Current tests of alternative gravity theories: the Modified Newtonian Dynamics case

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Abstract

We address the possibility of taking advantage of high accuracy gravitational space experiments in the Solar System and complementary cosmological tests to distinguish between the usual general relativistic theory from the alternative modified Newtonian dynamics paradigm.

Key words:
Dark matter, Modified Newtonian Dynamics, Tests of Gravity
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1 Introduction

The current observational status indicates that gravitational physics is in a considerable agreement with Einstein's theory of General Relativity (GR) (Will, 2005; Bertolami et al., 2006b); however, there are some theoretical and experimental issues that shed doubt on its status of the ultimate description of gravity.

Most theoretical difficulties are related to the strong gravitational field regime, together with the ensuing spacetime singularities. It is believed that a possible way of overcoming these troublesome questions lies in the quantization of gravity. Unfortunately, the success of modern gauge field theories in describing the electromagnetic, weak, and strong interactions does not yet extend to

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gravity and, in fact, the fundamental pillars of physics, Quantum Mechanics and General Relativity, are incompatible with each other. Furthermore, fundamental theories that attempt to include gravity lead to new long-range forces. Even at a purely classical level, Einstein's theory does not provide the most general way to establish the spacetime metric, and alternative metric theories that have been put forward allow for violations of the Equivalence Principle, modification of large-scale gravitational phenomena and variation of the fundamental "constants", which are fairly constrained phenomenologically. Thus, the derived predictions serve well to motivate new searches for very small deviations from General Relativity; these quests should clearly include further gravitational experiments in space, including laser astrometric measurements (LATOR Collaboration (Turyshev et al.), 2005; Turyshev et al., 2004a,b,c), high-resolution lunar laser ranging (LLR) (Murphy et al., 2002) and long range tracking of spacecraft using the formation flight concept, as proposed (Pioneer Colaboration (Dittus et a
2005) to test the Pioneer anomaly (Anderson et al., 2002); a broader review of fundamental physics experiments in space can be found elsewhere (Bertolami, 2006; Bertolami et al., 2006a).

From an observational standpoint, recent cosmological observations indicate that, if GR is indeed correct, then one must also assume that most of the energy content of the Universe lies in some unknown forms of dark matter and dark energy that may permeate much, if not all spacetime. Indeed, recent Cosmic Microwave Background Radiation (CMBR) WMAP three year data(Spergel et al., 2006) tells us that the Universe is well described by a flat Robertson-Walker metric, with an energy density of the Universe fairly close to its critical value, $\rho_c \equiv 3H_0^2/8\pi G \simeq 10^{-29} g/cm^3$, where $H_0 \simeq 71\ km\ s^{-1} Mpc^{-1}$ is the Hubble expansion parameter at present. Furthermore, CMBR, Supernova and large scale structure data are consistent with each other if, in the cosmic budget of energy, dark energy corresponds to about 73% of the critical density, while dark matter to about 23% and baryonic matter to only about 4%.

As is widely known, dark matter was firstly suggested in 1933 by Zwicky, from a study of the observed motion of the peripheral galaxies of the Coma cluster; this indicated a discrepancy between the total mass, as inferred from its brightness or total number of galaxies (Trimble, 1987), which appeared to indicate that the total amount of mass in the cluster is about 400 times more mass than expected. This led to the postulation of a yet although unaccounted, non-luminous matter, which became known as "dark matter". This hypothesis was also supported by the differential rotation of our galaxy, firstly discussed by Oort in 1927, the flatness of galactic rotation curves, large scale data and, more recently, by the observation of the so-called "bullet" cluster (Clowe et al., 2006).

Returning to current times, one finds that the amount of new data and related issues coincides with recent progress in high-precision measurement technologies for physics experiments in space, ranging from spacecraft navigation to time transfer, clock synchronization, weight and length standards. Hence, the concerned physicist is no longer restrained to mere speculation, but has a chance of probing crucial matters such as the nature of dark energy and dark matter, effects of intermediate range forces and, ultimately, the fundamental nature of gravity. Naturally, the evolving techniques should allow for an ever increasing precision in measurements, with associated refinement of viable models, and exclusion of unrealistic ones. Eventually, it is expected that an ultimate "theory of everything", engulfing both Quantum Mechanics and General Relativity will finally shed some light over the cosmological issues involving the origin and destiny of the Universe. In what follows, we discuss the current status of one of the most widely discussed alternatives to GR, namely the Modified Newtonian Dynamics and its underlying fundamental theory, the so-called Tensorial-Vector-Scalar theory. For the discussion of the implications of scalar tensor theories and modifications designed to mimic dark energy the reader is directed to Refs. (Bertolami et al., 2006b; Bertolami, 2006)

2 Modified Newtonian Dynamics and Tensorial-Vector-Scalar theory

2.1 Modified Newtonian Dynamics

Although the current paradigm of GR endowed with dark matter and dark energy components is highly successful in describing observations on larger scales, and seeding very enticing theoretical ideas, it is important to study competing alternatives, in particular those which attempt to account for observations without the dark matter component. For instance, a possibility involves the putative running off the gravitational coupling (Bertolami et al., 1993; Bertolami and García-Bellido, 1996). Another, well discussed alternative, goes by the name of Modified Newtonian Dynamics (MOND), and is based in a modification of Poisson's equation for the gravitational potential (Milgrom, 1983). Hence, instead of the traditional form, MOND postulates the following equation:

$$\nabla \cdot \left(\mu \left(\frac{\nabla \phi}{a_0} \right) \nabla \phi \right) = 4\pi G \rho \quad , \tag{1}$$

where ρ is the density of barionic matter and $a_0 \approx 10^{-10}\ m\ s^{-2}$ is the scale at which Poisson's equation fails; specifically, one regains the traditional form for accelerations $a = -\nabla\phi \gg a_0$, through the Milgrom function μ, which values

$\mu(x) \approx x$ for $x \ll 1$, and $\mu(x) \approx 1$ for $x \gg 1$.

This phenomenological prescription provides with an alternative solution for the flattening of the rotational curves of galaxies. Indeed, far away from the central region, the gravitational potential should be small enough so that MONDian behaviour takes over, and a simple algebraic computation shows that, instead of the inverse square law for gravity, one obtains the desired r^{-1} behaviour, yielding $v_c(r)$ constant. Furthermore, a direct consequence of this scenario is that the luminosity scales with v_C^4, which is in agreement with the empirical Tully-Fisher law (Milgrom, 1983; Zhao and Famaey, 2006).

Given that it was designed to solve these and other issues specifically, MOND is a purely phenomenological model, thus lacking a suitable support in the form of a fundamental theory, covariantly formulated in terms of the variation of a specific action functional. To avoid this caveat, several attempts to generalize MOND were undertaken, which have so far settled in what is known as Tensorial-Vector-Scalar theories.

2.2 Tensorial-Vector-Scalar theory

The proposed Tensorial-Vector-Scalar (TeVeS) (Bekenstein, 2004) theory assumes, asides from the metric $g_{\alpha\beta}$ and matter fields φ_i (with the associated energy-momentum tenor $T_{\alpha\beta}$, the presence of a vector field U^α and two scalar fields, σ and ϕ. The vector field is everywhere timelike, with the normalization condition $U_\alpha U^\alpha = -1$; furthermore, the vector field couples with normal matter through a physical metric $\tilde{g}_{\alpha\beta} = e^{-2\phi} g_{\alpha\beta} - 2U_\alpha U_\beta \; sinh(2\phi)$. The action of the theory is written as $S = S_G + S_V + S_S + S_M$, with

$$\begin{aligned}
S_G &= \int g^{\mu\nu} R_{\mu\nu} (-g)^{1/2} \; d^4x \quad , \qquad (2) \\
S_V &= -\frac{K}{32\pi G} \int \left[g^{\alpha\beta} g^{\mu\nu} U_{[\alpha,\mu]} U_{[\beta,\nu]} - \frac{2\lambda}{K} (g^{\mu\nu} U_\mu U_\nu + 1) \right] (-g)^{1/2} \; d^4x \quad , \\
S_S &= -\frac{1}{2} \int \left[\sigma^2 h^{\alpha\beta} \phi_{,\alpha} \phi_{,\beta} + \frac{G}{2l^2} \sigma^4 F(kG\sigma^2) \right] (-g)^{1/2} d^4x \quad , \\
S_M &= \int \mathcal{L}(\varphi_i) (-\tilde{g})^{1/2} \; d^4x \quad ,
\end{aligned}$$

and K, k and l are constants specific of the theory, λ is a Lagrange multiplier that sets the timelike normalization condition for the vector field, F is a free function and $h^{\alpha\beta} = g^{\alpha\beta} - U^\alpha U^\beta$. Notice that there is no kinetic term for the scalar field σ, whose equation of motion provides an algebraic relation between its value, the free function F and the dynamic scalar field ϕ.

There are a number of criticisms concerning the action of the theory: firstly, the presence of a non-dynamic σ is somewhat *ad hoc*, although it is argued that the scalar action resembles that of a self-interacting complex scalar field in a particular limit (Bekenstein, 2004). This suggests that σ is not a fundamental field, but only an artificial tweak of the theory in order to enforce some specific behaviour. The choice of physical metric $\tilde{g}^{\alpha\beta}$ is also somewhat odd, since it is assumed that the vector field U^α couples directly with matter. Furthermore, there is no coupling between the scalar fields and matter. Also, asides from the dynamic and physical metrics, a third tensor $h^{\alpha\beta}$ also appears, which is somewhat similar to the physical metric, but appears only in the scalar action; this reveals that the vector field couples to the non-dynamic scalar field only, another arbitrary feature. Finally, the free function F has no rationale apart from the fact that it is specified in order to obtain the desired features. In the overall, the action for TeVeS appears as an odd concatenation of different tensorial, vector and scalar quantities, with no apparent internal consistency, asides from the fact that it provides MOND with some more underlying supporting theory.

The full demonstration of the connection between TeVeS and MOND is outside the scope of this text, and can be found elsewhere (Bekenstein, 2004). It can be shown that MOND behaviour arises if one assumes a spherically symmetric, quasistatic case, that is, one endowed with a weak potential and where only slow motion occurs. For this identification to be complete, Milgrom's function μ is obtained from a specific choice for the free function F . This yields a rather complex form for the latter quantity, having an asymptote when its argument goes to unity, and diverging as it approaches infinity. The function $F(x)$ also lacks definition when its argument varies between $1 \leq x \leq 2$, thus dividing it into two different branches; the branch $x < 1$ is relevant in quasi–stationary systems, while $x > 2$ concerns cosmological issues. Given that F appears as a potential function for the non-dynamic field σ, this increases the difficulty associated with the latter, as previously discussed.

3 Testing MOND and TeVeS

3.1 Cosmology

In what follows, we briefly describe the current status of observations, and its implications in testing if MOND and TeVeS can be a viable alternative to the standard paradigm of GR and dark matter. Before dwelling into its natural testing grounds, we quickly address some cosmological issues, namely the current accelerated expansion of the Universe and the initial structure formation. These are implemented in the context of TeVeS by assuming that the vector field has only a temporal component (due to isotropy) and resorting to the $x > 2$ branch of the free function $F(x)$. Also, homogeneity requires that the scalar field depends solely of the cosmic time t (Bekenstein, 2004; Skordis et al., 2006).

Regarding the accelerated cosmic expansion, it is found that a cosmological constant term can be included in TeVeS by adding an appropriate integration constant to the free function F; this has no effect in an astrophysical context, but yields the required repulsive force that counteracts the expected deceleration of the rate of expansion (Bekenstein, 2004). However, its inclusion is purely phenomenological, and offers no explanation as to the value of this quantity, nor it provides any clue concerning the much debated cosmological constant problem.

The issue of initial structure formation is more evolved; indeed, standard cosmology assumes that structure formation occurred due to the condensation of initial fluctuations, enhanced by the presence of dark matter (Trimble, 1987). In the case of TeVeS, which does not assume this component to be present, the situation is still under study, but proponents of the theory usually rely on TeVeS also manifesting MOND behaviour in the early Universe scenario (Skordis et al., 2006). More specifically, instead of density fluctuations of the dark matter component, it is argued that fluctuations of the scalar field ϕ also give rise to baryonic structure formation. However, these results have been recently challenged by a numerical study (Pointecouteau, 2006). Concerning other evolutionary stages of the Universe, namely inflation, the radiation and matter eras, it is found that the scalar field does not interfere significantly with standard cosmology, as long as its magnitude does not change abruptly between the end of one era and the onset of the next, and is globally between $0.0007 < \phi \ll 1$ (Bekenstein, 2004).

3.2 Galactic Clusters

Recent observations of galactic clusters have been interpreted as a strong indication of the validity of the GR with dark matter paradigm. This assessment is based on the gravitational profile reconstructed from gravitational lensing data, which clearly shows two strong peaks, coinciding with the centers of the colliding galaxies (Gavazzi et al., 2003; Clowe et al., 2006). Indeed, most of the "visible" matter in a galaxy is in the form of plasma, with stars and dust accounting for only about 20% of the total; colliding plasma distributions tend to merge, yielding a gravitational profile with a single peak, while "normal" matter retains its integrity, so that its gravitational profile should show the individual peaks of the colliding objects. Given that the reconstituted profile for the two galaxy clusters is in accordance with the latter, one concludes that a dark matter component dominates each galaxy's mass distribution. Since known matter amounts only for a small proportion, dark matter is what accounts for the gravitational profile. There are, however, some tenacious efforts towards an alternative interpretation of this result, trying to implement the MOND hypothesis in this context. This resorts to an already known feature of MOND (and, fundamentally, from TeVeS): that this theory does not completely rule out the need for dark matter, but still enables a reduction of its proportion by a factor of five (with the remainder possibly being made of neutrinos or some unaccounted "normal" matter) (Sanders, 1998). Hence, it is suggested that MOND can also account for the twin peaks gravitational profile, by assuming the presence of neutrinos with a mass of approximately $2\ eV$ (Angus et al., 2006a,b). Two criticisms can be put forward: firstly, the assumed neutrino mass is very close to the known allowed upper bound of $0.07\ eV < m_\nu < 2.2\ eV$, so its validity must be reassessed as tighter results become available. Secondly, and possibly more crucially, the reconstruction of the observed gravitational profile based on MOND resorts to various linear combinations of "MONDian" potentials ϕ_i . This is somewhat troublesome, given that the inherent non-linearities of the underlying TeVeS theory may indicate that, even if each of the galaxies allows for a MONDian behaviour away from its central region, one cannot treat their collision as a simple superposition of individual potentials. Moreover, the chosen potentials are not fully accounted for, with no clear justification given for the values of the fitting parameters or the algebraic form of the potentials themselves. Also, uncertainties on the mass distribution, gravitational lensing statistics and spatial symmetry of galaxy mergers do not allow for a clear relation between the fitting parameter giving the strength of the scalar field ϕ, and the typical value $k \sim 0.03$ (Bekenstein, 2004), which appears in the free function F of TeVeS and sets the order of magnitude of this scalar field in a spherical symmetric case, through

$$\phi(r) = \phi_c - \frac{kGM}{4\pi r} \quad , \tag{3}$$

where ϕ_c is the cosmological value of ϕ, to which $\phi(r)$ goes asymptotically, as the metric goes to its Minkowski form. Furthermore, a thorough analysis of galaxy merging in the context of MOND should eventually resort to TeVeS as a more fundamental approach to tackle the complexity of the spatial superposition of two matter distributions. A mere fit of parameters based on plausible, but somewhat arbitrary potentials, offers only a superficial solution .

3.3 Solar System

It is our opinion that implications of MOND in the Solar System should be better examined. Most probably, this lack of interest is due to the preconception that the condition $\nabla\phi \ll a_0$ does not occur within the vicinity of the Sun (as can be checked by direct computation), and, as a result, MOND-like behaviour should be nonexistent or highly attenuated. However, there are two avenues of research that may be pursued: the first concerns the study of regions where the above condition may be realistically found, that is, by assuming not only a simple central body problem, but also take the remaining planets and objects into account. Specifically, one can deal with a two-body problem, and analyze the region close to the equilibrium point $\nabla\phi = 0$; a partial numerical study of this issue shows that there is a MONDian "bubble" of non-negligible size at such location, which could provide interesting clues as to the validity of the MOND paradigm (Bekenstein and Magueijo, 2006). Although direct effects are below current measurement capabilities, it is found that an object located within this bubble would experience an anomalous acceleration which, in principle, could be measured in the near future.

In parallel, one can assume a simpler central body problem and, while assuming that no MONDian regime occurs in its vicinity, perturbative effects should arise in the standard Scharzschild metric. This is clearly suitable for an analysis within the framework of the PPN formalism (Will, 2001). In short, a simplified version of this formalism states that, in a isotropic reference frame, the expansion of the metric to second order in the gravitational potential reads

$$g_{00} = -1 + \frac{2GM}{r} - \beta \left(\frac{2GM}{r}\right)^2 \quad , \quad g_{ii} = 1 + \gamma \frac{2GM}{r} \quad . \tag{4}$$

The PPN parameters β and γ are related to fundamental physical properties of the theory under scrutiny: the latter quantifies the spatial curvature produced by unit mass, while the former indicates the amount of non-linearity in the superposition law for gravity. These can be used to compare and distinguish GR from competing theories of gravitation, with the former displaying the reference values $\beta = \gamma = 1$.

It was first thought that such a procedure would yield results which made MOND indistinguishable from GR, that is, that the PPN parameters β and γ do not shift from unity (Bekenstein, 2004). However, this assumed that the vector field U^α had only a non-vanishing temporal component which, by virtue of its normalization condition, tracks the metric element g^{00} and, after some algebraic work, leads only to a redefinition of Newton's gravitational constant G; this prescription for the vector field is allowed by its equation of motion, and argued as a plausible choice for its form. However, a later work (Giannios,

2005) showed that one can also obtain another solution for the vector equation of motion, where U^α has also a non-vanishing radial coordinate, related to the temporal one through the normalization condition. Contrary to the previous null deviations from the GR values for the PPN parameters, this yields a case which differs from standard GR[1]:

$$\beta = 1 + \frac{k}{8\pi} + \frac{K}{4} + \phi_c \left(3 + \frac{k}{\pi K} \pm \sqrt{\frac{2k}{\pi K} + 5} \right) \quad , \quad \gamma = 1 \ . \tag{5}$$

It should be pointed out that, in our opinion, the result $\beta \neq 1$, asides from its clear theoretical interest, is more physically motivated that the previous $\beta = \gamma = 1$ case. This is so because the latter is based on the *Ansatz* $U^\alpha = (U^0, 0, 0, 0)$, while the former takes $U^\alpha = (U^0, U^r, 0, 0)$: while both are consistent with the vector field equation of motion, the purely temporal case seems somewhat artificial. Indeed, although the velocity–field of known matter admits only a temporal component, the scalar field ϕ, which pervades all space, has a velocity–field with both temporal and radial components. Hence, a sensible assumption seems to be that the vector field also has temporal and radial components. Moreover, if indeed one has $\beta \neq 1$, the superposition principle assumed in Angus et al. (2006a,b) for the summation of potentials is not exact, which further casts doubts concerning the ability of MOND to account for the gravitational profiles of galactic clusters.

Recall that the current experimental data shows an impressive agreement with General Relativity, with the most stringent bounds

$$\beta - 1 = (1.2 \pm 1.1) \times 10^{-4} \quad , \quad \gamma - 1 = (2.1 \pm 2.5) \times 10^{-5} \ , \tag{6}$$

arising respectively from the Cassini's 2003 radiometric experiment (Bertotti et al., 2003) and from limits on the Strong Equivalence violation parameter, $\eta \equiv 4\beta - \gamma - 3$, that are found to be $\eta = (4.4 \pm 4.5) \times 10^{-4}$, as inferred from LLR measurements (Williams et al., 2004).

Hence, the assumed value of $k \sim 0.03$ is immediately excluded, which poses an issue concerning the choice of the free function F, since this quantity specifies the order of magnitude of that parameter of TeVeS. Also, one obtains a weak bound on K, which must be smaller than 10^{-3}; unfortunately, there are no other tight constraints on this quantity to be measured against with, since the latter is related to the strength of the vector action in the full TeVeS action, and the vector field U^α does not play a dynamical role in cosmology or galactic structure. In any case, it is clear that more stringent bounds of β may render relevant constraints for TeVeS.

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[1] We have recently and independently obtained a very similar result, unaware that it had been derived earlier.

4 Discussion and Conclusions

In this study we have discussed the current status of the Modified Newtonian Dynamics alternative to General Relativity, based on the Tensorial-Vector-Scalar theory; we address both its theoretical foundations, as well as recent observations that question its applicability, attempting to cast a circumstanced criticism of both. We conclude that, although MOND still remains a logical possibility, it is at the verge of being excluded from the observations, unless one is willing to exploit some of its theoretical oddities, most of which unaccounted for. Hence, although it may still be too early for its utter dismissal as a viable alternative to the paradigm of GR, we believe that the case for MOND has grown increasingly weaker, favouring the more canonical approach that assumes the presence of a dark matter component in the Universe. Hopefully, the future will hold further experimental results and theoretical advances in both competing theories, leading to a more profound understanding of gravity.

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