azimuth with respect to the plane of incidence given by

$$\tan\beta = \tan 2i \cdot \frac{\sin\frac{r_1 - r_2}{2}}{M}.$$

VII.

In the same year (1850) Cauchy extended his method to the problem of crystalline reflection: the complete solution was given in a memoir presented to the French Academy on September 16, 1850*.

This memoir was never published, though it was announced† to appear in the 23rd volume of the *Mémoires de l'Académie*; and we have only slight indications of Cauchy's manner of dealing with the problem.

In accordance with the results of his theory of double refraction, Cauchy does not suppose the vibrations to be necessarily strictly transversal and longitudinal‡. In order to eliminate the amplitudes of the latter vibrations, he assumes as an approximation the strict transversality of the former, and thus obtains§ four equations between the quasi-transversal amplitudes, which contain three coefficients, whose values are known when coordinate axes are taken depending on the refracting surface and the plane of incidence.

A second memoir‖ is devoted to the determination of the value of these coefficients, when fixed directions in the crystal are taken as the axes. The value of this determination is lessened by the fact, that at the very commencement an approximation is made depending on the peculiar relation between the coefficients of elasticity, which we have considered above.

This is all that has been published, except some notes indicating a few of the results of his analysis; it is, however, *probable*¶ that Cauchy first obtained a solution on the assumption of the strict transversality of the luminous vibrations, and then proceeded to apply corrections to the values thus obtained, and it is *possible*** that he adopted in the solution MacCullagh's idea of uniradial directions.

There is no need to enter further into this part of Cauchy's work, as Briot†† has employed both these methods in his excellent adaptation of Cauchy's theory to the problem of Crystalline Reflection.

* *C. R.* xxxi. p. 422.
† *Tom. cit.* p. 509.
‡ *Tom. cit.* pp. 258, 299.
§ *Tom. cit.* p. 257.
‖ *Tom. cit.* p. 297.
¶ *Tom. cit.* p. 160.
** *Tom. cit.* p. 532.
†† *Liouv. Journ.* [2] xii. p. 185.

XXI. *On the Self-induction of Wires.*—Part VI. *By* Oliver Heaviside*.

The most important as well as most frequent application of Mr. S. H. Christie's differential arrangement, known at various times under the names of Wheatstone's parallelogram, lozenge, balance, bridge, quadrangle, and quadrilateral, is to balance the resistances of four conductors, when supporting steady currents due to an impressed force in a fifth, and is done by observing the absence of steady current in a sixth. But its use in other ways and for other purposes has not been neglected. Thus, Maxwell described three ways of using the Bridge to obtain exact balances with transient currents (these will be mentioned later in connection with other methods); Sir W. Thomson has used it for balancing the capacities of condensers†; and it has been used for other purposes. But the most extensive additional use has been probably in connection with duplex telegraphy; and here, along with the Bridge, we may include the analogous differential-coil system of balancing, which is in many respects a simplified form of the Bridge.

On the revival of duplex telegraphy some fifteen years ago, it was soon recognized that "the line" required to be balanced by a similar line, or artificial line, not merely as regards its resistance, but also as regards its electrostatic capacity—approximately by a single condenser; better by a series of smaller condensers separated by resistances; and, best of all, by a more continuous distribution of electrostatic capacity along the artificial line. The effect of the unbalanced self-induction was also observed. This general principle also became clearly recognized, at least by some,—that no matter how complex a line may be, considered as an electrostatic and electromagnetic arrangement, it could be perfectly balanced by means of a precisely similar independent arrangement; that, in fact, the complex condition of a perfect balance is identity of the two lines throughout. The great comprehensiveness of this principle, together with its extreme simplicity, furnish a strong reason why it does not require formal demonstration. It is sufficient to merely state the nature of the case to see, from the absence of all reason to the contrary, that the principle is correct.

Thus, if AB_1C and AB_2C be two identically similar independent lines (which of course includes similarity of environ-

* Communicated by the Author.
† Journal S. T. E. and E. vol. i. p. 394.

ment in the electrical sense in similar parts), joined in parallel, having the A ends connected, and also the C ends, and we join A to C by an external independent conductor in which is an impressed force e, the two lines must, from their similarity, be equally influenced by it, so that similar parts, as B_1 in one line and B_2 in the other, must be in the same state at the same moment. In particular, their potentials must always be equal, so that, if the points B_1 and B_2 be joined by another conductor, there will be no current in it at any moment, so far as the above-mentioned impressed force is concerned, however it vary. The same applies when it is not mere variation of the impressed force e, but of the resistance of the branch in which it is placed. And, more generally, B_1 and B_2 will be always at the same potential as regards disturbances originating in the independent electrical arrangement joining A to C externally, however complex it may be.

There is, however, this point to be attended to, that might be overlooked at first. Connecting the bridge-conductor from B_1 to B_2 must not produce current in it from other causes than difference of potential; for instance, there should be, at least in general, no induction between the bridge-wire and the lines, or some special relation will be required to keep a balance. This case might perhaps be virtually included under similarity of environment.

If we had sufficiently sensitive methods of observation, the statement that one line must be an exact copy of the other would sometimes have to be taken literally. But the word copy may practically be often used to mean copy only as regards certain properties, either owing to the balance being independent of other properties, or owing to our inability to recognize the effects of differences in other properties. Thus, in the steady resistance-balance, we only require AB_1 and AB_2 to have equal total resistances, and likewise B_1C and B_2C; resistances in sequence being additive. But evidently, if the balance is to be kept whilst B_1 and B_2 are shifted together from end to end of the two lines, the resistance must be similarly distributed along them.

If, now, condensers be attached to the lines, imitating a submarine cable, though of discontinuous capacity, we require that the resistance of corresponding sections shall be equal, as well as the capacities of corresponding condensers, in order that we shall have balance in the variable period as well as in the steady state; and the two properties, resistance and capacity, are the elements involved in making one line a copy of the other.

In case of electromagnetic induction, again, if AB_1C and AB_2C each consist of a number of coils in sequence, they will balance if the coils are alike, each for each, in the two lines, and are similarly placed with respect to one another. But the lines will easily balance under simpler conditions, coefficients of self-induction being additive, like resistances; and it is only necessary that the total self-inductions of AB_1 and AB_2 (including mutual induction of their parts) be equal, and likewise of B_1C and B_2C. Again, if a coil a_1 in the branch AB_1 have another coil b_1 in its neighbourhood (not in either line, but independent), and a_2 be a copy of a_1, in the branch AB_2, we can complete the balance by placing a coil b_2 which is a copy of b_1 in the neighbourhood of the coil a_2, so that the action between a_1 and b_1 is the same as that between a_2 and b_2. But it is not necessary for b_1 and b_2 to be copies of one another except in the two particulars of resistance and self-induction; whilst as regards their positions with respect to a_1 and a_2, we only require the mutual induction of a_1 and b_1 to equal that of a_2 and b_2.

On the other hand, if b_1 be a piece of metal, not a coil of fine wire, that is placed near the coil a_1, many more specifications are required to make a copy of it. The piece of metal is not a linear conductor; and, although no doubt only a small number (instead of an infinite number) of degrees of freedom allowed for would be sufficient to make a practical balance, yet, as we have not the means of simply analyzing pieces of metal (like coils) into a few distinct elements, we must generally make a copy of b_1 by means of a similar piece of the same metal, b_2, and place it with respect to a_2 as b_1 is to a_1, to secure a good balance. But very near balances may be sometimes obtained by using quite dissimilar pieces of metal, dissimilarly placed.

So far, copy signifies equality in certain properties. But one line need be merely a reduced copy of the other. It is only when we inquire into what makes one line a reduced copy of another, that we require to examine fully the mathematical conditions of the case in question. In the state of steady flow the matter is simple enough. If AB_1 has n times the resistance of AB_2, then must B_1C have n times the resistance of B_2C to keep the potentials of B_1 and B_2 equal. If condensers be connected to the lines, as before mentioned, we require, first, the resistance-balance of the last sentence applied to every section between a pair of condensers; and next, that the capacity of a condenser in the line AB_1C shall be, not n times (as patented by Mr. Muirhead, I believe), but $1/n$ of the capacity of the corresponding condenser in the line AB_2C*.

* "On Duplex Telegraphy," Phil. Mag. January 1876.

If the lines are representable by resistance, self-induction, electrostatic capacity, and leakage conductance (R, L, S, K of Parts IV. and V., per unit lengths), one line will be a reduced copy of the other if, when R and L in the first line are n times those in the second, S and K in the second are n times those in the first, in similar parts.

After these general remarks, and preliminary to the consideration of the quadrilateral, let us briefly consider the general theory of the conjugacy of a pair of conductors in a connected system, when an impressed force in either can cause no current in the other, either transient or permanent. The direct way is to seek the full differential equation of the current in either, when under the influence of impressed force in the other alone. Let $V=ZC$ be the differential equation of any one branch, C being the current in it, V the fall of potential in the direction of C, and Z the differential operator concerned, according to the notation of Parts III., IV., and V. If there be impressed force e in the branch, it becomes $e+V=ZC$. We have $\Sigma V=0$ in any circuit, by the potential property; therefore $\Sigma e=\Sigma ZC$ in any circuit. Also the currents are connected by conditions of continuity at the junctions. These, together with the former circuit equations, lead us to a set of equations:—

$$\left.\begin{array}{l} FC_1 = f_{11}e_1+f_{12}e_2+\dots \\ FC_2 = f_{21}e_1+f_{22}e_2+\dots \\ \dots\dots\dots \end{array}\right\} \quad . \quad . \quad . \quad . \quad (1c)$$

$C_1, C_2, \dots$, being the currents, and $e_1, e_2, \dots$ the impressed forces in branches 1, 2, &c.; F being common to all, and it and the f's being differential operators. We arrive at similar equations when the differential equation of a branch is not merely between the V and C of that branch, but between those of many branches; for instance when

$$V_1 = Z_{11}C_1+Z_{12}C_2+\dots \quad . \quad . \quad . \quad . \quad (2c)$$

is the form of the differential equation of branch 1.

Now let there be impressed force e in one branch only, and C be the current in a second, dropping the numbers as no longer necessary. We then have

$$FC = f(e). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3c)$$

Conjugacy is therefore secured by $f(e)=0$, making C independent of e. Therefore $f(e)=0$ is the complex condition of conjugacy. If, for example,

$$f(e) = a_0e+a_1\dot{e}+a_2\ddot{e}+\dots, \quad . \quad . \quad . \quad . \quad (4c)$$

where the a's are constants, functions of the electrical constants concerned, then, to ensure conjugacy, we require

$$a_0 = 0, \ a_1 = 0, \ a_2 = 0, \ \&c. \quad . \quad . \quad . \quad (5c)$$

separately; and if these a's cannot all vanish together we cannot have conjugacy.

What C may be then depends only upon the initial state of the system in subsiding, or upon other impressed forces that we have nothing to do with. As depending upon the initial state, the solution is

$$C = \Sigma A\epsilon^{pt}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6c)$$

the summation being with respect to the p's which are the roots of $F(p)=0$, p being put for d/dt in F; and the A belonging to a certain p is to be obtained by the conjugate property of the equality of the mutual electric to the mutual magnetic energy of the normal systems of any pair of p's.

As depending upon e, the impressed force in the conductor which is to be conjugate to the one in which the current is C, let e be zero before time $t=0$, and constant after. Then, by (3c),

$$C = \frac{f(d/dt)e}{F(d/dt)} = \Sigma \frac{f(p)e}{-pF'}(1-\epsilon^{pt})$$

$$= C_0 - \Sigma \frac{f(p)e}{-pF'}\epsilon^{pt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7c)$$

if C_0 is the final steady current, and $F'=dF/dp$, the summation being with respect to the p's.

If there is a resistance-balance, $a_0=0$, $C_0=0$, and

$$C = \Sigma \frac{f(p)e}{pF'}\epsilon^{pt}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8c)$$

Now, subject to (4c), calculate the integral transient current:—

$$\int_0^\infty C\,dt = \Sigma \frac{f(p)e}{-p^2F'},$$

$$= \text{value of } f(p)e/pF(p) \text{ when } p=0,$$

$$= a_1/F_0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9c)$$

if F_0 is the $p=0$ value of F. If then $a_1=0$ also, we prove that the integral transient current is zero.

Supposing both $a_0=0$, $a_1=0$, then

$$C = \Sigma \frac{a_2p^2+\dots}{pF'}\epsilon^{pt};$$

therefore

$$\int_0^t C\,dt = \Sigma \frac{a_2p+\dots}{-pF'}(1-\epsilon^{t}) = \Sigma \frac{a_2p+\dots}{pF'}\epsilon^{pt},$$

and therefore

$$\int_0^\infty dt \int_0^t \mathrm{C}dt = \Sigma \frac{a_2 + \dots}{-p\mathrm{F}'} = \frac{a_2}{\mathrm{F}_0}. \quad . \qquad (10c)$$

Thus, if $a_2 = 0$ also, we have

$$\int_0^\infty dt \int_0^t \mathrm{C}dt = 0. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (11c)$$

Similarly, if $a_3 = 0$ also, then

$$\int_0^\infty dt \int_0^t dt \int_0^t \mathrm{C}dt = 0, \quad . \quad . \quad . \quad . \quad . \qquad (12c)$$

and so on. The physical interpretation of $a_0 = 0$ and $a_1 = 0$ is obvious, but after that it is less easy.

If F contain inverse powers of p, the steady current may be zero. But in spite of that, it will be found that to secure perfect conjugacy for transient currents, we must have a true resistance-balance, or that relation amongst the resistances which would make the steady current zero, if we were to allow the possibility of a steady current by changing the value of other electrical quantities concerned. I will give an example of this later.

I have elsewhere* pointed out these properties of the function F, in the case where there is no mutual induction, or $\mathrm{V} = \mathrm{ZC}$ is the form of the differential equation of a branch. Let n points be united by $\frac{1}{2}n\,(n-1)$ conductors, whose conductances are K_{12}, K_{13}, &c., it being the points that are numbered 1, 2, &c. Then the determinant

$$\begin{vmatrix} \mathrm{K}_{11}, & \mathrm{K}_{12}, & \dots, & \mathrm{K}_{1n} \\ \mathrm{K}_{21}, & \mathrm{K}_{22}, & \dots, & \mathrm{K}_{2n} \\ . & . & . & . \\ \mathrm{K}_{n1}, & \mathrm{K}_{n2}, & \dots, & \mathrm{K}_{nn} \end{vmatrix}$$

is zero, and its first minors are numerically equal, if any K with equal double suffixes be the negative of the sum of the real K's in the same row or column†. Remove the last row and column, and call the determinant that is left F. It is the F required, and is the characteristic function of the combination, expressed in terms of the conductances. If every branch have self-induction, so that $\mathrm{R} + \mathrm{L}(d/dt)$ takes the place of K^{-1}, then $\mathrm{F} = 0$ is the differential equation of the combination, without impressed forces, and $\mathrm{F} = 0$ is always the differential equation subject to the condition of no mutual induction. In the paper referred to cores are placed in the coils, giving a special form to K.

* 'Electrician,' Dec. 20, 1884, p. 106.

† As in Maxwell, vol. i. art. 280.

When K is conductance merely, the characteristic function contains within itself expressions for the resistance between every two points in the combination, which can therefore be written down quite mechanically. For it is the sum of products each containing first powers of the K's, and therefore may be written

$$\mathrm{F} = \mathrm{K}_{12}\mathrm{X}_{12} + \mathrm{Y}_{12} = \mathrm{K}_{23}\mathrm{X}_{23} + \mathrm{Y}_{23} = \dots, \quad . \quad . \qquad (14c)$$

where X_{23}, Y_{23} do not contain K_{23}, and X_{12}, Y_{12} do not contain K_{12}. (It is to be understood that the diagonal K_{11}, K_{22}, ..., is got rid of.)

Then

$$\left.\begin{array}{l} \mathrm{R}'_{12} = \mathrm{X}_{12}/\mathrm{Y}_{12} = \text{resistance between points 1 and 2,} \\ \mathrm{R}'_{23} = \mathrm{X}_{23}/\mathrm{Y}_{23} = \quad \text{,,} \qquad \text{,,} \qquad \text{,,} \quad \text{2 and 3,} \end{array}\right\} \qquad (15c)$$

&c., it being understood that these resistances are not R_{12}, R_{23}, &c., but the resistances complementary to them, the combined resistance of the rest of the combination; thus, if e_{12} be the impressed force in the conductor 1, 2, the current (steady) in it is

$$\mathrm{C}_{12} = \frac{e_{12}}{\mathrm{R}_{12} + \mathrm{X}_{12}/\mathrm{Y}_{12}} = \frac{e_{12}}{\mathrm{R}_{12} + \mathrm{R}'_{12}}. \quad . \quad . \qquad (16c)$$

The proof by determinants is rather troublesome, using the K's, but, in terms of their reciprocals, and extending the problem, it becomes simple enough. Thus if we turn K to R^{-1} in F, and then clear of fractions, we may write $\mathrm{F} = 0$ as

$$\mathrm{R}_{12}\mathrm{X}'_{12} + \mathrm{Y}'_{12} = 0, \quad \mathrm{R}_{23}\mathrm{X}'_{23} + \mathrm{Y}'_{23} = 0, \quad \text{\&c.,} \quad . \quad . \qquad (17c)$$

where X'_{12}, Y'_{12}, do not contain R_{12}; &c. From this we see that the differential equation of the current C_{12} in 1, 2, subject to e_{12} only, is

$$(\mathrm{R}_{12} + \mathrm{R}'_{21})\,\mathrm{C}_{12} = e_{12}, \quad . \quad . \quad . \quad . \quad . \qquad (18c)$$

if $\mathrm{R}'_{21} = \mathrm{Y}'_{12}/\mathrm{X}'_{12}$. For this make the dimensions correct, and that is the only additional thing required, when we observe that it makes the fixed steady current

$$\mathrm{C}_{12} = e_{12}/(\mathrm{R}_{12} + \mathrm{R}'_{21}), \quad . \quad . \quad . \quad . \quad . \qquad (19c)$$

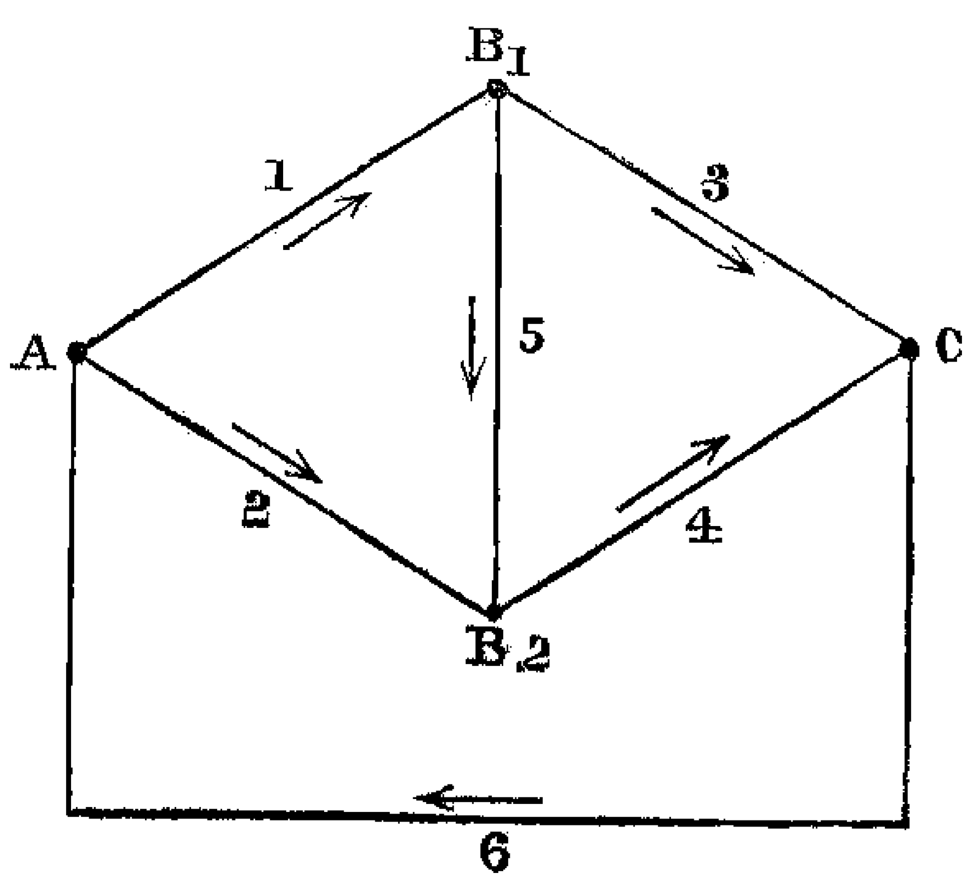
so that R'_{21} is the resistance complementary to R_{12}.

Although it is generally best to work in terms of resistances, yet there are times when conductances are preferable, and, to say nothing of conductors in parallel arc, the above is a case in point, as will be seen by the way the characteristic function is made up out of the K's. There is also less work in another way. Thus, $\frac{1}{2}n(n-1)$ conductors uniting n points give $\frac{1}{2}(n-1)(n-2)$ degrees of freedom to the currents. It is the least number of branches in which, when the currents in them are given, those in all the rest follow. Thus, if 10 conductors unite 5 points, the currents in at least 6 conductors must be given, and no four of them should meet at one point. The remaining conductors are $n-1$ in number, or one less than the number of points, and $n-1$ is the degree of the characteristic function in terms of the conductances. Now put $F=0$ in terms of the resistances, by multiplying by the product of all the resistances. It is then made of degree $\frac{1}{2}(n-1)(n-2)$ in terms of the resistances, which is the number of current freedoms. If $n=4$, the degree is the same, viz. three, whether in terms of conductances or resistances; but if $n=5$, it is of the sixth degree in terms of resistances and only of the fourth in terms of the conductances; and if $n=6$, it is of the tenth degree in terms of the resistances, but only of the fifth in terms of the conductances, and so on; so that F becomes enormously more complex in terms of resistances than conductances.

When every branch has self-induction, $Z=R+Lp$, and the degree of p in $F=0$ is the number of freedoms, so that there are $n-1$ fewer roots than the number of branches. It is the same when there is mutual induction. The missing roots belong to terms in the solutions for subsidence from an arbitrary initial state which instantaneously vanish, producing a jump from the initial state to another, which subsides in time.

On the other hand, if every branch (without self-induction) is shunted by a condenser of capacity S_1, S_2, &c., K becomes $K+Sp$, so that the degree of p in $F=0$ is the same as that of K, or $\frac{1}{2}(n-1)(n-2)$ fewer than the number of condensers *.

Coming next to the Wheatstone quadrilateral self-induction balance, let there be six conductors, 1, 2, &c., uniting the four points A, B_1, B_2, C in the figure. AB_1C and AB_2C are the lines referred to in the beginning. Let R be the resistance and L the inductance of a branch in which the current is C, reckoned positive in the direction of the arrow, and the fall of potential V in the same direction; thus R_1, L_1, V_1, C_1 for the first branch. The six branches may be conjugate in pairs, thus: 1 and 4, or 2 and 3, or 5 and 6. In the following 5 and 6 are selected always, the battery or other source being in 6, and the telephone or other indicator in 5. Mutual inductances will be denoted by M; thus, $M_{12}\,C_1$ is the electromotive impulse in 2 due to the stoppage of the current C_1 in 1; similarly $M_{12}\,C_2$ is the impulse in 1 due to stopping C_2.



* 'Electrician,' Jan. 1, 1886, p. 147.

Deferring mutual induction for the present, though not confining self-induction to be of the electromagnetic kind only, but to include electrostatic if required, the condition of conjugacy is that the potentials at B_1 and B_2 be always equal. Therefore

$$V_1=V_2, \quad \text{and} \quad V_3=V_4; \quad . \quad . \quad . \quad . \quad (20c)$$

so, if $V=ZC$,

$$Z_1C_1=Z_2C_2, \quad \text{and} \quad Z_3C_3=Z_4C_4. \quad . \quad . \quad . \quad (21c)$$

But, by continuity, $C_1=C_3$, and $C_2=C_4$ at every moment (including equality of all their differential coefficients); so that (21c) becomes

$$Z_1C_1=Z_2C_2, \qquad Z_3C_1=Z_4C_2; \quad . \quad . \quad . \quad . \quad (22c)$$

consequently

$$Z_1Z_4-Z_2Z_3=0=f \quad . \quad . \quad . \quad . \quad . \quad . \quad (23c)$$

is the complex condition of conjugacy. This function is the f of the previous investigation.

When the self-induction is of the electromagnetic kind, $Z=R+Lp$; so that, arranging f in powers of p,

$$=(R_1R_4-R_2R_3)+(R_1L_4+R_4L_1-R_2L_3-R_3L_2)p+(L_1L_4-L_2L_3)p^2. \quad (24c)$$

Therefore, if $x=L/R$, the time-constant of a branch, we have three conditions to satisfy, namely,

$$\left.\begin{aligned} R_1R_4&=R_2R_3, \\ x_1+x_4&=x_2+x_3, \\ L_1L_4&=L_2L_3. \end{aligned}\right\} \quad \begin{aligned} &. \quad . \quad . \quad . \quad . \quad . \quad (25c)\\ &. \quad . \quad . \quad . \quad . \quad . \quad (26c)\\ &. \quad . \quad . \quad . \quad . \quad . \quad (27c)\end{aligned}$$

"If the first condition is fulfilled, there will be no final current in 5 when a steady impressed force is put in 6. This is the condition for a true resistance balance.

"If, in addition to this, the second condition is also satisfied, the integral extra current in 5 on making or breaking 6 is zero, besides the steady current being zero, (25c) and (26c) together therefore give an approximate induction balance with a true resistance balance.

"If, in addition to (25c) and (26c), the third condition is satisfied, the extra current is zero at every moment during the transient state, and the balance is exact however the impressed force in 6 vary.

"Practically, take

$$R_1=R_2, \quad \text{and} \quad L_1=L_2 ; \quad . \quad . \quad . \quad . \quad (28c)$$

that is, let branches 1 and 2 be of equal resistance and inductance. Then the second and third conditions become identical; and, to get perfect balance, we need only make

$$R_3=R_4, \quad \text{and} \quad L_3=L_4. \quad . \quad . \quad . \quad . \quad (29c)$$

"This is the method I have generally used, reducing the three conditions to two, whilst preserving exactness. It is also the simplest method. The mutual induction, if any, of 1 and 2, or of 3 and 4, does not influence the balance when this ratio of equality $R_1=R_2$ is employed (whether $L_1=L_2$ or not) *. So branches 1 and 2 may consist of two similar wires wound together on the same bobbin, to keep their temperatures equal." †

Of the eight quantities, four R's and four L's, only five can be stated arbitrarily, of which not more than three may be R's, and not more than three may be L's. We may state the matter thus :—There must first be a resistance-balance. Then, if we give definite values to two of the L's, the corresponding time-constants become fixed, and it is required that the other two time-constants shall be equal to them ; thus

$$\text{either} \quad x_1=x_3 \quad \text{and} \quad x_2=x_4,$$
$$\text{or else} \quad x_1=x_2 \quad \text{and} \quad x_3=x_4.$$

Thus the remaining two L's become usually fixed. In fact, eliminating R_4 and L_4 from (26*c*) by (25*c*) and (27*c*), the second condition may be written

$$(x_1-x_2)(x_1-x_3)=0.$$

Suppose R_1, R_2, R_3 given, then R_4 is fixed by (25*c*). Two of the inductances may then be given, fixing the corresponding time-constants. If these inductances be L_1 and L_2, then we must have (unless $x_1=x_2$)

$$x_1=x_3, \quad x_2=x_4.$$

But if L_1 and L_3 be given, then we require (unless $x_1=x_3$)

$$x_1=x_2, \quad x_3=x_4.$$

These two cases present a remarkable difference in one respect. The absence of current in 5 allowing us to remove 5 altogether, we see by (18*c*) that the differential equation of C_6 is

$$e=\left\{Z_6+\frac{(Z_1+Z_3)(Z_2+Z_4)}{Z_1+Z_2+Z_3+Z_4}\right\}C_6,$$

manipulating the Z's like resistances. The absence of branch 5 thus reduces the number of free-subsidence systems to two. Now, if we choose $x_1=x_2$, we shall make

$$(L_1+L_3)/(R_1+R_3) = (L_2+L_4)/(R_2+R_4),$$

or the time-constants of the two branches 1+3 and 2+4 equal. Then one of the p's is

$$p_1=-\frac{R_1+R_3}{L_1+L_3} ;$$

and this is only concerned in the free subsidence of current in the circuit AB_1CB_2A. Consequently the second p, which is

$$p_2=-\frac{(R_1+R_3)R_2+R_6(R_1+R_2)}{(L_1+L_3)R_2+L_6(R_1+R_2)},$$

is alone concerned in the setting-up of current by the impressed force in 6 ; and the current divides between AB_1C and AB_2C in the ratio of their conductances, in the variable period as well as finally. In fact, the fraction in the above equation of C_6 will be found to contain Z_1+Z_3 as a factor in its numerator and denominator, thus excluding the p_1 root, so far as e is concerned. On the other hand, if we choose $x_1=x_3$, we do not have equality of time-constants of AB_1C and AB_2C, so that there are two p's concerned, which are not those given ; and the current C_6 does not, in the variable period, divide between AB_1C and AB_2C in the ratio of their conductances, but only finally.

In the above statement it was assumed that when L_1 and L_2 were chosen, it was not so as to make $x_1=x_2$. When this happens, however, it is only the ratio of L_3 to L_4 that becomes fixed, for we have $x_3=x_4$ = anything.

Similarly, when L_1 and L_3 are so chosen that $x_1=x_3$, we shall have $x_2=x_4$ = anything, so that only the ratio of L_2 to L_4 is fixed.

And if L_3, L_4 be so chosen that $x_3=x_4$, then $x_1=x_2$ = anything, only fixing the ratio of L_1 to L_2. But should x_3 not $=x_4$, then we require $x_1=x_3$ and $x_2=x_4$, thus fixing L_1 and L_2.

And if L_2, L_4 be so chosen that $x_2=x_4$, then $x_1=x_3$= anything, only fixing the ratio of R_1 to R_3. But if so that x_2 not $=x_4$, then $x_1=x_2$ and $x_3=x_4$ fix L_1 and L_3.

There are yet two other pairs that may be initially chosen, and with somewhat different results. Let it be L_1 and L_4 that are chosen ; if not so as to make $x_1=x_4$, there are two ways

* The words in the ( ) should be cancelled. The independence of M_{12} and M_{34}, which is exact when $L_1=L_2$, $L_3=L_4$, and sensibly true when the inequalities are small, becomes sensibly untrue when the inequalities L_1-L_2 and L_3-L_4 are great.

† 'Electrician,' April 30, 1886, p. 489.

of fixing L_2 and L_3, viz. either by $x_1=x_3$ and $x_2=x_4$, or by $x_1=x_2$ and $x_3=x_4$; but if so that $x_1=x_4$ in the first place, then they must also $=x_2=x_3$.

Similarly the choice of L_2 and L_3 so as not to make $x_2=x_3$ gives two ways of fixing L_1 and L_4, by vertical or by horizontal equality of time-constants, as before; whilst $x_2=x_3$ produces equality all round.

The special case of all four sides equal in resistance may be also noticed. Balance is given in two ways, either by horizontal or by vertical equality in the L's.

Leaving the mathematical treatment for a little while, I proceed to give a short general account of my experience of induction-balances. I did not originally arrive at the method of equal ratio just described through the general theory, (20*c*) to (27*c*), but simply by means of the general principle of balancing by making one line a copy of the other, of which I obtained knowledge through duplex telegraphy, and investigated the conditions (25*c*) to (27*c*) more from curiosity than anything else, though the investigation came in useful at last. In 1881 I wished to know what practical values to give to the inductances of various electromagnets used for telegraphic purposes, and to get this knowledge went to the quadrilateral. Not having coils of known inductance to start with, I employed Maxwell's condenser method *, with an automatic intermitter and telephone. Let 1, 2, and 3 be inductionless resistances, and 4 a coil having self-induction. Put the telephone in 5, the battery and intermitter in 6. We require first the ordinary resistance-balance, $R_1R_4=R_2R_3$. But the self-induction of the coil will cause current in 5 when 6 is made or broken. This will be completely annulled by shunting 1 by a condenser of capacity S_1, such that

$$R_1S_1 = L_4/R_4,$$

signifying that the time-constant of the coil on short-circuit and that of the condenser on short-circuit with the resistance R_1 are equal.

The method is, in itself, a good one. But the double adjustment is sometimes very troublesome, especially if the capacity of the condenser be not adjustable. For when we vary R_1, to approximate to the correct value of R_1S_1, we upset the resistance-balance, and have therefore to make simultaneous variations in some of the other resistances to restore it. But the method has the remarkable recommendation of giving us the value of the inductance of a coil at once in electromagnetic units.

* Maxwell vol. ii. art. 778.

In the course of these experiments I observed the upsetting of the resistance- and induction-balance by the presence of metal in the neighbourhood of the coils, which is manifested in an exaggerated form in electromagnets with solid cores. So, having got the information I wanted in the first place, I discarded the condenser method with its troublesome adjustments, and, to study these effects with greater ease, went to the equal-ratio method with the assistance that I had obtained by the condenser method, the values of the inductances of various coils, to be used as standards.

"To use the Bridge to speedily and accurately measure the inductance of a coil, we should have a set of proper standard coils, of known inductance and resistance, together with a coil of variable inductance, *i. e.* two coils in sequence, one of which can be turned round, so as to vary the inductance from a minimum to a maximum *. The scale of this coil could be calibrated by (12*a*), first taking care that the resistance-balance did not require to be upset. This set of coils, in or out of circuit according to plugs, to form say branch 3, the coil to be measured to be in branch 4. Ratio of equality. Branches 1 and 2 equal. Of course inductionless, or practically inductionless, resistances are also required to get and keep the resistance-balance. The only step to this I have made (this was some years ago) .... was to have a number of little equal coils, and two or three multiples; and get exact balance by allowing induction between two little ones, with no exact measurement of the fraction of a unit." †

Although rather out of order, it will be convenient to mention here that although I have not had a regular induction-box made (the coils, if close together, would have to be closed solenoids), yet shortly after making these remarks, I returned to my earlier experiments by calibrating the scale of the coil of variable inductance. As it then becomes an instrument of precision, it deserves a name; and as it is for the measurement of induction it may, I think, be appropriately termed an Inductometer. Of course, for many purposes no calibration is needed.

I found that the calibration could be effected with ease and rapidity by the condenser method more conveniently than by comparisons with coils. Thus, first ascertain the minimum and the maximum inductance, and that of the coils separately. Suppose the range is from 20 to 50 units (hundreds, thou-

* Prof. Hughes's oddly named Sonometer will do just as well, if of suitable size and properly connected up. It is the manner of connection and use that give individuality to my inductometer.

† 'Electrician,' April 30, 1886, p. 490.

sands, millions, &c. of centimetres, according to the quite arbitrary size of the instrument). It will then be sufficient to find the places on the scale corresponding to 20, 21, 22, &c., 49, 50. Starting at 21, set the resistance-balance so that L_4 should be 21 units; turn the movable coil till silence is reached, and mark the place 21. Then set the balance to suit 22, turn again till silence comes, and mark again; repeat throughout the whole range. Why this can be done rapidly is because the resistance-balance is at every step altered in the same manner. We have thus an instrument of constant resistance and variable known inductance, ranging from

$$l_1+l_2-2m_0 \quad \text{to} \quad l_1+l_2+2m_0,$$

if l_1 and l_2 are the separate inductances and m_0 the maximum mutual inductance. The calibration is thoroughly practical, as no table has to be referred to to find the value of a certain deflection.

I formerly chose 10^9 centim. as a practical unit of inductance, and called it a tom; the attraction this had for me arose from L toms÷R ohms equalling L/R seconds of time. But it was too big a unit, and millitoms and microtoms were wanted. Another good name is mac. 10^6 centim. might be called a mac. Since Maxwell made the subject of self-induction his own, and described methods of correctly measuring it, there is some appropriateness in the name, which, as a mere name, is short and distinctive.

The two coils of the inductometer need not be equal; but it is very convenient to make them so, before calibration, by the equal-ratio method, which, of course, merely requires us to get a balance, not to measure the values. Let 1 and 2 be any equal coils; put one coil of the inductometer in 3, the other in 4, and balance. It happened by mere accident that my inductometer had nearly equal coils; so I made them quite equal, to secure two advantages. First, there is facility in calculations; next, the inductometer may be used with its coils in parallel or in sequence, as desired. When in parallel, the effective resistance and inductance are each one fourth of the sequence values. Thus, let V=ZC be the differential equation of the coils in parallel, C being the total current, and V the common potential fall; it is easily shown that

$$Z=\frac{(r_1+l_1p)(r_2+l_2p_2)-m^2p^2}{r_1+r_2+(l_1+l_2-2m)p}, \quad . \quad . \quad . \quad (30c)$$

when the coils are unequal; r_1 and r_2 being their resistances, l_1 and l_2 their inductances, and m their mutual inductance in

any position. Now make $r_1=r_2$, and $l_1=l_2$; this reduces Z to

$$Z=\tfrac{1}{2}r+\tfrac{1}{2}(l+m)p\,; \quad . \quad . \quad . \quad . \quad . \quad (31c)$$

whilst, when in sequence, we have

$$Z=2r+2(l+m)p, \quad . \quad . \quad . \quad . \quad . \quad . \quad (32c)$$

thus proving the property stated. We may therefore make one inductometer serve as two distinct ones, of low or high resistance.

There does not seem to be any other way of making the two coils in parallel behave as a single coil as regards external electromotive force. Any number of coils whose time-constants are equal will, when joined up in parallel, behave as a single coil of the same time-constant; but there must be no mutual induction. (An example of the property * that any linear combination whose parts have the same time-constant has only that one time-constant.) This seriously impairs the utility of the property. This reservation does not apply in the case of the equal-coil inductometer.

Having got the inductometer calibrated, we may find the inductance of a given coil, or of a combination of coils in sequence, with or without mutual induction, nearly as rapidly as the resistance. Thus, 1 and 2 being equal, put the coil to be measured in 3, and the inductometer in 4. We have to make $R_3=R_4$ and $L_3=L_4$, or to get a resistance-balance, and then turn the inductometer till silence is reached, when the scale-reading tells us the inductance. This assumes that L_3 lies within the range of the inductometer. If not, we may vary the limits as we please by putting a coil of known inductance in sequence with branch 3 or 4 as required, putting at the same time equal resistance in the other branch.

Or, the inductometer being in 4, and 1, 2 being inductionless resistances, put the coil to be measured in 3. If it has a larger time-constant than the inductometer's greatest, insert resistance along with it to bring the time-constants to equality. The conditions of silence are $R_1R_4=R_2R_3$ and $L_3/R_3=L_4/R_4$. Here a ratio of equality is not required. The method is essentially the same as one of Maxwell's†, and is a good one for certain purposes.

Or, 1 and 2 being any equal coils, put one coil of the

* This property supplies us with induction-balances of a peculiar kind. Let there be any network of conductors, every branch having the same time-constant. Set up current in the combination, and then remove the impressed force. During the subsidence all the *junctions* will be at the same potential, and any pair of them may consequently be joined by an external conductor without producing current in it.

† Maxwell, vol. ii. art. 757.

inductometer in 6 and the other in 4, the coil to be measured being in 3. Then

$$L_3 = L_4 - 2M_{46} \quad . \quad . \quad . \quad . \quad . \quad . \quad (33c)$$

gives the induction-balance, L_4 being here the inductance of the coil of the inductometer in 4, and M_{46} the mutual inductance of the two coils, in the position giving silence. This is known in all positions, because the scale-reading gives the value of $l_1 + l_2 + 2m$ (or else $2(l+m)$ if the coils are equal), and $l_1 + l_2$ is known. If the range is not suitable, we may, as before, insert other coils of known inductance.

There are other ways; but these are the simplest, and the equal-ratio method is preferable for general purposes. I have spoken of coils always, where inductances are large and small errors unimportant. When, however, it is a question of small inductances, or of experiments of a philosophical nature, needing very careful balancing, then the equal-ratio method acquires so many advantages as to become *the* method.

"So long as we keep to coils we can swamp all the irregularities due to leading wires &c., or easily neutralize them, and can therefore easily obtain considerable accuracy. With short wires, however, it is a different matter. The inductance of a circuit is a definite quantity: so is the mutual inductance of two circuits. Also, when coils are connected together, each forms so nearly a closed circuit that it can be taken as such; so that we can add and subtract inductances, and localize them definitely as belonging to this or that part of a circuit. But this simplicity is, to a great extent, lost when we deal with short wires, unless they are bent round so as to make nearly closed circuits. We cannot fix the inductance of a straight wire, taken by itself. It has no meaning, strictly speaking. The return current has to be considered. Balances can always be got, but as regards the interpretation, that will depend upon the configuration of the apparatus.

"Speaking with diffidence, having little experience with short wires, I should recommend 1 and 2 to be two equal wires, of any convenient length, twisted together, joined at one end, of course slightly separated at the other, where they join the telephone wires, also twisted. The exact arrangement of 3 and 4 will depend on circumstances. But always use a long wire rather than a short one (experimental wire). If this is in branch 4, let branch 3 consist of the standard coils (of appropriate size), and adjust *them*, inserting, if necessary, coils in series with 4 also. Of course I regard the matter from the point of view of getting easily interpretable results."*

* 'Electrician,' April 6, 1886, p. 490.

Consider the equations (24*c*) to (27*c*). *Three* conditions have to be satisfied, in general, the resistance-balance (25*c*) and the balance of integral extra-current (26*c*) not being sufficient. To illustrate this in a simple manner, let 2 and 3 be equal coils, by previous adjustment, and 1 and 4 coils having the same resistance as the others, but of lower inductance, or else two coils whose total resistance in sequence is that of each of the others, but of lower inductance when separated. The resistance-balance is satisfied, of course. Now, if the next condition were sufficient to make an induction-balance, all we should have to do would be to make $L_1 + L_4 = 2L_3$. For instance, if L_1 is first adjusted to equal L_2 and L_3, then, by increasing either L_1 or L_4 to the right amount, silence would result. It does result when it is L_4 that is increased, but not when it is L_1. If the sound to be quenched is slight, the residual sound in the L_1 case is feeble and might be overlooked; but if it be loud, then the residual sound in the L_1 case is loud and is comparable with that to be destroyed, whilst in the L_4 case there is perfect silence.

The reason of this is that in the L_1 case we satisfy only the second condition, whilst in the L_4 case we satisfy the third as well.

Another way to make the experiment is to make 1, 2, and 3 equal, and 4 of the same resistance but of lower inductance, much lower. Then the insertion of a non-conducting iron core in 1 will lead to a loud minimum, but if put in 4 will bring us to silence, except as regards something to be mentioned later.

Supposing, however, we should endeavour to get silence by operating upon L_1, although we cannot do it exactly, yet by destroying the resistance-balance we may approximate to it. Thus we have a false resistance- and a false induction-balance, and the question would present itself, If we were to wilfully go to work in this way in the presence of exact methods, how should we interpret the results? As neither (25*c*) nor (26*c*) is true, it is suggested that we make use of the formula based upon the assumption that the currents are sinusoidal or pendulous, or S.H. functions of the time. Take $p^2 = -n^2$ in (24*c*), the frequency being $n/2\pi$, and we find

$$R_1R_4(x_1 + x_4) = R_2R_3(x_2 + x_3), \quad . \quad . \quad . \quad (34c)$$

$$(R_1R_4 - R_2R_3) = n^2(L_1L_4 - L_2L_3) \quad . \quad . \quad (35c)$$

are the two conditions to be satisfied; and we can undoubtedly, if we take enough trouble, correctly interpret the results, if the assumption that has been made is justifiable.

I should have been fully inclined to admit (and have no doubt it is sometimes true) that, with an intermitter making regular vibrations, we might regard the residual sound as due to the upper partials, and that $n/2\pi$ could be taken as the frequency of the intermitter, and (34 *c*), (35 *c*) employed safely, though not with any pretensions to minute accuracy, if circumstances compelled us to ignore the exact methods of true balances, were it not for the fact that this hypothesis sometimes leads to utterly absurd results when experimentally tested. Of this I will give an illustration, and, as we have only to test that intermittences may be regarded as S.H. reversals, simplify by taking $R_1=R_2$, $L_1=L_2$, which makes an exact equal-ratio balance, $R_3=R_4$, $L_3=L_4$.

Since a steady or slowly varying current does not produce sound in the telephone, if a battery could be treated as an ordinary conductor, we could put it in one of the sides of the quadrilateral and balance it, just like a coil, in spite of its electromotive force. So, let 1 and 2 be equal coils, 3 the battery to be tested, and 4 the balancing coils. I find that a good battery can be very well balanced, though not perfectly, with intermittences, as regards resistance, which is, however, far less with rapid intermittences than with a steady current*. Thus: steady, $2\frac{1}{2}$ ohms; intermittent (about 500), $1\frac{1}{2}$ ohm. Another battery: steady, 166 ohms; intermittent, 126 ohms. The steady resistances are got by cutting out the intermitter, using a make-and-break instead; the deflection of a galvanometer in 5 must be the same whether 6 is in or out. If we leave out the battery in 6, it becomes Mance's method. The sensitiveness is, however, far greater when the battery is not left out, although other effects are then produced.

So far regarding the resistance. As regards the inductance, or apparent inductance, of batteries, that is, I find, usually negative. That is to say, after bringing the sound to a minimum by means of resistance-adjustment, the residual sound (sometimes considerable) may be quenched by inserting equal coils in branches 3 and 4, and then increasing the inductance of the one containing the battery under test. I selected the battery which showed the greatest negative inductance, about $\frac{1}{2}$ mac, or 500,000 centim., got the best possible silence by adjustment of resistance and inductance, and then found the residual sound could be nearly quenched by allowing induction between the coil in 3 and a silver coin, provided, at the same time, R_4 were a little increased.

It was naturally suggested by the negative inductance and lower resistance that the battery behaved as a shunted condenser, or as a shunted condenser with resistance in sequence, or something similar; and I examined the influence of the frequency on the values of the effective resistance and inductance. The change in the latter was uncertain, owing to the complex balancing, but the apparent resistance was notably increased by increasing the frequency, viz. from 125 to 130 ohms, when the frequency was raised from about 500 to about 800, whilst there was a small reduction in the amount of the negative inductance. The effect was distinct, under various changes of frequency, but was the opposite (as regards resistance) of what I expected on the S.H. assumption. To see whereabouts the minimum apparent resistance was (being 165 steady), I lowered the speed by steps. The resistance went down to 113 with a slow rattle, and so there was no minimum at all. The S.H. assumption had not the least application to the apparent resistance, as regards the values 165 steady, 113 slow intermittences, although it no doubt is concerned in the rise from 113 to 130 at frequency 800. The balance (approximate) was some complex compromise, but was principally due to a vanishing of the integral extra-current. Of course in such a case as this we should employ a strictly S.H. impressed force; a remark that applies more or less in all cases where the combination tested does not behave as a mere coil of constant R and L.

* I am aware that Kohlrausch employs the telephone with intermittences to find the resistance of electrolytes, but have no knowledge of how he gets at the true resistance.

The other effects, due to using a battery in branch 6 as well, are complex. It made little difference when the current in the cell was in its natural direction; but on reversal (by reversing the battery in 6) there was a rapid fall in the resistance—for instance, from 46 ohms to 18 ohms in half a minute in the case of a rather used-up battery, but a comparatively small fall when the battery was good.

Besides the advantage of independence of the manner of variation of the impressed force (in all cases where the resistance and inductance do not vary with the frequency), and the great ease of interpretation, the equal-ratio method gives us independence of the mutual induction of 1 and 2 and of 3 and 4; and this, again, leads to another advantage of an important kind. If the arrangement is at all sensitive, the balance will continually vary, on account of temperature inequalities occurring in experimenting, caused by the breath, heat of hands, lamps, &c. Now, if the four sides of the quadrilateral consist of four coils, equal in pairs, it is a difficult matter to follow the temperature changes. To restore a resistance-balance is easy enough; but more than that is needed, viz. the preservation of the ratio of equality. But, by reason of the independence of the self-induction balance of M_{12}, we may, as before mentioned, wind them together, and thus ensure their equality at every moment. There is then only left the inequality between branches 3 and 4, which must, of course, be separated for experimental purposes, and that is very easily followed and set right. When a sound comes on, holding a coin over the coil of lower resistance will quench it, if it be slight and due to resistance inequality, and tell us which way the inequality lies. If it be louder, the cancelling will be still further assisted by an iron wire over or in the same coil, or by a thicker iron wire alone, for reasons to be presently mentioned.

On the other hand, a small inequality in the inductance may be at once detected by a fine iron wire, quenching the sound when over or in the coil of lower inductance; and when the resistance- and induction-balances are both slightly wrong, a combination of these two ways will show us the directions of departure. These facts are usefully borne in mind and made use of when adjusting a pair of coils to equality, during which process it is also desirable to handle them as little as possible, otherwise the heating will upset our conclusions and cause waste of time. But a pair of coils once adjusted to equality, and not distorted in shape afterwards, will practically keep equal in inductance; for the effect of temperature-variation on the inductance is small, compared with the resistance change.

Regarding the intermitter, I find that it is extremely desirable to have one that will give a pure tone, free from harsh irregularities, for two reasons: first, it is extremely irritating to the ear, especially when experiments are prolonged, to have to listen to irregular noises or grating and fribbling sounds; next, there is a considerable gain in sensitiveness when the tone is pure*.

Coming now to the effects of metal in the magnetic field of a coil, the matter is more easily understood from the theoretical point of view in the first instance than by the more laborious course of noting facts and evolving a theory out of them—a quite unnecessary procedure, seeing that we have a good theory already, and, guided by it, have merely to see whether it is obeyed and what the departures are, if any, that may require us to modify it.

First, there is the effect of inductive magnetization in increasing the inductance of a coil. Diamagnetic decrease is quite insensible, or masked by another effect, so that we are confined to iron and the other strongly magnetic bodies. The foundation of the theory is Poisson's assumption (no matter what his hypothesis underlying it was) that the induced magnetization varies as the magnetic force; and when this is put into a more modern form, we see that impressed magnetic force is related to a flux, the magnetic induction, through a specific quality, the inductivity, in the same manner as impressed electric force is related to electric conduction-current through that other specific quality, the conductivity of a body. Increasing the inductivity in any part of the magnetic field of a coil, therefore, always increases the inductance L, or the amount of induction through the coil per unit current in it, and the magnetic energy, $\frac{1}{2}LC^2$. The effect of iron therefore is, in the steady state, merely to increase the inductance of a coil, without influence on its resistance. I have, indeed, speculated* upon the existence of a magnetic conduction-current, which is required to complete the analogy between the electric and magnetic sides of electromagnetism; but whilst there does not appear to be any more reason for its existence than its suggestion by analogy, its existence would lead to phenomena which are not observed.

* *I. e.* pure in the common acceptation, not in the scientific sense of having a definite single frequency, which is only needed in a special class of cases, when no true balance could be got without it.

But this increase of L by a determinable amount—determinable, that is, when the distribution of inductivity is known, on the assumption that the only electric current is that in the coil—breaks down when there are other currents, connected with that in the coil, such as occur when the latter is varying, the induced currents in whatever conducting matter may be in the field. L then ceases to have any definite value. But in one case, that of S.H. variation, the mean value of the magnetic energy becomes definite, viz. $\frac{1}{4}L'C_0^2$, where L' is the effective L, and C_0 the amplitude of the coil-current, the change from $\frac{1}{2}$ to $\frac{1}{4}$ being by reason of the mean of the square of a sine or cosine being $\frac{1}{2}$. This definiteness must be, because the variation of the coil-current is S.H., as well as that of the whole field. That L' is less than L, the steady-flow value, may be concluded in a general though vague manner from the opposite direction of an induced current to that of an increasing primary, and its magnetic field in the region of the primary; or, more distinctly, from the power of conducting-matter to temporarily exclude magnetic induction.

In a similar manner, the resistance of a coil, if regarded as the R in RC^2, the Joulean generation of heat per second,

* 'Electrician,' January 4, 1885, p. 219 *et seq.*

ceases to have a definite value when the current is varying, if C be taken to be the coil-current, on account of the external generation of heat. But in the S.H. case, as before, the mean value is necessarily a definite quantity (at a given frequency), making $\frac{1}{2}R'C_0^2$ the heat per second, where R′ is the effective resistance. That R′ is always greater than R is certain, and obvious without mathematics; for the coil-heat is $\frac{1}{2}RC_0^2$, and there is the external heat as well. It is suggested that, in a similar manner, a non-mathematical and equally clear demonstration of the reduction of L is possible. The magnetic energy of the coil-current alone is $\frac{1}{4}LC_0^2$, and we have to show non-mathematically, but quite as clearly as in the argument relating to the heat, that the existence of induced external current reduces the energy without any reference to a particular kind of coil or kind of distribution of the external conductivity. Perhaps Lord Rayleigh's dynamical generalization* might be made to furnish what is required.

When the matter is treated in an inverse manner, not regarding electric current as causing magnetic force, but as caused by or being an affection of the magnetic force, there is some advantage gained, inasmuch as we come closer to the facts as a whole, apart from the details relating to the reaction on the coil-current. Magnetic force, and with it electric current, a certain function of the former, are propagated with such immense rapidity through air that we may, for present purposes, regard it as an instantaneous action. On the other hand, they are diffused through conductors in quite another manner, quite slowly in comparison, according to the same laws as the diffusion of heat, allowing for their being vector magnitudes, and that the current must be closed, thus producing lateral propagation. The greater the conductivity and the inductivity, the slower the diffusion. Hence a conductor brought with sufficient rapidity into a magnetic field is, at the first moment, only superficially penetrated by the magnetic disturbance to an appreciable extent; and a certain time—which is considerable in the case of a large mass of metal, especially copper, by reason of high conductivity, and more especially iron, by reason of high inductivity more than counteracting the effect of its lower conductivity—is required before the steady state is reached, in which the magnetic field is calculable from the coil-current and the distribution of inductivity. And hence a sufficiently rapidly oscillatory impressed force in the coil-circuit induces only superficial currents in a piece of metal in the field of the coil, the interior being comparatively free from the magnetic induction.

* Phil. Mag. May 1886.

The same applies to the conductor forming the coil-circuit itself; it also may be regarded as having the magnetic disturbance diffused into its interior from the boundary, and we have only to make the coil-wire thick enough to make the effect of the approximation to surface-conduction experimentally sensible. But in common fine-wire coils it may be wholly ignored, and the wires regarded as linear circuits. There is no distinction between the theory for magnetic and for non-magnetic conductors; we pass from one to the other by changing the values of the two constants, conductivity and inductivity. Nor is there any difference in the phenomena produced, if the steady state be taken in each case as the basis of comparison. But, owing to copper having practically the same inductivity as air, there seems to be a difference in the theory which does not really exist.

A fine copper wire placed in one (say in branch 3) of a pair of balanced coils in the quadrilateral, under the influence of intermittent currents, produces no effect on the balance. Its inductivity is that of the air it replaces, so that the steady magnetic field is the same; and it is too small for the diffusion effect to sensibly influence the balance. On the other hand, a fine iron wire, by reason of high inductivity, requires the inductance of the balancing-coil (say in 4) to be increased. The other effect is small in comparison, but quite sensible, and requires a small increase of the resistance of branch 4 to balance it. A thick copper wire shows the diffusion effect; and if we raise the speed and increase the sensitiveness of the balance, its thickness may be decreased as much as we please, if other things do not interfere, and still show the diffusion effect. If thick, so that the disturbance is considerable, the approximate balancing of it by change of resistance is insufficient, and the inductance of coil 4 requires a slight decrease or that of 3 a slight increase. A thick iron wire shows both effects strongly: the inductance and the resistance of branch 3 must be increased. These effects are greatly multiplied when big cores are used; then the balancing, with intermittences, at the best leaves a considerable residual sound. The influence of pole-pieces and of armatures outside coils in increasing the inductance, which is so great in the steady state, becomes relatively feeble with rapid intermittences. This will be understood when the diffusion effect is borne in mind.

If the metal is divided so that the main induced conduction currents cannot flow, but only residual minor currents, we destroy the diffusion effect more or less, according to the fineness of the division, and leave only the inductivity effect. In my early experiments I was sufficiently satisfied by finding that the substitution of a bundle of iron wires for a solid iron core, with a continuous reduction in the diameter of the wires, reduced the diffusion effect to something quite insignificant in comparison with the effect when the core was solid, to conclude that we had only to stop the flow of currents to make iron, under weak magnetizing forces, behave merely as an inductor. More recently, on account of some remarks of Prof. Ewing on the nature of the curve of induction under weak forces, I immensely improved the test by making and using nonconducting cores, containing as much iron as a bundle of round wires of the same diameter as the cores. I take the finest iron filings (siftings) and mix them with a black wax in the proportion of 1 of wax to 5 or 6 of iron filings by bulk. After careful mixture I roll the resulting compound, when in a slightly yielding state, under considerable pressure, into the form of solid round cylinders, somewhat resembling pieces of black poker in appearance. ($\frac{1}{2}$ inch diameter, 4 to 6 inches long.) That the diffusion effect was quite gone was my first conclusion. Next, that there was a slight effect, though of doubtful amount and character. The resistance-balance had to be very carefully attended to. But, more recently, by using coils containing a much greater number of windings, and thereby increasing the sensitiveness considerably, as well as the magnetizing force, I find there is a distinct effect of the kind required. Though small, it is much greater than could be detected; but whether it should be ascribed to the cause mentioned or to other causes, as dissipation of energy due to variations in the intrinsic magnetization, or to slight curvature in the line of induction, so far as the quasi-elastic induction is concerned, is quite debateable. To show it, let 1 and 2 be equal coils wound together ($L=3$ macs, $R=47$ ohms), 3 and 4 equal in resistance ($R_3=R_4=98$ ohms), but of very unequal inductances, that of coil 3 ($L_3=24$ macs) being so much greater than that of coil 4 that the iron core must be fully inserted in the latter to make $L_4=L_3$. (Coils 3 and 4; $1\frac{1}{4}$ inch external, $\frac{1}{2}$ inch internal diameter, and $\frac{3}{8}$ inch in depth. Frequency 500.) The balancing of induction is completed by means of an external core. Resistance of branch 6 a few ohms, E.M.F. 6 volts. There is, of course, immense sound when the core is out of coil 3, but when it is in there is merely a faint residual sound which is nearly destroyed by increasing R_3 by about $\frac{1}{600}$ part, a relatively considerable change. On the other hand, pure self-induction of copper wires gives perfect silence, and so does M_{64}, a method I have shown to be

exact*. [I may, however, here mention that in experiments with mere fine copper-wire coils there are sometimes to be found traces of variations of resistance-balance with the frequency of intermittence, of very small amount and difficult to elucidate owing to temperature-variations.] Balancing partly by M_{64} and partly by the iron cores, the residual sound increases from zero with M_{64} only, to the maximum with the cores only. Halving the strength of current upsets the induction-balance in this way. The auxiliary core must be set a little closer when the current is reduced. This would indicate a slightly lower inductivity with the smaller magnetizing force, and proves slight curvature in the line of induction. But, graphically represented, it would be invisible except in a large diagram.

It is confidently to be expected, from our knowledge of the variation of μ, that when the range of the magnetizing force is made much greater, the ability of nonconducting iron to act merely as an increaser of inductance will become considerably modified, and that the dissipation of energy by variations in the intrinsic magnetization will cease to be insensible. But, so far as weak magnetizing oscillatory forces are concerned, we need not trouble ourselves in the least about minute effects due to these causes. Under the influence of regular intermittences, the iron gets into a stationary condition, in which the variations in the intrinsic magnetization are insensible. It seems probable that μ must have a distinctly lower value under rapid oscillations than when they are slow. The values of μ calculated from my experiments on cores have been usually from 50 to 200, seldom higher. I should state that I define μ to be the ratio $\mathbf{B}/\mathbf{H}$, if $\mathbf{B}$ is the induction and $\mathbf{H}$ the magnetic force, which is to include $\mathbf{h}$, the impressed force of intrinsic magnetization. (See the general equations in Part I. †) It is with this μ, not with the ratio of the induction to the magnetizing force as ordinarily understood, that we are concerned with in experiments of the present kind.

Knowing, then, that iron when made a nonconductor acts merely as an inductor, when we remove the insulation and make the iron a solid mass, it requires to be treated as both a conductor and an inductor, just like a copper mass, in fact, of changed conductivity and inductivity. When the coil is a solenoid whose length is a large multiple of its diameter, and the core is placed axially, the phenomena in the core become amenable to rigorous mathematical treatment in a compara-

* 'Electrician,' April 30, 1886.

† Phil. Mag. August 1886.

tively simple manner. [In passing, I may mention that on comparing the measured with the calculated value of the inductance of a long solenoid according to Maxwell's formula (vol. ii. art. 678, equations (21) and (23)) in the first edition of his treatise, I found a far greater difference than could be accounted for by any reasonable error in the ohm (reputed) or in the capacity of the condenser, and therefore recalculated the formula. The result was to correct it, and reduce the difference to a reasonable one. On reference to the second edition (not published at the time referred to) I find that the formula has been corrected. I will therefore only give my extension of it. Let M be the mutual inductance of two long coaxial solenoids of length l, outer diameter c_2, inner c_1, having n_1 and n_2 turns per unit length. Then

$$M=4\pi^2 n_1 n_2 c_2^2(l-2\,c_1), \quad . \quad . \quad . \quad . \quad . \quad (46c)$$

where, if $\rho=c_1/c_2$,

$$2\alpha=1-\frac{\rho}{8}\Big(1+\frac{\rho}{8}\Big(1+\frac{5\rho}{16}\Big(1+\frac{7\rho}{16}\Big(1+\frac{21\rho}{40}\Big(1+\frac{33\rho}{56}+\ .\ .\ .\ . \qquad (47c)$$

When

$$c_1=c_2, \qquad 2\alpha=1-\cdot149=\cdot851.$$

As regards Maxwell's previous formula (22), art. 678, however, there is disagreement still.]

References to authors who have written on the subject of induction of currents in cores other than, and unknown to, and less comprehensively than, myself, are contained in Lord Rayleigh's recent paper *. So far as the effect on an induction-balance is concerned, when oscillatory currents are employed, it is to be found, as he remarks, by calculating the reaction of the core on the coil-current. This I have fully done in my article on the subject. Another method is to calculate the heat in the core, to obtain the increased resistance. This I have also done. When the diffusion effect is small, its influence on the amplitude and phase of the coil-current is the same as if the resistance of the coil-circuit were increased from the steady value R to †

$$\left.\begin{aligned} R' &= R+\tfrac{1}{2}L_1\pi\mu k n^2 c^2 \\ &= R+2l\pi k(\pi N c^2\mu n)^2=R+R_1 \text{ say} \end{aligned}\right\} \quad . \quad . \quad (48c)$$

Many phenomena which may be experimentally observed when rods are inserted in coils may be usefully explained in this manner. Here μ and k are the inductivity and conductivity of the core, of length l, the same as that of the coil, $n/2\pi$ the frequency, c the core's radius, and N the number of turns of wire in the coil per unit length; whilst

$$L_1=(2\pi N c)^2\mu l$$

is that part of the steady inductance of the coil circuit which is contributed by the core.

The full expression for the increased resistance due to the dissipation of energy in the core is to be got by multiplying the above R_1 by Y, which is given by *

$$Y=\frac{1+\dfrac{y}{6.8^2}\Big(1+\dfrac{y}{2.10.12^2}\Big(1+\dfrac{y}{3.14.16^2}\Big(1+\dfrac{y}{4.18.20^2}\Big(1+\ldots}{1+\dfrac{y}{2.4^2}\Big(1+\dfrac{y}{2.6.8^2}\Big(1+\dfrac{y}{3.10.12^2}\Big(1+\dfrac{y}{4.14.16^2}\Big(1+\ldots}, \quad (49c)$$

where $y=(4\pi\mu k n c^2)^2$. The value of R′ is therefore $R+R_1Y$. The series being convergent, the formula is generally applicable. The law of the coefficients is obvious. I have slightly changed the arrangement of the figures in the original to show it. We may easily make the core-heat a large multiple of the coil-heat, especially in the case of iron, in which the induced currents are so strong. When y is small enough, we may use the series obtained by division of the numerator by the denominator in (49c), which is

$$Y=1-\frac{11y}{16\,.\,24}+\frac{11\,.\,43y^2}{15\,.\,16^3\,.\,9}-\ .\ .\ .\ . \quad . \quad . \quad (50c)$$

Corresponding to this, I find from my investigation † of the phase-difference, that the decrease of the effective inductance from the steady value is expressed by

$$L_1\times\frac{y}{48}\Big(1-\frac{19y}{16\,.\,40}+\frac{229y^2}{16^3.63}+\ .\ .\ .\ .\Big). \quad . \quad . \quad . \quad (51c)$$

When the same core is used as a wire with current longitudinal, and again as core in a solenoid with induction longitudinal, the effects are thus connected. Let L_1 be the above steady inductance of the coil so far as is due to the core, and L'_1 its value at frequency $n/2\pi$, when it also adds resistance R'_1 to the coil. Also let R_2 be the steady resistance of the same when used as a wire, and R'_2 and L'_2 its resistance and inductance at frequency $n/2\pi$, the latter being what $\tfrac{1}{2}\mu$ then

* Phil. Mag. December 1886.
† 'Electrician,' May 31, 1884, p. 55.

* 'Electrician,' May 10, 1884 p. 606.
† Ibid. May 14, 1884, p. 103.

becomes. Then

$$\left.\begin{aligned}4\pi\mu N^2 l^2/k = R_2 L_1 = R'_1 L'_2 + R'_2 L'_1 \\ R'_1 R'_2 = L'_1 L'_2 n^2.\end{aligned}\right\} \quad . \quad . \quad . \quad (52c)$$

I did not give any separate development of the L'_1 of the core, corresponding to (48c) and (49c) above for R′, but merged it in the expression for the tangent of the difference in phase between the impressed force and the current in the coil-circuit. The full development of L'_1 is

$$\frac{L'_1}{L_1} = \frac{1+\frac{y}{6.16}\left(1+\frac{y}{2^3.10.16}\left(1+\frac{y}{3^3.14.16}\left(1+\frac{y}{4^3.18.16}\left(1+\dots\right.\right.\right.\right.}{.\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .}$$

the denominator being the same as in (49c).

The high-speed formulæ for R'_1 and L'_1 are

$$R'_1 = L'_1 n = \frac{L_1 n}{(2z)^{\frac{1}{2}}},$$

if $y=16z^2$. When z is as large as 10, this gives

$$R'_1 = L'_1 n = \cdot 2234\ L_1 n,$$

whereas the correct values by the complete formulæ are

$$R'_1 = \cdot 198\ L_1 n, \quad L'_1 = \cdot 225\ L_1.$$

It is therefore clear that we may advantageously use the high-speed formulæ when z is over 10, which is easily reached with iron cores at moderate speeds.

The corresponding fully developed formulæ for R'_2 and L'_2, when the current is longitudinal, are

$$\frac{R'_2}{R_2} = \frac{1+\frac{y}{6.16}\left(1+\frac{y}{2^3.10.16}\left(1+\frac{y}{3^3.14.16}\left(1+\dots\right.\right.\right.}{1+\frac{y}{2.6.16}\left(1+\frac{y}{3.2^2.10.16}\left(1+\frac{y}{4.3^2.14.16}\left(1+\dots\right.\right.\right.},$$

showing the laws of formation of the terms, and

$$\frac{L_2}{\frac{1}{2}\mu} = \frac{1+\frac{y}{2^2.6.16}\left(1+\frac{y}{2.3^2.10.16}\left(1+\frac{y}{3.4^2 14.16}\left(1+\dots\right.\right.\right.}{.\ .\ .\ .\ .\ .\ .\ .\ .\ .\ .},$$

the denominator being as in the preceding formula. At $z=10$, or $y=1600$, these give

$$R'_2 = 2\cdot 507\ R_2, \quad L'_2 = \tfrac{1}{2}\mu \times \cdot 442\ ;$$

whereas Lord Rayleigh's high-speed formulæ, which are

$$R'_2 = L'_2 n = R_2 (\tfrac{1}{2} x)^{\frac{1}{2}}$$

make

$$R'_2 = 2\cdot 234\ R_2, \quad L'_2 = \tfrac{1}{2}\mu \times \cdot 447.$$

This particular speed makes the amplitude of the magnetic force in the core case, and of the electric current in the other case, fourteen times as great at the boundary as at the axis of the wire or core (see Part I.). As, however, we do not ordinarily have very thick wires for use with the current longitudinal, the high-speed formulæ are not so generally applicable as in the case of cores, which may be as thick as we please, whilst by also increasing the number of windings the core heating per unit coil-current amplitude may be greatly increased.

If the core is hollow, of inner radius C_0, else the same, the equation of the coil-current is, if e be the impressed force and C the current in the coil-circuit whose complete steady resistance and inductance are R and L, whilst L_1 is the part of L due to the core and contained hollow (dielectric current in it ignored),

$$e = RC + (L-L_1)\dot{C} + \frac{2}{sc} \cdot \frac{J_1(sc) - qK_1(sc)}{J_0(sc) - qK_0(sv)} L_1 \dot{C}, \quad . \quad (53c)$$

when q depends upon the inner radius, being given by

$$q = \frac{\frac{1}{2}sc_0 J_0(sc_0) - J_1(sc_0)}{\frac{1}{2}sc_0 K_0(sc_0) - K_1(sc_0)} \quad . \quad . \quad . \quad . \quad . \quad (54c)$$

(whose value is zero when the core is solid), and

$$s^2 = -4\pi\mu k(d/dt).$$

There may be a tubular space between the core and coil, and R, L include the whole circuit. In reference to this (53c) equation, however, it is to be remarked that there is considerable labour involved in working it out to obtain what may be termed practical formulæ, admitting of immediate numerical calculations. The same applies to a considerable number of unpublished investigations concerning coils and cores that I made, including the effects of dielectric displacement; the analysis is all very well, and is interesting enough for educational purposes, but the interpretations are so difficult in general that it is questionable whether it is worth while either publishing the investigations or even making them.

Professor Hughes * has also devoted some attention to induction in cores, and has arrived at the remarkable conclu-

* Proc. Roy. Soc. 1886.

sion that he has obtained experimental evidence of the existence of induced currents therein. Now, although when it is considered that although induced currents in wires were known to exist, yet the possibility of their existing in metal not in the form of wires was only a matter of the wildest speculation, Professor Hughes's conclusion must be admitted to be very comforting and encouraging.

Leaving now the question of cores and the balance of purely electromagnetic self-induction, and returning to the general condition of a self-induction balance $Z_1Z_4=Z_2Z_3$, equation (23*c*), let the four sides of the quadrilateral consist of coils shunted by condensers. Then R, L, and S denoting the resistance, inductance, and capacity of a branch, we have

$$Z=\{Sp+(R+Lp)^{-1}\}^{-1}; \quad . \quad . \quad . \quad . \quad (55c)$$

so that the conjugacy of branches 5 and 6 requires that

$$\{S_1p+(R_1+L_1p)^{-1}\}\{S_4p+(R_4+L_4p)^{-1}\}$$
$$=\{S_2p+(R_2+L_2p)^{-1}\}\{S_3p+(R_3+L_3p)^{-1}\}, \quad (56c)$$

wherein the coefficient of every power of p must vanish, giving seven conditions, of which two are identical by having a common factor. It is unnecessary to write them out, as such a complex balance would be useless; but some simpler cases may be derived. Thus, if all the L's vanish, leaving condensers shunted by mere resistances, we have the three conditions

$$\left.\begin{array}{c} R_1R_4=R_2R_3, \\ S_1/R_4+S_4/R_1=S_2/R_3+S_3/R_2, \\ S_1S_4=S_2S_3, \end{array}\right\} \quad . \quad . \quad . \quad (57c)$$

which may be compared with the three self-induction conditions (25*c*) to (27*c*).

If we put $RS=y$, the time-constant, the second of (57*c*) may be written

$$y_1+y_4=y_2+y_3, \quad . \quad . \quad . \quad . \quad . \quad (58c)$$

which corresponds to (26*c*). If $S_2=0=S_4$, the single condition in addition to the resistance-balance is $y_1=y_3$. If $S_1=0=S_2$, it is $y_3=y_4$.

Next, let each side consist of a condenser and coil in sequence. Then the expression for Z is

$$Z=R+Lp+(Sp)^{-1}, \quad . \quad . \quad . \quad . \quad . \quad (59c)$$

which gives rise to five conditions,

$$\left.\begin{array}{c} S_1S_4=S_2S_3, \\ y_1+y_4=y_2+y_3, \\ S_1S_4(R_1R_4-R_2R_3)=L_2S_2+L_3S_3-L_1S_1-L_4S_4, \\ \dfrac{1}{x_1}+\dfrac{1}{x_4}=\dfrac{1}{x_3}+\dfrac{1}{x_2}, \\ L_1L_4=L_2L_3. \end{array}\right\} \quad . \quad (60c)$$

Here it looks as if the resistance-balance were unnecessary; and, as there can be no steady current, this seems a sufficient reason for its not being required. But, in fact, the third condition, by union with the others, eliminating S_3, L_3, S_4, and L_4 by means of the other four conditions, becomes

$$0=(R_1R_4-R_2R_3)\,\frac{S_1S_2(R_1S_1-R_2S_2)(L_1R_2-R_1L_2)-(L_2S_2-L_1S_1)^2}{(R_3S_1-R_4S_2)(L_1R_2-R_1L_2)}. \quad (61c)$$

So the obvious way of satisfying it is by the true resistance-balance.

If there are condensers only, without resistance-shunts, we have

$$Z=(Sp)^{-1}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (62c)$$

so that

$$S_1S_4=S_2S_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (63c)$$

as the sole condition of balance.

If two sides are resistances, R_1 and R_2, and two are condensers, S_3 and S_4, we obtain

$$R_1/R_2=S_4/S_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (64c)$$

as the sole condition. The multiplication of special kinds of balance is a quite mechanical operation, presenting no difficulties.

Passing now to balances in which induction between different branches is employed, suppose we have, in the first place, a true resistance-balance, $R_1R_4=R_2R_3$, but not an induction-balance, so that there is sound produced. Then, by means of small test coils placed in the different branches, we find that we may reduce the sound to a minimum in a great many ways by allowing induction between different branches. If the sound to be destroyed is feeble, we may think that we have got a true induction-balance; but if it is loud, then the minimum sound is also loud, and may be comparable to the original in intensity. We may also, by upsetting the resistance-balance by trial, still further approximate to silence, and it may be a very good silence, with a false resistance-balance. The question arises, Can these balances, or any of them, be made of service and be as exact as the previously described exact balances? and are the balances easily interpretable, so that we may know what we are doing when we employ them?

There are fifteen M's concerned, and therefore fifteen ways of balancing by mutual induction when only two branches at a time are allowed to influence one another, and in every case three conditions are involved, because there are three degrees of current-freedom in the six conductors involved. Owing to this, and the fact that in allowing induction between a pair of branches we use only one condition (*i. e.* giving a certain value to the M concerned), whilst the resistance-balance makes a second condition, I was of opinion, in writing on this subject before *, that all the balances by mutual induction, using a true resistance-balance, were imperfect, although some of them were far better than others. Thus, I observed experimentally that when a ratio of equality ($R_1=R_2$, $L_1=L_2$) was taken, the balances by means of M_{63} or M_{64} were very good, whilst that by M_{65} was usually very bad, the minimum sound being sometimes comparable in intensity to that which was to be destroyed.

I investigated the matter by direct calculation of the integral extra-current in branch 5 arising on breaking or making branch 6, due to the momenta of the currents in the various branches, making use of a principle I had previously deduced from Maxwell's equations †, that when a coil is discharged, through various paths, the integral current divides as in steady flow, in spite of the electromotive forces of induction set up during the discharge. This method gives us the second condition of a true balance.

But more careful observation, under various conditions, showing a persistent departure from the true resistance-balance in the M_{65} method (due to Professor Hughes), and that the M_{63} and M_{64} methods were persistently good and were not to be distinguished from true balances, led me to suspect that the second and third conditions united to form one condition when a ratio of equality was used (just as in (28*c*), (29*c*) above) in the M_{63} and M_{64} methods, but not in the M_{65} method. So I did what I should have done at the beginning: investigated the differential equations concerned, verified my suspicions, and gave the results in a Postscript. I have since further found that, when using the only practicable method of equal ratio, there are no other ways than those described in the paper referred to of getting a true balance of induction by variation of a single L or M, after the resistance-balance has been secured. This will appear in the following investigation, which, though it may look complex, is quite mechanical in its simplicity.

* 'Electrician,' April 30, 1886.

† Journal S. T. E. 1878, vol. vii. p. 303.

Write down the equations of electromotive force in the three circuits $6+1+3$, $1+5-2$, and $3-4-5$, when there is impressed force in branch 6 only. They are (p standing for d/dt),

$$\left.\begin{aligned}
e_6=&(R_6+L_6p)C_6+(R_1+L_1p)C_1+(R_3+L_3p)C_3\\
&+p(M_{61}C_1+M_{62}C_2+M_{63}C_3+M_{64}C_4+M_{65}C_5)\\
&+p(M_{12}C_2+M_{13}C_3+M_{14}C_4+M_{15}C_5+M_{16}C_6)\\
&+p(M_{31}C_1+M_{32}C_2+M_{34}C_4+M_{35}C_5+M_{36}C_6).\\
0=&(R_1+L_1p)C_1+(R_5+L_5p)C_5-(R_2+L_2p)C_2\\
&+p(M_{12}C_2+M_{13}C_3+M_{14}C_4+M_{15}C_5+M_{16}C_6)\\
&+p(M_{51}C_1+M_{52}C_2+M_{53}C_3+M_{54}C_4+M_{56}C_6)\\
&-p(M_{21}C_1+M_{23}C_3+M_{24}C_4+M_{25}C_5+M_{26}C_6).\\
0=&(R_3+L_3p)C_3-(R_4+L_4p)C_4-(R_5+L_5p)C_5\\
&+p(M_{31}C_1+M_{32}C_2+M_{34}C_4+M_{35}C_5+M_{36}C_6)\\
&-p(M_{41}C_1+M_{42}C_2+M_{43}C_3+M_{45}C_5+M_{46}C_6)\\
&-p(M_{51}C_1+M_{52}C_2+M_{53}C_3+M_{54}C_4+M_{56}C_6).
\end{aligned}\right\} \quad (65c)$$

Now, eliminate C_1, C_2, C_6 by the continuity conditions

$$C_1=C_3+C_5, \quad C_2=C_4-C_5, \quad C_6=C_3+C_4, \quad . \quad . \quad (66c)$$

giving us

$$\left.\begin{aligned}
e_6&=X_{11}C_3+X_{12}C_4+X_{13}C_5,\\
0&=X_{21}C_3+X_{22}C_4+X_{23}C_5,\\
0&=X_{31}C_3+X_{32}C_4+X_{33}C_5,
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (67c)$$

where the X's are functions of p and constants. Solve for C_5. Then we see that

$$X_{21}X_{32}=X_{22}X_{31} \quad . \quad . \quad . \quad . \quad . \quad (68c)$$

is the complex condition of conjugacy of branches 5 and 6. This could be more simply deduced by assuming $C_5=0$ at the beginning, but it may be as well to give the values of all the X's, although we want but four of them. Thus

$$\left.\begin{aligned}
X_{11} &= \quad R_1+R_3+R_6+(L_1+L_3+L_6+2M_{61}+2M_{63}+2M_{31})p,\\
X_{12} &= \quad R_6+(L_6+M_{62}+M_{64}+M_{12}+M_{14}+M_{16}+M_{32}+M_{34}+M_{36})p,\\
X_{13} &= \quad R_1+(L_1+M_{61}-M_{62}+M_{65}-M_{12}+M_{15}+M_{31}-M_{32}+M_{35})p,\\
X_{21} &= \quad R_1+(L_1+M_{13}+M_{15}+M_{16}+M_{53}+M_{56}-M_{21}-M_{23}-M_{26})p,\\
X_{22} &= -R_2+(-L_2+M_{12}+M_{14}+M_{16}+M_{52}+M_{54}+M_{56}-M_{24}-M_{26})p,\\
X_{23} &= \quad R_1+R_2+R_5+(L_1+L_2+L_5+2M_{15}-2M_{25}-2M_{12})p,\\
X_{31} &= \quad R_3+(L_3+M_{31}+M_{36}-M_{41}-M_{43}-M_{46}-M_{51}-M_{53}-M_{56})p,\\
X_{32} &= -R_4+(-L_4+M_{32}+M_{34}+M_{36}-M_{42}-M_{46}-M_{52}-M_{54}-M_{56})p,\\
X_{33} &= -R_5+(-L_5+M_{31}-M_{32}+M_{35}-M_{41}+M_{42}-M_{45}+M_{52}-M_{51})p.
\end{aligned}\right\} \quad (69c)$$

Now, using the required four of these in (68 *c*), and arranging in powers of p, it becomes

$$0=A_0+A_1p+A_2p^2. \quad . \quad . \quad . \quad . \quad . \quad (70c)$$

So $A_0=0$ gives the resistance-balance; $A_1=0$, in addition, makes the integral transient current vanish; and $A_2=0$, in addition, wipes out all trace of current.

There is also the periodic balance,

$$A_1=0, \qquad A_0=A_2n^2, \quad . \quad . \quad . \quad . \quad . \quad (71c)$$

if the frequency is $n/2\pi$.

The values of A_0 and A_1 are

$$A_0=R_2R_3-R_1R_4, \quad . \quad . \quad . \quad . \quad . \quad (72c)$$

$$\begin{aligned}
A_1= \quad & R_2L_3+R_3L_2-R_1L_4-R_4L_1\\
&+R_2(M_{31}+M_{36}-M_{41}-M_{43}-M_{46}-M_{51}-M_{53}-M_{56})\\
&+R_3(M_{24}+M_{26}-M_{12}-M_{14}-M_{16}-M_{52}-M_{54}-M_{56})\\
&+R_1(M_{32}+M_{34}+M_{36}-M_{42}-M_{46}-M_{52}-M_{54}-M_{56})\\
&+R_4(M_{21}+M_{23}+M_{26}-M_{13}-M_{16}-M_{15}-M_{53}-M_{56}). \quad . \quad (73c)
\end{aligned}$$

In this last, let the coefficients of R_2, R_3, R_1, R_4 in the brackets be q_2, q_3, q_1, q_4. Then the value of A_2 is

$$A_2=L_2L_3-L_1L_4+L_2q_2+L_3q_3+L_1q_1+L_4q_4+q_2q_3-q_1q_4. \quad . \quad (74c)$$

It is with the object of substituting one investigation for a large number of simpler ones that the above full expressions for A_1 and A_2 are written out.

If we take all the M's as zero, we fall back upon the self-induction balance (25 *c*) to (27 *c*). Next, by taking all the M's as zero except one, we arrive at the fifteen sets of three conditions. Of these we may write out three sets, or, rather,

the two conditions in each case besides the resistance-balance condition, which is always the same.

All M's $=0$, except M_{36}.

$$\left.\begin{aligned}
R_1R_4(x_1+x_4-x_2-x_3)&=(R_1+R_2)M_{36},\\
L_1L_4-L_2L_3&=(L_1+L_2)M_{36}.
\end{aligned}\right\} \quad . \quad . \quad (75c)$$

All M's $=0$, except M_{46}.

$$\left.\begin{aligned}
R_1R_4(x_1+x_4-x_2-x_3)&=-(R_1+R_2)M_{46},\\
L_1L_4-L_2L_3&=-(L_1+L_4)M_{46}.
\end{aligned}\right\} \quad . \quad . \quad (76c)$$

As these only differ in the sign of the M, we may unite these two cases, allowing induction between 6 and 3, and 6 and 4. The two conditions will be got by writing $M_{36}-M_{46}$ for M_{36} in (75*c*).

All M's $=0$, except M_{56} (Prof. Hughes's method).

$$\left.\begin{aligned}
0&=R_1R_4(x_1+x_4-x_2-x_3)+M_{56}(R_1+R_2+R_3+R_4),\\
0&=L_1L_4-L_2L_3+M_{56}(L_1+L_2+L_3+L_4).
\end{aligned}\right\} \quad (77c)$$

Now choose a ratio of equality, $R_1=R_2$, $L_1=L_2$, which is the really practical way of using induction-balances in general. In the M_{36} case the two conditions (75 *c*) unite to form the single condition

$$L_4-L_3=2M_{36}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (78c)$$

and in the M_{46} case (76*c*) unite to form the single condition

$$L_4-L_3=-2M_{46}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (79c)$$

We know already that the same occurs in the simple Bridge (29*c*), making

$$L_4=L_3; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (80c)$$

so that we have three ways of uniting the second and third conditions. Now examine all the other M's, one at a time, on the same assumption, $R_1=R_2$, $L_1=L_2$. With M_{12} we obtain

$$(L_4-L_3)(L_1-M_{12})=0, \quad \text{and} \quad L_4=L_3.$$

But L_1-M_{12} cannot vanish; so that

$$L_4=L_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (81c)$$

is the single condition. Similarly, in case of M_{43},

$$L_4=L_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (82c)$$

again. All these, (77 *c*) to (82 *c*), were given in the paper referred to; the last two mean that M_{12} and M_{34} have absolutely no influence on the balance of self-induction.

All the rest are double conditions. Thus, in A_1 and A_2

put $R_1=R_2$, $R_3=R_4$, and $L_1=L_2$; then the two conditions are

$$
\begin{aligned}
0=L_4-L_3+(1+R_4/R_1)&(M_{14}-M_{23}+M_{51}+M_{52}+M_{53}+M_{54}+2M_{56})\\
&+2(M_{46}-M_{36})+(1-R_4/R_1)(M_{24}-M_{13})+2(R_4/R_1)(M_{16}-M_{26}); \quad (83c)
\end{aligned}
$$

$$
\begin{aligned}
0=L_1(L_4-L_3)&+L_3(M_{12}+M_{14}+M_{16}+M_{52}+M_{54}+M_{56}-M_{24}-M_{26})\\
&+L_4(M_{13}+M_{15}+M_{16}+M_{53}+M_{56}-M_{12}-M_{23}-M_{26})\\
&+L_1(M_{41}+M_{42}+M_{51}+M_{52}+M_{53}+M_{54}-M_{31}-M_{32}+2M_{46}+2M_{56}-2M_{36})\\
&+(M_{13}+M_{15}+M_{16}+M_{53}+M_{56}-M_{21}-M_{23}-M_{26})\\
&\qquad\times(M_{42}+M_{46}+M_{54}+M_{52}+M_{56}-M_{32}-M_{34}-M_{36})\\
&+(M_{41}+M_{43}+M_{46}+M_{51}+M_{53}+M_{56}-M_{31}-M_{36})\\
&\qquad\times(M_{24}+M_{26}-M_{12}-M_{14}-M_{16}-M_{52}-M_{54}-M_{56}); \quad (84c)
\end{aligned}
$$

which are convenient for deriving the conditions when several M's are operative at the same time. Thus, one at a time, excepting the few already examined :—

$$M_{51} \;.\;.\; \begin{cases}0=L_4-L_3+M_{51}(1+R_4/R_1)\\ 0=L_4-L_3+M_{51}(1+L_4/L_1)\end{cases}, \quad . \;.\;.\;. \quad (85c)$$

$$M_{52} \;.\;.\; \begin{cases}0=L_4-L_3+M_{52}(1+R_4/R_1)\\ 0=L_4-L_3+M_{52}(1+L_3/L_1)\end{cases}, \quad . \;.\;.\;. \quad (86c)$$

$$M_{53} \;.\;.\; \begin{cases}0=L_4-L_3+M_{53}(1+R_4/R_1)\\ 0=L_4-L_3+M_{53}(1+L_4/L_1)\end{cases}, \quad . \;.\;.\;. \quad (87c)$$

$$M_{54} \;.\;.\; \begin{cases}0=L_4-L_3+M_{54}(1+R_4/R_1)\\ 0=L_4-L_3+M_{54}(1+L_3/L_1)\end{cases}, \quad . \;.\;.\;. \quad (88c)$$

$$M_{56} \;.\;.\; \begin{cases}0=L_4-L_3+2M_{56}(1+R_4/R_1)\\ 0=L_4-L_3+M_{56}\{2+(L_4+L_3)/L_1\}\end{cases}, \; . \quad (89c)$$

$$M_{16} \;.\;.\; \begin{cases}0=L_4-L_3+2M_{16}R_4/R_1\\ 0=L_4-L_3+M_{16}(L_3+L_4)/L_1\end{cases}, \quad . \;.\;.\;. \quad (90c)$$

$$M_{26} \;.\;.\; \begin{cases}0=L_4-L_3-2M_{26}R_4/R_1\\ 0=L_4-L_3-M_{26}(L_3+L_4)/L_1\end{cases}, \quad . \;.\;.\;. \quad (91c)$$

$$M_{13} \;.\;.\; \begin{cases}0=L_4-L_3-M_{13}(1-R_4/R_1)\\ 0=L_4-L_3-M_{13}(1-L_4/L_1)\end{cases}, \quad . \;.\;.\;. \quad (92c)$$

$$M_{24} \;.\;.\; \begin{cases}0=L_4-L_3+M_{24}(1-R_4/R_1)\\ 0=L_4-L_3+M_{24}(1-L_3/L_1)\end{cases}, \quad . \;.\;.\;. \quad (93c)$$

$$M_{14} \;.\;.\; \begin{cases}0=L_4-L_3+M_{14}(1+R_4/R_1)\\ 0=L_4-L_3+M_{14}(1+L_3/L_1)-M_{14}^2/L_1\end{cases}, \quad (94c)$$

$$M_{23} \;.\;.\; \begin{cases}0=L_4-L_3-M_{23}(1+R_4/R_1)\\ 0=L_4-L_3-M_{23}(1+L_4/L_1)+M_{23}^2/L_1\end{cases}. \quad (95c)$$

If we compare the two general conditions (83*c*), (84*c*), we shall see that whenever

$$q_1q_4-q_2q_3=0,$$

we may obtain the reduced forms of the conditions by adding together the values of L_3-L_4 given by every one of the M's concerned. We may therefore bracket together certain sets of the M's. To illustrate this, suppose that M_{13} and M_{24} are existent together, and all the other M's are zero. Then (92*c*) and (93*c*) give, by addition,

$$
\left.\begin{aligned}
L_3-L_4&=(M_{24}-M_{13})\left(1-\frac{R_4}{R_1}\right),\\
L_3-L_4&=M_{24}-M_{13}+M_{13}\frac{L_4}{L_1}-M_{24}\frac{L_3}{L_1},
\end{aligned}\right\}
$$

which are the conditions required.

Similarly M_{12} and M_{34} may be bracketed. Also M_{61}, M_{62}, M_{63}, M_{64}, and M_{65}. Also M_{51}, M_{52}, M_{53}, M_{54}, and M_{56}. But M_{14} and M_{23} will *not* bracket.

As already observed, the self-induction balance (28*c*) (29*c*) is independent of M_{12} and M_{34}, when these are the sole mutual inductances concerned; that is, when $R_1=R_2$, $L_1=L_2$, $R_3=R_4$, $L_3=L_4$. By (92*c*) and (93*c*) we see that independence of M_{13} and M_{24} is secured by making all four branches 1, 2, 3, 4 equal in resistance and inductance.

But it is unsafe to draw conclusions relating to independence when several coils mutually influence, from the conditions securing balance when only one pair of coils at a time influence one another. Let us examine what (83*c*) and (84*c*) reduce to when there is induction between all the four branches 1, 2, 3, 4, but none between 5 and the rest or between 6 and the rest. Put all M's=0 which have either $_5$ or $_6$ in their double suffixes, and put $L_4=L_3$. Then we may write the conditions thus :—

$$0=(1+R_4/R_1)(M_{14}-M_{23})+(1-R_4/R_1)(M_{24}-M_{13}), \quad . \;.\;.\;.\;. \quad (96$$

$$
\begin{aligned}
0=(L_1+L_4)&(M_{14}-M_{23})+(L_1-L_4)(M_{24}-M_{13})+M_{23}^2-M_{14}^2\\
&+(M_{24}-M_{13})(M_{34}-M_{12})+(M_{14}-M_{23})(M_{24}+M_{13}-M_{12}-M_{34}), \quad (97
\end{aligned}
$$

The simplest way of satisfying these is by making

$$M_{14}=M_{23} \quad \text{and} \quad M_{24}=M_{13}. \;.\;.\;.\;. \quad (98c)$$

If these equalities be satisfied, we have independence of M_{12} and M_{34}.

Now, if we make the four branches 1, 2, 3, 4 equal in resistance and inductance, so that in (96*c*) and (97*c*) we have $R_1=R_4$ and $L_1=L_4$, the first reduces to

$$0=M_{14}-M_{23}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (99c)$$

so that it is first of all absolutely necessary that $M_{14}=M_{23}$, if the balance is to be preserved; whilst, subject to this, the second condition reduces to

$$0=(M_{24}-M_{13})(M_{34}-M_{12}), \quad . \quad . \quad . \quad (100c)$$

so that either $M_{24}=M_{13}$, or else $M_{34}=M_{12}$. Thus there are two ways of preserving the balance when all four branches are equal, viz. $M_{14}=M_{23}$ and $M_{24}=M_{13}$, independent of the values of M_{12} and M_{34}; and $M_{14}=M_{23}$ and $M_{34}=M_{12}$, independent of the values of M_{24} and M_{13}.

The verification of these properties, (98*c*) and later, makes some very pretty experiments, especially when the four branches consist, not merely of one coil each, but of two or more. The meanings of some of the simpler balances are easily reasoned out without mathematical examination of the theory; but this is not the case when there is simultaneous induction between many coils, and their resultant action on the telephone-branch is required.

Returning to (96*c*) and (97*c*), the nearest approach we can possibly make to independence of the self-induction balance of the values of all the M's therein concerned, consistent with keeping wires 3 and 4 away from one another for experimental purposes, is by winding the equal wires 1 and 2 together. Then, whether they be joined up straight, which makes $M_{13}=M_{23}$ and $M_{14}=M_{24}$ identically, or reversed, making $M_{13}=-M_{23}$ and $M_{14}=-M_{24}$, we shall find that

$$M_{14}=M_{23}$$

is the necessary and sufficient condition of preservation of balance.

At first sight it looks as if M_{31} and M_{32} must cancel one another when wires 1 and 2 are reversed. But although 1 and 2 cancel on 3, yet 3 does not cancel on 1 and 2 as regards the telephone in 5. The effects are added. On the other hand, when wires 1 and 2 are straight, 3 cancels on them as regards the telephone, but 1 and 2 add their effects on 3. Similar remarks apply to the action between 4 and the equal wires 1 and 2 when straight or reversed; hence the necessity of the condition represented by the last equation.

On the other hand, M_{61} and M_{62} cancel when 1 and 2 are straight, and add their effects when they are reversed; whilst M_{51} and M_{52} cancel when 1 and 2 are reversed, and add their effects when they are straight, results which are immediately

evident. But wires 1 and 2 must be thoroughly well twisted, before being wound into a coil, if it is desired to get rid of the influence of, say, M_{61} and M_{62}, when it is a coil that operates in 6, and this coil is brought near to 1 and 2. [This leads me to remark that a simple way of proving that the mutual induction between iron and copper (fine wires) is the same as between copper and copper, which is immensely more sensitive than the comparison of separate measurements of the induction in the two cases, is to take two fine wires of equal length, one of iron, the other of copper, twist them together carefully, wind into a coil, and connect up with a telephone differentially. On exposure of the double coil to the action of an external coil in which strong intermittent currents or reversals are passing, there will be hardly the slightest sound in the telephone, if the twisting be well done, with several twists in every turn. But if it be not well done, there will be a residual sound, which can be cancelled by allowing induction between the external or primary coil and a turn of wire in the telephone-circuit. A rather curious effect takes place when we exaggerate the differential action by winding the wires into a coil without twists, in a certain short part of its length. The now comparatively loud sound in the telephone may be cancelled by inserting a nonconducting iron core in the secondary coil, provided it be not pushed in too far, or go too near or into the primary coil. This paradoxical result appears to arise from the secondary coil being equivalent to two coils close together, so that insertion of the iron core does not increase the mutual inductance of the primary and secondary in the first place, but first decreases it to a minimum, which may be zero, and later increases it, when the core is further inserted. Reversing the secondary coil with respect to the primary makes no difference. Of course insertion of the core into the primary always increases the mutual inductance and multiplies the sound. The fact that one of the wires in the secondary happens to be iron has nothing to do with the effect.]

Another way of getting unions of the two conditions of the induction-balance is by having branches 1 and 3 equal, instead of 1 and 2. Thus, if we take $R_1=R_3$, $L_1=L_3$, $R_2=R_4$ in A_1 and A_2 (73*c*) and (74*c*), we obtain fifteen sets of double conditions similar to those already given, out of which just four (as before) unite the two conditions. Thus, using M_{13} only, we have

$$L_2=L_4, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (101c)$$

and the same if we use M_{24} only, and the same when both M_{13}

and M_{34} are operative. That is, the self-induction balance is independent of M_{13} and M_{24}. This corresponds to (81*c*) and (82*c*).

The other two are M_{25} and M_{45}. With M_{25} we have

$$0=L_2-L_4-2M_{25}, \quad . \quad . \quad . \quad . \quad (102c)$$

and with M_{45},

$$0=L_2-L_4-2M_{45}. \quad . \quad . \quad . \quad . \quad (103c)$$

The remaining eleven double conditions corresponding to (85*c*) to (95*b*) need not be written down.

Several special balances of a comparatively simple kind can be obtained from the preceding by means of inductionless resistances, double-wound coils whose self-induction is negligible under certain circumstances, allowing us to put the L's of one, two, or three of the four branches 1, 2, 3, 4 equal to zero. We may then usefully remove the ratio of equality restriction if required. This vanishing of the L of a branch of course also makes the induction between it and any other branch vanish.

For instance, let $L_1=L_2=L_4=0$; then

$$0=R_2L_3+M_{36}(R_1+R_2) \quad . \quad . \quad . \quad . \quad (104c)$$

gives the induction-balance when M_{36} is used, subject to $R_1R_4-R_2R_3$. And

$$0=R_2L_3-M_{35}(R_2+R_4) \quad . \quad . \quad . \quad . \quad (105c)$$

is the corresponding condition when M_{35} is used. But M_{56} will not give balance, except in the special case of S.H. currents, with a false resistance-balance. The method (104*c*) is one of Maxwell's. His other two have been already described.

In the general theory of reciprocity, it is a force at one place that produces the same flux at a second as the same force at the second place does at the first. That the reciprocity is between the force and the flux, it is sometimes useful to remember in induction-balances. Thus the above-mentioned second way of having a ratio of equality is merely equivalent to exchanging the places of the force and the (vanishing) flux. We must not, in making the exchange, transfer a coil that is operative. For example, in the M_{64} method (79*c*), there is induction between branches 6 and 4; M_{45} (equation (88*c*)), on the other hand, fails to give balance. But if we exchange the branches 5 and 6, it is the battery and telephone that have to be exchanged; so that we now use M_{54}, which gives silence, whilst M_{64} will not.

I have also employed the differential telephone sometimes, having had one made some five years ago. But it is not so adaptable as the quadrilateral to various circumstances. I need say nothing as to its theory, that having been, I understand, treated by Prof. Chrystal. Using a pair of equal coils, it is very similar to that of the equal-ratio quadrilateral.

December 29th, 1886.

XXII. *Notices respecting New Books.*

The Origin of Mountain-Ranges, considered Experimentally, Structurally, Dynamically, and in Relation to their Geological History. By T. MELLARD READE, *C.E., F.G.S., F.R.I.B.A.* London: Taylor and Francis, 1886.

IT is now twenty years since Mr. George L. Vose published his 'Orographic Geology,' containing an admirable review of all that had, up to that time, been done in the way of explaining the structure and origin of mountain-chains. Strange to say, the author of the work now before us does not appear to be acquainted with the labours of his predecessor in the same field; but the large amount of original research bearing upon the subject in question, which has been carried on in the interval, fully justifies the preparation of this new book by one so competent to undertake it as Mr. Mellard Reade has shown himself to be.

The author aims at nothing less than framing a complete and consistent theory of the origin of mountain-ranges; and whatever divergences of opinion may arise as to the soundness of particular portions of that theory, or of the force or value of certain of the arguments by which they are supported, there can be no hesitation among candid readers in admitting the great value of the mass of facts relating to the question which have been obtained by the author by ingenious experiment and patient observation, or the interest attaching to the conclusions which he has founded upon those facts.

If the theory, as a whole, can lay no claim to absolute novelty, there are certain new and striking features introduced into it by the author, and the principles on which it is based are certainly exemplified and enforced by him with much freshness, ingenuity, and vigour.

Mr. Mellard Reade insists on the principle so well recognized by Hall, Rogers, Dana, Le Conte, and most recent authors who have treated on the subject, that the first stage in the origination of a mountain-chain consists in excessive sedimentation. After giving an outline of the main facts made known by recent researches concerning the Appalachians, the Rocky Mountains, the Andes, the Himalayas, the Alps, and the mountains of our own islands, he summarizes his conclusions as follows:—" No great range of mountains was ever ridged up excepting in areas of great previous sedimentation. Out of these sediments the mountains are mostly built and carved, but along with the newer and originally horizontal sedimentary beds, the older gneissic and Archæan rocks are usually thrust