out that cuprous chloride has only half the diffusibility of cupric chloride.

I am still prosecuting the investigation, and other interesting relationships may be detected, but the data as to the specific gravity and molecular volumes of compounds of many of the rarer elements are entirely wanting, and even in the case of those of well-known elements are incomplete.

II. *On the Self-induction of Wires.*—Part V.

By Oliver Heaviside*.

THE mathematical difficulties in the way of the discovery of exact solutions of problems concerning the propagation of electromagnetic disturbances into wires of other than circular section—or, even if of circular section, when the return current is not equidistantly distributed as regards the wire, or is not so distant that its influence on the distribution of the wire current throughout its section may be disregarded—are very considerable. As soon as we depart from the simple type of magnetic field which occurs in the case of a straight wire of circular section, we require at least two geometrical variables in place of the one, distance from the axis of the wire, which served before; and we may have to supplement the magnetic force "of the current," as usually understood, by a polar force, or a force which is the space-variation of a single-valued scalar, the magnetic potential, in order to make up the real magnetic force.

There are, however, some simplified cases which can be fully solved, viz. when the external magnetic field, that in the dielectric, is abolished, by enclosing the wire in a sheath of infinite conductivity. It is true that we must practically separate the wire from the sheath by some thickness of dielectric, in order to be able to set up current in the circuit by means of impressed force, so that we cannot entirely abolish the external magnetic field; but we may approximate in a great measure to the state of things we want for purposes of investigation. The wire, of course, need not be a wire in the ordinary sense, but a large bar or prism. The electrostatic induction will be ignored, requiring the wire to be not of great length; thus making the problem an electromagnetic one.

Consider, then, a straight wire or rod or prism of any symmetrical form of section, so that when a uniformly distributed current passes through it its axis is the axis of the magnetic field, where the intensity of force is zero. Let a steady current exist in the wire, longitudinal of course, and let the return conductor be a close-fitting infinitely conducting sheath. This stops the magnetic field at the boundary of the wire. The sudden discontinuity of the boundary magnetic force is then the measure and representative of the return current.

* Communicated by the Author.

The magnetic energy per unit length is $\frac{1}{2}LC^2$, where C is the current in the wire and L the inductance per unit length. As regards the diminution of the L of a circuit in general, by spreading out the current, as in a strip, instead of concentrating it in a wire, that is a matter of elementary reasoning founded on the general structure of L. If we draw apart currents, keeping the currents constant, thus doing work against their mutual attraction, we diminish their energy at the same time by the amount of work done against the attraction. Thus the quantity $\frac{1}{2}LC^2$ of a circuit is the amount of work that must be done to take a current to pieces, so to speak; that is, supposing it divided into infinitely fine filamentary closed currents, to separate them against their attractions to an infinite distance from one another. We do not need, therefore, any examination of special formulæ to see that the inductance of a flat strip is far less than that of a round wire of the same sectional area; their difference being proportional to the difference of the amounts of the magnetic energy per unit current in the two cases. The inductance of a circuit can, similarly, be indefinitely increased by fining the wire; that of a mere line being infinitely great. But we can no more have a finite current in an infinitely thin wire than we can have a finite charge of electricity at a point, in which case the electrostatic energy would be also infinitely great, for a similar reason; although by a useful and almost necessary convention we may regard fine-wire circuits as linear, whilst their inductances are finite.

Now, as regards our enclosed rod with no external magnetic field, we can in several cases estimate L exactly, as the work is already done, in a different field of Physics. The nature of the problem is most simply stated in terms of vectors. Thus, let $\mathbf{h}$ be the vector magnetic force when the boundary of the section perpendicular to the length is circular, and $\mathbf{H}$ what it becomes with another form of boundary; then

$$\mathbf{H}=\mathbf{h}+\mathbf{F}, \text{ and } \mathbf{F}=-\nabla\Omega. \quad . \quad . \quad . \quad (1a)$$

That is, the field of magnetic force differs from the simple circular type by a polar force $\mathbf{F}$ whose potential is Ω. This must be so because the curl of $\mathbf{H}$ and of $\mathbf{h}$ are identical, requiring the curl of $\mathbf{F}$ to be zero. To find $\mathbf{F}$ we have the datum that the magnetic force must be tangential to the boundary, and therefore have no normal component; or, if $\mathbf{N}$ be the unit vector normal drawn outward,

$$-\mathbf{FN}=\mathbf{hN} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

is the boundary condition. This gives $\mathbf{F}$, when it is remembered that $\mathbf{F}$ must have no convergence within the wire.

In another form, since we have $\mathbf{h}$ circular about the axis, and of intensity $2\pi r\Gamma_0$, at distance r from it, the current-density being Γ_0; or

$$\mathbf{h}=2\pi\Gamma_0 \mathrm{V}\mathbf{kr}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

if $\mathbf{r}$ is the vector distance from the axis in a plane perpendicular to it, and $\mathbf{k}$ a unit vector parallel to the current; we have

$$\mathbf{hN}=(2\pi\Gamma_0)(\mathbf{N}\mathrm{V}\mathbf{kr})=(2\pi\Gamma_0)(\mathbf{r}\mathrm{V}\mathbf{Nk})$$
$$=-\pi\Gamma_0\frac{d(r^2)}{ds}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4a)$$

if s be length measured along the bounding curve, in the direction of the magnetic force. The boundary condition (2 a) therefore becomes, in terms of the magnetic potential,

$$-\frac{d\Omega}{dp_1}=\pi\Gamma_0\frac{d(r^2)}{ds}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5a)$$

which, with $\nabla^2\Omega=0$, finds the magnetic potential. Here p_1 is length measured along the normal to the boundary outward.

Or we may use the vector-potential $\mathbf{A}$. It is parallel to the current, and consists of two parts; thus,

$$\mathbf{A}=\mathbf{A}'-(\mu\pi\Gamma_0 r^2)\mathbf{k}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (6a)$$

where the second part on the right side is, except as regards a constant, what it would be if the boundary were circular, its curl being $\mu\mathbf{h}$. To find $\mathbf{A}'$, let its tensor be A'; then

$$\nabla^2\mathrm{A}'=0, \text{ and } \mathrm{A}'=\mu\pi\Gamma_0 r^2, \quad . \quad . \quad . \quad . \quad (7a)$$

the latter being the boundary condition, expressing that $\mathbf{A}$ is zero at the boundary. Comparing with (5a), we see that (7a) is the simpler.

The magnetic energy per unit length of rod, say T, is

$$\mathrm{T}=\Sigma\mu\mathbf{H}^2/8\pi=\Sigma\mu(\mathbf{h}+\mathbf{F})^2/8\pi, \quad . \quad . \quad . \quad (8a)$$

the summation extending over the section. But $\Sigma\mathbf{FH}=0$, because $\mathbf{F}$ is polar and $\mathbf{H}$ is closed; so that

$$\mathrm{T}=\Sigma\mu\mathbf{h}^2/8\pi-\Sigma\mu\mathbf{F}^2/8\pi$$
$$=\Sigma\mu\mathbf{h}^2/8\pi+\Sigma\mu\mathbf{hF}/8\mu. \quad . \quad . \quad . \quad . \quad . \quad (9a)$$

Or, in Cartesian coordinates, let H_1 and H_2 be the x and y components of the magnetic force $\mathbf{H}$, z being parallel to the current; then

$$\mathrm{H}_1=-2\pi y\Gamma_0-\frac{d\Omega}{dx}, \quad \mathrm{H}_2=2\pi x\Gamma_0-\frac{d\Omega}{dy} \quad . \quad (10a)$$

express (1a), and (8a) is represented by

$$\left.\begin{aligned}\mathrm{T}&=\frac{\mu}{8\pi}\Sigma(\mathrm{H}_1^2+\mathrm{H}_2^2),\\&=\frac{\mu\pi}{2}\Gamma_0^2\Sigma(x^2+y^2)-\frac{\mu\Gamma_0}{4}\Sigma\Big(x\frac{d\Omega}{dy}-y\frac{d\Omega}{dx}\Big),\end{aligned}\right\} \quad (11a)$$

the latter form expressing (9a).

It will be observed that the mathematical conditions are identical with those existing in St. Venant's torsion problems. Thus, if $\mathfrak{a}$ and $\mathfrak{b}$ are the y and x tangential strain components in the plane x, y in a twisted prism, and γ the longitudinal displacement along z, parallel to the length of the prism, we have

$$\mathfrak{b}=-\tau y+\frac{d\gamma}{dx}, \quad \mathfrak{a}=\tau x+\frac{d\gamma}{dy}, \quad . \quad . \quad . \quad (12a)$$

where τ is the twist (Thomson and Tait, Part II. § 706, equation (9)). The corresponding forces are n times as great, if n is the rigidity (*loc. cit.* equation (10)); so that the energy per unit length is

$$\tfrac{1}{2}n\Sigma(\mathfrak{a}^2+\mathfrak{b}^2) \text{ over section.} \quad . \quad . \quad . \quad . \quad (13a)$$

Also, to find γ we have

$$\nabla^2\gamma=0, \quad \frac{d\gamma}{dp_1}=\tfrac{1}{2}\tau\frac{dr^2}{ds}, \quad . \quad . \quad . \quad . \quad (14a)$$

(*loc. cit.* equations (12) and (18)). Comparing (14 a) with (5a), (12a) with (10a), and (13a) with the first of (11a), we see that there is a perfect correspondence, except, of course, as regards the constants concerned. The lines of tangential stress in the torsion problem and the lines of magnetic force in our problem are identical, and the energy is similarly reckoned. We may therefore make use of all St. Venant's results.

It will be sufficient here to point out that the ratio of the inductance of wires of different sections is the same as the ratio of their torsional rigidities. Thus, as $\mathrm{L}=\frac{1}{2}\mu$ in the case of a round wire, that of a wire of elliptical section, semiaxes a and b, is $\mathrm{L}=\mu ab/(a^2+b^2)$; when the section is a square, it is $\cdot4417\mu$; when it is an equilateral triangle, $\cdot3627\mu$, &c.

That of a rectangle will be given later in the course of the following subsidence solution.

Consider the subsidence from the initial state of steady flow to zero, when the impressed force that supported the current is removed, in a prism of rectangular section. Let $2a$ and $2b$ be its sides, parallel to x and y respectively, the origin being taken at the centre. Let H_1 and H_2 be the x and y components of the magnetic force at the time t. Let E be the intensity of the magnetic-force vector **E**, which is parallel to z; then the two equations of induction ((6), (7), Part I.), or

$$\text{curl } \mathbf{H}=4\pi\mathbf{\Gamma}, \quad -\text{curl } \mathbf{E}=\mu\dot{\mathbf{H}},$$

are reduced to

$$-\frac{d\mathrm{E}}{dy}=\mu\dot{\mathrm{H}}_1, \quad \frac{d\mathrm{E}}{dx}=\mu\dot{\mathrm{H}}_2, \quad . \quad . \quad . \quad . \quad (15a)$$

$$\frac{d\mathrm{H}_2}{dx}-\frac{d\mathrm{H}_1}{dy}=4\pi k\mathrm{E}=4\pi\Gamma; \quad . \quad . \quad . \quad . \quad (16a)$$

if Γ is the current density, k the conductivity, μ the inductivity. [I speak of the intensity of a "force" and of the "density" of a flux, believing a distinction desirable.] The equation of $\mathbf{\Gamma}$ is therefore

$$\left(\frac{d^2}{dx^2}+\frac{d^2}{dy^2}\right)\Gamma=4\pi\mu k\dot{\Gamma}, \quad . \quad . \quad . \quad . \quad (17a)$$

of which an elementary solution is

$$\Gamma=\cos mx \cos ny\, \epsilon^{pt}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (18a)$$

if

$$4\pi\mu kp=-(m^2+n^2). \quad . \quad . \quad . \quad . \quad (19a)$$

At the boundary we have, during the subsidence, $\mathrm{E}=0$, or $\Gamma=0$; therefore

$$\cos mx \cos ny=0 \text{ at the boundary},$$

or

$$\cos ma=0, \quad \cos nb=0, \quad . \quad . \quad . \quad . \quad . \quad (20a)$$

or $ma=\frac{1}{2}\pi, \frac{3}{2}\pi, \frac{5}{2}\pi$, &c., $nb=$ditto. The general solution is therefore the double summation over m and n,

$$\Gamma=\Sigma\Sigma \mathrm{A} \cos mx \cos ny\, \epsilon^{pt},$$

if we find A to make the right member represent the initial state. This has to be $\Gamma=\Gamma_0$, a constant.

Now

$$1=\Sigma(2/ma)\sin ma \cos mx, \quad \text{from } x=-a \text{ to } +a,$$
$$1=\Sigma(2/nb)\sin nb \cos ny, \quad \text{from } y=-b \text{ to } +b.$$

Hence the required solution is

$$\Gamma=\frac{4}{ab}\Gamma_0\Sigma\frac{\sin ma}{m}\cos mx\, \epsilon^{-\frac{m^2t}{4\pi\mu k}}.\Sigma\frac{\sin nb}{n}\cos ny\, \epsilon^{-\frac{n^2t}{4\pi\mu k}},$$

or

$$\Gamma=\frac{4}{ab}\Gamma_0\Sigma\Sigma\frac{\sin ma \sin nb}{mn}\cos mx \cos ny\, \epsilon^{pt}. \quad . \quad . \quad (21a)$$

From this derive the magnetic force by (15a). Thus

$$\left.\begin{aligned}\mathrm{H}_1&=-\frac{16\pi}{ab}\Gamma_0\Sigma\Sigma\frac{\sin ma}{m}\sin nb \cos mx \sin ny\frac{\epsilon^{pt}}{m^2+n^2}\\ \mathrm{H}_2&=\frac{16\pi}{ab}\Gamma_0\Sigma\Sigma\frac{\sin nb}{n}\sin ma \sin mx \cos ny\frac{\epsilon^{pt}}{m^2+n^2}.\end{aligned}\right\} \quad (22a)$$

The total current, say C, in the prism is given by

$$4\pi\mathrm{C}=2\int_{-b}^{b}\mathrm{H}_2 dy_{(x=a)}-2\int_{-a}^{a}\mathrm{H}_1 dx_{(y=b)},$$
$$=\frac{64\pi}{ab}\Gamma_0\Sigma\Sigma\frac{\epsilon^{pt}}{m^2n^2},$$

by line integration round the boundary. Or

$$\mathrm{C}=\frac{4}{a^2b^2}\mathrm{C}_0\Sigma\Sigma\frac{\epsilon^{pt}}{m^2n^2}, \quad . \quad . \quad . \quad . \quad (23a)$$

if $\mathrm{C}_0=4ab\Gamma_0$, the initial current in the prism.

Since the current is longitudinal, and there is no potential difference, the vector potential is given by $\mathbf{E}=-\dot{\mathbf{A}}$; or, A being the tensor of **A**, A is got by dividing the general term in the Γ solution (21a) by $-pk$; giving

$$\mathrm{A}=\frac{16\pi\mu}{ab}\Sigma\Sigma\frac{\sin ma \sin nb}{mn(m^2+n^2)}\cos mx \cos ny\, \epsilon^{pt}. \quad . \quad (24a)$$

Since the magnetic energy is to be got by summing up the product $\frac{1}{2}\mathrm{A}\Gamma$ over the section, we find, by integrating the square of Γ, that the amount per unit length is

$$\mathrm{T}=\frac{2\pi\mu\mathrm{C}_0}{a^3b^3}\Sigma\Sigma\frac{\epsilon^{2pt}}{m^2n^2(m^2}. \quad . \quad . \quad (25a)$$

By the square of the force method the same result is reached, of course. We may also verify that $\mathrm{Q}+\dot{\mathrm{T}}=0$, during the subsidence, Q being the dissipativity per unit length of prism.

The steady-flow resistance per unit length is the L in

$T=\frac{1}{2}LC_0^2$, which (25*a*) becomes when $t=0$; this gives

$$L=4\pi\mu\Sigma\Sigma\frac{1}{(ma)^2(nb)^2\left\{\frac{a}{b}(nb)^2+\frac{b}{a}(ma)^2\right\}}. \qquad (26a)$$

The lines of magnetic current are also the lines of equal electric-current density. That is, a line drawn in the plane x, y through the points where Γ has the same value is a line of magnetic current. For, if s be any line in the plane x, y,

$$\frac{dE}{ds}=\text{component of } \mu\dot{H} \text{ perpendicular to } s,$$

so that $\dot{H}$ is parallel to s, when $dE/ds=0$. The transfer of energy is, as usual, perpendicular to the lines of magnetic force and electric force.

The above expression (26*a*) for L may be summed up either with respect to *ma* or to *nb*, but not to both, by any way I know. Thus, writing it

$$L=4\pi\mu\Sigma\frac{1}{(ma)^2}\Sigma\frac{1}{(nb)^2}\frac{1}{\frac{a}{b}(nb)^2+\frac{b}{a}(ma)^2}, \quad . \quad (27a)$$

we may effect the second summation, with respect to *nb*, regarding *ma* as constant in every term. Use the identity

$$\frac{l-x}{h^2}-\frac{\epsilon^{h(l-x)}-\epsilon^{-h(l-x)}}{h^3(\epsilon^{hl}+\epsilon^{-hl})}=\frac{2}{l}\Sigma\frac{\cos(i\pi x/2l)}{(i\pi/2l)^2\{(i\pi/2l)^2+h^2\}},$$

where i has the values 1, 3, 5, &c. Take $x=0$, $i\pi/2l=nb$, $h=(b/a)(ma)$, $l=1$, and apply to (27*a*), giving

$$L=4\pi\mu\Sigma\frac{1}{(ma)^2}\left\{\frac{a}{2b}\frac{1}{(ma)^2}\left(1-\frac{a}{b}\cdot\frac{1}{ma}\cdot\frac{\epsilon^{\frac{b}{a}(ma)}-\epsilon^{-\frac{b}{a}(ma)}}{\epsilon^{\frac{b}{a}(ma)}+\epsilon^{-\frac{b}{a}(ma)}}\right)\right\} \quad (28a)$$

where the quantity in the { } is the value of the second Σ in (27*a*). The first part of (28*a*) is again easily summed up, and the result is

$$L=4\pi\mu\frac{a}{b}\left\{\frac{1}{12}-\frac{1}{2}\frac{a}{b}\Sigma\frac{1}{(ma)^5}\frac{\epsilon^{2\frac{b}{a}(ma)}-1}{\epsilon^{2\frac{b}{a}(ma)}+1}\right\}, \quad . \quad (29a)$$

in which summation, we may repeat, *ma* has the values $\frac{1}{2}\pi$, $\frac{3\pi}{2}$, $\frac{5}{2}\pi$, &c. The quantities a and b may be exchanged; that is, a/b changed to b/a, without altering the value of L. This follows by effecting the *ma* summation in (26*a*) instead of the *nb*, as was done.

When the rod is made a flat sheet, or a/b is very small, we have $L=\frac{1}{3}\pi\mu(a/b)$.

Compare (29*a*) with Thomson and Tait's equation (46) § 707, Part II. Turn the nab^2 outside the [ ] to nab^3, and multiply the Σ by 2. These corrections have been pointed out by Ayrton and Perry. When made, the result is in agreement with the above (29*a*), allowing, of course, for changed multiplier. [I also observe that the $-\tau$ in their equation (44) should be $+\tau$, and the $+\tau$ in (45), (the second τ) should be $-\tau$.] Such little errors will find their way into mathematical treatises; there is nothing astonishing in that; but a certain collateral circumstance renders the errors in their equation (46) worthy of being long remembered. For the distinguished authors pointedly called attention to the astonishing theorems in pure mathematics to be got by the exchange of a and b, such as rarely fall to the lot of pure mathematicians. They were miraculous.

I now pass to a different problem, viz. the solution in the case of a periodic impressed force situated at one end of a homogeneous line, when subjected to any terminal conditions of the kind arising from the attachment of apparatus. The conditions that obtain in practice are very various, but valuable information may be arrived at from the study of the comparatively simple problem of a periodic impressed force, of which the full solution may always be found. In Part II. I gave the fully developed solution when the line has the three electrical constants R, L, and S (resistance, inductance, and electrostatic capacity), of which the first two may be functions of the frequency, but without any allowance for the effect of terminal apparatus. If we take $L=0$ we get the submarine-cable formula of Sir W. Thomson's theory; but although the effect of L on the amplitude of the current at the distant end becomes insignificant when the line is an Atlantic cable, its omission would in general give quite misleading results.

There are some *à priori* reasons against formulating the effect of the terminal apparatus. They complicate the formulæ considerably in the first place; next, they are various in arrangement, so that it might seem impracticable to formulate generally; and, again, in the case of a very long submarine cable, we may divide the expression of the current-amplitude into factors, one for the line and two more for the terminal apparatus, of which the first, for the line, is always the same, whilst the apparatus-factors vary, and are less important than the line-factor. But in other cases the terminal apparatus may be of far greater importance than the line, in their influence on the current-amplitude, whilst the resolution into independent factors is no longer possible.

The only serious attempt to formulate the effect of the terminal apparatus with which I am acquainted is that of the late Mr. C. Hockin (Journal S. T. E. and E., vol. v. p. 432). His apparatus arrangement resembled that usually occurring then in connection with long submarine cables, including, of course, many derived simpler arrangements; and from his results much interesting information is obtainable. But the results are only applicable to long submarine cables, on account of the omission of the influence of the self-induction of the line. The work must, therefore, be done again in a more general manner. It is, besides, independently of this, not easy to adapt his formulæ, in so far as they show the influence of terminal apparatus, to cases that cannot be derived from his. For instance, the effect of electromagnetic induction in the terminal arrangements was omitted. I have therefore thought it worth while to take a far more general case as regards the line, and at the same time have endeavoured to put it in such a form that it can be readily reduced to simpler cases, whilst at the same time the results apply to any terminal arrangements we choose to use.

The general statement of the problem is this. A homogeneous line, of length l, whose steady-flow resistance is R, inductance L, electrostatic capacity S, and conductance of insulator K, all per unit length of line, is acted upon by an impressed force $V_0 \sin nt$ at one end, or in the wire attached to it; whilst any terminal arrangements exist. Find the effect produced; in particular, the amplitude of the current at the end remote from the impressed force. If the line consist of two parallel wires, R must be the sum of their resistances per unit length.

Let C be the current in the line and V the potential difference at distance z from the end where the impressed force is situated. Then

$$-\frac{dC}{dz}=\left(K+S\frac{d}{dt}\right)V, \quad -\frac{dV}{dz}=R''C, \quad . \quad . \quad (1b)$$

are our fundamental line equations. Here $R''=R+L\,(d/dt)$ to a first approximation, and $=R'+L'(d/dt)$ in the periodic case, where R′ and L′ are what R and L become at the given frequency. Let the terminal conditions be

$$\left.\begin{aligned} V&=Z_1C \quad \text{at } z=l \text{ end,}\\ -V_0\sin nt+V&=Z_0C \quad \text{at } z=0 \text{ end,}\end{aligned}\right\} \quad . \quad . \quad . \quad (2b)$$

so that $V=Z_0C$ would be the $z=0$ terminal condition if there were no impressed force.

The solution is a special case of the second of (162*b*), Part IV., which we may quote. In it take

$$S''=K+Sp, \quad R''=R'+L'p, \quad . \quad . \quad . \quad (3b)$$

p meaning d/dt so far. Also put $z_2=0$, $e_2=V_0 \sin nt$, and

$$-m^2=F^2=(K+Sp)(R'+L'p), \quad . \quad . \quad . \quad (4b)$$

and put the equation referred to in the exponential form. Thus,

$$C=\frac{(F/S''+Z_1)\epsilon^{F(l-z)}+(F/S''-Z_1)\epsilon^{-F(l-z)}}{\epsilon^{Fl}(F/S''+Z_1)(F/S''-Z_0)-\epsilon^{-Fl}(F/S''-Z_1)(F/S''+Z_0)}V_0\sin nt. \quad (5b)$$

This is the differential equation of C in the line. Now in F, S″, Z_0, and Z_1, let $d^2/dt^2=-n^2$. It is then reducible to

$$C=\frac{P'+Q'\frac{d}{dt}}{A'+B'\frac{d}{dt}}V_0\sin nt=\frac{(A'P'+B'Q'n^2)+(A'Q'-B'P')\frac{d}{dt}}{A'^2+B'^2n^2}V_0\sin nt, \quad (6b)$$

giving the amplitude and phase-difference anywhere; and the amplitude is

$$C_0=V_0(A'^2+B'^2n^2)^{-\frac{1}{2}}(P'^2+Q'^2n^2)^{\frac{1}{2}}; \quad . \quad . \quad . \quad (7b)$$

A′ and B′ are functions of z, whilst P′ and Q′ are constants.

Put

$$\left.\begin{aligned} F&=P+Qi,\\ Z_1&=R'_1+L'_1ni,\\ -Z_0&=R'_0+L'_0ni,\end{aligned}\right\}\text{ where } i=(-1)^{\frac{1}{2}}, \text{ or } p=ni. \quad . \quad . \quad (8b)$$

The values of P and Q are

$$\left.\begin{aligned} P&=(\tfrac{1}{2})^{\frac{1}{2}}\{(R'^2+L'^2n^2)^{\frac{1}{2}}(K^2+S^2n^2)^{\frac{1}{2}}+(KR'-L'Sn^2)\}^{\frac{1}{2}}\\ Q&=\cdot\cdot\;\{\;\cdots\cdots\cdots\;-\;\cdots\cdots\}^{\frac{1}{2}}\end{aligned}\right\}, (9b)$$

possessing the following properties, to be used later,

$$\left.\begin{aligned} P^2+Q^2&=(K^2+S^2n^2)^{\frac{1}{2}}(R'^2+L'^2n^2)^{\frac{1}{2}},\\ P^2-Q^2&=KR'-L'Sn^2,\\ 2PQ&=(R'S+KL')n.\end{aligned}\right\} \quad . \quad (10b)$$

The expressions of R_0', R_1', L_0', L_1' can only be stated when the terminal conditions are fully given. Their structure will be considered later. P and Q depend only upon the line.

Let

$$\left.\begin{aligned} A&=R'-Sn^2(R_0'L_1'+R_1'L_0')+K(R_0'R_1'-L_0'L_1'n^2);\\ B&=L'n+Sn(R_0'R_1'-L_0'L_1'n^2)+Kn(R_0'L_1'+R_1'L_0');\\ a&=P(R_0'+R_1')-Qn(L_0'+L_1');\\ b&=Q(R_0'+R_1')+Pn(L_0'+L_1').\end{aligned}\right\}(11b)$$

The effect of making the substitutions ($8b$) in ($5b$) is to express C in terms of the P, Q of ($9b$) and the A, B, a, b of ($11b$); thus:—

$$
\begin{aligned}
C = \Big[& \big\{ (P - L_1'Sn^2 + KR_1')\cos Q(l-z) - (Q + R_1'Sn + KL_1'n)\sin Q(l-z) \big\}\epsilon^{P(l-z)} \\
& + \big\{ (\ldots + \ldots - \ldots)\ \ldots\ldots + (\ldots - \ldots - \ldots)\ \ldots\ldots \big\}\epsilon^{-P(l-z)} \\
& + i\big\{ (\ldots - \ldots + \ldots)\sin Q(l-z) + (\ldots + \ldots + \ldots)\cos Q(l-z) \big\}\epsilon^{P(l-z)} \\
& + i\big\{ -(\ldots + \ldots - \ldots)\ \ldots\ldots + (\ldots - \ldots - \ldots)\ \ldots\ldots \big\}\epsilon^{-P(l-z)} \Big] \\
& \times V_0 \sin nt \\
\div \Big[& \big\{ (A+a)\epsilon^{Pl}\cos Ql - (A-a)\epsilon^{-Pl}\cos Ql - (B+b)\epsilon^{Pl}\sin Ql - (B-b)\epsilon^{-Pl}\sin Ql \big\} \\
& + i\big\{ (B+b)\ \ldots\ -(B-b)\ \ldots\ +(A+a)\ \ldots\ +(A-a)\ \ldots \big\} \Big]. \qquad (12b)
\end{aligned}
$$

The dots indicate repetition of what is immediately above them. Here we see the expressions for the four quantities A', B', P', Q' of ($6b$), which we require. ($12b$) therefore fully serves to find the phase-difference, if required. I shall only develop the amplitude expression ($7b$). It becomes, by ($12b$),

$$
\begin{aligned}
\left(\frac{C_0}{V_0}\right) = & \\
\Big[& \epsilon^{2P(l-z)} \big\{(P^2+Q^2) + (K^2+S^2n^2)(R_1'^2 + L_1'^2n^2) + 2Qn(R_1'S + KL_1') + 2P(KR_1' - L_1'Sn^2)\big\} \\
& + \epsilon^{-2P(l-z)}\big\{ \ldots + \ldots\ldots\ldots - \ldots\ldots\ldots - \ldots\ \ldots \big\} \\
& + 2\cos 2Q(l-z)\,\big\{(P^2+Q^2) - (K^2+S^2n^2)(R_1'^2 + L_1'^2n^2)\big\} \\
& - 4\sin 2Q(l-z)\,\big\{Pn(R_1' + KL_1') + Q(L_1'Sn^2 - KR_1')\big\}\Big]^{\frac{1}{2}} \\
\div \Big[& \epsilon^{2Pl}\big\{(A+a)^2 + (B+b)^2\big\} + \epsilon^{-2Pl}\big\{(A-a)^2 + (B-b)^2\big\} \\
& - 2\cos 2Ql\,.\,(A^2+B^2-a^2-b^2) + 4\sin 2Ql\,.\,(Ab - aB)\Big]^{\frac{1}{2}}, \qquad (13b)
\end{aligned}
$$

in terms of A, B, a, b of ($11b$).

This referring to any point between $z=0$ and l, a very important simplification occurs when we take $z=l$. It reduces the numerator to $2(P^2+Q^2)^{\frac{1}{2}}$. It only remains to simplify the denominator as far as possible, to show as explicitly as we can the effect of the terminal apparatus, which is at present buried away in the functions of A, B, a, b occurring in ($13b$).

First of all, we may show that the product of the coefficients of ϵ^{2Pl} and ϵ^{-2Pl} equals half the square of the amplitude of the circular part in the denominator. This is an identity, independent of what A, B, a, b are. ($13b$) therefore takes the form

$$C_0 = 2V_0(P^2+Q^2)^{\frac{1}{2}} \div \big[G\epsilon^{2Pl} + H\epsilon^{-2Pl} - 2(GH)^{\frac{1}{2}}\cos 2(Ql+\theta)\big]^{\frac{1}{2}}. \qquad (14b)$$

The following are the expansions of the quantities occurring in the denominator of ($13b$):—

Let

$$I^2 = R'^2 + L'^2n^2,\quad I_0^2 = R_0'^2 + L_0'^2n^2,\quad I_1^2 = R_1'^2 + L_1'^2n^2. \qquad (15b)$$

Then

$$
\left.
\begin{aligned}
A^2+B^2 &= I^2 + (K^2+S^2n^2)I_0^2I_1^2 + 2(R_0'R_1' - L_0'L_1'n^2)(KR' + L'Sn^2) \\
&\quad + 2(R_1'L_0' + R_0'L_1')n^2(KL' - R'S), \\
a^2+b^2 &= (P^2+Q^2)\big\{(R_0'+R_1')^2 + (L_0'+L_1')^2n^2\big\}, \\
Aa+Bb &= (R_0'+R_1')(R'P + L'nQ) + (L_0'+L_1')n(L'nP - R'Q) \\
&\quad + (R_0'I_1^2 + R_1'I_0^2)(KP + SnQ) + (L_0'I_1^2 + L_1'I_0^2)n(KQ - SnP), \\
Ab-aB &= (R_0'+R_1')(R'Q - L'nP) + (L_0'+L_1')n(R'P + L'nQ) \\
&\quad + (R_0'I_1^2 + R_1'I_0^2)(KQ - SnP) - (L_0'I_1^2 + L_1'I_0^2)n(KP + SnQ).
\end{aligned}
\right\} \quad (16b)
$$

These may be used direct in the denominator of ($14b$), which is the same as that of ($13b$). But G and H may be each resolved into the product of two factors, each containing the apparatus-constants of one end only. Noting therefore that the θ in ($14b$) is given by

$$\tan 2\theta = \frac{2(Ab - aB)}{A^2+B^2-a^2-b^2}, \qquad (17b)$$

whose numerator and denominator are given in ($16b$), it will clearly be of advantage to develop these factors. First observe that the expansion of H is to be got from that of G, using ($16b$), by merely turning P to $-P$ and Q to $-Q$. We have therefore merely to split up one of them, say G. If we put $R_1'=0$, $L_1'=0$ in G it becomes

$$I^2 + (P^2+Q^2)I_0^2 + 2P(R_0'R' + L_0'L'n^2) + 2Q(L'nR_0' - R'nL_0'). \qquad (18b)$$

If, on the other hand, we put $R_0'=0$, $L_0'=0$ in G it becomes the same function of R_1' L_1' as ($18b$) is of R_0', L_0'. It is then suggested that G is really the product of ($18b$) into the similar function of R_1', L_1'; when the result is divided by I^2. This may be verified by carrying out the operation described. But I should mention that it is not immediately evident, and requires some laborious transformations to establish it, making use of the three equations ($10b$). When done, the final result is that ($14b$) becomes

$$
\begin{aligned}
C_0 = 2V_0\left[\frac{K^2+S^2n^2}{R'^2+L'^2n^2}\right]^{\frac{1}{2}} & \\
\div \big[G_0G_1\epsilon^{2Pl} + H_0H_1\epsilon^{-2Pl} - 2(G_0G_1H_0H_1)^{\frac{1}{2}}\cos 2(Ql+\theta)\big]^{\frac{1}{2}}, & \qquad (19b)
\end{aligned}
$$

wherein G_0 and H_0 contain only constants belonging to the apparatus at $z=0$, and G_1 and H_1 those belonging to $z=l$, besides the line-constants. Only one of the four need be written; thus

$$G_0=1+\frac{1}{I^2}\{(P^2+Q^2)I_0^2+2P(R'R_0'+L'L_0'n^2)+2Qn(R_0'L'-R'L_0')\}. \quad (20b)$$

From this get H_0 by changing the signs of P and Q. Then, to obtain G_1 and H_1, the corresponding functions for the $z=l$ end, change R_0' to R_1' and L_0' to L_1'. These functions have the value unity when the line is short-circuited at the ends, ($Z_0=0$, $Z_1=0$). They may therefore be referred to as the terminal functions. Their form is invariable. We only require to find the R' and L', or the effective resistance and inductance of the terminal arrangements, and insert in (20*b*) and its companions.

Thus, let the two conductors at the $z=l$ end be joined through a coil. Then R_1' is its resistance, L_1' its inductance, the steady-flow values, and the accents may be dropped, except under very unusual circumstances, and I_1 is its impedance at the given frequency, when on short circuit. But if the coil contain a core, especially if it be of iron, neither R_1 nor L_1 can have the steady-flow values, on account of the induction of currents in the core. Their approximate values at a given frequency may be experimentally determined by means of the Wheatstone bridge. Of course R_1 and L_1 are really somewhat changed in a similar manner by allowing any induction between the coil and external conductors, the brass parts of a galvanometer, for instance; L going down and R going up, though this does not materially affect I.

If, instead of a coil, it be a condenser of capacity S_1 that is inserted at $z=l$; then, since

$$C=S_1\dot{V}=S_1pV,$$

we have

$$Z_1=(S_1p)^{-1}=-p/(S_1n^2).$$

Therefore take

$$R_1'=0, \quad \text{and} \quad L_1'=-(S_1n^2)^{-1}.$$

The condenser behaves, so far as the current is concerned, as a coil of no resistance and negative inductance, the latter decreasing as the frequency is raised, and as the capacity is increased; tending to become equivalent to a short circuit, though this would require a great speed in general, as the *quasi*-negative inductance is large. [Thus $n=100$, $S=10^{-15}$ (one microfarad), makes $L_1'=-10^{11}$. To get the inductance of a coil to be 10^{11} it must contain a very large number of turns of fine wire.] Thus, whilst the condenser stops slowly periodic or steady currents, it tends to readily pass rapidly periodic currents, a property which is very useful in telephony, as in Van Rysselberghe's system.

On the other hand, the coil passes the slowly periodic, and tends to stop the rapidly periodic, a property which is also very useful in telephony. A very extensive application of this principle occurs in the system of telephonic intercommunication invented and carried out by Mr. A. W. Heaviside, known as the Bridge System, from the telephones at the various offices being connected up as bridges across from one to the other of the two conductors which form the line. Whilst all stations are in direct communication with one another, one important desideratum, there is no overhearing, which is another. For all stations except the two which are in correspondence at a certain time have electromagnets of high inductance inserted in their bridges, which electromagnets will not pass the rapid telephonic currents in appreciable strength, so that it is nearly as if the non-working bridges were non-existent, and, in consequence, a far greater length of buried wire can be worked through than on the Sequence system, wherein the various stations have their apparatus in sequence with the line, whilst at the same time a balance is preserved against inductive interferences. When the two stations have finished correspondence, they insert their own electromagnets in their bridges. As these electromagnets are used as call instruments, responding to slowly periodic currents, we have the direct intercommunication. Of course there are various other details, but the above sufficiently describes the principle.

As regards the property of the self-induction of a coil in stopping or greatly decreasing the amplitude of rapidly periodic currents, or acting as an insulation at the first moment of starting a current, its influence was entirely overlooked by most writers on telegraphic technics before 1878, when I wrote on the subject (Journ. S. T. E. & E. vol. vii.). A knowledge of the important quantity $(R^2+L^2n^2)^{\frac{1}{2}}$, which is now the common property of all electrical schoolboys (especially by reason of the great impetus given to the spread of a scientific knowledge of electromagnetism by the commercial importance of the dynamo), was, before then, confined to a few theorists.

If the coil R, L, and the condenser S_1 be in parallel, we have

$$C=\left(S_1p+\frac{1}{R+Lp}\right)V,$$

or

$$\frac{V}{C}=\frac{R+\{L-S_1(R^2+L^2n^2)\}}{(1-LS_1n^2)^2+(RS_1n)^2},$$

which show the expressions of R_1' and L_1', the second being the coefficient of p, the first the rest.

Similarly in other simple cases. And, in general, from the detailed nature of the combination inserted at the end of the line, write out the connections between the current and potential difference in each branch, and eliminate the intermediates so as to arrive at $V=Z_1C$, the differential equation of the combination, wherein Z is a function of p or d/dt. Put $p^2=-n^2$, and it takes the form $Z_1=R_1'+L_1'p$, wherein R_1' and L_1' are functions of the electrical constants and of n^2, and are the required effective R_1' and L_1' of the combination, to be used in (20*b*), or rather in its $z=l$ equivalent G_1.

As regards the $z=0$ end, it is to be remarked that, owing to the current being reckoned positive the same way at both ends, when we write $V=Z_0C$ as the terminal equation, it is $-Z_0$ that corresponds to Z_1. Thus $-Z_0=R_0'+L_0'p$, where, in the simplest case, R_0' and L_0' are the resistance and inductance of a coil.

So far sufficiently describing how to develope the effective resistance and inductance expressions to be used in the terminal functions G and H, we may now notice some other peculiarities in connection with the solution (19*b*). First short-circuit the line at both ends, making the terminal functions unity and $\theta=0$. The solution then differs from that given in Part II., equation (82), in the presence of the quantity K, the former Sn now becoming $(K^2+S^2n^2)^{\frac{1}{2}}$, whilst P and Q differ from the former P and Q of (78), Part II., by reason of K, which, when it is made zero, makes them identical. If we compare the old with the new P and Q, we find that

$$\left.\begin{aligned} L' \text{ becomes } & L'-KR'/Sn^2, \\ R' \text{ becomes } & R'+KL'/S, \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (21b)$$

in passing from the old to the new. Then the function

$$\frac{R'^2+L'^2n^2}{S^2n^2} \text{ becomes } \frac{(R'+KL'/S)^2+(L'-KR'/Sn^2)^2n^2}{K^2+S^2n^2}=\frac{R'^2+L'^2n^2}{S^2n^2}$$

or is unaltered by the leakage. It follows that the equation (85) Part II. is still true, with leakage, if in it we make the changes (21 *b*) just mentioned, or put

$$\frac{1}{v'^2}=S\left(L'-\frac{KR'}{Sn^2}\right), \quad h=\frac{(R'S+KL')^2}{n^2}v'^4, \quad . \quad . \quad . \quad . \quad (22b)$$

instead of using the v' and h expressions of Part II.

At the particular speed given by $n^2=KR'/L'S$, we shall have

$$P=Q=(\tfrac{1}{2})^{\frac{1}{2}}(R'^2+L'^2n^2)^{\frac{1}{4}}(K^2+S^2n^2)^{\frac{1}{4}}=(\tfrac{1}{2})^{\frac{1}{2}}(R'S+KL')n, (23b)$$

making

$$\frac{V_0}{C_0}=(R'^2+L'^2n^2)^{\frac{1}{2}}l\left\{1+2\left(\frac{(2Pl)^4}{\lfloor 6}+\frac{(2Pl)^8}{\lfloor 10}+\frac{(2Pl)^{12}}{\lfloor 14}+\ldots\right)\right\}^{\frac{1}{2}}. (24b)$$

If we should regard the leakage as merely affecting the amplitude of the current at the distant end of a line, we should be overlooking an important thing, viz. its remarkable effect in accelerating changes in the current, and thereby lessening the distortion that a group of signals suffers in its transmission along the line. If there is only a sufficient strength of current received for signalling purposes, the signals can be far more distinct and rapid than with perfect insulation, as I have pointed out and illustrated in previous papers. Thus the theoretical desideratum for an Atlantic cable is not high, but low insulation, the lowest possible consistent with having enough current to work with. Any practical difficulties in the way form a separate question.

Regarding this quickening effect, or partial abolition of electrostatic retardation, I have ('Electrician,' Dec. 18, 1885, and Jan. 1, 1886) pushed it to its extreme in the electromagnetic scheme of Maxwell. In a medium whose conductivity varies in any manner from point to point, possessed of dielectric capacity which varies in the same manner, so that their ratio, or the electrostatic time-constant, is everywhere the same, but destitute of magnetic inertia ($\mu=0$, no magnetic energy), I have shown that electrostatic retardation is entirely done away with, except as regards imaginable preexisting electrification, which subsides everywhere according to the common time-constant, without true electric current, by the discharge of every elementary condenser through its own resistance. This being over, if any impressed force act, varying in any manner in distribution and with the time, the corresponding current will everywhere be the steady-flow distribution appropriate to the impressed force at any moment, in spite of the electric displacement and energy; and, on removal of the impressed force, there will be instantaneous disappearance of the current and the displacement. This seems impossible, but the same theory applies to combinations of shunted condensers, arranged in a suitable manner, as described in the paper referred to.

Of course this extreme state of things is quite imaginary, as we cannot really overlook the electromagnetic induction in such a case. If we regard it as the limiting form of a real problem, in which inertia occurs, to be afterwards made zero, we find that the instantaneous subsidence of the electrostatic problem becomes an oscillatory subsidence of infinite frequency but finite time-constant, about the mean value zero; which is mathematically equivalent to instantaneous non-oscillatory subsidence.

The following will serve to show the relative importance of R, S, K, and L in determining the amplitude of periodic currents at the distant end of a long submarine cable, of fairly high insulation: resistance:—

$$4 \text{ ohms per kilom. makes } R=40^4,$$
$$\tfrac{1}{4} \text{ microf. ,, ,, } S=\frac{1}{40^{20}},$$
$$100 \text{ megohms ,, ,, } K=10^{-22}.$$

Here, it should be remembered, K is the conductance of the insulator per centim. The least possible value of L would be such that $LS=v^{-2}$, where $v=30^{10}$; this would make $L=4/9$ only. But it is really much greater, requiring to be multiplied by the dielectric constant of the insulator in the first place, making $L=2$ say. It is still further increased by the wire, and considerably by the sheath and by the extension of the magnetic field beyond the sheath, to an extent which is very difficult to estimate, especially as it is a variable quantity; but it would seem never to become a very large number, as of course an iron wire for the conductor is out of the question. But leaving it unstated, we have, by (9*b*), taking $R'=R$, $L'=L$,

$$P=(\tfrac{1}{2})^{\frac{1}{2}}\left\{(160^8+L^2n^2)^{\frac{1}{2}}\left(\frac{1}{10^{44}}+\frac{n^2}{160^{40}}\right)^{\frac{1}{2}}+\left(\frac{4}{10^{18}}-\frac{Ln^2}{40^{20}}\right)\right\}^{\frac{1}{2}}$$
$$=\frac{(\frac{1}{2})^{\frac{1}{2}}}{10^{10}}\left\{(160^8+L^2n^2)^{\frac{1}{2}}\left(\frac{1}{10^4}+\frac{n^2}{16}\right)^{\frac{1}{2}}+\left(400-\frac{Ln^2}{4}\right)\right\}^{\frac{1}{2}}.$$

Now $n/2\pi$ is the frequency, necessarily very low on an Atlantic cable. We see then that the first L^2n^2 is quite negligible in its effect upon P, even when we allow L to increase greatly from the above $L=2$. The high insulation also makes the $(RK-LSn^2)$ part negligible, making approximately

$$P=Q=(\tfrac{1}{2}n)^{\frac{1}{2}}\,.\,10^{-8},$$

P being a little greater than Q, at least when L is small. Now this is equivalent to taking $L=0$, $K=0$, when

$$P=Q=(\tfrac{1}{2}RSn)^{\frac{1}{2}}, \quad . \quad . \quad . \quad . \quad . \quad (25b)$$

reducing (19*b*) to

$$C_0=2V_0(Sn/R)^{\frac{1}{2}}\div\{G_0G_1\epsilon^{2Pl}+H_0H_1\epsilon^{-2Pl}-2(G_0G_1H_0H_1)^{\frac{1}{2}}\cos 2Pl\}^{\frac{1}{2}}. \quad (26b)$$

which is, except as regards the terminal functions I introduce, quite an old formula. It is what we get by regarding the line as having only resistance and electrostatic capacity. But, still regarding the line as an Atlantic or similar cable, worked nearly up to its limit of speed, Pl is large, say 10 at most, so that we may take this approximation to (26*b*),

$$C_0=2V_0(Sn/R)^{\frac{1}{2}}\epsilon^{-Pl}\times G_0^{-\frac{1}{2}}\times G_1^{-\frac{1}{2}}, \quad . \quad . \quad . \quad (27b)$$

where the first of the three factors is the line-factor, the second that due to the apparatus at the $z=0$ end, and the third to that at the $z=l$ end of the line; thus, by (20 *b*) and (25*b*), with $L'=0$ and $R'=R$ in the former,

$$\left.\begin{aligned} G_0&=1+\frac{1}{R^2}\{2PR(R_0'-L_0'n)+2P^2(R_0'^2+L_0'^2n^2)\},\\ G_1&=1+\frac{1}{R^2}\{2PR(R_1'-L_1'n)+2P^2(R_1'^2+L_1'^2n^2)\}.\end{aligned}\right\}. \quad (28b)$$

This reduction to (27 *b*) is of course not possible when the line is very far from being worked up to its possible limit; in fact, all three terms in the { } of (26 *b*), or, more generally, of (19 *b*), require to be used in general. For this reason a full examination of the effect of terminal apparatus is very laborious. Most interesting results may be got out of (19 *b*), especially as regards the relative importance of the line and terminal apparatus at different speeds, complete reversals taking place as the speed is varied whilst the line and apparatus are kept the same. The general effect is that, as the speed is raised, the influence of the apparatus increases much faster than that of the line. For instance, to work a land-line of, say, 400 miles up to its limit, we must reduce the inertia of the instruments greatly to make it even possible. In fact electromagnets seem unsuitable for the purpose, unless quite small, and chemical recording has probably a great future before it. But it would be too lengthy a digression to go into the necessarily troublesome details.

The following relates to some properties of the terminal function G, which have application when (27 *b*) is valid. Consider the G_1 of (28 *b*). Let it be simply a coil that is in question. Then R_1 is its resistance and L_1 its inductance, dropping the accent. Keep the resistance constant, whilst varying the inductance so as to make G_1 a minimum, and therefore the current amplitude a maximum. The required value of L_1 is

$$L_1=R/2Pn, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29b)$$

depending only upon the line-constants and the speed, independently of the resistance of the coil. Taking $Pl=10$, this makes $L_1=Rl/20n$, where Rl is the resistance of the line. The relation (29 *b*) makes

$$G_1=\tfrac{1}{2}+\frac{2PR_1}{R}+\frac{2P^2R_1^2}{R^2}. \quad . \quad . \quad . \qquad (30b)$$

If the coil had no inductance, but the same resistance, G_1 would have the same expression, but with 1 instead of $\frac{1}{2}$ in (30 *b*). The effect of the inductance has therefore increased the amplitude of the current, and it is conceivable that G_1 could be made less than unity, though not practicable.

Now the G_1/R_1 of (30 *b*) is a minimum, with R_1 variable, when $R=2PR_1$, and this will make $G_1=2$, or the terminal factor to be $G_1^{-\frac{1}{2}}=\cdot 7$. Now if we vary the number of turns of wire in the coil, keeping it of the same size and shape, the magnetic force will vary as $(R_1/G)^{\frac{1}{2}}$, so it at first sight appears that $R_1=R/2P$ and $L_1=R/2Pn$ make the magnetic force a maximum for a fixed size and shape of coil. There is, however, a fallacy here, because varying the size of the wire as stated varies L_1 nearly in the same ratio as R_1, whilst (30 *b*) assumes L_1 to be a constant, given by (29 *b*). It is perhaps conceivable to keep L_1 constant during the variation of R_1, by means of iron, and so get $(R_1/G)^{\frac{1}{2}}$ to be a maximum; but then, on account of the iron, this quantity will not represent the magnetic force.

If, on the other hand, we vary R_1 in the original G_1 of (28 *b*), keeping L_1/R_1 constant (size and shape of coil fixed, size of wire variable), G_1/R_1 is made a minimum by

$$R_1^2+L_1^2n^2=R^2/2P^2, \quad . \quad . \quad . \quad . \quad . \qquad (31b)$$

giving a definite resistance to the coil of stated size and shape to make the magnetic force a maximum. Now G_1 becomes

$$G_1=2+\frac{2P}{R}(R_1-L_1n), \quad . \quad . \quad . \quad . \qquad (32b)$$

where L_1/R_1 has been constant. If this constant have the value n^{-1}, we have $G_1=2$ again, and R_1, L_1 have the same values as before. There is thus some magic about $G_1=2$.

Again, if the terminal arrangement consist of a coil R_1, L_1 and a condenser of capacity S_1 and conductance K_1 joined in sequence, we shall have

$$V/C=(R_1+L_1p)+(K_1+S_1p)^{-1},$$

$$=\left(R_1+\frac{K_1}{K_1^2+S_1^2n^2}\right)+\left(L_1-\frac{S_1}{K_1^2+S_1^2n^2}\right)p, \quad . \qquad (33b)$$

$$=R_1'+L_1'p, \quad \text{say,}$$

if R_1', L_1' are the effective resistance and inductance, to be used in G_1, making

$$G_1=1+\frac{2P}{R}\left\{R_1-L_1n+\frac{K_1+S_1n}{K_1^2+S_1^2n^2}\right\}$$
$$+\frac{2P^2}{R^2}\left\{R_1^2+L_1^2n^2+\frac{1}{K_1^2+S_1^2n^2}+2\frac{R_1K_1-S_1L_1n^2}{K_1^2+S_1^2n^2}\right\}. \qquad (34b)$$

Variation of L_1 alone makes G_1 a minimum when

$$L_1n=\frac{S_1n}{K_1^2+S_1^2n^2}+\frac{R}{2P}; \quad . \quad . \quad . \quad . \qquad (35b)$$

and if we take $K_1=0$ (condenser non-leaky, and not shunted), we have the value of G_1 given by (30 *b*) again, independent of the condenser. Similarly we can come round to the same $G_1=2$ again. These relations are singular enough, but it is difficult to give them more than a very limited practical application to the question of making the magnetic force of the coil a maximum, although the (30 *b*) relation is not subject to any indefiniteness.

[In Part III. Equation (103), ϕ represents or reduces to a negative resistance. In Part IV., for greater convenience, ϕ is always a positive resistance.

Errata, p. 350. Equation (135), put the $-$ sign before the Σ. Equation (137), *for* E *read* M.]

III. *The peculiar Sunrise-Shadows of Adam's Peak in Ceylon.* *By the* Hon. Ralph Abercromby, *F.R. Met. Soc.**

THERE are certain peculiarities about the shadows of Adam's Peak which have long attracted the attention of travellers: a good deal has been written about them, and several theories have been proposed to explain the observed phenomena. In the course of a meteorological tour round the world, the author stopped in Ceylon for the express purpose of visiting the Peak, and was fortunate enough to see the shadow under circumstances which could leave no doubt as to the true explanation, and which also entirely disproved certain theories which have been propounded on the subject.

The following account is taken from a paper by the Rev. R. Abbay, many years resident in the island, entitled "Remarkable Atmospheric Phenomena in Ceylon," which was

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