

It was considered advisable to make a new determination of the standard fork. Lord RAYLEIGH ('Phil. Trans.,' 1883, p. 320) found that its speed at  $t^{\circ}$  C was

$$128\cdot140 \{1 - (t - 16) \times \cdot00011\}.$$

We have taken Lord RAYLEIGH's value of the temperature coefficient.

We determined the speed of the standard in the following way. At one end of the commutator an endless screw was fixed with a cog-wheel geared into it. The cog-wheel makes one revolution for 30 made by the commutator. A pin fixed to the wheel touches a spring once in every revolution, and completes an electric circuit



which causes a mark to be made on the tape of the recording apparatus. The laboratory clock is also arranged so as to complete a circuit once every second. A paper tape is pulled along at an approximately uniform rate between guides on a block of wood. Two electro-magnets are fixed at right angles to the guides. One of these is excited when contact is made by the cog-wheel of the commutator, the other when contact is made by the clock. The armatures are kept away from their magnets by springs fixed to one end.

The other end of the armature carries a fork in which a small disc can rotate. When the magnet is not excited the disc presses against a roller covered with printer's ink. When the magnet is excited the disc presses on the paper tape as it passes underneath it. The disc is kept inked by revolving the inking roller.

The general disposition of the apparatus will be seen from the figure.

As the tape is drawn along we get two series of marks, one due to the commutator, the other to the clock. By comparing the position of the marks made by the commutator with those made by the clock we get a very accurate method of timing the commutator. We can find the time at which say 15 commutator contacts are made, and then, after the lapse of 5 or 10 minutes, find the time at which another 15 commutator contacts are made. Subtracting the one set of times from the others we get 15 intervals which should, of course, be equal. Taking the average we get a very accurate value for the time occupied by (say) 600 revolutions of the cog wheel.

The paper tape was at first drawn along by a Morse receiver, but as this did not act very uniformly it was replaced by a small winding arrangement driven by the same motor as the commutator. This ran perfectly uniformly.

The method of experimenting was as follows: when the observing forks had been running for a few minutes, so that everything had got steady, the beats between the standard fork and the auxiliary fork were counted. The forks were so arranged that 20 beats occurred in about 65 seconds. The motor was then started and regulated so that the commutator could be kept at half the speed of the fork, *i.e.*, about 32 revolutions per second.

The recording apparatus was then started and allowed to run for 5 or 10 minutes, the commutator being kept at a constant speed by observing the stroboscopic pattern through the fork. The apparatus was then stopped and the beats again counted.

Our reason for adopting this method was that we had originally intended to measure the speed of the commutator while we were making the electrical observations. This was, however, found to be too laborious, and had to be given up. The Laboratory clock was compared with the clock belonging to the Cambridge Philosophical Society, which is regularly rated from the Observatory.

The last three observations, when the apparatus was working very satisfactorily give for the rate of the standard fork at 16° C.

December 19 (tape running for 6 minutes)	. . .	128·1081
February 14 ( " " 10 " )	. . .	128·0909
" 15 " " " "	. . .	128·1146
Mean	. . . . .	<u>128·1045</u>

Thus according to our observations the fork is slightly slower than when used by Lord RAYLEIGH. This is what might be expected from the secular softening of the steel.

*The Galvanometer.*

This was one made in the Laboratory. It has two coils, each with about 16,000 turns of wire. The resistance of the coils when in series is 17,380 legal ohms. Great care was taken with the insulation, which, when tested by means of a gold leaf electroscope, was found to be practically perfect.

*The Battery.*

We used 36 very small storage cells, two sets of 18 being placed in parallel. The battery had thus an electromotive force of about 36 volts. The small size of the battery enabled us to insulate it with ease. The insulation was tested by a gold leaf electroscope, but no leak could be detected.

*The Resistances.*

The resistances used were contained in three boxes.

- I. A Wheatstone's bridge box, No. 1256, ELLIOTT. Legal ohms, with coils ranging from 1 to 5000 ohms, and proportional arms, each containing 10, 100, 1000 ohms.
- II. A box by ELLIOTT, containing 4 coils, 10,000, 20,000, 30,000, 40,000 B.A. units.
- III. A box by MUIRHEAD, containing originally 10 coils, each 100,000 B.A. units.

The box I. was provided with an aperture for a thermometer. The other boxes had no such provision, but as they were always kept permanently on the same table as box I. we may hope that their temperature did not differ much from that of I.

The resistances of the coils in II. and III. were always ascertained by means of I. It was, therefore, necessary to obtain some definite knowledge of I. with reference to some coils whose values are accurately known.

The standard coils used were

No. 141, C.L.C. No. 102	. . .	10·00103	legal ohms	at 16°·7 C.
No. 143, C.L.C. No. 104	. . .	99·9977	„ „	16°·05 C.
No. 145, C.L.C. No. 106	. . .	1000·306	„ „	17°·4 C.

The Legal ohm being assumed to be 1/·9889 B.A. unit.

See 'British Association Report,' 1885.

Mr. GLAZEBROOK informs us that the temperature coefficient of these coils may be taken as ·0003 per 1° C. We shall take the same temperature coefficient for the coils in the boxes.

Calling the coils in the "proportional arm" which joins the rest of the box 10<sub>a</sub>, 100<sub>a</sub>, 1000<sub>a</sub>, those in the other arm 10<sub>b</sub>, 100<sub>b</sub>, 1000<sub>b</sub>, we found

$$\frac{1000_b}{1000_a} = 1 - \frac{\cdot 3}{10000}$$

$$\frac{1000_b}{100_a} = 10 \left\{ 1 - \frac{1\cdot 8}{10000} \right\}$$

$$\frac{1000_b}{10_a} = 100 \left\{ 1 + \frac{\cdot 76}{10000} \right\}.$$

Correcting for the inequalities of  $1000_a$  and  $1000_b$  we found by comparison with the Standard coils the following values at  $16^\circ \text{C}$ ., for the various coils in box I.

	Legal ohms.
$100_a$ . . . . .	100·144
$100_\beta$ . . . . .	100·144
100 . . . . .	100·113.

The last resistance was made up from all the coils from 1 to 50,

	Legal ohms.
$1000_a$ . . . . .	1001·06
$1000_\beta$ . . . . .	1001·01
1000 . . . . .	1001·06.

The last was made up of all the coils from 1 to 500,

	Legal ohms.
2000 . . . . .	2002·22
5000 . . . . .	5004·88.

We see from this that the coils are very consistent, and on the average greater than legal ohms, in the ratio of 1·0011 to 1.

The box II. was measured as a whole, using the “proportional” coils  $1000_b$  and  $10_a$ . The apparent value was 98765. Hence the true value is

$$98765 \times 1\cdot 0011 \left( 1 + \frac{\cdot 76}{10000} \right) = 98870 \text{ legal ohms at } 16^\circ \text{C}.$$

The coils of box III. were tested in the same way and were found to be

	Resistance as measured by box I.	True resistance in legal ohms.
1.	98723	98878
2.	98617	98732
3.	98690	98805
4.	98727	98842
5.	98717	98832
6.	98768	98883

The box III. had one of its coils wrong when we began to use it, and during part of our work when we were attempting to use much greater battery power than we finally adopted, three more of the coils gave way. For this reason we could only use six out of the ten coils. These six coils, as well as box II., kept their resistances very constant during the whole of the investigation. They were tested several times during a whole year, and no perceptible change was detected.

The resistance boxes were placed on blocks of paraffin, and the insulation tested by a gold leaf electroscope.

*Method of making the Observations.*

The battery, resistance boxes, galvanometer, and condenser were connected as in the diagram (3), the commutator being placed so near to the condenser that a very short wire sufficed to make the connection. The insulation of the whole system when connected up was tested from time to time by a gold leaf electroscope.

The electrically driven fork was set going and the beats of the auxiliary fork with the standard fork observed. The water supply was then adjusted so that the motor drove the commutator at the required speed. One observer (G. F. C. S.) observed the stroboscopic disc of the commutator through the slits of the fork, and kept the speed steady by means of the controlling arrangement already described. The other observer (J. J. T.) observed the galvanometer. When the speed of the commutator had got steady, resistances were taken out of the Wheatstone's bridge box in the arm CD until no deflection was produced when the galvanometer circuit was broken. The sensitiveness of the galvanometer was such that the effect produced by altering the resistance in CD by 2 ohms in 3000 could be detected, and the speed of the commutator was kept so steady that the light reflected from the galvanometer mirror did not move over more than half a division; the deflection produced by the condenser when not balanced was more than 500 scale divisions.

The battery was then reversed and the operation repeated. The wire connecting the condenser to the commutator was then detached from the condenser, and the same operation repeated. In this way the capacity of the wire was determined. The temperatures of the fork and resistances were then read, and the beats of the auxiliary fork with the standard fork again determined.

The results of these observations are exhibited in the annexed Table.

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Date.	Speed.	Beats per minute.	Temperature of fork.	$b$ .	$\beta$ .	Value of $\gamma$ for condenser + wire.	Value of $\gamma$ for wire.	Value of $\gamma$ for condenser.	Reduced to 64.	Temperature of resistances.	Correction to 16° for resistances.	Correction to 21 beats per minute.	Correction to 16° for fork.	Correction to $\alpha C = \frac{\gamma}{b\beta}$ .	Correction to legal ohms.	Correction for difference of potential between g. r. and condenser.	Corrected value of $\gamma$ .	Electromagnetic measure of capacity, $\times 10^{-21}$ .
Dec. 17	64	21.4	17.5	197753	494089	2797.5	39.5	2758	2758	16.5	-.41	+ .14	+ .45	- 1.37	+ 3.04	+ 1.72	2761.6	443.448
"	32	21.4	17.5	"	"	1399.5	20	1379.5	2759	16.5	-.41	+ .14	+ .45	-.68	+ 3.04	+ .86	2762.4	443.476
"	48	21.5	17.5	"	"	2099	30	2069	2758.7	16.5	-.41	+ .18	+ .45	- 1.07	+ 3.04	+ 1.29	2762.1	443.528
"	80	21.5	17.5	"	"	3498	50.25	3447.75	2758.2	16.5	-.41	+ .18	+ .45	- 1.80	+ 3.04	+ 2.15	2761.8	443.480
"	64	21.8	18	"	"	2798	40.5	2757.5	2757.5	17.2	- 1	+ .29	+ .60	- 1.37	+ 3.04	+ 1.72	2760.8	443.319
"	55	21.8	18	"	"	2399	34.5	2364.5	2758.6	17.2	- 1	+ .29	+ .60	- 1.15	+ 3.04	+ 1.49	2761.9	443.496
"	42	21.8	18	"	"	1866	27	1839	2758.5	17.2	- 1	+ .29	+ .60	-.91	+ 3.04	+ 1.14	2761.6	443.448
Dec. 18	64	21.7	17.5	296585	395257	3355	49	3306	3306	17.2	- 1.2	+ .29	+ .54	- 1.66	+ 3.64	+ 3.20	3310.3	443.112
"	48	21.7	17.5	"	"	2517.5	37.5	2480	3306.7	17.2	- 1.2	+ .29	+ .54	- 1.32	+ 3.64	+ 2.40	3311.1	443.153
"	32	21.7	17.5	"	"	1679.5	25	1654.5	3309	17.2	- 1.2	+ .29	+ .54	-.84	+ 3.64	+ 1.60	3313.0	443.407
"	80	21.7	17.5	"	"	4193	60.5	4132.5	3306	17.4	- 1.4	+ .29	+ .54	- 2.38	+ 3.64	+ 4.00	3310.2	443.032
"	64	21.8	18.2	"	"	3359.5	49	3310.5	3310.5	17.2	- 1.2	+ .35	+ .79	- 1.66	+ 3.64	+ 3.20	3315.6	443.754
"	55	21.8	18.2	"	"	2831.5	43	2838.5	3311.6	17.5	- 1.5	+ .35	+ .79	- 1.54	+ 3.64	+ 2.74	3315.7	443.768
"	48	21.8	18.2	"	"	2520	38	2482	3309.3	18	- 2	+ .35	+ .79	- 1.32	+ 3.64	+ 2.40	3313.2	443.434
"	32	21.6	18.2	"	"	1680.5	25	1655.5	3311	18	- 2	+ .26	+ .79	-.84	+ 3.64	+ 1.60	3314.4	443.594
"	16	21.6	18.2	"	"	841	13	828	3312	18	- 2	+ .26	+ .79	-.41	+ 3.64	+ .80	3315.0	443.675
"	80	21.6	18.2	"	"	4196	61.5	4134.5	3307.6	18.1	- 2.1	+ .26	+ .79	- 2.38	+ 3.64	+ 4.00	3311.8	443.246
Dec. 20	64	21.6	16.6	197748	494094	2799	41.5	2757.5	2757.5	16.2	-.16	+ .22	+ .18	- 1.37	+ 3.04	+ 1.72	2761.2	443.390
"	32	21.8	16.8	"	"	1399.5	21	1378.5	2757	16.4	-.33	+ .29	+ .24	-.68	+ 3.04	+ .86	2760.4	443.262
"	48	21.8	16.8	"	"	2101	30.75	2070.25	2760.3	16.4	-.33	+ .29	+ .24	- 1.07	+ 3.04	+ 1.29	2763.7	443.792
"	80	21.6	16.6	"	"	3500	51.5	3448.5	2758.8	16.4	-.33	+ .22	+ .18	- 1.80	+ 3.04	+ 2.15	2762.2	443.551
"	55	21.6	16.6	"	"	2401.5	35.25	2336.25	2760.6	16.4	-.33	+ .22	+ .18	- 1.15	+ 3.04	+ 1.49	2764.0	443.840
"	64	21.6	16.6	"	"	2800.5	41.3	2759.2	2759.2	16.7	-.57	+ .22	+ .18	- 1.37	+ 3.04	+ 1.72	2762.4	443.583



*Explanation of Table of Results and Methods of Reduction.*

Column 2 gives the approximate speed of the commutator.

Column 3 gives the beats per minute between the standard fork and the auxiliary fork driven by the electrically maintained fork. The auxiliary fork vibrated twice as fast as the driving fork, and was slightly slower than the standard.

Column 4. The temperature of the fork given by a thermometer hung a short distance from the fork.

Columns 5 and 6. The resistances of the arms AC and BD in legal ohms at 16° C.

Column 7 gives the value of the resistance in CD required for the balance when both the condenser and the connecting wire were joined to the key of the commutator. We shall denote this by  $\gamma_1$ . Each number is the mean of two observations made with the current from the battery flowing first in one direction and then in the opposite. The difference between the two readings very seldom amounted to more than 1 ohm.

Column 8 gives the value of the resistance in CD required for the balance when the wire alone is joined to the key of the commutator. We shall denote this by  $\gamma_2$ . Each is the mean of two readings corresponding to the two directions of the battery current.

Column 9 gives the difference between the last two columns. We must remark that, since the formula  $nC = \gamma/\beta b$  is not sufficiently accurate for our purpose, that this difference does not strictly represent the value of  $\gamma$ , which would be required to balance the condenser alone. With the resistances employed we may write the formula (7)

$$nC = \frac{\gamma}{\beta b} \left\{ 1 - \frac{\gamma(\gamma + \alpha)}{(\alpha + b)\beta} \right\},$$

so that if, as in our case, we keep  $\beta$  and  $b$  constant,  $nC$  is not quite proportional to  $\gamma$ . Strictly, we should find the capacity of the combination of the condenser and wire, and then subtract the capacity of the wire. What we have done is to calculate as if  $\gamma_1 - \gamma_2$  represented the capacity of the condenser. The difference amounts to writing in the small term  $\gamma(\gamma + \alpha)/(\gamma + b)\beta$ ,  $\gamma_1 - \gamma_2$  instead of  $\gamma$ , since the correcting term inside the bracket, in the case of the wire, is too small to be appreciable. Since, however, this term is already very small, and does not affect the result by much more than one part in 2000, and the change we have made only alters its value by 1 per cent. at most, it is evident that it will produce no appreciable effect on the value of the capacity.

Column 10 is headed "(reduced to 64)." Since the value of the capacity is given by

$$C = \frac{\gamma}{n\beta b} \left\{ 1 - \frac{\gamma(\gamma + \alpha)}{(\alpha + b)\beta} \right\},$$

or with sufficient accuracy for our purpose by

$$C = \frac{\gamma}{nb\beta} \left\{ 1 - \frac{\gamma^\alpha}{b\beta} \right\};$$

and if  $\gamma_a, \gamma_b$  are the values of  $\gamma$  corresponding to the speeds  $n_a, n_b$ , then

$$\frac{\gamma_a}{n_a} \left\{ 1 - \frac{\gamma_a^\alpha}{b\beta} \right\} = \frac{\gamma_b}{n_b} \left\{ 1 - \frac{\gamma_b^\alpha}{b\beta} \right\},$$

or

$$\gamma_a \left( 1 - \frac{\gamma_a^\alpha}{b\beta} \right) = \frac{n_a}{n_b} \gamma_b \left\{ 1 - \frac{\alpha}{b\beta} \gamma_b \right\}.$$

Hence we see that we may take as the corresponding uncorrected value of  $\gamma_a$  the value  $\gamma_b n_a / n_b$ , if we apply to this the correction  $1 - \frac{\alpha}{b\beta} \gamma_b$  instead of the correction  $\left( 1 - \frac{\alpha}{b\beta} \gamma_a \right)$ , this is what we have done in compiling this table, and this column contains the values of  $\gamma_b n_a / n_b$ .

Column 11 gives the temperature of the box from which the resistance was taken. The temperature of the other two boxes are assumed to be the same. Since they were always on the same table as the  $\gamma$  box, and the temperature of this box varied very little, this assumption will not lead to any serious error.

Column 12. "Correction to  $16^\circ$  for resistances" from the formula

$$nC = \frac{\gamma}{b\beta}$$

we see that if we take the temperature coefficients to be the same for the three resistances and equal to  $\cdot 0003$ , then we may throw all the effect of the variation on  $\gamma$  if for each degree above  $16$  we *subtract* from  $\gamma \cdot 0003 \gamma$ . In the first and third sets this amounts to  $\cdot 82$  ohm for each degree, in the second to  $1$  ohm per degree.

Column 13. "Correction to 21 beats per minute." The auxiliary fork always made rather more than 21 beats per minute with the standard. When making 21 beats the speed of the auxiliary fork is  $127\cdot 7545$  complete vibrations per second, and 1 additional beat per minute indicates a diminution in the speed in the ratio  $1 - \frac{1}{128 \times 60}$  to 1. Since we wish to treat the observations as if the speed were constant we must for each additional beat increase  $\gamma$  in the ratio  $1 + \frac{1}{128 \times 60}$  to 1, *i.e.*, in the first and third set observations we must add  $\cdot 36$  ohm to  $\gamma$ , and in the second set  $\cdot 43$  ohm.

Column 14. "Correction to  $16^\circ$  for fork." The fork has a temperature coefficient of  $- \cdot 00011$ , so that if we regard the speed as constant we must increase  $\gamma$  in the proportion  $1\cdot 00011$  to 1 for each degree the temperature of the fork exceeds  $16^\circ$  C. In the first and third sets this amounts to  $\cdot 30$ , and in the second set to  $\cdot 36$  ohm for each degree of excess of temperature above  $16^\circ$  C.

Column 15. "Correction to  $nC = \gamma/b\beta$ ." We see from equation (7), page 606, that this is only an approximation to the correct value for  $nC$ , which is

$$nC = \frac{\gamma}{b\beta} \frac{\left\{ 1 - \frac{\gamma^3}{(\gamma + \beta)(b + \gamma + \alpha)} \right\}}{\left\{ 1 + \frac{\gamma\alpha}{(b + \alpha + \gamma)\beta} \right\}}$$

or with sufficient accuracy for our purpose

$$nC = \frac{\gamma}{b\beta} \left\{ 1 - \frac{\gamma(\gamma + \alpha)}{\beta(b + \alpha + \gamma)} \right\},$$

thus, if we wish to use the formula  $nC = \gamma/b\beta$  we must diminish  $\gamma$  in the ratio  $1 - \frac{\gamma(\gamma + \alpha)}{\beta(b + \alpha + \gamma)}$  to 1. The amount by which  $\gamma$  is to be diminished is given in column 15.

Column 16. "Correction to legal ohms." The values of  $b$  and  $\beta$  given in columns (5) and (6) are already expressed in terms of legal ohms, but the values of  $\gamma$  are expressed in terms of the resistances in the Wheatstone bridge box, which are greater than legal ohms in the proportion 1.0011 to 1. We have then to add 3.04 ohms to  $\gamma$  in the first and third sets of experiments, and 3.64 ohms in the second.

Column 17. "Correction for the difference of potential between the middle cylinder and the guard ring." By equation (6), page 9, if the difference of potential between the outer cylinder and the guard ring is less by  $\delta V$  than that between the outer cylinder and the inner cylinder, the capacity is greater than it would be if  $\delta V = 0$  in the ratio

$$1 + \frac{\delta V}{V} \frac{h}{l} \left\{ \frac{t}{c} - \frac{2}{\pi} \log \frac{4c}{h\epsilon} \right\} \text{ to } 1,$$

where  $V$  is the difference of potential between the cylinders,  $t$  the thickness of the guard ring,  $2c$  the thickness of the pieces of ebonite, and  $h$  the distance between the cylinders.

Since  $t = 1$ ,  $h = 1$ ,  $c = .145$  and  $\delta V/V = -b/\left\{ \alpha + \beta \frac{(b + \alpha + \gamma)}{\gamma} \right\}$  the capacity is less than it would be if the guard ring and the cylinder were at the same potential in the ratio of

$$1 - \frac{7.5}{61} \frac{b\gamma}{\alpha\gamma + \beta(b + \alpha + \gamma)} \text{ to } 1,$$

so that in order to get the corresponding value of  $\gamma$  when the guard ring and cylinder are at the same potential, we must add to  $\gamma$

$$\frac{7.5}{61} \frac{b\gamma^2}{\alpha\gamma + \beta(b + \alpha + \gamma)},$$

and it is this correction which is given in column 17.

Column 18 contains the values of  $\gamma$  which result when all these corrections are made.

Column 19 contains the values of the electromagnetic measure of the capacity calculated from the formula

$$C = \frac{\gamma}{n\beta b}.$$

Since the auxiliary fork makes 21 beats per minute with the standard fork, which makes 128.1045 vibrations per second, and since the observing fork makes half the number of vibrations of the auxiliary fork

$$n = 63.8773.$$

Assuming that the B.A. unit =  $.9867 \times 10^9$  in absolute measure which corresponds to the legal ohm being  $.9978 \times 10^9$ , we have

$$C = \frac{\gamma \times 10^{-9}}{b\beta \times 63.8773 \times .9977}$$

We have supposed that the resistance to the variable current which passes through the resistance coils during the charging of the condenser, is the same as the resistance to a steady current. This is justified by the following investigation. Though in our experiments the resistances were so large that the charging was not oscillatory, let us suppose that it is oscillatory and has the maximum frequency  $1/\sqrt{LC}$ . Then if  $r'$  is the resistance to this oscillatory current,  $r$  the resistance to a steady current

$$r' = r \{1 + kn^2\},$$

where  $k$  is a small numerical coefficient and

$$n = \frac{\pi a^2}{\sqrt{LC} \sigma},$$

where  $a$  is the radius of the cross section of the wire, and  $\sigma$  its specific resistance. The coefficient of self induction of a galvanometer similar to the one we used was found some time ago to be  $5 \times 10^9$ ,  $\sigma = 2 \times 10^4$  and  $a = .023$  for 32 B.W.G.; substituting these numbers we find  $n = 10^{-3}$  about, and

$$n^2 = 10^{-6},$$

so that the correction will only amount to one part in a million, and may be neglected.

The mean of the first set of observations	=	$443.471 \times 10^{-21}$ .
„ second „	=	$443.417 \times 10^{-21}$ .
„ third „	=	$443.569 \times 10^{-21}$ .
The mean of all the observations	=	$443.486 \times 10^{-21}$ .

The means of the observations at different speeds are given in the following table.

Speed.	Electromagnetic measure of capacity $\times 10^{-21}$ .
80	443.327
64	443.434
55	443.701
48	443.478
42	443.418
32	443.460
16	443.675

The means for the different speeds thus agree very well together, the greatest difference from the mean being about one part in 2000. There does not seem any indication of an effect depending on the number of times the condenser is charged per second, such as was very marked in Professor ROWLAND'S experiments ('Phil. Mag.,' vol. 28, 1889).

Since the electrostatic measure of the capacity is  $397.927$ , and the electromagnetic measure  $443.454 \times 10^{-21}$

$$\begin{aligned} "v" &= \sqrt{\left(\frac{397.927}{443.486 \times 10^{-21}}\right)} \\ &= \mathbf{2.9955} \times \mathbf{10^{10}} \text{ cm. sec}^{-1}. \end{aligned}$$

The value of the B.A. unit is taken to be  $.9867 \times 10^9$  in absolute measure. This value of " $v$ " agrees very nearly indeed with the value obtained by the most recent experiments for the velocity of light in air, these are

CORNU (1878)	. . . . .	$3.003 \times 10^{10}$ cm. sec <sup>-1</sup> .
MICHELSON (1879)	. . . . .	$2.9982 \times 10^{10}$ .
MICHELSON (1882)	. . . . .	$2.9976 \times 10^{10}$ .
NEWCOMB (1885)	. . . . .	2.99615
" "	. . . . .	2.99682
" "	. . . . .	2.99766

}  $\times 10^{10}$ .

In conclusion we desire to express our thanks to Mr. R. S. COLE, of Emmanuel College, who has given us valuable assistance on several occasions.

The following table taken from Mr. E. B. Rosa's paper on the Determination of "v" ('Phil. Mag.,' vol. 28, 1889, p. 315), gives the results of previous determinations of "v":—

1856.	WEBER and KOHLRAUSCH . . . . .	3·107	$\times 10^{10}$ .
1869.	W. THOMSON and KING . . . . .	2·808	$\times 10^{10}$ .
1868.	MAXWELL . . . . .	2·842	$\times 10^{10}$ .
1872.	M'KICHAN . . . . .	2·896	$\times 10^{10}$ .
1879.	AYRTON and PERRY . . . . .	2·960	$\times 10^{10}$ .
1880.	SHIDA . . . . .	2·955	$\times 10^{10}$ .
1883.	J. J. THOMSON . . . . .	2·963	$\times 10^{10}$ .
1884.	KLEMENČIČ . . . . .	3·019	$\times 10^{10}$ .
1888.	HIMSTEDT . . . . .	3·009	$\times 10^{10}$ .
1889.	W. THOMSON . . . . .	3·004	$\times 10^{10}$ .
1889.	E. B. ROSA . . . . .	2·9993	$\times 10^{10}$ .