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GRAVITY AS THE AFFINE GOLDSTONE PHENOMENON AND BEYOND

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Abstract

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The two-phase structure is imposed on the world continuum, with the graviton emerging as the tensor Goldstone boson during the spontaneous transition from the affinely connected phase to the metric one. The physics principle of metarelativity, extending the respective principle of special relativity, is postulated. The theory of metagravitation as the general nonlinear model $GL(4,R)/SO(1,3)$ in the arbitrary background continuum is built. The concept of Metauniverse as the ensemble of the regions of the metric phase inside the affinely connected phase is introduced, and the possible bearing of the emerging multiple universes to the fine tuning of our Universe is conjectured.

Аннотация

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На мировой континуум наложена двухфазная структура, так что гравитон возникает как тензорный голдстоуновский бозон при спонтанном переходе от фазы с аффинной связностью к метрической фазе. Постулируется физический принцип метаотносительности, расширяющий соответствующий принцип специальной относительности. Построена теория метагравитации как общая нелинейная модель $GL(4,R)/SO(1,3)$ в произвольном фоновом континууме. Вводится концепция Метавселенной как ансамбля областей с метрической фазой внутри фазы с аффинной связностью, и указывается на возможное отношение возникающей множественности вселенных к проблеме тонкой настройки нашей Вселенной.

Introduction

The General Relativity (GR) is the well-stated theory attributing the gravity to the Riemannian geometry of the space-time. Nevertheless, the ultimate nature of gravity awaits, conceivably, its future explanation. In this respect, it is of great interest the approach to gravity as the Goldstone phenomenon corresponding to the broken global affine symmetry [1,2]. Originally, it was realized as the nonlinear model $GL(4,R)/SO(1,3)$ in the Minkowski background space-time, as distinct from the geometrical framework of the GR.

In the present paper, we adhere to the viewpoint that the above construction is more than just the mathematical one, but has a deeper physics foundation underlying it. In this respect, the new insights motivating and extending the Goldstone approach to gravity are put forward. Of principle, we go beyond the framework of the Riemannian geometry. Namely, we start with the world continuum considered as the affinely connected manifold without metric and end up in the space-time with the effective Riemannian geometry. Our main results are threefold.

(i) The physics principle of the extended relativity, or *the metarelativity*, is introduced as a substitution for the special relativity. It states the physics invariance, at an underlying level, relative to the transformations within the extended set of the local coordinates, including the inertial ones. The principle justifies the pattern of the affine symmetry breaking $GL(4,R) \to SO(1,3)$ required for the Goldstone approach to gravity.

(ii) The extended theory of gravity, or *the metagravitation*, with the GR as the lowest approximation, is built as the proper nonlinear model in an arbitrary background continuum with the affine connection. The natural hierarchy of the possible GR extensions, according to their mode of the affine symmetry realization, is put forward.

(iii) The extended Universe, or *the Metauniverse*, as the ensemble of the Riemannian metric universes inside the affinely connected world continuum is considered. It is conjectured that the multiple universes may clarify the fine tuning problem of our Universe.

The contents of the paper are as follows. In sec. 1, the principle of metarelativity is introduced. The spontaneous breaking of the respective global symmetry, the affine one, with the residual Poincare symmetry and the emerging tensor Goldstone boson is then considered. In sec. 2, the nonlinear realization of the broken affine symmetry is studied. In sec. 3, the respective nonlinear model in the tangent space is developed. Its prolongation, as the metagravitation, to the arbitrary world coordinates is presented in sec. 4. Finally, the concept of the Metauniverse is discussed in sec. 5, with some remarks in conclusion.

1. Metarelativity

1.1. Affine symmetry

Conventionally, the GR starts by postulating that the world continuum, i.e., the set of the world events (points), is the Riemannian manifold. In other words, a metric is imposed on the world *ab initio*. The metric specifies all the fine properties of the continuum converting the latter into the space-time. Nevertheless, not all of the properties of the space-time depend crucially on the metric [1]. To appreciate the deeper meaning of the gravity and the very space-time, one needs possibly go beyond the Riemannian geometry.

To this end, consider the space-time not as *a priori* existing but as emerging in the processes of the world structure formation. Namely, suppose that at an underlying level the continuum is endowed only with the topological structure (without metric, yet). More particularly, it is the affinely connected manifold. The affine connection supports such the detailed continuity properties, as the parallel transport of the tensor fields, their covariant derivatives, etc. In particular, the connection defines the curvature tensor as a result of the parallel transport of a vector around the infinitesimal closed contour. But, there is yet no geometrical structures which would be inherent in the metric, such as the interval, distances, angles, etc.

Let x^μ, $\mu = 0, \ldots, 3$ be the world coordinates, generally, in the patches [2]. There being, in absence of the metric, no partition of the continuum onto the space and time, the index 0 has yet no particular meaning and is just the notational one. Call all the structures related to the underlying level of the world continuum as the background ones. Let $\bar\psi^\lambda{}_{\mu\nu}(x)$ be the background affine connection and let $\bar\xi^\alpha$ be the background related coordinates where the connection have a particular, to be defined, form $\bar\psi^\gamma{}_{\alpha\beta}(\bar\xi)$. The connections are related as usually:

$$\bar\psi^\gamma{}_{\alpha\beta}(\bar\xi) = \frac{\partial x^\mu}{\partial\bar\xi^\alpha}\frac{\partial x^\nu}{\partial\bar\xi^\beta}\left(\frac{\partial\bar\xi^\gamma}{\partial x^\lambda}\,\bar\psi^\lambda{}_{\mu\nu}(x) - \frac{\partial^2\bar\xi^\gamma}{\partial x^\mu\partial x^\nu}\right). \tag{1}$$

It follows thereof, that the symmetric and antisymmetric parts transform independently, the first one transforming inhomogeneously, whereas the second one homogeneously. In particular, being zero in a point in some coordinates, the antisymmetric part (the torsion) $\bar\tau^\lambda{}_{\mu\nu} \equiv 1/2\,(\bar\psi^\lambda{}_{\mu\nu} - \bar\psi^\lambda{}_{\nu\mu})$ remains to be zero independent of the coordinates. For reason stated later on in this section, put the background torsion to be absent identically, $\bar\tau^\lambda{}_{\mu\nu} = 0$. As for the symmetric part, one is free to choose the special coordinates to make the physics description as simple as possible, not affecting the physics content.

So, let P be a fixed but otherwise arbitrary point (the reference point) with the world coordinates X^μ. One can nullify the symmetric part of the connection in the reference point by adjusting the proper coordinates to the point. To this end, let us put

$$\bar\xi^\alpha_P = \bar\Xi^\alpha + \bar e^\alpha_\lambda(X)\Big((x-X)^\lambda + \frac{1}{2}\bar\psi^\lambda{}_{\mu\nu}(X)\,(x-X)^\mu(x-X)^\nu\Big) + \mathcal{O}((x-X)^3), \tag{2}$$

with $\alpha = 0, \ldots, 3$ and $\bar\Xi^\alpha = \bar\xi^\alpha_P(X)$. In the above, $\bar e^\alpha_\lambda(X) \equiv \partial\bar\xi^\alpha_P/\partial x^\lambda|_{x=X}$ is the tetrad, with $\bar e^\lambda_\alpha(X)$ being the inverse one. The tetrad remains still liable to further determination. It is seen

[1] Cf. the reflections on the space-time structure due to E. Schrödinger [3].

[2] At this stage, the coordinates are merely of the mathematical nature. The procedure of their physical prescription is to be clarified after the metric emerges.

that the affine connection vanishes in the reference point: $\bar\psi^\gamma{}_{\alpha\beta}(\bar\Xi)=0$ [3, 4]. In the vicinity of P, it looks like:

$$\bar\psi^\gamma{}_{\alpha\beta}(\bar\xi_P)=\frac{1}{2}\bar\rho^\gamma{}_{\alpha\delta\beta}(\bar\Xi)\,(\bar\xi_P-\bar\Xi)^\delta+\mathcal{O}((\bar\xi_P-\bar\Xi)^2), \tag{3}$$

with $\bar\rho^\gamma{}_{\alpha\delta\beta}(\bar\Xi)$ being the background curvature tensor in the reference point.

Let us consider the whole set of the coordinates $\{\bar\xi^\alpha_P\}$ with the property $\bar\psi^\alpha{}_{\beta\gamma}(\bar\Xi)=0$. The allowed group of transformations of the respective coordinates is the inhomogeneous general linear group $IGL(4,R)=T_4\odot GL(4,R)$ (the affine one):

$$(A,a)\;:\;\bar\xi^\alpha_P\to\bar\xi'^\alpha_P=A^\alpha{}_\beta\bar\xi^\beta_P+a^\alpha, \tag{4}$$

with A being an arbitrary nondegenerate matrix [5]. Under these, and only under these transformations, the affine connection is still equal to zero. The group is the global one in the sense that it transforms the local, i.e., P-related coordinates in the global manner, i.e., for all the continuum at once (at least, in the patch containing P). The respective coordinates will be called the local affine ones [6]. In these coordinates, the continuum is approximated by the affinely flat one in a neighbourhood of the reference point. Particularly, the covariant derivative in the affine coordinates in the point P coincides with the ordinary one.

The affine group $IGL(4,R)$ is 20-parametric and extends the 10-parameter Poincare group by the dilatation, varying the scales [4], and the nine special affine transformations, preserving the volumes. They belong, respectively, to the multiplicative group R of the positive real numbers:

$$R\;:\;\bar\xi^\alpha_P\to\bar\xi'^\alpha_P=e^{-\lambda}\bar\xi^\alpha_P, \tag{5}$$

with λ any real, and to the special linear group $SL(4,R)\ni A_0$, with $\det A_0=1$,

Conventionally, the affine group (except for the Poincare subgroup and, sometimes, the dilatation) was considered as nothing but a part of the general covariance group, resulting in no special physics content. We propose to raise the status of the affine group and to consider the latter as the physics invariance symmetry. To this end, let us introduce the principle of the extended relativity, or *the metarelativity*, stating that the physics laws, at the underlying level, are invariant relative to the choice of the affine coordinates. This extends the physics symmetry from the Poincare symmetry to the affine one. Stress that this extension concerns the physics content. It determines the structure of the theory and is in no way liable to elimination by means of the general covariance.

1.2. Spontaneous symmetry breaking

According to the special relativity, the local physics is invariant just under the Poincare symmetry. There is known no exact affine symmetry. Thus, the latter should be broken in transition from the underlying level to the effective one. We postulate that this is achieved due

[3] It is clearly impossible to fulfill this conditions identically in the whole continuum untill the latter is the affinely flat one.

[4] Here and in what follows, the bar sign means the background affiliation. The Greek indices $\alpha,\beta,\ldots$ are those of the special coordinates nullifying locally the background connection, while the indices $\lambda,\mu,\ldots$ are the arbitrary world ones.

[5] Note that the nonzero background torsion, being changing under A, would violate explicitly the affine covariance of the theory. Just to abandon this, the torsion is put to be zero.

[6] Being tacitly understood for the affine coordinates, the term "local" will be omited for short.

to the spontaneous emergence of the background metric in the world continuum. The metric is assumed to be symmetric with the Minkowskian signature $(+ - --)$ and correlated with the affine connection so that it looks in the affine coordinates as

$$\bar{\varphi}_{\alpha\beta}(\bar{\xi}_P) = \bar{\eta}_{\alpha\beta} - \frac{1}{2}\bar{\rho}_{\gamma\alpha\delta\beta}(\bar{\Xi})\,(\bar{\xi}_P - \bar{\Xi})^\gamma(\bar{\xi}_P - \bar{\Xi})^\delta + \mathcal{O}((\bar{\xi}_P - \bar{\Xi})^3). \tag{6}$$

Here one puts $\bar{\eta}_{\alpha\beta} \equiv \bar{\varphi}_{\alpha\beta}(\bar{\Xi})$ and $\bar{\rho}_{\gamma\alpha\delta\beta}(\bar{\Xi}) = \bar{\eta}_{\gamma\gamma'}\,\bar{\rho}^{\gamma'}{}_{\alpha\delta\beta}(\bar{\Xi})$. The metric (6) is such that the Christoffel connection $\bar{\phi}^\gamma{}_{\alpha\beta}(\varphi)$, determined by the metric, matches with the affine connection $\bar{\psi}^\gamma{}_{\alpha\beta}$ in the sense, that the connections coincide locally, up to the first derivative: $\bar{\phi}^\gamma{}_{\alpha\beta} = \bar{\psi}^\gamma{}_{\alpha\beta} + \mathcal{O}((\bar{\xi}_P - \bar{\Xi})^2)$. This is quite reminiscent of the well-known fact that the metric in the Riemannian manifold may be approximated locally, up to the first derivative, by the Euclidean metric. In the wake of the emerging background metric, there appears the (yet primordial) partition of the world continuum onto the space and time.

Under the linearly realized affine symmetry, the background metric ceases, in general, to be invariant. But it still possesses an invariance subgroup. To find it note that, without any loss of generality, one can choose among the affine coordinates the particular ones with $\bar{\eta}_{\alpha\beta}$ being in the Minkowskian form $\eta = \mathrm{diag}\,(1, -1, -1, -1)$. The respective coordinates will be called the background inertial ones [7]. Under the general linear transformations, one has

$$(A, a)\ :\ \eta \to \eta' = A^{-1T}\eta A^{-1} \neq \eta, \tag{7}$$

whereas the Lorentz transformations $A = \Lambda$ still leave η invariant:

$$(\Lambda, a)\ :\ \eta \to \eta' = \Lambda^{-1T}\eta\Lambda^{-1} = \eta. \tag{8}$$

It follows that the group of invariance is isomorphous to the Lorentz group $SO(1,3) \in GL(4,R)$ for any fixed $\bar{\eta}_{\alpha\beta}$. Physically, the spontaneous symmetry breaking corresponds to fixing, modulo the Lorentz transformations, the class of the distinguished coordinates among the affine ones. These coordinates correspond to the particular choice for $\bar{\eta}_{\alpha\beta}$. Of course, the fact that the distinguished coordinates are precisely those with the Minkowskian $\eta_{\alpha\beta}$ is no more than the matter of convention corresponding to the proper redefinition of the basis of the affine group.

Thus under the appearance of the metric, the affine symmetry is broken spontaneously up to the residual Poincare group $ISO(1,3)$:

$$IGL(4,R) \xrightarrow{M_A} ISO(1,3). \tag{9}$$

For the symmetry breaking scale M_A, one expects *a priori* $M_A \sim M_{Pl}$, with M_{Pl} being the Planck mass. More particularly the relation between the scales will be discussed in sec. 4 [8].

1.3. Poincare symmetry

Generically, the group $GL(4,R)$ possesses the 16 affine generators $\sigma^\alpha{}_\beta$. By means of $\eta_{\alpha\beta}$, one can redefine the generators in the background inertial coordinates as $\sigma^{\alpha\beta} \equiv \sigma^\alpha{}_\gamma\eta^{\gamma\beta}$ and

[7] This is to distinguish them from the effective inertial coordinates, to be defined in sec. 4.

[8] Note, in particular, that at the level of $GL(4,R)$, there are only the infinite dimensional spinor representations. Thus the physics at the underlying level shoud be quite unusual. Only at the level of $SO(1,3)$, there appear the finite dimensional spinors to be associated with the ordinary matter.

substitute the later ones by the symmetric and antisymmetric combinations $\sigma_{\pm}^{\alpha\beta} = \sigma^{\alpha\beta} \pm \sigma^{\beta\alpha}$. The proper commutation relations read as follows:

$$\begin{aligned} \frac{1}{i}[\sigma_{\pm}^{\alpha\beta}, \sigma_{\pm}^{\gamma\delta}] &= \eta^{\alpha\gamma}\sigma_{-}^{\beta\delta} \pm \eta^{\alpha\delta}\sigma_{-}^{\beta\gamma} \pm (\alpha \leftrightarrow \beta), \\ \frac{1}{i}[\sigma_{-}^{\alpha\beta}, \sigma_{+}^{\gamma\delta}] &= \eta^{\alpha\gamma}\sigma_{+}^{\beta\delta} + \eta^{\alpha\delta}\sigma_{+}^{\beta\gamma} - (\alpha \leftrightarrow \beta). \end{aligned} \tag{10}$$

The generators $\sigma_{-}^{\alpha\beta}$ correspond to the residual Lorentz symmetry, whereas $\sigma_{+}^{\alpha\beta}$ to the broken affine symmetries. In particular, one has in the adjoint representation $(\sigma^{\alpha}{}_{\beta})^{\gamma}{}_{\delta} = 1/i\, \delta^{\alpha}_{\delta}\delta^{\gamma}_{\beta}$, so that the generators $\sigma_{\pm}$ in this representation are as follows:

$$(\sigma_{\pm}^{\alpha\beta})^{\gamma}{}_{\delta} = \frac{1}{i}(\delta^{\alpha}_{\delta}\eta^{\beta\gamma} \pm \delta^{\beta}_{\delta}\eta^{\alpha\gamma}). \tag{11}$$

The ten broken generators contain, in turn, the dilatation one $\sim i\eta_{\alpha\beta}\sigma_{+}^{\alpha\beta}$. The latter commutes with all the generators and is thus proportional to unity in any irreducible representation.

Due to the spontaneous breaking, the affine symmetry should be realized in the nonlinear manner [5], with the nonlinearity scale M_A, the Poincare symmetry being still realized linearly. The unitary linear representations of the latter correspond to the matter, as usually. The broken part $IGL(4,R)/ISO(1,3)$ should be realized in the Nambu-Goldstone mode. Accompanying the spontaneous emergence of the metric, there should appear the 10-component Goldstone boson which corresponds to the ten generators of the broken affine transformations. The effective field theory of the boson is given by the relevant nonlinear model to be studied in the next two sections. First, we study the three kinds of the substance, i.e., the affine Goldstone boson, matter and radiation, which are characterized by the three distinct types of the nonlinear realization. With these building blocks, we then construct the nonlinear model itself.

2. Nonlinear realization

2.1. Affine Goldstone boson

Let $\bar{\xi}_P^{\alpha}$ be the background inertial coordinates adjusted to the space-time point P. Attach to this point the auxiliary linear space T_P, the tangent space in the point. T_P is isomorphous to the Minkowski space-time. By definition, the tangent space is the structure space of the theory, whereupon the realizations/representations of the physics space-time symmetries, the affine and the Poincare one, are defined. Introduce in T_P the coordinates ξ_P^{α}, the counterpart of the background inertial coordinates $\bar{\xi}_P^{\alpha}$ in the space-time. By construction, the connection in the tangent space is zero identically. For the connection in the space-time in the the point P to be zero, too, the coordinates are to be related as $\xi_P^{\alpha} = \bar{\xi}_P^{\alpha} + \mathcal{O}((\bar{\xi}_P - \bar{\Xi})^3)$ for $\bar{\xi}_P^{\alpha}$ in the patch containing P. In accord with the affine symmetry breaking, we consider the coordinates ξ_P^{α} as the preferred ones, wherein all the constructions in T_P are built [9].

According to ref. [5], the nonlinear representation (the realization) of the symmetry G spontaneously broken to the symmetry $H \subset G$ can be built on the quotient space $K = G/H$, the residual subgroup H serving as the classification group. We are interested in the pattern $GL(4,R)/SO(1,3)$, with the quotient space consisting of all the broken affine transformations.

[9] The point P being fixed, the coordinates in T_P are designated in what follows simply as ξ^{α}, until stated otherwise.

Let $k \in K$, be the coset function on the tangent space, i.e., $k(\xi)$ is the group element defined up to the Lorentz transformation, $k \sim k\Lambda^{-1}$, with $\Lambda \in SO(1,3)$.

Under the arbitrary affine transformation $\xi \to \xi' = A\xi + a$, the coset is to be transformed as

$$(A, a) \ : \ k(\xi) \to \ k'(\xi') = Ak(\xi)\Lambda^{-1}, \tag{12}$$

where Λ is the appropriate transformation of the residual group, here the Lorentz one. One has similarly: $k^{-1} \to \Lambda k^{-1} A^{-1}$. In the same time, by the very construction, the Minkowskian η stays invariant under the nonlinear realization:

$$(A, a) \ : \ \eta \to \eta' = \Lambda^{-1T}\eta\Lambda^{-1} = \eta, \tag{13}$$

in distinction with the linear realization eq. (7). Accounting for eq. (13), one gets in the other terms:

$$(A, a) \ : \ k(\xi)\eta \to \ k'(\xi')\eta = Ak(\xi)\eta\Lambda^{T}. \tag{14}$$

To restrict k by the quotient space K, one should impose some auxiliary condition. E.g., impose the requirement that k is pseudosymmetric in the sense that $k\eta = (k\eta)^T$ (and similarly for k^{-1}). This ensures that k has ten independent components, indeed, in accord with the ten broken generators. Under the affine transformations, this results in the restriction $Ak\eta\Lambda^T = \Lambda\eta k^T A^T$. This entails implicitly the dependence of the Lorentz transformation on the Goldstone boson: $\Lambda = \Lambda(A, k)$. Hereof, the term "nonlinear" follows. This construction implements the realization of the whole broken group $IGL(4, R)$. The residual Poincare subgroup $ISO(1, 3)$ is still realized linearly, i.e., $\Lambda(A, k)|_{A=\Lambda} \equiv \Lambda$. And what is more, the dilatation R being Abelian, one gets $\Lambda(R, k) = 1$, so that $\Lambda(A, k) = \Lambda(\pm A_0, k)$, with $A \equiv \pm RA_0$ and $A_0 \in SL(4, R)$.

By doing as above, one looses the explicit local Lorentz symmetry. For this reason, we will not impose any gauge fixing condition. This could be considered as the linearization of the nonlinear model, with the extra Goldstone degrees of freedom being eliminated by the gauge Lorentz transformations $\Lambda(\xi)$, independent of A. Now, for quantities in the tangent space, one should distinguish two types of indices: the affine ones, acted on by the global affine transformations A, and the Lorentz ones, acted on by the local Lorentz transformations $\Lambda(\xi)$. To make this difference explicit, designate the affine indices in the tangent space as α, β, etc, while the Lorentz ones as a, b, etc. Choose k in the adjoint representation, so that it looks now like $k^{\alpha}{}_{b}$ (respectively, $k^{-1b}{}_{\alpha}$). In what follows, it is understood that the Lorentz indices are manipulated by means of the Minkowskian η_{ab} (respectively, η^{ab}). So, in the component notation, $k\eta$ looks like $k^{\alpha b}$ (similarly, ηk^{-1} is $k^{-1}{}_{a\beta}$).

2.2. Matter

The affine symmetry contains the Abelian, though broken, subgroup of dilatation. For this reason, the physical matter fields ϕ_s (the subscript s designating the species) may additionally be classified by their scale dimensions. One puts

$$(A, a) \ : \ \phi_s(\xi) \to \phi'_s(\xi') = e^{l_s\lambda}\rho_s(\Lambda)\phi_s(\xi), \tag{15}$$

with l_s being the scale dimension of the species ϕ_s and $\rho_s(\Lambda)$ taken in the proper Lorentz representation. Recall that the canonical dimension of the integer-spin particles is $l_s = 1$ and that of the half-integer spin particles is $l_s = 3/2$. With account for the transformation $\det k \to e^{-4\lambda}\det k$ under dilatation, one can rescale the matter fields to the effective ones

$$\hat{\phi}_s = (\det k)^{l_s/4}\phi_s. \tag{16}$$

The new fields are scale invariant, i.e. correspond to $\hat{l}_s = 0$, and transform simply as the Lorentz representations. They are to be used in constructing the nonlinear model [10].

2.3. Radiation

From the point of view of the nonlinear realization, the gauge bosons constitutes one more separate kind of the substance, the radiation. By definition, the gauge boson fields V_α transform under A linearly as the derivative $\partial_\alpha \equiv \partial/\partial\xi^\alpha$:

$$(A, a) \ : \ V(\xi) \to V'(\xi') = A^{-1T}V(\xi), \tag{17}$$

corresponding thus to the scale dimension $l_V = 1$. For this reason, redefine the gauge fields as $\hat{V}_a = k^\alpha{}_a V_\alpha$. The new fields transform as the Lorentz vectors

$$\hat{V}(\xi) \to \Lambda^{-1T}\hat{V}(\xi) \tag{18}$$

and correspond to $\hat{l}_V = 0$. These redefined gauge fields are to be used in the model building. Altogether, this exhausts the description of all the three kinds of the substance: the affine Goldstone boson, matter and radiation.

3. Nonlinear model

3.1. Nonlinear connection

To explicitly account for the residual symmetry, here the Lorentz one, it is convenient to start with the objects transforming only under the latter symmetry. Clearly, any nontrivial combinations of k and k^{-1} alone transform explicitly under A. Thus the derivative terms are inevitable. To describe the latter ones, let us introduce the Maurer-Cartan one-form chosen as follows:

$$\Omega = \eta k^{-1}dk, \tag{19}$$

with dk being the ordinary differential of k. Under the affine coordinate transformation $\xi \to \xi' = A\xi + a$ with the ensuing transformation $k \to k' = Ak\Lambda^{-1}$, the one-form transforms as the Lorentz quantity:

$$\Omega(\xi) \to \Omega'(\xi') = \Lambda^{-1T}\Omega(\xi)\Lambda^{-1} + \Lambda^{-1T}\eta d\Lambda^{-1}, \tag{20}$$

with $d\Lambda$ being the differential of $\Lambda(\xi)$. Here use is made of the relation $\eta\Lambda\eta = \Lambda^{-1T}$ for the Lorentz transformations.

In the component notation, the so defined one-form looks like Ω_{ab}. Decompose it as

$$\Omega_{ab} \equiv \sum_\pm \Omega^\pm_{ab} = \sum_\pm [\eta k^{-1}dk]^\pm_{ab}, \tag{21}$$

[10] In the above, the scale dimension of k was chosen to be $l_k = -1$. One could also conceive the more general transformation law for k under R which would include the additional factor $e^{\Delta l_k \lambda}$, with Δl_k being the extra scale dimension of k. For simplicity, we restrict ourselves by the case $\Delta l_k = 0$.

where $[\ldots]^{\pm}$ means the symmetric and antisymmetric parts, respectively. One sees that $\Omega^{\pm}_{ab}$ transform independently as

$$\Omega^{\pm}(\xi) \to \Omega'^{\pm}(\xi') = \Lambda^{-1T}\Omega^{\pm}(\xi)\Lambda^{-1} + \delta^{\pm}, \tag{22}$$

where

$$\begin{aligned} \delta^{-} &= \Lambda^{-1T}\eta d\Lambda^{-1}, \\ \delta^{+} &= 0. \end{aligned} \tag{23}$$

Transforming homogeneously, the symmetric part Ω^{+} can naturally be associated with the nonlinear covariant differential of the Goldstone field. At the same time, the antisymmetric part Ω^{-} transforms inhomogeneously and allows one to define the nonlinear covariant differential of the matter fields: $D\hat{\phi}_s = (d + i/2\,\Omega^{-}_{ab}\,\Sigma^{ab}_s)\hat{\phi}_s$, with Σ^{ab}_s being the Lorentz generators in the proper representation ρ_s. The so defined $D\hat{\phi}_s$ transforms homogeneously as the Lorentz representation, like $\hat{\phi}_s$ itself.

The generic nonlinear covariant derivative $D_\alpha \equiv D/d\xi^\alpha$ transforms as the affine vector of the global $GL(4,R)$. The effective covariant derivative, which transforms as the Lorentz vector, can be constructed as follows:

$$\hat{D}_a \equiv k^{\alpha}{}_a D_\alpha = k^{\alpha}{}_a D/d\xi^\alpha. \tag{24}$$

Thus one gets for the covariant derivative of the one-form:

$$\hat{\Omega}^{\pm}_{abc} = k^{\gamma}{}_c\,\Omega^{\pm}_{ab}/d\xi^\gamma = [\eta k^{-1}\hat{\partial}_c\,k]^{\pm}_{ab}, \tag{25}$$

where

$$\hat{\partial}_c \equiv k^{\gamma}{}_c\partial_\gamma = k^{\gamma}{}_c\partial/\partial\xi^\gamma \tag{26}$$

is the effective (Goldstone boson dependent) partial derivative. It follows that $\hat{\Omega}^{-}_{abc}$ could be used as the connection for the nonlinear realization. Note, that this expression precisely corresponds to the case of the nonlinear realization of the spontaneously broken internal symmetry, where this connection is determined uniquely, being thus universal. But in the present case of the space-time symmetry, the coordinates transform under the same group as the fields. This results in the possible ambiguity of the nonlinear connection.

Namely, the transformation properties of the covariant derivative do not change if one adds to the above minimal connection the properly modified terms $\hat{\Omega}^{+}_{abc}$, the latter ones transforming homogeneously. For reason justified later on in this section, we choose for the nonminimal connection the following universal combination:

$$\begin{aligned} \hat{\omega}_{abc} &= \hat{\Omega}^{-}_{abc} + \hat{\Omega}^{+}_{cab} - \hat{\Omega}^{+}_{cba} \\ &= [\eta k^{-1}\hat{\partial}_c\,k]^{-}_{ab} + [\eta k^{-1}\hat{\partial}_b\,k]^{+}_{ca} - [\eta k^{-1}\hat{\partial}_a\,k]^{+}_{cb}. \end{aligned} \tag{27}$$

The nonlinear covariant derivative of the matter fields now becomes

$$\hat{D}_c\hat{\phi}_s = \Big(\hat{\partial}_c + \frac{i}{2}\,\hat{\omega}_{abc}\,\Sigma^{ab}_s\Big)\,\hat{\phi}_s. \tag{28}$$

The so defined $\hat{D}_c\hat{\phi}_s$ transforms under $GL(4,R)$ homogeneously as the Lorentz vector times the Lorentz representation ρ_s.

Containing $\hat{\Omega}^+_{abc}$, the connection $\hat{\omega}_{abc}$ will be used to describe the tensor part of the affine Goldstone boson through the generally covariant Lagrangian (see later on in this section). On the other hand, the contraction $\hat{\Omega}^{+\,b}_{ab}$ will serve to describe the scalar dilaton as the part of the affine Goldstone boson. As for another independent contraction $\hat{\Omega}^{+b}{}_{ba}$, it can be shown to correspond to the scalar-vector part of the boson with the wrong norm, violating thus the microscopic causality. One could conceive also the additional kinetic terms for the tensor part of the affine Goldstone boson, generically, in the form $\hat{\Omega}^+_{abc}\hat{\Omega}^{+abc}$ and/or $\hat{\Omega}^+_{abc}\hat{\Omega}^{+bca}$. The latter ones violate the general covariance and thus could endanger the fine properties of the GR. For safety, all these additional contributions will be omitted [11].

3.2. Gauge interactions

Let $\hat{V}_a$ be the generator valued gauge field for an internal gauge symmetry. Interacting universally, the gauge fields are supposed to be coupled with the universal nonlinear connection eq. (27). Taking into account the Lorentz generators in the adjoint representation eq. (11) (with the opposite common sign and the obvious substitution for the indices), one gets the respective nonlinear derivative as follows

$$\hat{D}_c\hat{V}_d = \left(\delta^f_d\hat{\partial}_c + \hat{\omega}^f{}_{dc}\right)\hat{V}_f. \tag{29}$$

It follows thereof, in particular, that $\hat{D}_c\eta_{ab} = 0$. Define the gauge strength as

$$\hat{F}_{cd} = \left(\hat{D}_c + i\hat{V}_c\right)\hat{V}_d - (c \leftrightarrow d). \tag{30}$$

Under the special choice eq. (27) for the nonlinear connection, the so defined gauge strength takes the form $\hat{F}_{cd} = k^\gamma{}_c k^\delta{}_d F_{\gamma\delta}$ with

$$F_{\gamma\delta} = \left(\partial_\gamma + iV_\gamma\right)V_\delta - (\gamma \leftrightarrow \delta). \tag{31}$$

Thus, $\hat{F}_{\gamma\delta}$ does possess the correct transformation properties with respect to both the affine symmetry and the internal gauge symmetry.

Further, consider the local Lorentz symmetry as the gauge one with the connection $\hat{\omega}_c \equiv 1/2\,\hat{\omega}_{abc}\,\Sigma^{ab}$, where Σ^{ab} are some generic Lorentz generators. Define the corresponding gauge strength for the tensor Goldstone boson as

$$\hat{G}_{cd} = (\hat{\partial}_c + i\,\hat{\omega}_c)\,\hat{\omega}_d - (c \leftrightarrow d) \equiv \frac{1}{2}\hat{R}_{abcd}\Sigma^{ab}. \tag{32}$$

This gives

$$\hat{R}_{abcd} = \hat{\partial}_c\,\hat{\omega}_{abd} - \hat{\omega}^f{}_{ac}\,\hat{\omega}_{fbd} - (c \leftrightarrow d). \tag{33}$$

This quantity transforms homogeneously as the Lorentz tensor (and similarly for its partial contraction $\hat{R}_{bd} \equiv \hat{R}^a{}_{bad}$). The total contraction

$$\hat{R} \equiv \hat{R}^{ab}{}_{ab} = 2\hat{\partial}_a\hat{\omega}^{ab}{}_b - \hat{\omega}^{fa}{}_a\hat{\omega}_f{}^b{}_b + \hat{\omega}^{fab}\hat{\omega}_{fba} \tag{34}$$

is the Lorentz scalar and can be used in building the Lagrangian for the Goldstone boson.

[11] In principle, one could not exclude some small microscopic causality violation not contradicting, of course, the observations. Remarkably, these and other similar violations could be done as small as necessary by the choice of the respective parameters. This is insured by the fact that in the limit when the proper parameters vanish, the symmetry of the theory increases up to the general covariance.

3.3. Lagrangian

(i) **Lorentz invariant form** The constructed objects can serve as the building blocks for the nonlinear model $GL(4,R)/SO(1,3)$ in the tangent space. Postulate the equivalence principle in the sense that the tangent space Lagrangian should not depend explicitly on the tangent space counterpart of the background curvature $\bar\rho_{\gamma\alpha\delta\beta}$ given by eq. (6). Thus, the Lagrangian may be written as the general Lorentz invariant function built of $\hat R$, $\hat F_{ab}$, $\hat D_a\hat\phi_s$ and $\hat\phi_s$. As usually, we restrict ourselves by the terms containing two derivatives at the most [12].

The generic Lorentz (and, thus, affine) invariant Lagrangian in the tangent space is

$$L = L_g(\hat R) + L_r(\hat F_{ab}) + L_m(\hat D_a\hat\phi_s, \hat\phi_s). \tag{35}$$

In the above, the Goldstone boson Lagrangian L_g is chosen as follows

$$L_g = c_g M_A^2 \hat R(\hat\omega_{abc}), \tag{36}$$

with c_g being some dimensionless constant. Generically, the radiation Lagrangian L_r is as follows

$$L_r = -\frac{1}{4}\,\mathrm{tr}(\hat F^{ab}\hat F_{ab}), \tag{37}$$

whereas L_m is the proper matter Lagrangian. As for the radiation and matter, the proper Lagrangians could well be the affine invariant Lagrangian of the Standard Model or of any its extension. In fact, the given nonlinear model provides the shell for any field theory.

(ii) **Affine invariant form** The Lagrangian above gives the basic dynamical description of the affine Goldstone boson, radiation and matter. The Lorentz quantities are necessary to construct the Lagrangian. The latter being built, one can rewrite it in terms of the affine quantities. This allows one to make explicit the geometrical structure of the theory and to relate it with the gravity. This is achieved by the proper regrouping the factors $k^\alpha{}_a$ and $k^{-1a}{}_\alpha$ so that to make, where possible, the affine indices to be explicit. Under the choice (27) for the nonlinear connection, the Lagrangian becomes [13]

$$L = c_g M_A^2 R(\gamma_{\alpha\beta}) + L_r(F_{\alpha\beta}) + \mathcal{L}_m(D_\alpha\phi_s, \phi_s). \tag{38}$$

Here

$$\gamma_{\alpha\beta} = k^{-1a}{}_\alpha \eta_{ab} k^{-1b}{}_\beta \tag{39}$$

transforms as the affine tensor

$$\gamma_{\alpha\beta} \to \gamma'_{\alpha\beta} = A^{-1\gamma}{}_\alpha \gamma_{\gamma\delta} A^{-1\delta}{}_\beta. \tag{40}$$

It proves that $R(\gamma_{\alpha\beta}) = \hat R(\hat\omega_{abc})$ can be expressed as the contraction $R = R^{\alpha\beta}{}_{\alpha\beta}$ of the tensor $R^\gamma{}_{\alpha\delta\beta} \equiv k^{\gamma c} k^{-1a}{}_\alpha k^{-1d}{}_\delta k^{-1b}{}_\beta \hat R_{cadb}$, the latter in turn being related with $\gamma_{\alpha\beta}$ as the Riemann-Christoffel curvature tensor with the metric. In this, all the contractions of the affine indices are understood with $\gamma_{\alpha\beta}$ (respectively, $\gamma^{\alpha\beta}$).

[12]For reason stated in the next section, the additional contributions to the Goldstone boson Lagrangian are disregarded. Similarly, the additional couplings of the boson with the matter are neglected, too.

[13]If the affine symmetry is exact, only the rescaled matter fields eq. (16) enter. Thus for notational simplicity, they will be designated in what follows simply as ϕ_s, instead of $\hat\phi_s$, until stated otherwise.

Similarly, D_α looks like the covariant derivative for the matter fields

$$D_\gamma \phi_s = \Big(\partial_\gamma + \frac{i}{2}\,\omega_{ab\gamma}\Sigma_s^{ab}\Big)\phi_s, \tag{41}$$

with the spin-connection

$$\omega_{ab\gamma} \equiv \hat\omega_{abc}\, k^{-1c}{}_\gamma = k^\beta{}_a \nabla_\gamma k^{-1}{}_{b\beta} - (a \leftrightarrow b). \tag{42}$$

In the above, $\nabla_\gamma k^{-1}{}_{b\beta} \equiv (\delta^\alpha_\beta \partial_\gamma - \Gamma^\alpha{}_{\beta\gamma})k^{-1}{}_{b\alpha}$ is the covariant derivative calculated with the Christoffel connection

$$\begin{aligned}\Gamma^\alpha{}_{\beta\gamma} &= k^{\alpha a} k^{-1b}{}_\beta k^{-1c}{}_\gamma\, \hat\omega_{abc} + k^\alpha{}_a \partial_\gamma k^{-1a}{}_\beta \\ &= \frac{1}{2}\,\gamma^{\alpha\delta}\Big(\partial_\beta \gamma_{\delta\gamma} + \partial_\gamma \gamma_{\delta\beta} - \partial_\delta \gamma_{\beta\gamma}\Big).\end{aligned} \tag{43}$$

In particular, one gets $\nabla_\gamma \gamma_{\alpha\beta} = 0$ as the affine counterpart of the Lorentz relation $\hat D_c \eta_{ab} = 0$. For the radiation Lagrangian one has the usual expression

$$L_r = -\frac{1}{4}\,\mathrm{tr}(F^{\alpha\beta}F_{\alpha\beta}), \tag{44}$$

with $F_{\alpha\beta}$ given by eq. (31). Finally, the matter Lagrangian is obtained straightforwardly from L_m, eq. (35), with account for eq. (39) and the relation $\hat D_a = k^\alpha{}_a D_\alpha$, eq. (24).

Clearly, L_g looks like the GR Lagrangian in the tangent space considered as the effective [14] Riemannian manifold with the metric $\gamma_{\alpha\beta}$, the Christoffel connection $\Gamma^\gamma{}_{\alpha\beta}$, the Riemann-Christoffel curvature $R^\gamma{}_{\alpha\delta\beta}$, the Ricci curvature $R_{\alpha\beta}$, the Ricci scalar R and the tetrad $k^\alpha{}_a$ (the inverse one $k^{-1b}{}_\beta$). This is in no way accidental. Namely, as it is shown in ref. [2], under the choice of the nonlinear connection eq. (27), the Lagrangian becomes conformally invariant, too. In this, the dilaton of the conformal symmetry coincides with the affine dilaton, while the vector Goldstone boson of the conformal symmetry, proving to be the derivative of the dilaton, is auxiliary. Now, according to the theorem due to Ogievetsky [6], it follows that the theory which is invariant both under the conformal symmetry and the global affine one is generally covariant, as well. After the proper choice of the metric, this imposes the effective Riemannian structure onto the tangent space [15]. In the world coordinates, this will result in the GR. Precisely this property justifies the above choice for the universal nonlinear connection. The affine Goldstone boson proves to be nothing but the graviton in disguise. The additional terms in the Lagrangian would violate the general covariance (see the next section).

4. Metagravitation

4.1. General covariance

The preceding construction referred to the tangent space T_P in the given reference point P. In the space-time, this construction makes generally sense only in the infinitesimal vicinity of P.

[14] In what follows, the term "effective" will be tacitly understood and omitted as a rule, while that "background" will, in contrast, be retained.

[15] Remind that all the constructions in the tangent space T_P are done really in the coordinates which are the counterpart of the background inertial ones. The transitions to the general coordinates in T_P are just the virtual ones, and allow one to tame the otherwise arbitrary theory.

The same holds for any other reference point. Due to the equivalence principle, by restricting to the tangent space one introduces the separation between the infinitesimal local domain and the rest of the world. The bundling of the different domains into a single whole, the world, is to be achieved by independently attaching the tangent spaces to all the space-time points. Thus, let the point P varies on the whole space-time. By construction, equate the affine quantities in the point Ξ of the tangent space T_P with the respective quantities in the point $\bar{\Xi}$ of the space-time. The Lorentz indices a, b, etc, refer now to any of the tangent spaces.

Accept the so defined Lagrangian as that for the space-time, being valid in the background inertial coordinates in the infinitesimal neighbourhood of the point $\bar{\Xi}$. After the subsequent multiplication of the Lagrangian by the affine invariant volume element $(-\gamma)^{1/2}\, d^4\bar{\Xi}\,,\, \gamma \equiv \det\gamma_{\alpha\beta}$, one gets the contribution into the action of the infinitesimal neighbourhood of the space-time point P. This is valid in the background inertial coordinates. Now the problem is to convert this contribution into the arbitrary world coordinates and to sum over all the space-time points P.

The relation between the background inertial and world coordinates is achieved by means of the background tetrad $\bar{e}^\alpha_\mu(X)$ eq. (2) (and the inverse one $\bar{e}^\mu_\alpha(X)$) transforming as

$$A \;:\; \bar{e}^\alpha_\mu \to \bar{e}'^\alpha_\mu = A^\alpha{}_\beta \bar{e}^\beta_\mu. \tag{45}$$

Introduce the effective metric as follows:

$$g_{\mu\nu}(X) = \bar{e}^\alpha_\mu(X)\gamma_{\alpha\beta}(\bar{\Xi})\bar{e}^\beta_\nu(X). \tag{46}$$

With account for eq. (40), this metric is invariant under the affine transformations

$$A \;:\; g_{\mu\nu} \to g_{\mu\nu}, \tag{47}$$

in line with the world coordinates:

$$A \;:\; X^\mu \to X^\mu. \tag{48}$$

On the contrary, construct the background metric

$$\bar{g}_{\mu\nu}(X) = \bar{e}^A_\mu(X)\eta_{AB}\bar{e}^B_\nu(X), \tag{49}$$

where the generic index A means α or a, as appropriate (and similarly for B, etc). With account for eq. (13), the so defined metric is clearly noninvariant under the arbitrary A:

$$A \;:\; \bar{g}_{\mu\nu} \to \bar{g}'_{\mu\nu} = \bar{e}^T_\mu A^T \eta A \bar{e}_\nu \neq \bar{g}_{\mu\nu}, \tag{50}$$

though being invariant under $A = \Lambda$. Likewise, the background interval $d\bar{s}^2 = \bar{g}_{\mu\nu} dX^\mu dX^\nu$ is not invariant relative to the arbitrary affine transformations, whereas the effective one $ds^2 = g_{\mu\nu} dX^\mu dX^\nu$ is invariant. Clearly, it is ds, not $d\bar{s}$, to which the geometrical meaning is to be attributed in the affine invariant theory.

The metric $\bar{g}_{\mu\nu}$ is the world counterpart of the primordial background metric $\bar{\varphi}_{\mu\nu}$ given in the affine coordinates by eq. (6). The metric $\bar{g}_{\mu\nu}$ approximates $\bar{\varphi}_{\mu\nu}$ as closely as possible in the lack of the knowledge of the primordial background curvature $\bar{\rho}^\gamma{}_{\alpha\delta\beta}$. The latter, according to the equivalence principle, does not enter the tangent space Lagrangian. Respectively, the Christoffel connection $\bar{\Gamma}^\lambda{}_{\mu\nu}(\bar{g})$ approximates with the same accuracy the primordial Christoffel connection $\bar{\phi}^\lambda{}_{\mu\nu}(\bar{\varphi})$ and thus the primordial affine connection $\bar{\psi}^\lambda{}_{\mu\nu}$, i.e., $\bar{\Gamma}^\lambda{}_{\mu\nu} \simeq \bar{\phi}^\lambda{}_{\mu\nu} \simeq \bar{\psi}^\lambda{}_{\mu\nu}$.

Now, introduce the effective tetrad related with the background one as

$$e^a_\mu(X) = k^{-1a}{}_\beta(\bar\Xi)\,\bar e^\beta_\mu(X). \tag{51}$$

The effective tetrad transforms independently of A as the Lorentz vector:

$$e_\mu(X) \to e'_\mu(X) = \Lambda(X)\,e_\mu(X). \tag{52}$$

Due to the local Lorentz transformations $\Lambda(X)$, one can eliminate six components out of e^a_μ, the latter having thus ten physical components. In this terms, the effective metric is

$$g_{\mu\nu}(X) = e^a_\mu(X)\eta_{ab}e^b_\nu(X). \tag{53}$$

In other words, this tetrad defines the effective inertial coordinates. Physically, eq. (51) describes the disorientation of the effective inertial and background inertial frames depending on the distribution of the affine Goldstone boson (and thus the gravity) [16].

With account for the relation $d\,\bar\Xi^\alpha = \bar e^\alpha_\mu dX^\mu$ between the displacements of the point P in the background inertial and world coordinates, and thus $\partial\,\bar\Xi^\alpha/\partial X^\mu = \bar e^\alpha_\mu$, one has

$$\Gamma^\lambda{}_{\mu\nu} = \bar e^\lambda_\alpha \bar e^\beta_\mu \bar e^\gamma_\nu \Gamma^\alpha{}_{\beta\gamma} + \bar e^\lambda_\alpha \partial_\mu \bar e^\alpha_\nu, \tag{54}$$

where $\partial_\mu = \partial/\partial X^\mu$. This can be rewritten as usually:

$$\Gamma^\lambda{}_{\mu\nu} = \frac{1}{2}\,g^{\lambda\rho}\Big(\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}\Big). \tag{55}$$

By construction, the world indices are manipulated via $g_{\mu\nu}$ and $g^{\mu\nu}$. The spin-connection looks in the world coordinates as follows:

$$\omega_{ab\mu} = \omega_{ab\gamma}\bar e^\gamma_\mu = e^\nu_a \nabla_\mu e_{\nu b} - (a \leftrightarrow b), \tag{56}$$

with the generally covariant derivative ∇_μ defined via the Christoffel connection $\Gamma^\lambda{}_{\mu\nu}$, as usually. Respectively, the covariant derivative of the matter fields looks like

$$D_\mu \phi_s = \Big(\partial_\mu + \frac{i}{2}\,\omega_{ab\mu}\Sigma^{ab}_s\Big)\phi_s. \tag{57}$$

In the similar way, one finds the usual expressions for the Riemann-Christoffel tensor $R^\lambda{}_{\mu\rho\nu}(g)$, the Ricci tensor $R_{\mu\nu} = R^\lambda{}_{\mu\lambda\nu}$, the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$, as well as for the gauge strength

$$F_{\mu\nu} = (\partial_\mu + iV_\mu)V_\nu - (\mu \leftrightarrow \nu). \tag{58}$$

Plugging the above modified objects into the Lagrangians for the affine Goldstone boson, radiation and matter and integrating with the invariant volume element one gets the total action including the Einstein-Hilbert one [17]:

$$S = \int \Big(\frac{1}{2}M^2_{Pl}R(g_{\mu\nu}) + \,L_r(F_{\mu\nu}) + L_m(D_\mu\phi_s, \phi_s)\Big)\,(-g)^{1/2}\,d^4X, \tag{59}$$

[16] This could be considered as the modified Mach's principle.

[17] If desired, one could add to the Lagrangian the constant term, not yet violating the affine symmetry. This would correspond to the cosmological constant.

with $g \equiv \det g_{\mu\nu}$. In the above, the constant c_g in eq. (38) is chosen so that $c_g M_A^2 = 1/(16\pi G_N) \equiv 1/2\, M_{Pl}^2$, with G_N being the Newton's constant and M_{Pl} being the Planck mass. Note, that due to the term $\sqrt{-g}$, the affine Goldstone boson enters the action also with the derivativeless couplings, even in absence of the explicit affine symmetry violation. This differs principally from the case of the internal symmetries. Varying the action with respect to the metric $g^{\mu\nu}$ one arrives at the well-known equation of motion for gravity:

$$R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = M_{Pl}^{-2} T_{\mu\nu}. \tag{60}$$

In the above, $T_{\mu\nu}$ is the conventional energy-momentum tensor of the radiation and matter, produced by L_r and L_m.

Let us clarify the physics peculiarity of eqs. (59) and (60) from the point of view of the Goldstone approach to gravity. Namely, to build the theory in the world coordinates, one should bundle the tangent spaces in the neighbouring space-time points. To this end, one should identify the points Ξ^α and $\Xi^\alpha + d\,\Xi^\alpha$ in the neighbour tangent spaces T_P and T_{P+dP}, respectively, via equating the displacements $d\,\Xi^\alpha = d\,\bar\Xi^\alpha \equiv \bar e^\alpha_\mu dX^\mu$. To allow this to be done unambiguously in the generally covariant fashion, the theory in the tangent space proves to be required to be generally covariant, too. This is the reason for the choice eq. (36) for the Goldstone boson Lagrangian in the tangent space. Doing so and properly choosing the field variables, one can express the theory exclusively in the internal dynamical terms. Otherwise, the matching of the tangent spaces will depend on a number of the background parameter-functions. The next-of-kin to the general covariance is the unimodular covariance, i.e., covariance with respect to the transformations preserving the volume element. It proves to result in the minimal violation of the general covariance in the space-time by means of the background scalar density $\bar g \equiv \det \bar g_{\mu\nu}$ only (see the next item). Further weakening the requirement of the uniqueness of bundling extends the set of the admissible theories in the tangent space, but results, instead, in the dependence of the theory in the world coordinates on the more elaborate properties of the background manifold. In the reasonable assumptions, it suffices to know only the background metric $\bar g_{\mu\nu}$.

4.2. General covariance violation

(i) Affine symmetry preservation In the GR, after the choice of the Lagrangian the theory becomes unique, independent of the choice of the coordinates. Under extension of the tangent space Lagrangian beyond the generally covariant one, the theory in the space-time ceases to be generally covariant and thus unique. It depends not only on the Lagrangian but on the choice of the coordinates, too. Relative to the general coordinate transformations, the admissible theories divide into the inequivalent classes, each of which is characterised by the particular set of the background parameter-functions. *A priori*, no one of the sets is preferable. Which one is suitable, should be determined by observations. Each class consists of the equivalent theories related by the residual covariance group. The latter consists of the coordinate transformations leaving the background parameter-functions invariant. On the contrary, one class can be obtained from another by the coordinate transformations changing these parameters.

To be more specific, consider the minimal possible extension of the tangent space Lagrangian by means of the terms depending on $\hat\sigma_a \equiv -\hat\Omega^{+b}{}_{ba}$. With account for eqs. (25) and (43), one gets for $\sigma_\alpha \equiv k^{-1b}{}_\alpha \hat\sigma_b$:

$$\sigma_\alpha = \Gamma^\beta{}_{\beta\alpha} = \partial_\alpha \sigma, \tag{61}$$

where $\sigma \equiv 1/2\ln(-\gamma)$ and $\gamma \equiv \det\gamma_{\alpha\beta}$. In these terms, one can add to the minimal Lagrangian the additional kinetic piece for the affine Goldstone boson:

$$L'_g = c'_g M_{Pl}^2 \gamma^{\alpha\beta}\partial_\alpha\sigma\partial_\beta\sigma, \tag{62}$$

where c'_g is a dimensionless constant [18]. This Lagrangian violates the conformal symmetry in the tangent space (more particularly, the local dilatation) though not violating the global affine symmetry. As a result, it violates the general covariance, too, but preserves the unimodular covariance, i.e., that leaving γ invariant.

It follows from eq. (46) that $g = -\gamma\bar{g}$, where $\bar{g} = \det\bar{g}_{\mu\nu} = -(\det\bar{e}^\alpha_\mu)^2$. In the world coordinate, the Lagrangian L'_g becomes

$$L'_g = c'_g M_{Pl}^2\, g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma, \tag{63}$$

with

$$\sigma \equiv \frac{1}{2}\ln(g/\bar{g}). \tag{64}$$

Thus, all the background dependence of the nonminimal terms in the Lagrangian is determined only by the scalar density $\bar{g}$. Note that $\partial_\mu\sigma$ transforms homogeneously and thus can not be eliminated by the general coordinate transformations, though each one of the terms $1/2\,\partial_\mu \ln g = \Gamma^\lambda{}_{\lambda\mu}$ and $1/2\,\partial_\mu\ln\bar{g} = \bar{\Gamma}^\lambda{}_{\lambda\mu}$ could be separately nullified.

Clearly, the Lagrangian L'_g, though not being generally covariant, is consistent with the unimodular covariance leaving $\bar{g}$ invariant. Altogether, the total Lagrangian depends on e^a_μ as the ten dynamical variables and the one background scalar density $\bar{g}$ [19]. Due to the unimodular covariance, this minimal GR extension describes the three physical degrees of freedom: the massless graviton and the dilaton σ as the respective parts of the affine Goldstone boson. In this, the dilaton becomes massive at the quantum level due to the dilatation anomaly [7]. In the case $c'_g = 0$, the general covariance is restored, so that the local dilatation can eliminate one more degree of freedom, leaving just two of them with helicities $\lambda = \pm 2$, as it should be for the massless spin-2 particle [20].

(*ii*) **Affine symmetry violation** The additional terms in the Lagrangian would activate the rest of the latent degrees of freedom of the gravity field. Thus, it is conceivable another way of the general covariance violation by adding to the tangent space Lagrangian the potential $V_g(k)$, which contains only the derivativeless couplings of the Goldstone boson. Of necessity, this would explicitly violate the affine symmetry, too [21]. To preserve nevertheless the Lorentz symmetry, the potential V_g should depend only on $\gamma_{\alpha\beta}$ (and/or $\gamma^{\alpha\beta}$). More particularly, the potential should be a scalar function of $\det\gamma$ and $\mathrm{tr}\,(\gamma\eta)^n$, with any degree n. At $n < 0$, one uses $(\gamma\eta)^{-1} = \eta\gamma^{-1}$, with γ^{-1} given by $\gamma^{\alpha\beta}$. In the above, one puts $(\gamma\eta)_A{}^B \equiv \gamma_{AA'}\eta^{A'B}$, were as before $A = \alpha$ or a, etc, as appropriate.

[18] We disregard the possible nonuniversal interactions of the Goldstone field σ with the matter.

[19] Under $\bar{g} = -1$ and a proper choice for c'_g, the given GR extension reduces to that of ref. [7], with σ being the dilaton in disguise as the part of the metric.

[20] Adding to the Lagrangian eq. (35), without the affine symmetry violation, the quadratic term depending on the another independent contraction $\hat{\Omega}^{+}_{ab}{}^{b}$ one would produce the scalar-vector contribution of the gravity field, which would depend on the background quantity $\bar{\Gamma}^\lambda \equiv \bar{g}^{-1\mu\nu}\bar{\Gamma}^\lambda{}_{\mu\nu}$ (see the next item).

[21] Remind, that in the case of the affine symmetry violation, there becomes distinguishable the rescaling factor for the matter fields ϕ_s. Thus one should return in this case to the full notation $\hat{\phi}_s$, again.

It follows that in the world coordinates, the potential should depend on $g\bar{g}^{-1}$ as

$$V_g = V_g\Big(\det(g\bar{g}^{-1}), \mathrm{tr}\,(g\bar{g}^{-1})^n\Big), \tag{65}$$

were $\bar{g}^{-1\mu\nu} \equiv \bar{e}^\mu_A \eta^{AB} \bar{e}^\nu_B$. Generally, one has $\bar{g}^{-1\mu\nu} \neq \bar{g}^{\mu\nu} \equiv g^{\mu\mu'} g^{\nu\nu'} \bar{g}_{\mu'\nu'}$ (and similarly, $\bar{g}^{-1}_{\mu\nu} \neq \bar{g}_{\mu\nu}$)). At the negative n, one puts $(g\bar{g}^{-1})^n = (\bar{g}g^{-1})^{-n}$, with $g^{-1\mu\nu} \equiv g^{\mu\nu}$. The potential corresponds to the mass terms for the gravity field. In these, the terms depending only on $\det(g\bar{g}^{-1})$ are unimodular covariant. Choosing the different terms one can vary the relation between the masses of the scalar and tensor gravitons [22].

With addition of the potential, the only modification of the GR equation of motion is the the appearance of the energy-momentum tensor $T_{g\mu\nu}$ in the r.h.s. of eq. (60). This contribution corresponds to the graviton mass. Due to the Bianchi identity, stating the covariant divergenceless of the l.h.s. of eq. (60) and thus the covariant conservation of the total energy-momentum (the contribution of the graviton mass including), there appear the four collective constraints on the gravity, matter and radiation fields. They substitute the Lorentz-Hilbert gauge condition. Thus at the level of the equation of motion, the theory describes six physical degrees of freedom, supposedly, the massive tensor and scalar gravitons. Not to collide explicitly with the principle of the microscopic causality, it is to be shown that, under the given $\bar{g}$ and $V_g(g\bar{g}^{-1})$, one can eliminate for the gravity field all the components with the negative norm. In the limit of the zero mass, the general covariance is restored and one recovers smoothly the GR with the two-component graviton.

Clearly, the mass term is noninvariant under the general coordinate transformations. Violating the affine invariance, the potential V_g is expected naturally to be highly suppressed (if any). Thus, the affine symmetry gives the *raison d'etre* for the graviton mass to be tiny [23]. This is in distinction with the extra terms depending on the derivatives of the affine Goldstone boson. The latter terms also result in the general covariance violation. Nevertheless, being affine invariant, they are not required *a priori* to be small [24].

Altogether, this exhausts the foundations of the effective field theory of the gravity, radiation and matter based on the affine symmetry. The above theory, embodying the GR within the extended framework, may be called *the metagravitation.*

5. Metauniverse

5.1. World continuum

The ultimate goal of the Goldstone approach to gravity is to go beyond the effective metric theory and to build the underlying premetric one. In what follows, we present some hints of the respective scenario. Of necessity, we will be very concise, just to indicate the idea.

The forebear of the space-time is supposed to be the world continuum. At the very least, the latter is to be endowed with the defining structure, the continuity in the topological sense. Being covered additionally with the patches of the smooth real coordinates x^μ, $\mu = 0, 1, \ldots, d-1$ (index 0 having yet no particular meaning), the continuum acquires the structure of the

[22] For the theory of the massive tensor field in the Minkowski background space-time see, e.g., ref. [8]. For the phenomenology of the graviton mass and for further references on the item cf., e.g., ref. [9].

[23] If, for some reason, the true cosmological constant would be forbidden, being only mimicd by the graviton mass, then this would justify the smallness of the effective cosmological constant.

[24] Another source of the general covariance violation could be due to the nonuniversal couplings of the affine Goldstone boson with the matter. One more source could be the small background torsion.

differentiable manifold of the dimension d (4, for definiteness). There exist in the continuum the tensor densities, in particular, the volume element. Thus, the integration over the manifold is allowed. But this does not suffice to define the covariant derivative and thus to get the covariant differential equations, etc. Suppose now, that the continuum can exist in two phases with the following affinity properties.

(i) **Affine connection** Being endowed with the primordial affine connection $\bar{\psi}^{\lambda}{}_{\mu\nu}$, the continuum becomes the affinely connected manifold. Generally, the connection is the 64-parametric structure. It defines the parallel transport of the world tensors, as well as their covariant derivatives. The parallel transport along the infinitesimal closed contour defines, in turn, the background curvature tensor $\bar{\rho}^{\lambda}{}_{\mu\rho\nu}$ and thus its contraction $\bar{\rho}_{\mu\nu} = \bar{\rho}^{\lambda}{}_{\mu\lambda\nu}$ (but not yet the scalar $\bar{\rho}$). To every point P, there can be attached the coordinates $\bar{\xi}^{\alpha}_{P}$, where the symmetric part of the connection locally nullifies, the manifold becoming thus locally affinely flat. This defines the global affine symmetry. For the symmetry to be exact, the antisymmetric part of the connection, the torsion, should be trivial, with the connection being just 40-parametric. In this phase, there is no metric and thus no space and time directions, even no definite space-time signature, no lengths and angles, no preferred Lorentz group and thus no finite dimensional spinors, no preferred Poincare group and thus no conventional particles, no invariant intervals, no quadratic invariants, no causality, etc. Though there can be implemented the principle of the least action with the simplest invariant Lagrangians, the world structure is still rather dull. Nevertheless, it should ultimately lead to the spontaneous transition from the given phase to the metric one.

(i) **Metric** Further, being endowed spontaneously with the primordial metric $\bar{\varphi}_{\mu\nu}$ with the Minkowskian signature, the continuum becomes the metric space, i.e., the space-time. The metric is much more restrictive 10-parametric structure. It defines the background Riemannian geometry. Under the emergence of the metric and the spontaneous breaking of the affine symmetry, there appears the affine Goldstone boson serving as the graviton in disguise. This results in the effective Riemannian geometry with the effective metric $g_{\mu\nu}$, etc. Now there appear the preferred time and space directions, the lengths and angles, the definite Lorentz group and thus the finite dimensional spinors, the definite Poincare group and thus the particles, the invariant intervals, the quadratic invariants, the causality, etc.

The world structure becomes now very flourishing. In the wake of the gravity, there appear the radiation and matter. The spontaneous breaking of the affine symmetry to the Poincare one reflects the appearance of the preferable particle structure, among a lot of *a priori* possible ones corresponding to the various choices of the Poincare subgroup. Formally, the effective Riemannian geometry is to be valid at all the space-time intervals. Nevertheless, its accuracy worsen when diminishing the intervals, requiring more and more terms in the decomposition over the ratio of the energy to the symmetry breaking scale M_A, as it should be for the effective theory. Thus, the scale M_A (or, rather, the Planck mass M_{Pl}) is a kind of the inverse minimal length in the nature.

5.2. The Universe

Supposedly, the formation of the Universe is the result of the actual transition between the two phases of the continuum. This transition is thus the “Grand Bang”, the origin of not only the Universe but of the very space-time. At this stage, there appears the world “arrow of time” as the reflection of the spontaneous ordering of the chaotic local times. The residual dependence of the structure of the Universe on the background parameter-functions could result in the primordial anisotropy and inhomogeneity.

And what is more, there is conceivable the appearance (as well as disappearance and coalescence) of the various regions of the metric phase inside the affinely connected one. These regions are to be associated with the multiple universes. One of the latter ones happens to be ours. Call the ensemble of the universes *the Metauniverse*. Within the concept of the Metauniverse, there becomes sensible the notion of the wave function of the Universe. Hopefully, this may clarify the long-standing problem of the fine tuning of our Universe (see, e.g., ref. [10]).

Conclusion

To conclude, the theory proposed realizes consistently the approach to gravity as the Goldstone phenomenon. It proceeds, in essence, from the two basic symmetries: the global affine one and the general covariance. The affine symmetry is the structure symmetry which defines the theory in the small. The general covariance is the bundling symmetry which terminates the *a priory* admissible local theories according to their ability to be prolongated onto the whole world. The theory embodies the GR as the lowest approximation. Its distinction with the GR are twofold. At the effective level, the present theory predicts the natural hierarchy of the conceivable GR extensions according to their mode of the affine symmetry realization. At the underlying level, it presents the new look at the gravitation, the Universe and the very spacetime. These topics are to be further studied in the future.

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References

[1] C. Isham, A. Salam and J. Strathdee, Ann. Phys. (NY) **62**, 98 (1971).

[2] A. B. Borisov and V. I. Ogievetsky, Teor. Mat. Fiz. (in Russian) **21**, 329 (1974).

[3] E. Schrödinger, *Space-time structure* (Cambridge Univ. Press, Cambridge, 1950).

[4] S. R. Coleman, *Dilatations*, in *Proc. of the Intern. Summer School of Physics Ettore Majorana*, ed. A. Zichichi (Erice, 1971).

[5] S. R. Coleman, J. Wess and B. Zumino, Phys. Rev. **177**, 2239 (1969);
D. V. Volkov, Particles and Nuclei (in Russian) **4**, 3 (1973).

[6] V. I. Ogievetsky, Lett. Nuovo Cim. **8**, 988 (1973).

[7] W. Buchmüller and N. Dragon, Nucl. Phys. B **321**, 207 (1989).

[8] V. I. Ogievetsky and I. V. Polubarinov, Ann. Phys. (NY) **35**, 167 (1965).

[9] M. Visser, Gen. Rel. Grav. **30** 1717 (1998), gr-qc/9705051.

[10] P. C. W. Davies, *The accidental Universe* (Cambridge Univ. Press, Cambridge, 1982).

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