X. *A Determination of "v," the Ratio of the Electromagnetic Unit of Electricity to the Electrostatic Unit.*

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THE experiments made by one of us in 1883 having given a value of "*v*" considerably smaller than the one found by several recent researches, it was thought desirable to repeat those experiments. The method used in 1883 was to find the electrostatic and electromagnetic measures of the capacity of a condenser; the electrostatic measure being calculated from the dimensions of the condenser, the electromagnetic measure determined by finding the resistance which would produce the same effect as that produced by the repeated charging of the condenser placed in one arm of a Wheatstone's Bridge. In the experiments of 1883 the condenser used in determining the electromagnetic measure of the capacity was not the same as the one for which the electrostatic measure had been calculated, but an auxiliary one, without a guard ring, the equality of the capacity of this condenser and that of the guard ring condenser being tested by the method given in MAXWELL's 'Electricity and Magnetism,' vol. 1, p. 324.

In repeating the experiment we adopted at first the method used before, using, however, a key of different design for testing the equality of the capacity of the two condensers by MAXWELL's method. We got very consistent results, practically identical with the previous ones. We may mention here, since it has been suggested that the capacity of the leads might account for the small values of "*v*" obtained, that this capacity is allowed for by the way the comparison between the capacities of the auxiliary and guard ring condensers is made, for the same leads are used both in this comparison and in the determination of the electromagnetic measure of the capacity of the auxiliary condenser; the capacity of the auxiliary condenser, plus that of its leads, is made equal to the capacity of the guard ring condenser, and it is the capacity of the auxiliary condenser, plus its leads, which is determined in electromagnetic measure. As the introduction of the auxiliary condenser introduced increased possibilities of error, we endeavoured to determine directly the electromagnetic measure of

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the capacity of the guard ring condenser, by using a complicated commutator which worked both the guard ring and the condenser. At first we tried one where the contacts were made by platinum styles attached to a tuning fork, but as the results were not so regular as we desired, we replaced the tuning fork commutator by a rotating one driven by a water motor. A stroboscopic arrangement was fixed to this commutator so that its speed might be kept regular and measured. With this arrangement, which worked perfectly, we got values for the electromagnetic measure of the capacity of the condenser distinctly less than those obtained by the old method. We then endeavoured to find out the cause of this difference, and after a good deal of trouble discovered that in the experiments by which the equality of the capacities of the guard ring and auxiliary condensers was tested by MAXWELL'S method, the guard ring did not produce its full effect. When the guard ring of the standard condenser was taken off, and its capacity made equal by MAXWELL'S method to the capacity of the auxiliary condenser, the two methods gave identical results; but the effect of adding the guard ring was less in the old method than in the new. We found also, by calculation, that the effect produced by the guard ring in the old method was distinctly too small, while that determined by the new method agreed well with its calculated value. As the new method was working perfectly satisfactorily, and as it possesses great advantages over the old one, inasmuch as we get rid entirely of the auxiliary condenser, and can also alter the speed of the rotating commutator with very much greater ease and considerably greater accuracy than in any arrangement where the speed is governed by a tuning fork, we discarded the old method and adopted the new one which we now proceed to describe, beginning by considering the errors to which this method is liable.

Advantages of the Method of Determining "v."

The best way of discussing the advantages of this method is to consider the quantities which have to be measured and the accuracy which can be obtained in their measurement. The investigation naturally divides into two parts (1) the determination of the capacity of a condenser in electrostatic measure; (2) the determination of the capacity of the same condenser in electromagnetic measure. Let us begin by considering the first part. The condenser consisted of two co-axial cylinders, the inner cylinder being provided with a guard ring. If the distribution of electricity on the middle part of the inner cylinder were the same as that on an equal length, l, of an infinite cylinder whose radius is a, surrounded by a co-axial infinite cylinder of radius b, the electrostatic measure of the capacity would be $\frac{1}{2}\,l/\log b/a$. The actual case may differ from this ideal one in some or all of the following ways. (1) The two cylinders may not be quite co-axial; this, however, is not important if we know the distance betwen the axes, as we can find the capacity of the system got by placing one cylinder anywhere inside another. (2) The cross sections of the cylinders may not be accurately circles. The effect on the capacity of a slight departure from circularity is calculated below, so that this effect may be corrected. (3) The conductors may not be true cylinders but swell or contract slightly as we proceed along their lengths; we show, however, below, how to correct for an effect of this kind. (4) The existence of the air space between the guard ring and the middle cylinder will cause the distribution of electricity near the ends of this cylinder to be irregular, and there will also be some electricity on the cross section of the cylinder; we have, therefore, found the distribution of electricity in a case so nearly resembling this as to allow us to use the result as a correction. In the arrangement we used the potential of the guard ring differed slightly from that of the middle cylinder, the very small correction due to this is, however, easily calculated. Since we know the corrections, the capacity of the condenser can be calculated in terms of its dimensions, and the only errors to which we are liable are those which may be made in the determination of these dimensions. The lengths which have to be measured with great accuracy are the length of the middle cylinder, its radius and that of the outer cylinder, and the distance between the cylinders. The first three of these are long enough to be measured by the ordinary methods of measuring length, without danger of an error greater than one part in 3000, the fourth, however, is too small to be measured with so great an accuracy by these methods, it was determined, therefore, by finding v, the volume of water required to fill the space between the two cylinders, then d the distance between the cylinders is given by the formula

$$d = \frac{v}{\pi l (a + b)}$$

where l is the length of the middle cylinder and a and b the radii of the two cylinders. In this way the percentage error of d was not greater than those of a, b, and l. Since an accuracy of one part in 3000 can be obtained in the measurements of the dimensions of the cylinders, and since the electrostatic measure of the capacity is of the dimensions of a length, this measure of the capacity can be obtained correct to one part in 3000.

We now pass on to the determination of the capacity in electromagnetic measure. This was determined by balancing, in a Wheatstone's bridge, a discontinuous current produced by rapidly charging the condenser against a steady current derived from the battery which charged the condenser. In order to calculate the electromagnetic measure of the capacity it is necessary to know accurately the number of times per second the condenser is charged, and to keep this number constant. The charging and discharging of the condenser were effected by a commutator driven by a Thirlmere Water Motor, the water being obtained, not from the main, but from a cistern at the top of the Laboratory. The number of revolutions per second made by the commutator was compared by a stroboscopic arrangement with the frequency of an electrically driven tuning fork. The observer (G.F.C.S.) was able, after practice, to govern the speed of the commutator so efficiently that when the condenser was in action the spot of light reflected from the mirror of the galvanometer did not move over more than half a millimetre.

The accuracy of measurement of the number of times the condenser was charged per second is thus practically the same as the accuracy of the determination of the frequency of the tuning-fork ; this frequency could be determined (see *infra*) to less than one part in 10,000.

The limit which is practically put on the determination of the electromagnetic measure of the capacity of the condenser is that imposed by the galvanometer. With the galvanometer we employed, which was one made in the laboratory, having about 30,000 turns and a resistance of 17,400 legal ohms, when the resistance of the variable arm of the Wheatstone's bridge was 2500 ohms, an alteration of 2 ohms could be detected; thus the measurement of the resistance equivalent to the repeatedly charged condenser could be made to one part in 1250 : an error of this magnitude would cause an error of one part in 2500 in the value of "v," and as all the other measurements were more accurate than this, there seems no reason why this method should not give as accurate a value of "v" as that obtained for the ohm.

The electromagnetic way of measuring the capacity affords us the means of testing the accuracy of the corrections applied to the electrostatic measure of the capacity ; we availed ourselves of this in the case of the correction for the effect of the air space between the middle cylinder and the guard-ring; we altered the thickness of this air space and found that the effect of this alteration was accurately represented by the correction we employed. One great advantage of the method is the ease with which the number of times per second the condenser is charged can be altered ; this affords a valuable means of detecting any leakage or any effect due to self-induction.

Calculation of the Electrostatic Measure of the Capacity of the Condenser.

Description of the Condenser.—The condenser, which was designed some years ago by Lord RAYLEIGH, is represented in section in fig. 1, and in plan in fig. 2. BHPD is a thick ebonite board, placed in an approximately horizontal position ; in this board two concentric circular grooves are cut. A cylindrical brass ring, HP, whose external diameter is about 23 cm., and whose height is about 10 cm., fits into the smaller of these grooves. Three pieces of ebonite carefully ground down to the same thickness (about 3 mm. in most of the experiments), with V-shaped grooves cut in them to increase the distance over which the electricity would have to leak are placed at equal ntervals on the top of this ring. On these the brass cylinder FGMN is placed ; this cylinder is of exactly the same diameter as the cylindrical ring HP, and is about 60 cm. long. The cylinders GFMN and HP are placed so that their axes are coincident. On the top of this cylinder three pieces of ebonite similar to those on HP are placed, and upon the top of these a cylindrical ring EL, similar to the ring at the bottom. Another brass cylinder, ABCD, made in three pieces, two rings somewhat similar in height to the rings HP, EL, and a long middle piece of the same length as the cylinder FGMN, is then fitted over the other cylinders, the bottom ring fitting into the outer groove in the ebonite board; the internal diameter of this cylinder is about 25 cm.

Fig. 1.

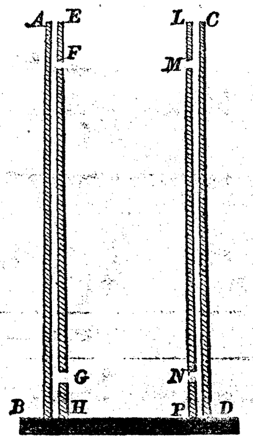
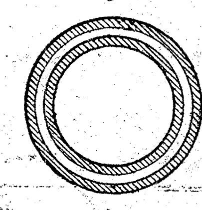


Fig. 2.



The cylinders are made co-axial by means of three pieces of ebonite worked down to the same thickness (the difference between the radii of the cylinders) pushed by rods attached to them down between the cylinders, the cylinders are adjusted until these three pieces of ebonite arranged symmetrically round the cylinder are each just in contact with the two cylinders; the rods were then removed. The insulation between the inner and outer cylinders and between the inner cylinder and its guard rings was tested by connecting one of these to earth, and the other to a charged gold leaf electroscope; the condenser was not used unless there was no appreciable loss of electricity shown by the electroscope in five minutes.

Calculation of the Capacity.—The capacity of the system regarded as two co-axial cylinders of circular section with a uniform distribution of electricity over them is

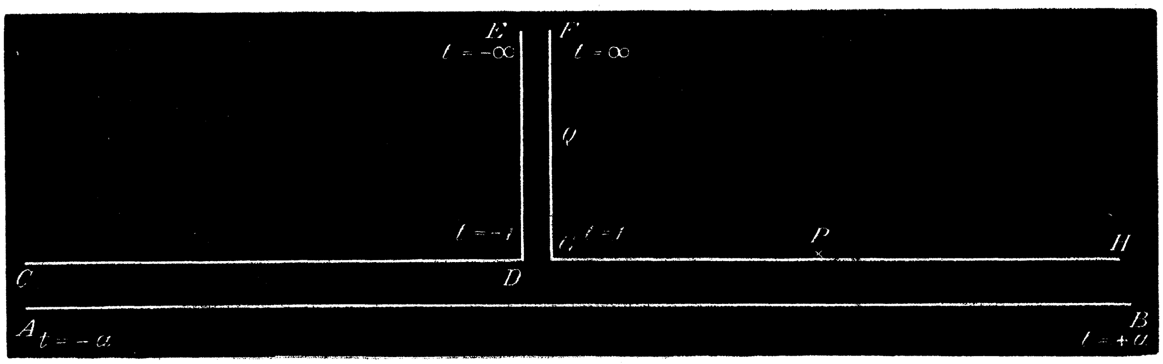
$$\tfrac{1}{2}\, l/\log\frac{a}{b},$$

where a is the radius of the outer cylinder, b that of the inner, and l the length of the cylinder FGMN.

Correction for want of coincidence between the Axes.—It is shown in a paper by J. J. THOMSON, "On the Determination of the number of Electrostatic units in the Electromagnetic unit of Electricity" ('Phil. Trans.,' 1883, p. 714), that if c be the small distance between the axes of the cylinders, the capacity is

$$\tfrac{1}{2}\,\frac{l}{\log\frac{a}{b}}\left\{1+\frac{c^2}{(a^2-b^2)\log b}\right\}.$$

Correction for the want of equality in the distribution produced by the air spaces between the inner cylinder and the guard rings.—To find this correction we shall find the distribution of electricity on the system represented in the figure, when AB is the section by the plane of the paper of an infinite horizontal metal plane, and CDE, FGH sections of conductors, CD and GH being horizontal and at the same distance from AD; and DE and FG vertical. Let h be the distance between the planes CD, and AB, and $2c$ the breadth of the slit DE FG.



Let us take AB as the axis of x and the vertical line midway between ED and FG as the axis of y. Then writing z for $x + \iota y$, and supposing that ϕ and ψ are the stream and potential functions respectively, we find by using SCHWARZ's method that the solution of the problem is given by the equations

$$dz = -\mathrm{A}\frac{\{1 - t^2\}^{\frac{1}{2}}}{t^2 - a^2}\,dt. \quad a < 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\phi + \iota\psi = \mathrm{B}\log\frac{t - a}{t + a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

t being supposed to have all real values from $-\infty$ to $+\infty$.

For putting $t = \sin\theta$, $a = \sin\alpha$, and integrating (1) we find

$$z = \mathrm{A}\left(\theta - \tfrac{1}{2}\cot\alpha\log\frac{\sin(\alpha - \theta)}{\sin(\alpha + \theta)}\right)$$

or

$$x + \iota y = \mathrm{A}\left(\theta - \tfrac{1}{2}\cot\alpha\log\frac{\sin(\alpha - \theta)}{\sin(\alpha + \theta)}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

as θ goes from 0 to α, the right hand side of this equation is real so that $y = 0$ and x ranges from 0 to $+\infty$, this gives the positive half of the plane AB. As θ goes from α to $\frac{1}{2}\pi$

$$x + \iota y = \mathrm{A}\left(\theta - \tfrac{1}{2}\cot\alpha\log\frac{\sin(\theta - \alpha)}{\sin(\alpha + \theta)} + \tfrac{1}{2}\iota\pi\cot\alpha\right)$$

so that $y = \frac{1}{2} \mathrm{A}\pi \cot \alpha$, and x ranges from ∞ to $\frac{1}{2} \mathrm{A}\pi$, so if

$$h = \tfrac{1}{2} \mathrm{A}\pi \cot \alpha \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$c = \tfrac{1}{2} \mathrm{A}\pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

this will give the portion GH of the diagram. When $\sin \theta$ is greater than unity we may put

$$\theta = \tfrac{1}{2} \pi + \iota\vartheta,$$

the right hand side of (3) now equals

$$\mathrm{A}\left\{\frac{\pi}{2} + \iota\vartheta - \tfrac{1}{2} \cot \alpha \log \frac{\cos \alpha\, (\epsilon^{\vartheta} + \epsilon^{-\vartheta}) + \iota \sin \alpha\, (\epsilon^{\vartheta} - \epsilon^{-\vartheta})}{\cos \alpha\, (\epsilon^{\vartheta} + \epsilon^{-\vartheta}) - \iota \sin \alpha\, (\epsilon^{\vartheta} - \epsilon^{-\vartheta})} + \tfrac{1}{2}\, \iota\pi \cot \alpha\right\}$$

calling the quantity under the logarithm $\mathrm{P} + \iota\mathrm{Q}$, we see since

$$\mathrm{P} + \iota\mathrm{Q} = \frac{\cos \alpha\, (\epsilon^{\vartheta} + \epsilon^{-\vartheta}) + \iota \sin \alpha\, (\epsilon^{\vartheta} - \epsilon^{-\vartheta})}{\cos \alpha\, (\epsilon^{\vartheta} + \epsilon^{-\vartheta}) - \iota \sin \alpha\, (\epsilon^{\vartheta} - \epsilon^{-\vartheta})}$$

that

$$\mathrm{P} - \iota\mathrm{Q} = \frac{\cos \alpha\, (\epsilon^{\vartheta} + \epsilon^{-\vartheta}) - \iota \sin \alpha\, (\epsilon^{\vartheta} - \epsilon^{-\vartheta})}{\cos \alpha\, (\epsilon^{\vartheta} + \epsilon^{-}) + \iota \sin \alpha\, (\epsilon^{\vartheta} - \epsilon^{-\vartheta})};$$

multiplying these together we see that $\mathrm{P}^2 + \mathrm{Q}^2 = 1$. So that $\log (\mathrm{P} + \iota\mathrm{Q})$ is wholly imaginary. Thus we see that as ϑ ranges from 0 to ∞, and t therefore from 1 to ∞, $x = \frac{1}{2}\mathrm{A}\pi$ and y ranges from $\frac{1}{2}\mathrm{A}\pi \cot \alpha$ to ∞, thus this range of values of t gives the portion GF of the figure. Since the real part of the right hand side of the equation (3) changes sign with θ or t, we see that the other portions of the figure are given by the negative values of t.

Since

$$\phi + \iota\psi = \mathrm{B} \log \frac{t - a}{t + a}$$

we see that as long as t is between $-a$ and a, that is for the portion AB of the figure,

$$\phi + \iota\psi = \iota\pi\mathrm{B} + \text{real quantities},$$

so that $\psi = \pi\mathrm{B}$, and, therefore, the potential is constant over AB; when t is not between these values, that is for the other portion of the figure $\mathrm{B} \log \{(t - a)/(t + a)\}$ is real, and therefore $\psi = 0$.

Thus these equations give us the solution of the problem when AB is maintained at the potential $\pi\mathrm{B}$ and CDE, FGH are at zero potential.

The quantity of electricity on the conductor FGP when P is a point on GH

$$= \frac{1}{4\pi}(\phi_{P} - \phi_{F})$$

$$= \frac{B}{4\pi}\log\frac{t-a}{t+a},$$

where t is the value of t at P. If we represent the increase in the quantity of electricity, due to the irregularity of the distribution, by supposing a strip of breadth d to be added to the conductor GH, and the distribution of electricity to be regular, and the same as if the air space were not present; the equation to find d is, if x is the value of x at P and V, the difference of potential between AB and GH.

$$\frac{V}{4\pi h}\{x - c + d\} = -\frac{B}{4\pi}\log\frac{t-a}{t+a},$$

substituting for x and t their values in terms of θ, and remembering that

$$V = \pi B, \qquad c = \tfrac{1}{2}A\pi \qquad h = \tfrac{1}{2}A\pi\cot\alpha$$

we get

$$A\left(\theta - \tfrac{1}{2}\cot\alpha\log\frac{\sin(\theta-\alpha)}{\sin(\alpha+\theta)}\right) - c + d = -\tfrac{1}{2}A\cot\alpha\log\frac{(\sin\theta - \sin\alpha)}{(\sin\alpha + \sin\theta)};$$

or

$$d = c\left\{1 - \frac{2}{\pi}\theta\right\} - \frac{h}{\pi}\log\frac{(\sin\theta - \sin\alpha)\sin(\alpha+\theta)}{(\sin\alpha+\sin\theta)\sin(\theta-\alpha)}.$$

Now if P be some distance from G we may put $\theta = \alpha$ and we get

$$d = c\left\{1 - \frac{2}{\pi}\alpha\right\} - \frac{h}{\pi}\log\cos^2\alpha,$$

from equations (4) and (5) we see that $\tan\alpha = c/h$, so that

$$d = c\left\{1 - \frac{2}{\pi}\tan^{-1}\frac{c}{h}\right\} + \frac{1}{\pi}h\log\left\{1 + \frac{c^2}{h^2}\right\}.$$

To deduce the corresponding solution for the cylinders from this we must multiply by the correction for curvature $1 + \frac{1}{4}h/a$, where a is here the radius of the inner cylinder, so that we have finally, if D be the whole breadth to be added for the two air spaces,

$$D = 2\left[c\left\{1 - \frac{2}{\pi}\tan^{-1}\frac{c}{h}\right\} + \frac{1}{\pi}h\log\left(1 + \frac{c^2}{h^2}\right)\right]\left(1 + \frac{1}{4}\frac{h}{a}\right).$$

Now in our condenser l was about 60, $2c = \cdot 3$, and $h = 1$, so that if we put $D = 2c$ the value of the capacity will be correct to 1 part in 2000.

Correction for a small difference of potential between the guard ring and the middle cylinder.—The arrangement we used necessitates the existence of a small difference of potential between the cylinder and the guard ring, a slight modification of the preceding investigation will enable us to find the correction for this. If we put

$$\phi + \iota\psi = \frac{V}{\pi}\log\frac{t-a}{t+a} + \frac{\delta V}{\pi}\log(t+a),$$

then the potential over FGH = 0, that over EDC = δV, and over AB = V.

The breadth of the strip which must be added to compensate for the electricity on the portion QGH due to the difference of potential δV between CD and GH is

$$\frac{h}{\pi}\frac{\delta V}{V}\log\frac{(t_Q + a)}{(t_H + a)}.$$

Now, if QG is large,

$$GQ = A\log\frac{2t_Q}{\epsilon},$$

where ϵ is the base of the Napierian logarithms. Hence, since $t_H = a = \sin\alpha$, the breadth of the additional strip is

$$\frac{h}{\pi}\frac{\delta V}{V}\left\{\frac{QG}{A} - \log\frac{4\sin\alpha}{\epsilon}\right\};$$

but $A = c\,2/\pi$ and $\sin\alpha = c/h$ approximately, hence the breadth of the strip for the two guard rings is

$$h\frac{\delta V}{V}\left\{\frac{QG}{c} - \frac{2}{\pi}\log\frac{4c}{h\epsilon}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

In our experiments $h/c = 6{\cdot}6$, QG = 1, so that the correction amounts to a strip whose breadth is about

$$7{\cdot}5\frac{\delta V}{V}.$$

The value of δV/V depended on the speed, the usual value was about $\frac{1}{183}$. In this case the breadth of the strip would be about $\frac{1}{30}$ of a centimetre, and, since the length of the cylinder was about 60 cm., the correction amounts to about 1 part in 1800.

To test the accuracy of these corrections, determinations of the capacity of the condenser were made when the top guard ring was separated from the middle cylinder (1) by pieces of ebonite ·504 cm. thick, (2) by pieces ·067 cm. thick. The capacity of the cylinders with the thick ebonite was greater than that with the thin by about 11 parts in 2760. According to the results we have obtained for these corrections the effect of increasing the thickness of the ebonite would be to add a breadth ·218 cm. to the cylinder in consequence of the increased air space, and ·06 in consequence of the difference of potential, thus the two would add ·28 to the length of the cylinder, and would increase the capacity by $\frac{\cdot 28}{61} \times 2760$, or 12 parts in 2760. Thus the observed and calculated results agree well together.

Correction for ellipticity of the cross section.—Let us consider the case of a cylinder whose cross section is represented by the equation

$$r = b\{1 + e\cos 2\theta\},$$

placed inside one whose cross section is represented by

$$r = a\{1 + \alpha\cos 2\theta + \beta\sin 2\theta\},$$

where, since the measurements of the cylinder show that e, α, β are less than $\frac{1}{2000}$, we can neglect the squares of these quantities.

Let the potential between the cylinders be given by

$$V = A\log r + \frac{C\cos 2\theta}{r^2} + Dr^2\cos 2\theta + \frac{E\sin 2\theta}{r^2} + Fr^2\sin 2\theta.$$

Then, neglecting the squares of e, α, β, the difference of potential between the cylinders is

$$A\log\frac{a}{b},$$

and to the same approximation the charge per unit length is $\frac{1}{2}A$, thus the capacity per unit length is $\frac{1}{2}\log a/b$. Here a and b are the means of any two radii of the cylinders at right angles to one another. If we take these values as the radii of the cylinders the only correction required will be one of the order of one part in $(2000)^2$, which may be neglected.

Correction of Conicality.—We may see how to get rid of this correction by considering the electrical distribution on two infinite conductors, the one a plane perpendicular to the axis of y, the other a corrugated plane represented by the equation

$$y = h + \beta\sin\frac{2\pi x}{l},$$

the other plane being taken as the plane of xz. Let V the potential between the planes be given by

$$V = Ay + C\sin\frac{2\pi x}{l}\{\epsilon^{2\pi y/l} - \epsilon^{-2\pi y/l}\}$$

putting $y = h + \beta\sin 2\pi x/l$, and making the potential constant and equal to V_0

$$\left.\begin{aligned} V_0 &= Ah \\ C &= -\frac{A\beta}{\epsilon^{2\pi h/l} - \epsilon^{-2\pi h/l}} \end{aligned}\right\} \text{neglecting } \beta^2.$$

Thus σ, the surface density on the plane of xz,

$$= \frac{V_0}{4\pi h}\left\{1 - \frac{\beta \sin \frac{2\pi x}{l} \frac{2\pi}{l}}{\epsilon^{2\pi h/l} - \epsilon^{-2\pi h/l}}\right\}$$

$$= \frac{V_0}{4\pi h}\left\{1 - \frac{(y-h)\frac{2\pi}{l}}{\epsilon^{2\pi h/l} - \epsilon^{-2\pi h/l}}\right\}.$$

Thus, if we choose h so that it is the *mean* distance between the plates, for the breadth on which we wish to find the charge, the second term will vanish in our integration, and we get for Q the quantity of electricity on a breadth x

$$Q = \frac{V_0}{4\pi h} x.$$

Thus we can use the ordinary formula even when the plates are slightly inclined, provided h is the mean distance. Any correction to this will be the order of the square of the inclination at least, and in our case may be neglected.

Measurement of Dimensions of Condenser.

The dimensions are all referred to the standard metre of the Cavendish Laboratory which has been compared with the standard of the Board of Trade. The errors of the divisions are too small to affect the measurements given below. The comparison of the lengths with the standard metre was made by means of a pair of reading microscopes with micrometer screws. The pitch of the screws is accurately $\frac{1}{50}$th of an inch, and the head of the screw is divided into 100 parts, so that one division of the screw-head corresponds to ·0002 inch. The tenths of divisions are easily read and are recorded. The screws were tested by Mr. FITZPATRICK when working with Mr. GLAZEBROOK at the Specific Resistance of Mercury, and were found to be free from sensible error in either pitch or uniformity.

The standard metre is correct at 0° C., and its temperature coefficient is ·000017 per 1° C.

We require the dimensions of the condenser at 16° C. The metal of which the condenser is made is much the same as that of the standard metre, so that if we assume that the temperatures of the condenser and standard metre are the same at the time of comparison we shall simply have to correct the metre to 16° C. The temperature of the room never differed from 16° by more than 2°, so that no appreciable error can be introduced on this account.

External Diameter of Inner Cylinder.

The sliding calipers of the laboratory were used to measure this. The bar of the calipers rested on the flat top of the cylinder, so that the calipers could be moved backwards and forwards along the top. The jaws are supposed to be at right angles to the bar along which the sliding one moves, but this was found not to be exactly the case. To obviate this difficulty a small piece of brass was fastened to the end of one jaw, so that the contacts were made at the ends of both jaws. The calipers were then placed under the microscope and two definite marks read off. The standard metre was then placed beneath the microscopes and treated in the same way. The distance between the marks when the jaws of the calipers were in contact was determined by the micrometer screw alone.

The readings of the screws are given in terms of $\frac{1}{5}$ inch.

MAXIMUM Diameter of top end of Cylinder.

	Calipers.		23·8 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
1	1·1765	·6643	1·2308	·5970
2	1·1930	·6420	1·1939	·6280
3	1·8100	·5642	1·7980	·5648
4	1·8676	·5134	1·8793	·4965

The numbers in (4) are the mean of three readings.

These measurements gave as the distance between the marks—

	23·8 cm.	—·00260 in.	(1)
	,, ...	—·00262 ,,	(2)
	,, ...	—·00220 ,,	(3)
	,, ...	—·00104 ,,	(4)
Mean	23·8 cm	—·00211 in.	
		= 23·7946 cm.	

Distance between the marks with jaws of calipers closed. Read with left-hand screw—

(1.) 1·2779	(2.) 1·5778	(3.) 2·0561
·7538	1·0430	1·5192
·5241	·5348	·5369
(4.) 2·3553	(5.) 2·2183	(6.) 2·2187
1·8136	1·6891	1·6895
·5417	·5292	·5292
(7.) 2·3550	(8.) 2·5410	(9.) 2·5330
1·8172	2·0096	1·9998
·5378	·5314	·5332

The mean of these is

$$\frac{\cdot 53314}{5} \text{ in.} = \cdot 10662 \text{ in.} = \cdot 2708 \text{ cm.}$$

Thus we find that the maximum diameter at the top of the cylinder is

$$23\cdot 7946 - \cdot 2708 = 23\cdot 5238 \text{ cm.,}$$

referred to the standard at 16° C.

To reduce the standard to 0° C we must multiply by $(1 + 16 \times \cdot 000017)$ and we get as the true diameter

$$23\cdot 5302 \text{ cm.}$$

MINIMUM diameter of top end of Cylinder.

	Calipers.		23·8 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
1. (Mean of 4 readings)	2·2158	0·4338	2·1822	0·4279
2. (Mean of 4 „)	1·9271	0·7133	1·8625	0·7406

Giving as minimum diameter of top end

(1.) 23·8 cm. — ·2708 cm. — ·00790 in.
(2.) 23·8 cm. — ·2708 cm. — ·00746 in.

Correcting for temperature, we find

Minimum diameter of top end = 23·5161 cm.

BOTTOM end of Cylinder.

MAXIMUM DIAMETER.

	Calipers.		23·8 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
1. (Mean of 4 readings)	1·4078	1·2042	1·4171	1·1950
2. (Mean of 4 „)	1·5469	1·0671	1·5910	1·0196

MINIMUM DIAMETER.

	Calipers.		23·8 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
1. (Mean of 4 readings)	1·6915	·9584	1·6814	0·9318
2. (Mean of 4 „)	1·6916	·9581	1·6495	0·9634

Thus, maximum diameter of bottom end equals

(1.) 23·8 cm. − ·2708 cm. + ·00002 in.
(2.) 23·8 cm. − ·2708 cm. − ·00068 in.

Correcting for temperature, we find

Maximum diameter of bottom end = 23·5348 cm.

The minimum diameter of bottom end equals

(1.) 23·8 cm. − ·2708 cm. − ·00734 in.
(2.) 23·8 cm. − ·2708 cm. − ·00736 in.

Correcting for temperature

Minimum diameter of bottom end = 23·5169 cm.

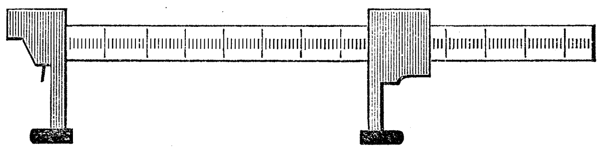
Collecting these results we have for the inner cylinder

Top end .	Maximum diameter	= 23·5302.
	Minimum diameter	= 23·5161.
Bottom end	Maximum diameter	= 23·5348.
	Minimum diameter	= 23·5169.
Mean of these		= 23·5245.

The corresponding ends of the measured diameters were found to be almost exactly on the same generating line, so that though the cylinder is slightly elliptical and conical, it is free from anything of the nature of helicality.

Measurement of the Internal Diameter of the Outer Cylinder.

This was found to be a good deal more troublesome than the measurement of the external diameter of the inner cylinder, the plan finally adopted was to fix two pieces of hardened steel to the ends of the jaws of the sliding calipers, thus



One side of each piece was polished, and the end was then ground and polished on a fine oilstone so as to form a good edge with the polished face. The shape of the end was semi-circular. In this way the edges made contact with the cylinder, and the cross wires (one of which was set carefully perpendicular to the line of travel of the microscopes) could be easily focussed on to the end of the steel.

It was not found practicable to determine exactly when contact was made in the same way as was done for the inner cylinder, since when the calipers were set to nearly the size of the cylinder scarcely any movement was possible. The sharp edges were also an impediment to the motion. We found, however, that by insulating one of the steel contact pieces we could determine accurately by the aid of a telephone when contact was made. As the cylinder was found to be nearly circular, and the formula for a slightly elliptical cylinder outside a circular one indicates that the lengths of two diameters at right angles to each other are required, two such diameters were measured. The following are the details of the measurements, each of the numbers being the mean of four observations:—

TOP end.

DIAMETER A.

	Calipers.		25·4 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
(1)	1·3821	1·1721	1·3859	1·1798
(2)	1·3319	1·2060	1·3255	1·2324
(3)	1·1822	1·3445	1·1462	1·3993

DIAMETER B.

	Calipers.		25·4 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
(1)	1·1701	1·3843	1·1793	1·3787
(2)	1·1873	1·3762	1·1614	1·4032
(3)	1·1268	1·4415	1·1150	1·4485
(4)	1·8703	0·6891	1·8875	0·6620

BOTTOM end.

DIAMETER A.

	Calipers.		25·4 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
(1)	1·4352	1·0999	1·4289	1·1209
(2)	1·4734	1·0699	1·4918	1·0586

DIAMETER B.

	Calipers.		25·4 cm.	
	Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.
(1)	1·6109	0·9270	1·6131	0·9385
	1·6978	0·8427	1·7106	0·8371

Taking the mean of these and correcting for temperature we find

Top end . Diameter A = 25·4154 cm.
Diameter B = 25·4056 cm.

Bottom end Diameter A = 25·4125 cm.
Diameter B = 25·4122 cm.

Mean of these . . . . = 25·4114 cm.

Measurement of the Length of the Cylinder.

The length of the cylinder was transferred from the cylinder to the reading microscopes by means of the beam compasses of the laboratory; care being taken to keep the bar of the compasses parallel to the length of the cylinder while setting the compasses to the length of the cylinder.

On account of the length of the cylinder it was found difficult to ascertain by moving the beam compasses just when contact was complete. A small piece of thin sheet steel (about ·03 cm. thick) was interposed between the end of the cylinder and the point of the beam compasses. The compasses were considered adjusted when a slight resistance to the motion of the feeling piece was perceived. The beam compasses were then placed under the microscopes, and the distance between two definite marks on their points determined. The points were then placed close together, so that the same resistance to the motion of the sliding piece was felt as in the former case. The distance between the marks was then ascertained by means of one of the microscopes and its screw. The distance being so small it seems unnecessary to compare it with the divisions of the standard metre.

DISTANCE between the Marks when the Compasses were Closed.

	Right-hand mark.	Left-hand mark.	
(1)	2·3156	1·7220	Mean of 4 observations.
(2)	2·1283	1·5308	,, 4 ,,
(3)	2·1287	1·5290	,, 5 ,,
(4)	2·9120	2·3154	,, 6 ,,

Giving, as the distance between the marks, ·11937 in., or ·3032 cm.

DISTANCE between the Marks when the Compasses were Open.

Calipers.		61·3 cm.		
Left-hand screw.	Right-hand screw.	Left-hand screw.	Right-hand screw.	
2·1297	·5698	2·0656	·5532	Mean of 4 observations.
2·4270	·2537	2·3819	·2415	,, ,,

Giving as the distance betwen the marks when open, 61·3 cm. — ·0138 in.

Hence the length of the cylinder when corrected for temperature equals 60·9784 cm.

The Distance between the Inner and Outer Cylinders.

Since the difference of the mean diameters is only about 1·09 cm., and since, on account of the difficulties of measurement and the irregularities in the shape of the cylinders, it is impossible to arrive at any satisfactory result by subtracting the mean diameter of one cylinder from that of the other, we had to apply some other method. We adopted that used in the experiment of 1883, which was to ascertain the amount of water required to fill the space between the two cylinders This amount was determined by weighing. The water employed was distilled, and was boiled a few hours previous to its use to enable it to absorb air bubbles more readily.