III. "The Resistance of Electrolytes to the Passage of very rapidly alternating Currents, with some Investigations on the Times of Vibration of Electrical Systems." By J. J. THOMSON, M.A., F.R.S., Cavendish Professor of Experimental Physics, Cambridge. Received January 9, 1889.

The electromagnetic effect of the currents induced in a conducting plate by alternations in a primary electromagnetic system in its neighbourhood, is, at a point on the side of the plate opposite to the primary system, in the contrary direction to the electromagnetic effect of the primary. Such a plate, therefore, tends to shield off from a secondary system the induction due to the primary, the diminution it produces in the current induced in the secondary depending upon the conductivity and thickness of the plate and the rate of reversal of the primary current. If the rate of reversal is infinitely rapid, a thin plate of very badly conducting substance will be sufficient to screen off from the secondary circuit all the induction arising from the primary, while, if the rate is very slow, a thick plate of the best conducting metal will hardly be sufficient to do this. When the current in the primary is reversed a few hundred times per second, a metal plate of very moderate thickness will completely shield off all induction. If the thickness of the plate exceeds this limit, the currents induced in the layers next the primary will shield off all electromotive force from those layers which are more remote, so that in these layers no currents will be formed, the induced currents will thus be confined to the skin of the conductor, the thickness of the skin varying inversely as the conductivity of the plate and the rate of reversal of the current.

In Hughes' induction balance this screening effect of metal plates is made use of to compare the resistances of two metals, but with that apparatus it is hardly possible to make the alternations sufficiently rapid to produce appreciable effects with substances which conduct so badly as electrolytes; we can, however, by employing the vibrations of electrical systems such as those used by Hertz in his recent experiments on the rate of propagation of electrodynamic action get oscillations sufficiently rapid to make the shielding effect of moderately thin plates of electrolytes quite appreciable.

Before describing the experiments made on this point, we shall consider the theory of the screening effect of a slab of a conductor bounded by two parallel planes. Let us suppose that these planes are represented by the equations $x = 0$, $x = -h$; let F_1, G_1, H_1 represent the components parallel to the axes of x, y, z respectively of the vector potential on the side of the slab on which the primary system is situated; F_2, G_2, H_2 their values in the conductor, and F_3, G_3, H_3 their values on the side of the slab remote from the primary system. Let ϕ be the electrostatic potential, and let us suppose that all the quantities vary as $e^{\iota pt}$, then

$$F = F' + \frac{\nu}{\iota p}\frac{d\phi}{dx},$$

$$G = G' + \frac{\nu}{\iota p}\frac{d\phi}{dy},$$

$$H = H' + \frac{\nu}{\iota p}\frac{d\phi}{dz},$$

$$\frac{dF'}{dx} + \frac{dG'}{dy} + \frac{dH'}{dz} = 0,$$

where ν is a constant which depends on the theory of electricity we adopt. If we assume Maxwell's theory, $\nu = 1$, and as we shall see reason later on for believing that ν has always this value, we shall henceforth in this investigation assume this value for it. F', G', H' represent transverse disturbances propagated in a dielectric with the velocity of propagation of electrodynamic action.

Let

$$\left.\begin{aligned} F_1 &= B_1 e^{\iota(ax+by+cz+pt)} + B_1' e^{\iota(-ax+by+cz+pt)} + \frac{1}{\iota p}\frac{d\phi}{dx} \\ G_1 &= G e^{\iota(ax+by+cz+pt)} + G' e^{\iota(-ax+by+cz+pt)} + \frac{1}{\iota p}\frac{d\phi}{dy} \\ H_1 &= D_1 e^{\iota(ax+by+cz+pt)} + D_1' e^{\iota(-ax+by+cz+pt)} + \frac{1}{\iota p}\frac{d\phi}{dz} \end{aligned}\right\} \ldots (1),$$

where the terms of the type $B_1 e^{\iota(ax+by+cz+pt)}$ represent the disturbance proceeding from the primary, and those of the type $B_1' e^{\iota(-ax+by+cz+pt)}$ the disturbance reflected from the plate.

Let

$$F_2 = B_2 e^{\iota(a'x+by+cz+pt)} + B_2' e^{\iota(-a'x+by+cz+pt)} + \frac{1}{\iota p}\frac{d\phi}{dx},$$

$$G_2 = C_2 e^{\iota(a'x+by+cz+pt)} + C_2' e^{\iota(-a'x+by+cz+pt)} + \frac{1}{\iota p}\frac{d\phi}{dy},$$

$$H_2 = D_2 e^{\iota(a'x+by+cz+pt)} + D_2' e^{\iota(-a'x+by+cz+pt)} + \frac{1}{\iota p}\frac{d\phi}{dz},$$

$$F_3 = B_3 e^{\iota(ax+by+cz+pt)} + \frac{1}{\iota p}\frac{d\phi}{dx},$$

$$G_3 = C_3 e^{\iota(ax+by+cz+pt)} + \frac{1}{\iota p}\frac{d\phi}{dy},$$

$$H_3 = D_3 e^{\iota(ax+by+cz+pt)} + \frac{1}{\iota p}\frac{d\phi}{dz}.$$

The boundary conditions are that F, G, H are continuous as we cross from one medium to another, that the magnetic induction at right angles to the bounding surface $dG/dz - dH/dy$ is also continuous, and that the magnetic force parallel to the surface, the components of which along the axes of y and z are respectively

$$\frac{1}{\mu}\left\{\frac{dH}{dx}-\frac{dF}{dz}\right\},$$

$$\frac{1}{\mu}\left\{\frac{dF}{dy}-\frac{dG}{dx}\right\},$$

where μ is the magnetic permeability, should also be continuous.

Let us first consider the special case where the electromotive force is everywhere parallel to the conducting plate, as this is the case which is most important for the interpretation of our experiments. In this case, B_1, B_1', B_2, B_2', and $B_3 = 0$, and we have, since G is continuous at the surface $x = 0$,

$$C_1 + C_1' = C_2 + C_2',$$

since it is continuous at $x = -h$,

$$C_3 e^{-\iota a h} = C_2 e^{-\iota a' h} + C_2' e^{\iota a' h},$$

since $dG/\mu dx$ is continuous, we have if μ' is the magnetic permeability of the plate,

$$a(C_1 - C_1') = \frac{a'}{\mu'}(C_2 - C_2'),$$

$$a C_3 e^{-\iota a h} = \frac{a'}{\mu'}(C_2 e^{-\iota a' h} - C_2' e^{\iota a h}).$$

Solving these equations we get

$$C_1 = \frac{C_3 e^{-\iota a h}}{4 a a'}\mu'\left\{\left(a+\frac{a'}{\mu'}\right)^2 e^{\iota h a'} - \left(\frac{a'}{\mu'}-a\right)^2 e^{-\iota h a'}\right\} \quad \ldots\ldots \ (2),$$

$$C_1 = \frac{C_2' e^{\iota a' h}}{a\left(\frac{a'}{\mu}-a\right)}\left\{\left(a+\frac{a'}{\mu'}\right)^2 e^{\iota h a'} - \left(\frac{a'}{\mu'}-a\right)^2 e^{-\iota h a'}\right\} \quad \ldots\ldots \ (3),$$

$$C_1 = \frac{C_2 e^{-\iota a' h}}{a\left(\frac{a'}{\mu'}+a\right)}\left\{\left(a+\frac{a'}{\mu'}\right)^2 e^{\iota h a'} - \left(\frac{a'}{\mu'}-a\right)^2 e^{-\iota h a'}\right\} \quad \ldots\ldots \ (4),$$

$$C_1 = \frac{C_1'}{e^{-\iota h a'} - e^{\iota h a'}}\left\{\frac{\left(a+\frac{a'}{\mu'}\right)}{\left(\frac{a'}{\mu'}-a\right)} e^{\iota h a'} - \frac{\left(\frac{a'}{\mu'}-a\right)}{\left(\frac{a}{\mu'}+a\right)} e^{-\iota h a'}\right\} \ldots\ldots \ (5).$$

There will be equations of exactly similar form connecting the D coefficients.

Equation (2) may be written

$$C_1 = \frac{C_3 e^{-\iota a h}\mu'}{4aa'}\left\{\left(a^2+\frac{a'^2}{\mu'^2}\right)(e^{\iota h a'}-e^{-\iota h a'})+\frac{2aa'}{\mu'}(e^{\iota h a'}+e^{-\iota h a'})\right\},$$

and if the plate is so thin that ha' is small, this may be written

$$C_1 = C_3 e^{\iota a h}\left\{\iota\frac{h}{2}\left(\mu' a+\frac{a'^2}{a\mu'}\right)+1\right\} \quad \ldots\ldots\ldots\ldots \quad (6).$$

Now the transverse disturbances satisfy in the dielectric equations of the form

$$\frac{d^2F'}{dx^2}+\frac{d^2F'}{dy^2}+\frac{d^2F'}{dz^2} = -\frac{p^2}{v^2}F',$$

where v is the velocity of propagation of the electrodynamic action; in the plate they satisfy equations of the form

$$\frac{d^2F'}{dx^2}+\frac{d^2F'}{dy^2}+\frac{d^2F'}{dz^2} = \frac{4\pi\mu'}{\sigma}\frac{dF'}{dt},$$

where σ is the specific resistance of the substance of which the plate is made.

From these equations we see that

$$a^2+b^2+c^2 = \frac{p^2}{v^2},$$

and

$$a'^2+b^2+c^2 = \frac{-4\pi\mu\iota p}{\sigma}.$$

Now if the primary system is a circular coil whose plane is parallel to the plane of the plate, b and c will be of the order π/R, where R is the radius of the coil; hence if as in our experiments $4\pi\iota p/\sigma$ is large compared with π^2/R^2, we may put

$$a'^2 = \frac{-4\pi\mu'\iota p}{\sigma}.$$

Since p^2/v^2 was small compared with b^2+c^2 for the vibrations used, we have approximately

$$a^2 = -(b^2+c^2),$$

and, therefore, a^2 is small compared with a'^2; hence from equation (6) we get

$$C_1 = -C_3 e^{-\iota a h}\left(1-\frac{2\pi h p}{a\sigma}\right)$$

or $$C_1/C_3e^{-\iota ah} = 1 - \frac{2\pi hp}{a\sigma} = \frac{2\pi\iota hp}{\sqrt{(b^2+c^2)}\sigma} + 1 \quad \ldots\ldots\ldots (7).$$

But $C_1/C_3e^{-\iota ah}$ is the proportion in which the electromotive force is reduced by the conducting plate; hence we see that if this is considerable $2\pi hp/\sqrt{(b^2+c^2)}\sigma$ must be large, and in this case the reduction is proportional to the thickness of the plate, the number of reversals in the direction of the current per second, and the specific resistance. The term b^2+c^2 will not change if the primary remains undisturbed. We see from the above investigation that if with the same rate of reversal two different plates produce the same effect upon the induced current, their thicknesses must be proportional to their specific resistances, or, in other words, the resistance of slabs of the same area to currents parallel to their bounding surfaces must be the same.

The above case is the one that is most generally useful; there is no difficulty, however, in writing down the solution of the most general case when the vector potential is not assumed to be parallel to the plate.

Using the same notation as before we have

$$C_1 + C_1' = C_2 + C_2',$$
$$C_3e^{-\iota ah} = C_2e^{-\iota a'h} + C_2'e^{\iota a'h},$$
$$D_1 + D_1' = D_2 + D_2',$$
$$D_3e^{-\iota ah} = D_2e^{-\iota a'h} + D_2'e^{\iota a'h},$$

$$c(B_1+B_1') - a(D_1 - D_1') = \frac{1}{\mu'}\{c(B_2+B_2') - a'(D_2 - D_2')\},$$

$$b(B_1+B_1') - a(C_1 - C_1') = \frac{1}{\mu'}\{b(B_2+B_2') - a'(C_2 - C_2')\},$$

$$cB_3e^{-\iota ha} - aD_3e^{-\iota ha} = \frac{1}{\mu'}\{c(B_2e^{-\iota ha'} + B_2'e^{\iota ha'}) - a'(D_2e^{-\iota a'h} - D_2'e^{\iota a'h})\},$$

$$bB_3e^{-\iota ha} - aC_3e^{-\iota ha} = \frac{1}{\mu'}\{b(B_2e^{-\iota ha'} + B_2'e^{\iota ha'}) - a'(C_2e^{-\iota a'h} - C_2'e^{\iota a'h})\},$$

$$aB_1 + bC_1 + cD_1 = 0,$$
$$-aB_1' + bC_1' + cD_1' = 0,$$
$$a'B_2 + bC_2 + cD_2 = 0,$$
$$-a'B_2' + bC_2' + cD_2' = 0,$$
$$aB_3 + bC_3 + cD_3 = 0.$$

The solutions of these equations are

$$bD_1-cC_1 = (bD_3-cC_3)\frac{e^{-\iota ah}\mu'}{4aa'}\left\{\left(a+\frac{a'}{\mu}\right)^2 e^{\iota h'a}-\left(\frac{a'}{\mu}-a\right)^2 e^{-\iota h'a}\right\},$$

$$bC_1+cD_1 = (bC_3+cD_3)\frac{e^{-\iota ah}}{2\gamma^2}\{(\gamma^2+1)^2 e^{\iota h'a}-(\gamma^2-1)^2 e^{-\iota h'a}\},$$

where $$\gamma^2 = \frac{4\pi\mu\iota p}{\sigma p^2/v^2}.$$

From these equations we can at once find C_3 and D_3, and hence the screening effect of the plate; exactly the same conclusions hold for this as for the special case previously considered; if the screening effect of two plates is the same their thicknesses must be proportional to their specific resistance.

The rapidly alternating currents, which in the experiments were screened by the plates, were those resulting from the electrical vibrations which are set up when the electrical equilibrium of a system is disturbed. We shall now proceed to give a somewhat detailed investigation of the periods of such vibrations, as the ordinary expression for the time of vibration of a condenser, whose plates are connected by an induction coil, is not applicable to this case, and, in addition, I think the result of these investigations taken in conjunction with some experiments by Hertz, will enable us to decide the vexed question as to whether the currents flow like an incompressible fluid, and to show that Maxwell's hypothesis on this point is correct.

The case we shall investigate is that of a straight wire connecting two spherical balls. Let us take the axis of the wire as the axis of z, and let F, G, H be the components of the vector potential, ϕ the electrostatic potential.

Then $$F = F'+\frac{\nu}{\iota p}\frac{d\phi}{dx},$$

$$G = G'+\frac{\nu}{\iota p}\frac{d\phi}{dy},$$

$$H = H' + \frac{\nu}{\iota p}\frac{d\phi}{dz},$$

where $$\frac{dF'}{dx}+\frac{dG'}{dy}+\frac{dH'}{dz} = 0,$$

and where ν is a constant. According to Maxwell's theory $\nu = 1$, while according to v. Helmholtz's more general theory $\nu = k\omega^2$, where ω is the velocity of propagation of the electrostatic potential, and k a quantity which may be determined by the equation

$$\frac{dF}{dx}+\frac{dG}{dy}+\frac{dH}{dz} = -k\frac{d\phi}{dt}.$$

On this theory ν is also equal to $1+\frac{1}{4\pi\epsilon}$ where ϵ is a quantity such that the effect of the polarisation produced in a parallelipipedal element of dielectric by an electromotive force X may be represented by distributions of electricity of surface densities plus and minus ϵ over the faces of the parallelipipedon at right angles to X.

Let all the variable quantities be proportional to $e^{\iota(mz+pt)}$. Then in the conductor since

$$\frac{d^2H'}{dx^2}+\frac{d^2H'}{dy^2}+\frac{d^2H'}{dz^2} = \frac{4\pi\mu}{\sigma}\frac{dH'}{dt},$$

where μ is the magnetic permeability and σ the specific resistance of the conductor, we have

$$\frac{d^2H'}{dx^2}+\frac{d^2H'}{dy^2}-\left(m^2+\frac{4\pi\mu\iota p}{\sigma}\right)H' = 0,$$

or, since the axis of the wire is an axis of symmetry, if r be the distance of a point in the wire from this axis

$$\frac{d^2H'}{dr^2}+\frac{1}{r}\frac{dH'}{dr}-\left(m^2+\frac{4\pi\mu\iota p}{\sigma}\right)H' = 0;$$

and if

$$n^2 = m^2+\frac{4\pi\mu\iota p}{\sigma},$$

the solution of the equation is

$$H' = AJ_0(\iota nr)e^{\iota(mz+pt)},$$

where $J_0(x)$ represents the Bessel's function of zero order which is finite when $x = 0$.

In the dielectric surrounding the wire H′ satisfies the differential equation

$$\frac{d^2H'}{dx^2}+\frac{d^2H'}{dy^2}+\frac{d^2H'}{dz^2} = \frac{1}{v^2}\frac{d^2H'}{dt^2},$$

where v is the velocity of propagation of electrodynamic action through the dielectric. Transforming this as before, this may be written

$$\frac{d^2H'}{dr^2}+\frac{1}{r}\frac{dH'}{dr}-\kappa^2H' = 0,$$

where

$$\kappa^2 = m^2-\frac{p^2}{v^2}.$$

The solution of this is

$$H' = BI_0(\iota\kappa r)e^{\iota(mz+pt)},$$

where $I_0(x)$ is the Bessel's function of zero order which vanishes when x is infinite. We may by symmetry, since there is no current in a plane at right angles to the wire, put—

$$F' = \frac{d\chi}{dx}, \quad G' = \frac{d\chi}{dy},$$

where since

$$\frac{dF'}{dx}+\frac{dG'}{dy}+\frac{dH'}{dz} = 0,$$

and F′, G′, H′ all satisfy differential equations of the same form we have in the wire

$$\chi = -\frac{\iota m}{n^2}AJ_0(\iota nr),$$

and in the dielectric

$$\chi = -\frac{\iota m}{\kappa^2}BI_0(\iota\kappa r).$$

Again if ω and ω' are the velocities of propagation of the electrostatic potential in the wire and dielectric respectively, we have in the wire

$$\phi = CJ_0(\iota qr),$$

where

$$q^2 = m^2-\frac{p^2}{\omega^2};$$

and in the dielectric

$$\phi = DI_0(\iota q'r),$$

where

$$q'^2 = m^2-\frac{p^2}{\omega'^2}.$$

Since ϕ is continuous as we cross from the wire to the dielectric, we have if a be the radius of the wire

$$CJ_0(\iota qa) = DI_0(\iota q'a) \quad \ldots\ldots\ldots\ldots\ldots\ldots (8),$$

and since H is continuous, we have

$$AJ_0(\iota na)-BI_0(\iota\kappa a) = \frac{(\nu'-\nu)}{\iota p}\iota mCJ_0(\iota qa) \ldots\ldots\ldots (9),$$

where ν and ν' are the values of ν in the wire and dielectric respectively. Since F and G are continuous, we have

$$-\frac{\iota m}{n^2}\iota nAJ_0'(\iota na)+\frac{\nu}{\iota p}\iota qCJ_0'(\iota qa) = -\frac{\iota m}{\kappa^2}\iota\kappa BI_0'(\iota\kappa a)+\frac{\nu'}{\iota p}Di q'DI_0'(\iota q'a),$$

or

$$m\left\{\frac{A}{n}J_0'(\iota na)-\frac{B}{\kappa}I_0'(\iota\kappa a)\right\}=\frac{1}{p}\{\nu'q'DI_0'(\iota q'a)-\nu qCJ_0'(\iota qa)\}\ldots(10).$$

Since the magnetic force parallel to the surface of the wire is continuous,

$$\frac{1}{\mu}\left\{\frac{d}{dz}\frac{d\chi}{dr}-\frac{dH}{dr}\right\}$$

is continuous, and therefore

$$\frac{\iota m}{\mu n^2}\cdot\iota m\,.\,\iota nAJ_0'(\iota na)-\frac{1}{\mu}\iota nAJ_0'(ma)=\frac{\iota m}{\kappa^2}\iota m\,\iota\kappa\,BI_0'(\iota\kappa a)-\iota\kappa\,BI_0'(\iota\kappa a),$$

or

$$AJ_0'(\iota na)\frac{(m^2-n^2)}{\mu n}=BI_0'(\iota\kappa a)\frac{m^2-\kappa^2}{\kappa}\ldots\ldots\ldots(11).$$

From equations (9) and (11) we get

$$A\left(J_0(\iota na)-J_0'(\iota na)\frac{I_0(\iota\kappa a)}{I_0'(\iota\kappa a)}\frac{m^2-n^2}{\mu(m^2-\kappa^2)}\frac{\kappa}{n}\right)=\frac{\nu'-\nu}{p}mCJ_0(\iota qa)\ldots(12),$$

and from (10) and (11) we get

$$AJ_0'(\iota na)\frac{m}{n}\left\{1-\frac{(m^2-n^2)}{\mu(m^2-\kappa^2)}\right\}=\frac{C}{p}\left(\nu'q'\frac{I_0'(\iota q'a)}{I_0(\iota q'a)}J_0(\iota qa)-\nu qJ_0'(\iota qa)\right)$$

(13).

Hence, eliminating A and C from these equations, we get

$$\frac{n}{m}\frac{J_0(\iota na)}{J_0'(\iota na)}-\frac{I_0(\iota\kappa a)}{I_0'(\iota\kappa a)}\frac{(m^2-n^2)}{\mu(m^2-\kappa^2)}\frac{\kappa}{m}=\frac{(\nu'-\nu)m\left\{1-\frac{(m^2-n^2)}{\mu(m^2-\kappa^2)}\right\}}{\nu'q'\frac{I_0'(\iota q'a)}{I_0(\iota q'a)}-\nu q\frac{J_0'(\iota qa)}{J_0(\iota qa)}}\ldots(14).$$

In the cases dealt with in these experiments the rate of vibration was so rapid that na was very large. In this case $J_0'(\iota na)=\iota J_0(\iota na)$. If Maxwell's theory is true, the right hand side vanishes, since $\nu'=\nu$, and we have

$$\frac{\iota n}{m}=\frac{I_0(\iota\kappa a)}{I_0'(\iota\kappa a)}\frac{4\pi\iota p v^2}{\sigma p^2}\frac{\kappa}{m},$$

or

$$\kappa\frac{I_0(\iota\kappa a)}{I_0'(\iota\ a)}=\frac{\sigma pn}{v^2 4\pi}\ldots\ldots\ldots\ldots\ldots\ldots(15).$$

The right hand side of this equation is very small, so κ must be very small. In this case

$$I_0(\iota\kappa a)=\log\gamma\iota\kappa a\text{ approximately},$$

where $$\log \gamma = 0{\cdot}577 - \log 2,$$

so that equation (14) becomes

$$\iota\kappa^2 a \log \gamma\iota\kappa a = \frac{\sigma p n}{4\pi v^2},$$

the solution of which (see 'London Math. Soc. Proc.,' vol. 17, p. 316) is

$$\kappa^2 = \frac{p\sigma}{4\pi v^2 a}\sqrt{\frac{2\pi\mu p}{\sigma}}\frac{(1-\iota)}{\log \frac{p\sigma a\gamma^2}{2\pi v^2}\left\{\frac{2\pi u p}{\sigma}\right\}^{\frac{1}{2}}},$$

and therefore

$$m^2 = \frac{p^2}{v^2} + \frac{p\sigma}{4\pi v^2 a}\sqrt{\frac{2\pi\mu p}{\sigma}}\frac{(1-\iota)}{\log\left[\frac{p\sigma a\gamma^2}{2\pi v^2}\left\{\frac{2\pi\mu p}{\sigma}\right\}^{\frac{1}{2}}\right]}.$$

Thus in this case, since the second term on the right-hand side is small compared with the first, the disturbance is propagated along the wire with the same velocity as that of electrodynamic action through the dielectric. The amplitude of the vibration will sink to $1/e$ of its original value after traversing a distance

$$8va\left\{\frac{\pi}{2\mu\sigma p}\right\}^{\frac{1}{2}}\log\left[\frac{ap^{\frac{3}{2}}\gamma^2}{v^2}\left(\frac{\sigma}{4\pi\mu}\right)^{\frac{1}{2}}\right].$$

If, however, $\nu' - \nu$ does not vanish, and if we suppose qa small, which will be the case unless the velocity of propagation of the electrostatic potential is exceedingly small compared with that of electrodynamic action, since in this case

$$\nu' q' \frac{I_0'(\iota q' a)}{I_0(\iota q a)} - \nu q \frac{J_0'(\iota q' a)}{J_0(\iota q a)} = \frac{\nu'}{\iota a \log(\gamma\iota q a)},$$

and since $\dfrac{m^2 - n^2}{\mu(m^2 - \kappa^2)}$ is very large compared with unity, equation (14) becomes, remembering that na is large,

$$\frac{\iota n}{m} - \frac{I_0(\iota\kappa a)}{I_0'(\iota\kappa a)}\frac{4\pi\iota v^2}{\sigma p}\frac{\kappa}{m} = \frac{4\pi\iota v^2}{\sigma p}\frac{m(\nu' - \nu)}{\nu'}\iota a \log \gamma\iota q a \quad \ldots.(16);$$

and unless $(\nu' - \nu)/\nu$ be very small, the right hand side in this equation is very large compared with the first term on the left, and the equation becomes

$$\frac{I_0(\iota\kappa a)}{I'_0(\iota\kappa a)}\frac{\kappa}{m} = \frac{-m(\nu' - \nu)}{\nu'}\iota a \log \gamma\iota q a.$$

The right hand side is small so that κa will be small, and the equation to determine it

$$\kappa^2 a \log \gamma\iota\kappa a = -m^2\frac{(\nu'-\nu)}{\nu'}\iota a \log \gamma\iota q a,$$

or

$$\kappa^2 \log \gamma\iota\kappa a = -m^2\frac{(\nu'-\nu)}{\nu'} \log \gamma\iota q a.$$

The solution of this equation is approximately

$$\kappa^2 = \frac{-m^2\frac{(\nu'-\nu)}{\nu'}\log \gamma\iota q a}{\log\left(2\gamma^2 m^2 a^2\frac{(\nu'-\nu)}{\nu'}\log \gamma\iota q a\right)},$$

or say

$$\kappa^2 = -\beta m^2;$$

but

$$\kappa^2 = m^2 - \frac{p^2}{v^2},$$

therefore

$$m^2(1+\beta) = \frac{p^2}{v^2},$$

and the velocity of propagation of the disturbance through the wire is p/m or $v\sqrt{(1+\beta)}$. Since the imaginary part of m does not involve σ, and a only occurs under the logarithm, the rate at which the vibrations die away will in this case be practically independent of the resistance and size of the wire. Thus, unless Maxwell's theory is true, the rate of propagation of a very rapidly alternating disturbance through a conductor is not the same as that of the electrodynamic action through the surrounding dielectric; if β is positive it goes faster through the wire than through the dielectric, while if β is negative it goes more slowly. The rate of propagation through the wire is almost though not quite independent of the size and conductivity of the wire and of the rapidity of the vibrations. Thus, if it could be proved that the velocity of a disturbance through a conducting wire differed appreciably from the velocity of electrodynamic action, and that the rate at which the vibrations die away did not depend upon the resistance, it would be sufficient to show that Maxwell's assumption is untenable. Hertz's experiments would seem to show that the rate of propagation through a metallic wire is less than that of electrodynamic action through the dielectric; but I believe he has lately found that the former rate increases rapidly with rapidity of the vibrations, which is inconsistent with the above result, if ν' and ν are independent of p. No experiments seem to have been made on the rate at which the vibrations die away, though this would be one of the best ways of distinguishing between the theories.

If we suppose that the rate of propagation of the electrostatic potential is exceedingly small, q and q' will be very large, so that unless $\nu' q' = \nu q$, the denominator of the right hand of (15) will be exceedingly large, so that the case is the same as when $\nu' = \nu$, and therefore the rate of propagation of a disturbance through a wire the same as that of electrodynamic action through air.

We shall now investigate the time of vibration of a system consisting of a straight wire connecting two spherical balls. Let us take the middle of the wire as the origin, and suppose that the flow of electricity is symmetrical about this point; at points equidistant from the origin the electrostatic potential will be equal and opposite.

Using the same notation as before, let

$$\phi = \mathrm{C}(e^{\iota mz} - e^{-\iota mz})e^{\iota pt}\,\mathrm{J}_0(\iota qr) \text{ in the wire,}$$

$$= \mathrm{D}(e^{\iota mz} - e^{-\iota m}\ )e^{\iota pt}\mathrm{I}_0(\iota q'r) \text{ in the dielectric,}$$

$$\mathrm{H} = \mathrm{A}(e^{\iota mz} + e^{-\iota mz})e^{\iota pt}\mathrm{J}_0(\iota nr) + \frac{\nu}{\iota p}\frac{d\phi}{dz} \text{ in the wire,}$$

$$= \mathrm{B}(e^{\iota mz} + e^{-\iota mz})e^{\iota pt}\mathrm{I}_0(\iota\kappa a) + \frac{\nu'}{\iota p}\frac{d\phi}{dz} \text{ in the dielectric.}$$

If w is the intensity of the current parallel to the axis of z,

$$\sigma w = -\frac{d\mathrm{H}}{dt} + \frac{d\phi}{dz}$$

$$= -\mathrm{A}\iota p(e^{\iota mz} + e^{-\iota mz})e^{\iota pt}\,\mathrm{J}_0\ (\iota nr) - (\nu - 1)\mathrm{C}\iota m\,(e^{\iota mz} + e^{-\iota mz})$$
$$e^{\iota pt}\,\mathrm{J}_0(\iota qr).$$

The quantity of electricity Q which has passed across any section at right angles to the axis is given by

$$\frac{d\mathrm{Q}}{dt} = \int_0^a 2w\pi r dr,$$

since

$$\frac{d^2\mathrm{J}_0(\iota nr)}{dr^2} + \frac{1}{r}\frac{d\mathrm{J}_0(\iota nr)}{dr} = n^2\mathrm{J}_0\,(\iota nr),$$

$$\int_0^a r\mathrm{J}_0(\iota nr)dr = \frac{1}{n^2}\,a\iota n\mathrm{J}_0'(\iota na) = \frac{\iota a}{n}\,\mathrm{J}_0'(\iota na),$$

we see that

$$\frac{\sigma d\mathrm{Q}}{dt} = -2\pi\mathrm{A}\iota p(e^{\iota mz} + e^{-\iota mz})e^{\iota pt}\ \frac{\iota a}{n}\,\mathrm{J}_0(\iota na\,)$$
$$-(\nu - 1)2\pi\mathrm{C}\iota m(e^{\iota mz} + e^{-\iota mz})e^{\iota pt}\,\frac{\iota a}{q}\,\mathrm{J}_0'(\iota qa).$$

If the ends of the wire are given by $z = \pm l$, the rate at which electricity flows across the end is given by

$$\frac{\sigma dQ}{dt} = 2Aa.\frac{p}{n}\cos ml\, e^{\iota pt} J_0'(\iota na) - (\nu - 1)\, 2C\,.\,\frac{ma}{q}\cos ml\, e^{\iota pt} J_0'(\iota qa),$$

or by equation (13)

$$\sigma\frac{dQ}{dt} = \frac{4\pi Cae^{\iota pt}\cos ml}{m\left\{1 - \dfrac{(m^2-n^2)}{\mu(m^2-\kappa^2)}\right\}}\left\{\nu' q' \frac{I_0'(\iota q'a)}{I_0(\iota q'a)} J_0(\iota qa) - \nu q J_0'(\iota qa)\right\}$$
$$- (\nu - 1) 4\pi C m \frac{a}{q} e^{\iota pt}\cos ml\, J_0'(\iota qa)\,;$$

if, however, α is the capacity (in electromagnetic measure) of the condenser at the end $z = l$

$$Q = \alpha C(e^{\iota ml} - e^{-\iota ml}) e^{\iota pt} J_0(\iota qa)\,;$$

so that

$$\sigma\frac{dQ}{dt} = \sigma\alpha\iota p C(e^{\iota ml} - e^{-\iota ml}) e^{\iota pt} J_0(\iota qa)$$
$$= -2\alpha\sigma p \sin ml\; C e^{\iota pt} J_0(\iota qa).$$

Equating these expressions for $\sigma\dfrac{dQ}{dt}$ we get

$$-2\alpha\sigma p \sin ml\; Ce^{\iota pt} J_0(\iota qa) = \frac{4\pi Cae^{\iota pt}\cos ml}{m\left\{1 - \dfrac{(m^2-n^2)}{\mu(m^2-\kappa^2)}\right\}}\left\{\nu' q' \frac{I_0'(\iota q'a)}{I_0(\iota q'a)} J_0(\iota qa)\right.$$
$$\left. - \nu q\; J_0'(\iota qa)\right\} - 4\pi(\nu - 1)\frac{Cma}{q} e^{\iota pt}\cos ml\, J_0'(\iota qa)$$

$$-p\sigma m\alpha \tan ml\left\{1 - \frac{(m^2-n^2)}{\mu(m^2-\kappa^2)}\right\} = \frac{2\pi\nu' q' a I_0'(\iota q'a)}{I_0(\iota q'a)} - \frac{2\pi\nu qa J_0'(\iota qa)}{J_0(\iota qa)}$$
$$- 2\pi(\nu - 1)\frac{m^2 a}{q}\frac{J_0'(\iota qa)}{J_0(\iota qa)}\left\{1 - \frac{(m^2-n^2)}{\mu(m^2-\kappa^2)}\right\} \quad \ldots\ldots (17).$$

Since

$$\frac{m^2-n^2}{\mu(m^2-\kappa^2)} = -\frac{4\pi\iota p}{\sigma\dfrac{p^2}{v^2}},$$

it is very large compared with unity, and if qa and $q'a$ are small,

$$\nu' q' a \frac{I_0'(\iota q'a)}{I_0(\iota q'a)} - \nu qa \frac{J_0'(\iota qa)}{J_0(\iota qa)} = \frac{\iota\nu'}{\log(\gamma\iota q'a)},$$

and

$$\frac{a}{q}\frac{J_0'(\iota qa)}{J_0(\iota qa)} = -\tfrac{1}{2}a^2\iota.$$

This equation (17) reduces to

$$-4\pi\iota v^2\alpha m \tan ml = \frac{2\pi\iota\nu'}{\log(\gamma\iota q'a)} - \frac{(\nu-1)m^2a^2}{\sigma p}v^2 4\pi^2 \quad \ldots\ldots (18),$$

if $\nu = 1$, that is, if Maxwell's theory is true,

$$2v^2\alpha m \tan ml = -\frac{1}{\log(\gamma\iota q'a)}.$$

Now $v^2\alpha$ is the electrostatic measure of the capacity, so that if we denote this by $\{\alpha\}$,

$$ml \tan ml = \frac{l}{2\{\alpha\}\log(1/\gamma'q'a)}.$$

The form of the solution will depend upon the magnitude of $l/2\{\alpha\}\log(1/\gamma q'a)$. If this is small then ml will be small, and we have

$$m^2l^2 = \frac{l}{2\{\alpha\}\log(1/\gamma\iota'q'a)},$$

or

$$m = \frac{1}{\sqrt{2l\{\alpha\}\log(1/\gamma'ma)}},$$

since, if Maxwell's theory be true, $q' = m$.

This result, however, is only true when l is not large compared with α, in this case ml tan ml will be large, and m therefore will be approximately $\frac{1}{2}\pi$, $\frac{3}{2}\pi$, and so on. Thus in this case the ends of the wire are nodes of the electrical vibrations, and the gravest mode of vibration is that in which the wave-length is twice the length of the wire; here the wave-length, and therefore the rapidity of vibration, will be independent of the capacities of the condensers at the ends.

If $\nu - 1$ is finite, since the second term on the right hand side of equation (17) will in this case be large compared with the first, since pa^2/σ is large, the equation reduces to—

$$\iota v^2\alpha m \tan ml = \frac{(\nu-1)m^2a^2v^2\pi}{\sigma p};$$

or since $p = (1+\beta)vm$,

$$\iota\{\alpha\}m \tan ml = \frac{(\nu-1)pa^2\pi}{\sigma(1+\beta)^2}.$$

Now in the cases we are considering $p\pi a^2/\sigma$ is very large, amounting to 10^4 or 10^5 in the C.G.S. system of units, so that unless $\{\alpha\}$ is comparable with 1/10 of a microfarad ml will equal $\pi/2$, the ends of the wire will again be nodes, and the wave-length of the gravest vibration will be twice the length of the wire. Thus in this case, except the capacity of the condenser were exceedingly large, much greater than that requisite for the same purpose in the preceding case, the time of vibration would be independent of the capacities of the ends; and conversely, if we could prove that the time of vibration depends upon the capacity, we should prove that $\nu = 1$. Now Hertz in his experiments seems to have been able to bring two circuits into resonance by altering the capacity of the ends, though these capacities were exceedingly small compared with 1/10 of a microfarad. This, therefore, is exceedingly strong testimony in favour of the truth of Maxwell's theory, at any rate for conductors.

[*Note added February* 15, 1889.—We can find the ratio of ν_1 to ν_2, the values of ν for a dielectric and conductor respectively, by considering the reflection of an electromagnetic disturbance at a metallic surface. Using the notation of the beginning of the paper, let the incident waves of the vector potential be expressed by

$$F' = Ae^{\iota(ax+by+cz)},$$
$$G' = Be^{\iota(ax+by+cz)},$$
$$H' = Ce^{\iota(ax+by+cz)};$$

the reflected waves by

$$F_1' = A'e^{\iota(-ax+by+cz)},$$
$$G_1' = B'e^{\iota(-ax+by+cz)},$$
$$H_1' = C'e^{\iota(-ax+by+cz)}.$$

Then assuming that $4\pi\mu\iota p/\sigma$, b^2+c^2 are large compared with p^2/v^2, we find—

$$\frac{A'}{A} = \frac{\nu_2}{\nu_1},$$
$$B'+B = \frac{\nu_2-\nu_1}{\nu_1} A \frac{b}{\sqrt{(b^2+c^2)}},$$
$$C'+C = \frac{\nu_2-\nu_1}{\nu_1} A \frac{b}{\sqrt{(b^2+c^2)}}.$$

Thus the electromotive force parallel to the surface of the reflector does not vanish at the surface unless $\nu_2 = \nu_1$. Hertz ('Wied. Ann.,' 34, 615) found that when the plane of the secondary circuit was parallel to the reflecting surface, the sparks vanished at the reflecting surface, thus showing that $\nu_2-\nu_1$ is at any rate small. The method founded on the law of decay of the vibrations is more delicate, as it shows whether or not $(\nu_2-\nu_1)na$ is small and na is a large quantity.]

In the above work we have assumed that qa is small, but if qa be large, as would be case if the rate of propagation of the electrostatic potential were exceedingly small compared with that of electrodynamic action, the first term on the right hand side of equation (14) would be very large, so that in this case again tan ml would be large and $ml = \frac{1}{2}\pi$ approximately, and the same arguments would apply as in the case when $\nu-1$ was finite.

If $\nu = 1$ for all substances, then since the electromotive force parallel to the axis of x is $-dF/dt-d\phi/dx$, and since $F = F'+d\phi/dx.\iota p$, the x component of the electromotive force is $-dF'/dt$. Similarly, the y and z components are $-dG'/dt$, $-dH'/dt$. Thus the electromotive force is propagated with the velocity of the transverse vibrations (see "Report on Electrical Theories," 'Brit. Assoc. Report,' Aberdeen, 1885, p. 138), and since F', G', H' satisfy the solenoidal condition, there is no condensation.

The rate of propagation of a disturbance through a conductor is only equal to that of the electrodynamic action through a dielectric when $\sigma/\pi p a^2 \log p\sigma\gamma^2/\pi\mu v^2$ is small, and though this will be so for the rapid vibrations we are dealing with when the conductor is metallic, it would not be so if the conductor were a dilute electrolyte or a rarefied gas. In this case the rate of propagation of the disturbance through the conductor would not be the same as that through the dielectric. In this case the action propagated along the conductor, and that propagated through the dielectric, would when they met interfere and set up standing vibrations, so that along the conductor there would be a series of stationary nodes at which the current vanished, in other words, the current along the conductor would be striated. In the discharge of electricity through rarefied gases we have the current passing through a conductor of high resistance, and it seems possible that the striations which are observed in the case may be due to the interference of the disturbance propagated through the conducting gas and that passing through the dielectric. The widening of the striæ on rarefaction, and on increasing the diameter of the discharge tube, are consistent with this view.

The resistances of the electrolytes to the very rapidly alternating currents were compared in the following way:—

A, B, C are three coils, two of which (B and C) are approximately of the same dimensions, and are nearly but not quite closed. Spherical balls are fastened to the ends of these coils. The two balls of the coil C are supported in an ebonite frame provided with an ebonite screw, by means of which the two balls can be brought very near together and kept so as long as is necessary.

The coils B and C are placed on shelves of glass coated with shellac. The shelves are supported on a framework with supports at different levels, as in an ordinary book-case, so as to enable the distance between the primary and secondary coils B and C to be altered if necessary. The coil A is connected to an induction coil which, when in good order, will give sparks 5 or 6 inches long. The coil is worked by a slow mercury break, the speed of which can be regulated by altering the inclination of the arms of a fan whose motion resists that of the break: in the actual experiments the circuit was broken every few seconds. When the coil works sparks pass between the points e and f, electrical vibrations are started in the coil B; in other words, there are alternating currents in B whose period is that of its electrical vibration, and given by equation (18). The currents in B will induce currents in C, and these latter will be rendered evident by the production of a minute spark between the two balls at its extremities. These sparks, though small, are so bright that they can be readily observed without darkening the room.

The production of sparks in the secondary circuit is much affected by what are, apparently, slight alterations in the conditions of the primary. Thus, for example, it is very much facilitated by placing the balls of a pair of discharging tongs between e and f, and allowing the spark to jump from e to the discharging tongs, and then from the discharging tongs to f. This change did not seem to be due to the resonance between the coils B and C being improved by the presence of the tongs, for unless they were placed in the way of the spark they produced no effect; again, it was not altogether due to an increase in the quality of electricity which passed from A to B at each discharge, as this was measured by placing a specially insulated galvanometer in the circuit, and it was sometimes found that the quantity of electricity which passed when the tongs were not interposed and when no spark was produced in the secondary circuit, was greater than the amount which passed when the tongs were interposed and when sparks were produced. The character of the spark which passes between A and B has also great influence—the best sparks are those which are perfectly straight, and accompanied by a sharp snap; zigzag sparks in the primary very rarely produce any sparks in the secondary.

A conducting plate placed between B and C ought, as we have seen, to diminish the induction between them, and therefore the electromotive force in the circuit C, and since the diminution in the induction increases with the rapidity with which the current in the primary is reversed, it ought in this case to be very marked. This was found to be the case; thin sheet metal and tin-foil when placed between the coils were found to completely stop the sparks in C. I then coated a plate of glass, which of itself had no effect upon the sparks, with a film of Dutch metal about $\frac{1}{1700}$ of a centimetre in thickness, and found that it completely stopped the sparks, and I have not been able to get a film of metal thin enough to allow sufficient induction to pass through to produce sparks in the secondary.

This is in accordance with the results of our investigation on the screening effect of conducting plates, for we saw by equation (7) that when a screen of thickness h was interposed the electromotive force is only

$$\frac{\sqrt{(b^2+c^2)}\,\sigma}{2\pi hp},$$

when σ is the conductivity of the metal.

Since the electromotive force in the plane of the screen, which is taken as the plane of yz, is of the form

$$\Sigma \cos by \cos cz,$$

$\sqrt{(b^2+c^2)}$ will be of the order $2\pi/\mathrm{R}$ where R is the radius of the primary coil; several coils were used whose radii varied from 13 to 23 c., so that $\sqrt{(b^2+c^2)}$ will be of the order 1/2. The length of the coils varied from 81 to 140 c., and the balls at the extremities from 1 to 2 c. in diameter, so that the length divided by the capacity is large, and, therefore, by equation (18) the wave-length will be twice the length of the coil, or for the largest coil about 3 metres; thus p will be about $2\pi \times 10^8$, and if we suppose the film is $\frac{1}{2000}$ of a millimetre thick, h will be 5×10^{-4}, we may take σ to be 10^4. A film of this kind will, by the above formula, diminish the induction about 800 times, and we should, therefore, not expect the electromotive force acting on the secondary to be sufficient to produce sparks.

A thick plate of ebonite was next placed between the coils but did not produce any appreciable diminution in the sparks in the secondary; thus ebonite, though opaque to vibrations as rapid as those of light, still allows vibrations of which 10^8 take place in a second to pass through without interruption.

The effect of interposing a film of electrolyte was next tried. A large square glass trough was placed between the coils B and C and carefully levelled, the electrolyte was then poured in; when only a very small quantity of electrolyte was in the trough the sparks still passed, but they got feebler and feebler as the quantity of electrolyte in the trough increased, until finally, when the electrolyte was dilute sulphuric acid, they ceased altogether when the depth of the electrolyte in the trough amounted to 3 or 4 millimetres. The criterion adopted for the disappearance of the sparks was to allow 60 sparks to pass into the primary, stopping and starting the coils several times; if, during this time, no sparks passed in the secondary, the sparks were considered stopped. A variation of 5 per cent. in the quantity of electrolyte present would cause the system to pass this point one way or another in a marked way.

The balls at the extremity of the secondary were adjusted so that sparks passed freely before the electrolyte was put in, after each experiment the electrolyte was removed, and care was taken to ascertain that sparks still passed as freely as before so as to guard against any accidental disarrangement of the secondary during the experiment.

Three sets of coils were used which we shall describe by I, II, III. Set I consisted of two circular brass coils, 140·8 c. in circumference. The diameter of the brass rod of which they were made was about 0·6 c.; the balls at the extremities were 2 c. in diameter. The time of vibration of this coil, calculated by equation (18), is about 10^{-8} seconds.

Set II consisted of two circular copper coils 81·2 c. in circumference, the rod of which they were made being about 0·5 c. in diameter; the balls at the extremities were 1 c. in diameter. The time of vibration is about 5×10^{-9} seconds. With these small coils the balls of the secondary had to be exceedingly close together in order to get sparks, but when the micrometer screw was properly adjusted the sparks were very bright and the indications quite definite. The coils were about 9 c. apart.

Set III consisted of two rectangular coils made of sheet lead, one side was 30 c., the other 40, the breadth of the sides was 5 c., and the diameter of the balls at the extremity 2. The distance between the coils was 15 c. The time of vibration about 10^{-8} seconds.

The electrolytes used were solutions of—

Sulphuric acid,	specific gravity of solution		....	1·175
Ammonium chloride	,,	,,	....	1·072
Sodium ,,	,,	,,	....	1·185
Potassium ,,	,,	,,	....	1·155
Ammonium nitrate	,,	,,	....	1·175
Potassium carbonate	,,	,,	....	1·280

In the following table the relative thickness of the films of these substances required to stop the spark is given, each number being the mean of several observations. The thickness of the H_2SO_4 film was taken as unity. An observation with sulphuric acid was made before and after the observation with any other electrolyte.

	H_2SO_4.	NH_4Cl.	NaCl.	KCl.	NH_4NO_3.	K_2CO_3.
Coil I	1	1·45	2·4	2·5	1·6	3·1
Coil II	1	1·5	2·5	3·3	1·9	3·1
Coil III	1	1·63	2·75	3·2	2·0	3·3
Mean............	1	1·53	2·55	3·0	1·8	3·2
Relative resistance with very slow reversal	1	1·65	3·0	3·4	1·8	2·8

The thicknesses of the films are by equation (7) proportional to the specific resistances, so that the numbers in the fourth line give the relative resistance of the electrolytes to currents whose directions are reversed from 10^8 to 2×10^8 times per second. In order to see whether these resistances are the same as those with an almost infinitely slower reversal, I determined the resistance of the electrolytes by using a commutator which reversed the current through the electrolyte about 120 times a second, and kept the direction of the current through the galvanometer constant. The electrodes were platinised. and no polarisation could be detected. The numbers are given in the last line of the above table, and agree sufficiently well to enable us to say that the relative resistance of electrolytes is the same when the current is reversed a hundred million times a second as for steady currents.

It was not possible to compare in this way the resistances of electrolytes and metals, as the thinnest metallic film which could be obtained was evidently much thicker than was necessary to completely stop all induction. I succeeded, however, in comparing by this method the resistances of graphite and sulphuric acid. The graphite film was prepared by placing a sheet of glass at the bottom of a trough filled with water, holding a large quantity of finely powdered graphite in suspension; after the graphite had deposited itself uniformly on the glass plate, the water was syphoned off, and the graphite film allowed to dry gradually. When quite dry it was hard and compact, and could be rubbed down by emery to any required thickness. By diminishing the thickness of the film and adjusting the distance between the coils, a film of graphite was obtained which just stopped the sparks; a film of H_2SO_4 was then substituted for the graphite, and its thickness adjusted until it, too, just stopped the sparks. In this case, by formula (7) the resistance of equal and similar areas of the two films to currents parallel to their surface must be the same, the currents being reversed 10^8 times per second. I determined the resistances to steady currents parallel to the surface, and found that the resistance of the graphite film was 6·7 ohms, and that of the sulphuric acid 7·2 ohms; thus we may say that the ratio of the specific resistances of graphite and sulphuric acid is the same when the currents are steady as when they are reversed 10^8 times per second. Since the ratio of the resistances of such dissimilar things as graphite and electrolytes remains the same, we may conclude that the resistances themselves remain unaltered. The method described above for comparing the resistances of electrolytes is one that can be very easily and quickly applied, and only requires the simplest apparatus: an induction coil, or, if that is not available, an electrical machine, being all that is required. The method has the advantage of avoiding the use of electrodes, as all the circuits in the electrolyte are closed.

Since electrolytes are transparent, they must, if the electromagnetic theory of light is true, act as insulators when the currents are reversed as often as light vibrations, or about 10^{15} times per second. We have seen, however, that they conduct as well when the currents are reversed 10^8 times a second as when they are steady; thus the molecular processes which cause electrolytic conduction must occupy a time between 10^{-8} and 10^{-15} seconds.

Another point which can be settled by this method is, whether a vacuum is a conductor or an insulator. According to one view the great resistance which a highly exhausted vessel offers to the passage of electricity is due to the difficulty of getting the current from the electrode into the rarefied gas: when once the current has got there, there is, according to this theory, no further resistance to its passage: if this theory is correct, a highly exhausted receiver placed between the primary and secondary circuits ought to stop the sparks in the latter, as since all the circuits are closed there ought to be no obstacle to the passage of the induced currents. In order to test this I took a box, 50 c. by 50 c. by 4 c., the top and bottom of which were sheets of plate-glass fitting into wooden sides; the sheets of glass were also supported by five ebonite pillars placed at equal intervals over their surface. The box was repeatedly dipped into a bath filled with melted paraffin until it was surrounded by a coating of paraffin about 2 c. thick. The paraffin was then smoothed over with a hot soldering iron, and then covered with a layer of shellac varnish. The box was then exhausted by a mercury-pump, and it was found that the pressure could be reduced to about 1 mm. of mercury, but no further. When this vacuum was placed between the primary and secondary coils it did not produce the slightest effect upon the sparks, so that its conductivity must be very small indeed compared with that of the electrolytes used in the preceding experiments. I am having an earthenware vessel made with which I hope to repeat the experiment at much higher exhaustions.

I also tried whether the conductivity of the electrolyte was altered by sending a current through it; for this purpose a layer of sulphuric acid was placed between the primary and secondary coils of such thickness that it almost but not quite stopped the sparks in the latter; a current of about 2 ampères, which was reversed about 500 times a second, was then sent through the sulphuric acid, but the passage of the current did not seem to produce any effect whatever upon the sparks in the secondary. I conclude, therefore that the resistance of an electrolyte is not affected by the passage of a current.

I wish to express my thanks to my assistant, Mr. E. Everett, for the zeal and skill he has displayed in these experiments.

[*Note added February* 15, 1889.—I have recently tried the effect of a very high vacuum in stopping the sparks. The primary circuit consisted of two straight wires with spheres fastened to one end of each; these wires were connected with the poles of an induction coil, and the sparks passed between the spheres. The secondary consisted of two similar wires, with smaller balls at the ends, the distance between the balls being very small. The length of the wires of the secondary was altered until it was in resonance with the primary. The secondary was placed in a hollow cylinder formed of two coaxial glass tubes, sealed on to a mercury pump, by means of which a very high vacuum was obtained in the space between them, which surrounded the secondary. This vacuum, however, did not produce the slightest effect on the sparks.]

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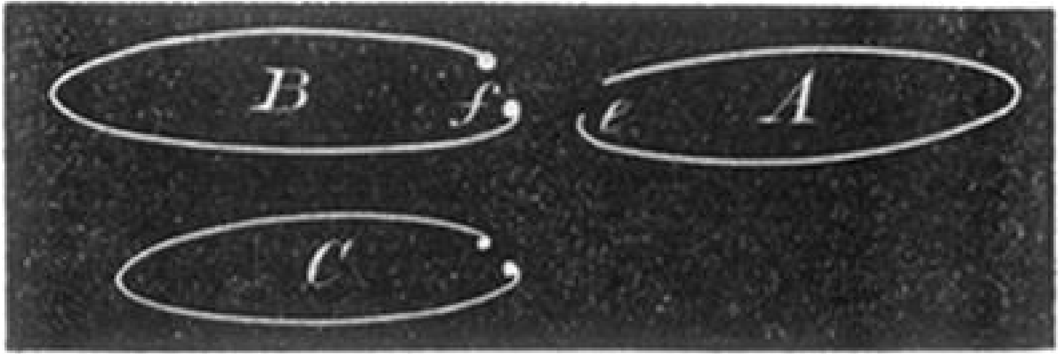
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