CLOCKS AROUND-THE-WORLD COMPARISONS VIA GPS SATELLITES AS A WAY OF RESEARCH OF RADIOPULSE VELOCITY IN MOVING SUBSTANCE

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When ground clocks are compared via GPS satellites, radiopulses pass through the atmosphere, which moves due to diurnal rotation together with the Earth. Radiopulse dragging by moving atmosphere leads to group-delay positive correction when pulse goes eastwards and to negative one when it goes westwards, that results in corresponding systematic error. This effect could manifest itself under around-the-world closure of comparisons loop. According to approach presented herein the estimate for around-the-world error in experiments performed till now is of the order of several nanoseconds. This value is in agreement with discrepancy obtained, its source being not entirely clear. If this discrepancy is indeed the consequence of radiopulse dragging by atmosphere, one could, by taking this effect into account, ensure more high accuracy in experiments, dealing with radiopulse one-way travel along "space-ground" paths.

From 1984 till now there were performed several around-the-world comparisons of three clocks (sited, for example, in USA, Germany and Japan) via GPS satellites [1-6]. If systematic effects were taken into account correctly, clocks mutual differences should add to zero. The source of several nanoseconds discrepancy obtained is not entirely clear.

Note that these experiments may be considered as research of radiopulse velocity in moving substance [7]. Indeed, in non-rotating frame of reference, connected with the Earth center, the radiopulse, travelling from satellite to ground antenna, goes through atmosphere which moves due to diurnal rotation together with the Earth. Pulse dragging by atmosphere leads to group-delay positive correction when pulse goes eastwards and to negative one when it goes westwards, that results in corresponding error when clocks are compared in common-view mode. When closing the around-the-world loop of such comparisons, these errors are accumulated and total discrepancy can be detected.

The size of this discrepancy depends on law of addition of pulse velocity in resting substance and substance velocity. The experimental research of this law, unlike the case with light phase velocity in moving substance, is difficult because effects to be measured are tiny, so till now theory of the subject was not urgent. But discrepancies in around-the-world comparisons can be considered as first empiric data on this problem.

Herein the approach, permitting one to write an expression for electromagnetic pulse velocity in moving substance with first-order accuracy, is presented, and this approach agreement with experimental results is discussed.

It is known that Fizeau expression, i.e. linear part of light phase velocity in a substance moving at speed $V \ll c$ in the same (opposite) direction, looks like

$$c' = \frac{c}{n} \pm \left(1 - \frac{1}{n^2}\right)V,\tag{1}$$

where *n* is the index of refraction. Let us write, by analogy with this expression, the linear part of electromagnetic pulse velocity in moving substance in a form:

$$u' = \frac{c}{n_g} \pm \alpha V , \qquad (2)$$

where n_g is the group index of refraction and α is the dimensionless parameter, which we are searching for. As the pulse velocity in resting substance far from absorption lines coinsides with velocity of electromagnetic energy movement, we shall consider an elementary act of this process in substance, i.e. energy quantum transfer between some pair of molecules. Let these molecules be in mutual rest. According to first postulate of special relativity [8], the quantum transfer duration in the frame of reference, connected with this pair of molecules, does not depend on velocity of their own direct and uniform motion. In the laboratory frame of reference, where the pair moves directly and uniformly, that duration does not change with first-order accuracy, as the relativistic transformations are the second-order effects. Then it can be shown, that in case of solid transparent substance, where molecules are believed to be moveless each relative to other, the expression (2) has, with first-order accuracy, classical form, i.e. $\alpha = 1$.

One can interpret in favour of this conclusion the results [9] of research of velocity aberration in satellite laser ranging. The velocity aberration consists in reply radiation deflection off the "satellite-locator" line at the angle proportional to apparent tangential velocity of satellite, so that level of signal recieved is decreased. The authors of [9] have found that, unlike the case with hollow retroreflector, in case with quartz prism retroreflector such a decreasing takes practically no place. As noted in [10], this result could be explained under assumption that in locator's frame of reference the light pulse velocity inside moving prism retroreflector obeys the expression (2), where $\alpha = 1$, then the velocity aberration is absent with (V/c) first-order accuracy.

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Now let us consider the question about velocity of electromagnetic pulse transfer by moving substance, in which a chaotic motion of molecules takes place, e.g., by a gas. It can be shown that thermal motion of molecules has no influence on parameter α . However, there exists α dependence on gas rarefaction degree, as with rarefaction increasing the parameter α tends, obviously, to zero. It is known that incontinuous structure of a substance begins manifest itself when the mean time between molecule collisions τ_{col} becomes comparable with some characteristic duration. In our task such a characteristic duration is the duration of molecule interaction with pulse τ_{int} . So we shall describe the rarefaction degree by ratio τ_{col}/τ_{int} , by analogy with Knudsen number [11] in gas dynamics. How could the parameter α depend on this ratio? As mentioned above, in case of solid substance, when $\tau_{col}=0$, parameter α is equal to the one, and when τ_{col} grows, the parameter α permanently diminishes so that $\alpha \to 0$ when $\tau_{col} \to \infty$. Taking that into account, we shall assume α dependence on τ_{col}/τ_{int} to be an exponential one, then (2) looks like

$$u' = \frac{c}{n_g} \pm V \, exp(-\tau_{col}/\tau_{int}). \tag{3}$$

As for duration of interaction τ_{int} (we consider linear interaction), in case of wideband pulse it is simply the pulse duration, i.e. inverse width of its band B. In case of narrowband pulse transfer by substance, the duration τ_{int} can't exceed, in our opinion, the inverse width of inhomogeneous broadening in carrier region, i.e. the Doppler inverse width $(\Delta v_{Dop})^{-1}$ in case of a gas. Theoretically, the pulse bandwidth lower limit in our task is the homogeneous broadening width Δv_{hom} in carrier region, because near this limit the concept of the pulse loses its physical sense. In practice, however, the pulse bandwidths used are far away from this limit. Thus, in case of pulse of continuous spectrum, so that $\Delta v_{hom} << B << v_{carrier}$, we have:

$$\tau_{\text{int}} = \begin{cases} \left(\Delta v_{Dop} \right)^{-1}, \Delta v_{\text{hom}} << B \leq \Delta v_{Dop}, \\ B^{-1}, B > \Delta v_{Dop}. \end{cases}$$
(4)

As follows from (3), at the length dL the group-delay correction $d\tau$ for substance motion, compared with case of substance being in rest, is

$$d\tau = \mp dL \frac{V}{c^2} n_g^2 exp(-\tau_{col}/\tau_{int}). \tag{5}$$

To calculate a correction $\Delta \tau$ for pulse dragging by atmosphere at "space-ground" path, one should integrate the expression (5) along the path segment in atmosphere. Let us introduce an auxiliary value, the thickness T_{eq} of such an equivalent atmosphere, which, keeping the condition $\alpha=1$ through all the thickness, would result in the same integral correction $\Delta \tau$, as real

atmosphere does. The expression for T_{eq} , with n_g , dependence on height being neglected, is the following:

$$T_{eq} = \int_{0}^{h_{up}} exp(-\tau_{col}(h)/\tau_{int})dh \qquad (6)$$

where h is current height, $\tau_{col}(h) = \tau_{col}(0) \exp(h/h_0)$, $h_0 = 8.5$ km in model of exponential atmosphere, and upper limit of integration h_{up} is chosen to obtain accuracy desired. The T_{eq} dependence on the pulse bandwidth under conditions of standard atmosphere is shown on Fig.1 for various carriers.

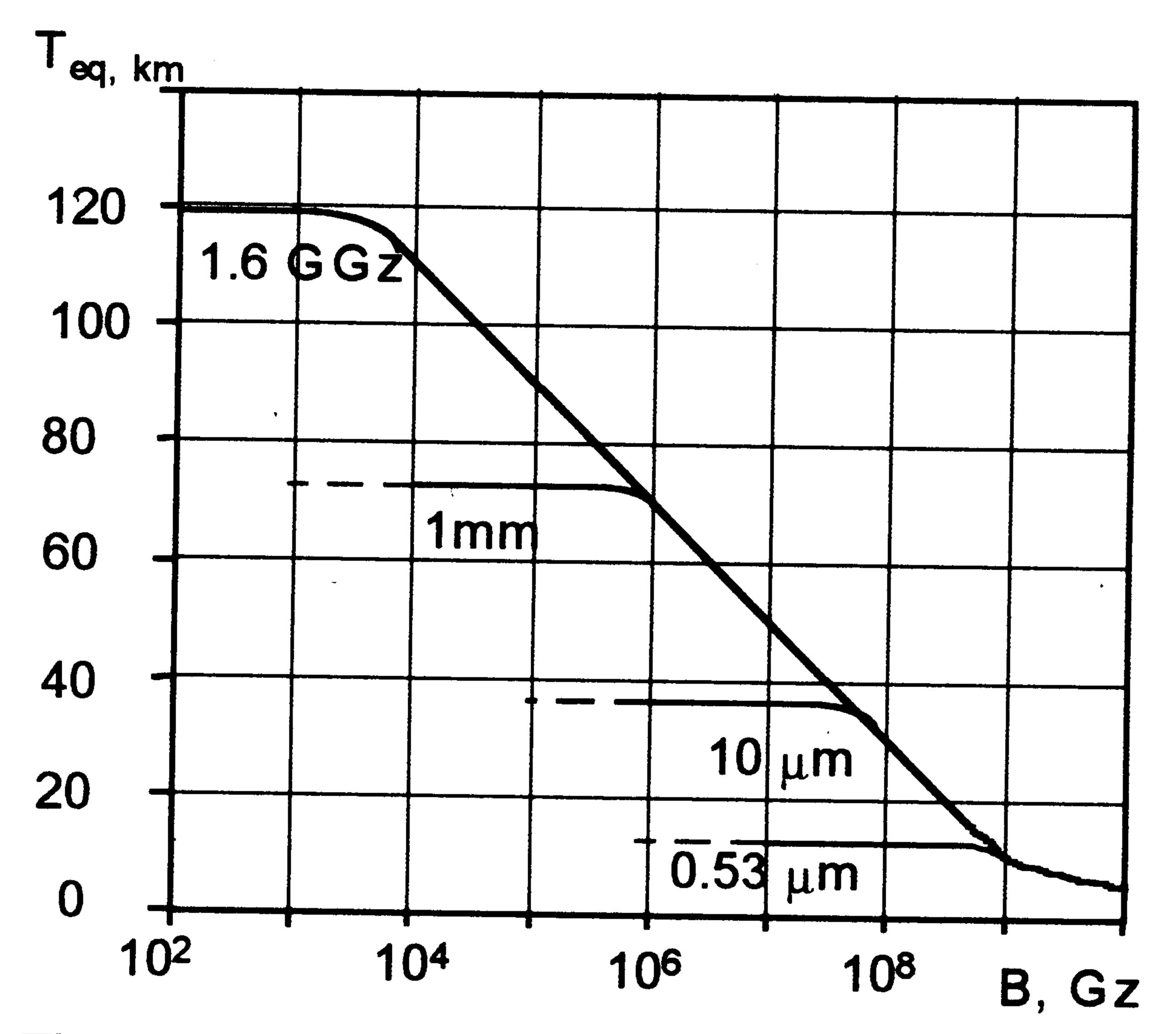


Fig.1. Calculated thickness of equivalent atmosphere as a function of pulse bandwidth for various carriers. Dotted segments indicate areas where the concept of the pulse loses its physical sense.

Using the equivalent atmosphere thickness, one can write, in flat ground approximation, the correction for dragging by atmosphere, when ground receiver is at latitude φ , and pulse moves from satellite, observed at zenith angle ζ and azimuth η , in the form:

$$\Delta \tau \cong -T_{eq} \frac{v}{c^2} tg \zeta \sin \eta \cos \varphi, \qquad (7)$$

where V=464 m/s is linear velocity of atmosphere rotation at equator.

Correction (7) at any atmospheric segment of the loop of around-the-world comparisons decreases the difference between eastwards and westwards travel times, caused by the rotation of geocentric frame of reference. Let us estimate the magnitude τ of corrections (7) total effect. The bandwidth of GPS-signals (in C/A-code) is 2 MGz, then the equivalent atmosphere thickness is 63 km, and factor $(T_{eq}V/c^2)$ in expression (7) is equal to 0.32 ns. Taking into account the geometry of experiments [1-6], we obtain for "around-the-world" correction τ a value of about 2 ns, if working satellites were observed from corresponding laboratories at approximately equal elevations.

Asymmetry of the elevations increases that value; for example, if minimal elevation is 10° , τ can reach 4 ns. Compare: if law of addition of pulse velocity and substance velocity is relativistic one, like in case of phase velocity, then around-the-world correction would be three orders smaller.

Now let us consider how the approach presented agrees with experimental results. In first experiment [1], performed in 1984, a discrepancy of 5.2 ns was found. However, its accuracy was not high enough, that was demonstrated in next experiment [2], where discrepancy spread was of about one hundred nanoseconds. Later, number of laboratories refined algorithms of GPS-data processing, and also examined in more detail various systematic effects [3,4]. In experiment of 1987 [3] two independent sets of three laboratories took part; discrepancies obtained were 9.3 ns and 18.9 ns. The work [4] reports the results of comparisons performed with use of more accurate values of receivers coordinates, ionospheric delays and satellites ephemerides, uncertainties of which make main contributions into systematic error [5,6]. Average discrepancies in every of eight 20-days-sessions hit the range from 2.1 to 8.1 ns, while standard deviation for every session did not exceed 1 ns [4]. One can make a conclusion, together with the authors of [6], that there exists unknown systematic effect of several nanoseconds, the sign of which being constant [12].

Though the estimates presented herein agree with this conclusion, today it is difficult to say, whether the effect discovered is indeed consequence of radiopulse dragging by atmosphere. Note that discrepancies quoted above are the averages which don't reflect current changes in geometry of the experiment. But the data available could be reprocessed in order to search for geometrydiscrepancy correlations. Besides, the following experiments could give us more information on the subject. First, one can perform clocks around-theworld comparisons with the help of very long base interferometry (VLBI). But till now clocks comparisons was not the aim of VLBI around-theworld experiments (see, e.g., Annual Reports of International Earth's Rotation Service). Moreover, clocks preliminary synchronization in these experiments was made using just GPS-technique [13]. Second, one can receive GPS-signals at same location simultaneously in C/A-code (B=2 MGz, $T_{eq}=63$ km) and in P-code (B=20 MGz, $T_{eq}=43$ km), and search for differences between corresponding pseudo-ranges to satellites observed at east and at west. Third, one can use same satellite to compare clocks by two methods simultaneously: by common-view technique and by unidirectional exchange technique, when satellite is used as re-emitter. By the latter technique pulse passes through atmosphere twice, not once, like by the former one, that may lead to a difference in results of these two methods. Fourth, favourable conditions for investigation of various systematic

effects at "space-ground" paths are expected in "H-maser in space" project [14,15]. In this project it is planned to use two-way laser channel for satellite precise ranging and one-way microwave channel for time transfer with the help of PRARE system having 15 MGz bandwidth. That could make it possible to observe the effect of radiopulse dragging by atmosphere for one pass of satellite. With the orbit low inclination being planned, the correction (7) variation, while satellite passes from east $(\zeta=70^{\circ})$, would be about 1.3 ns for an observer sited at equator.

To summarize, today it should not be excluded, that in state-of-the-art experiments on GPS time transfer the effect of radiopulse dragging by moving atmosphere has been revealed. If so, one could, by taking this effect into account, ensure more high accuracy not only in clocks global synchronization via GPS satellites, but also in VLBI, in pulsar precise timing, etc. To accomplish that, one needs further theoretical and experimental development of the subject.

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