High Energy Elastic Scattering of Electrons on Protons

M. N. ROSENBLUTH
Stanford University, Stanford, California
(Received March 28, 1950)

The theory of the elastic scattering of electrons on protons at very high energies is discussed in detail. A formula is given for the cross section. This formula contains certain parameters which depend on the action of the virtual photon and meson fields. In particular, curves have been calculated on the assumption of scalar and pseudoscalar meson theory. While these perturbation theory calculations are not very trustworthy, and the results depend on the choice of coupling constants, it is felt that qualitative features can be checked with experiment. It is concluded that at low relativistic energies ($E < 50$ Mev) the experiment provides a valuable check on quantum electrodynamics. At higher energies it should yield data on the nature of the meson cloud of the proton.

I. INTRODUCTION

THE Stanford linear electron accelerator program is expected to make available large currents of relativistic electrons with various energies ranging from 6 to 1000 Mev. Among the experiments of considerable interest which may then be performed is the elastic scattering of electrons on protons. This may be done on a hydrogen gas or liquid target. Despite the smallness of the cross section at high energies, the expected large intensity of the beam should render the experiments possible.

It is the purpose of this paper to show that, at appropriate energies and angles, the experiment should give considerable information both about the validity of the "quantum electrodynamical radiative corrections" to scattering, and about the structure of the meson cloud associated with the proton.

Processes competing with electron-proton elastic scattering can be grouped into two classes: (a) those arising from electron-electron interactions; (b) other electron-proton processes. The electron-electron interactions have a much larger cross section at high energies than the electron-proton interactions. Background from the electron-electron interactions may be eliminated by (1) angular coincidences between the scattered particles, (2) energy selection of the scattered electron at a given angle [or a combination of (1) and (2)], or (3) direct observation of the recoil protons by photographic plates.

The competing electron-proton processes are bremsstrahlung and meson production. They will have cross sections comparable to the corrections to the elastic scattering which are discussed below. Methods (1) and (2), discussed above, would also eliminate background from these processes. If the proton is observed directly

a determination of its energy by grain counting and a correlation of energy and angle could be used to eliminate these processes. At very high energies it may prove experimentally impossible to separate the different electron-proton processes, in which case the bremsstrahlung and meson production must be added to the elastic scattering which is calculated in this paper.

II. ELASTIC SCATTERING OF AN ELECTRON AND PROTON

The elastic scattering of an electron and a proton can be represented schematically on a Feynman[1] diagram as in Fig. 1.

Figure 1 shows a proton of 4-momentum $\mathbf{p}_1$ and an electron of 4-momentum $\mathbf{p}_2$ exchanging a virtual quantum of 4-momentum $\mathbf{q} = \mathbf{p}_3 - \mathbf{p}_1 = \mathbf{p}_2 - \mathbf{p}_4$ and being scattered to momenta $\mathbf{p}_3$ and $\mathbf{p}_4$, respectively. Here M is the proton rest-mass, e'' is the effective charge of the electron, e' the effective charge of the proton, and $\kappa' e'/2M$ its effective anomalous magnetic moment. The effective

FIG. 1. Diagram for the elastic scattering of a physical proton and a physical electron. (The letter "q" with the bar through it in this figure is the same as the German letter, q, used in the text.)
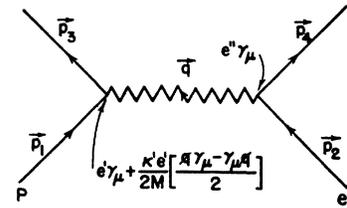
charges and magnetic moments are functions of $q^2 = q_4^2 - q_3^2 - q_2 - q_1^2$ as discussed below. The notation of a German letter, q, means $q_4\gamma_4 - q_3\gamma_3 - q_2\gamma_2 - q_1\gamma_1$, where the γ's are given in terms of the usual Dirac matrices by $(\gamma, \gamma_4) = (\beta\alpha, \beta)$.

The cross section for this process is computed by standard spur techniques to be

$$\sigma d\Omega = \left(\frac{e'e''}{2E}\right)^2 \operatorname{ctn}^2\frac{\theta}{2} \csc^2\frac{\theta}{2}\left\{\frac{1 + 2(E/M)\sin^2(\theta/2) + (E^2/M^2)[2(1+\kappa')^2\tan^2(\theta/2)\sin^2(\theta/2) + \kappa'^2\sin^2(\theta/2)]}{[1 + (2E/M)\sin^2(\theta/2)]^2}\right\} d\Omega. \quad (1)$$

[1] R. P. Feynman, Phys. Rev. **76**, 749 and 769 (1949). The methods of calculation and the notation used in this paper are just those of Feynman unless otherwise indicated. We also use natural units, $\hbar = c = 1$.

Here E is the energy of the incident electron and θ the angle through which it is scattered, both as measured in the system where the proton is initially at rest. The rest-mass of the electron has been neglected compared to its energy. We also introduce the useful parameter

$$q^2 = \frac{-4E^2 \sin^2(\theta/2)}{1+2(E/M)\sin^2(\theta/2)}. \qquad (2)$$

For electron energies small compared to the proton rest-mass Eq. (1) reduces just to the usual Mott-Rutherford formula for scattering of an electron by a fixed electrostatic potential. Since κ', the effective anomalous proton magnetic moment, may be larger than 1 (for $\mathbf{q}=0$, $\kappa'=\kappa_0=1.79$) the magnetic moment

FIG. 2. Diagram showing the effect of a virtual charged pseudoscalar meson on electron-proton scattering.

will play an important role in the very high energy region.

That the effective electron charge e'' differs from its "natural" value e is due to the so-called radiative correction to scattering,[2] i.e., to the possibility that the electron may emit and reabsorb a virtual quantum, or emit a low energy real quantum, during the scattering process. This modification has been treated extensively by Schwinger.[3] His formula is valid under the assumption that the proton acts as a fixed electrostatic potential. This is the case in the low energy region in which this is the most important correction term. At higher electron energies, the more exact expression could be derived by a modification of the radiative correction to Møller scattering, which has been calculated at Cornell.[4] Here we restrict ourselves to the remark that the Schwinger correction is a slowly varying function of angle and energy and corresponds to a decrease of the order of magnitude of five percent in the effective electron charge for the region of interest.

---

[2] Strictly speaking, we should also give the electron an anomalous magnetic moment, but this is quite small and decreases rapidly at high energy.
[3] J. Schwinger, Phys. Rev. **76**, 813 (1949). In our notation $(e''/e)^2 = e^{-\delta}$ where δ is given by Schwinger in his Eq. (2.105).
[4] R. P. Feynman (private communication).

III. MESON FIELD CORRECTIONS TO ELECTRON-PROTON SCATTERING

The modification of the proton charge and anomalous magnetic moment is here assumed to be caused by the action of a virtual meson field. At electron energies small compared with the meson rest-mass these modifications will be small. Thus, at low energies the scattering will give us information concerning chiefly the radiative corrections to scattering; at higher energies we may expect to learn something of the nature of the meson cloud which surrounds the proton.

The action of a scalar meson field in modifying the effective proton charge and magnetic moment can be understood qualitatively by assuming that during a fraction R of the time the proton exists as a neutron and a positive meson. Its charge and anomalous moment then will be spread out like $e^{-2\mu}/r^2$ (the square of the meson wave function) where μ is the mass of the meson. Thus a high energy electron is able to penetrate the meson cloud and hence see a smaller effective charge and magnetic moment. Under these assumptions Schiff[5] has given the effective charge and magnetic moment to be

$$(e'/e) = [(1-R)+R2\mu/(-q^2)^{\frac{1}{2}}\tan(-q^2)^{\frac{1}{2}}/2\mu], \qquad (3)$$
$$(\kappa'e'/\kappa_0 e) = 2\mu/(-q^2)^{\frac{1}{2}}\tan^{-1}(-q^2)^{\frac{1}{2}}/2\mu,$$

where q^2 is given by Eq. (2).

We will here calculate in the covariant manner of Feynman the effective charge and magnetic moment of the proton as given by four theories: Neutral and charged scalar mesons with scalar-coupling, neutral and charged pseudoscalar mesons with pseudoscalar coupling. The results for symmetrical theories may be obtained simply by adding the results for charged and neutral theories. Other meson theories lead to divergent results.

To illustrate the method, we will discuss briefly the case of charged pseudoscalar theory. The effect of the virtual mesons on the scattering is shown in Fig. 2.

Figure 2(a) shows the usual electromagnetic interaction between two Dirac-type particles of charge e and e''. Figure 2(b) shows the proton emitting a positive meson which absorbs the virtual photon and is then reabsorbed by the neutron. Figure 2(c) shows the meson being emitted and reabsorbed before the scattering takes place. Figure 2(d) shows the virtual emission and reabsorption taking place after the scattering. Here g is the meso-nuclear coupling constant; $\gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_4$; the factor 2 is inserted for simplicity in later discussing symmetrical theory; and the $2k_\mu+q_\mu$ at the meson-quantum vertex reflects the fact that a Klein-Gordon particle interacts with the electromagnetic field through the terms $i\partial(A_\mu\psi)/\partial x_\mu+iA_\mu\partial\psi/\partial x_\mu$ where ψ is the meson wave function, and A_μ the electromagnetic potential.

We endeavor to show that adding the diagrams 2(a) to 2(d) produces a situation like that in Fig. 1, and to

---

[5] L. I. Schiff, Stanford Microwave Laboratory Report No. 102, p. 8 (1949).

deduce the values of e' and κ'. For the case of $\mathbf{q}$, the photon momentum, equal to zero, diagrams (b), (c), and (d) are found to add to zero as might be expected, since there is then no scattering process. (There are non-essential mass renormalization terms but they do not concern us.) Moreover, the value of $\mathbf{q}$ does not affect the proton-meson parts of diagrams (c) and (d). Therefore we can obtain the final proton-meson portion of the amplitude by adding the proton-meson parts of diagrams (a) and (b) and subtracting off the value of diagram (b) for $\mathbf{q}=0$.

The proton-meson amplitude matrix from diagram (a) is simply $e\gamma_\mu$, that from diagram (b) can be written:[6]

$$e\frac{2g^2}{\pi i}\int d^4\mathbf{k}\frac{\gamma_5(\mathfrak{p}_1-\mathfrak{k}+M)\gamma_5(2k_\mu+q_\mu)}{[(\mathbf{p}_1+\mathbf{k})^2-M^2][k^2-\mu^2][(\mathbf{k}+\mathbf{q})^2-\mu^2]}.$$

Here μ is the meson rest-mass; the integration is to be performed over all virtual mesons; and we are interested in the element between initial and final proton states of this matrix.

After the integral over the virtual mesons is performed, and the amplitudes from diagrams (a) and (b) added, with the $\mathbf{q}=0$ value of diagram (b) subtracted, we obtain as final amplitude:

$$e\gamma_\mu\left\{1-\frac{g^2}{2\pi}\int_0^1 dx\int_0^1 dy\left[\frac{1}{2}\ln\left(1+\frac{u}{a}\right)\right.\right.$$
$$\left.\left.-\frac{u[3y^3-(5-(a/2))y^4+2y^5]}{[(1-y)^2+uy^2+ay][(1-y)^2+ay]}\right]\right\}$$
$$+\frac{e}{2M}\left[\frac{\mathfrak{q}\gamma_\mu-\gamma_\mu\mathfrak{q}}{2}\right]\frac{g^2}{\pi}\int_0^1 dx\int_0^1 dy\left(\frac{y(1-y)^2}{(1-y)^2+uy^2+ay}\right). \quad (4)$$
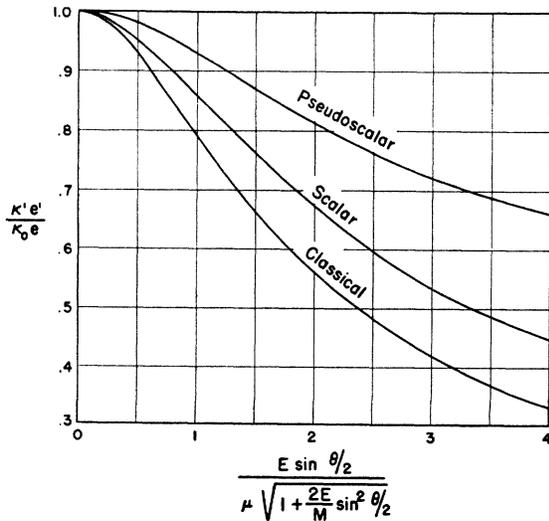
FIG. 3. Ratio of effective anomalous proton magnetic moment to its zero-energy value for charged meson theories.

[6] See Feynman, reference 1, for a full discussion of the method notation, and calculation techniques. In particular, the Appendix, p. 785, gives a full discussion of the evaluation of the radiative correction to scattering integral which is very analogous to our case.
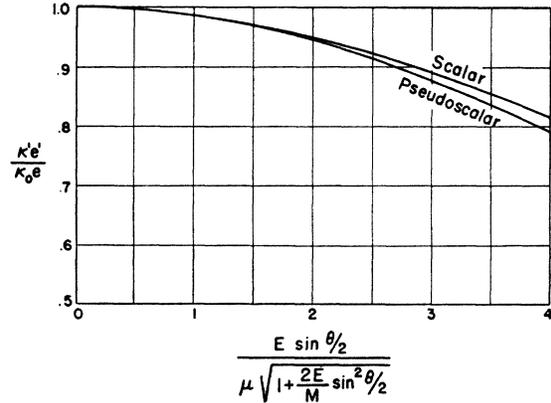
FIG. 4. Ratio of effective anomalous proton magnetic moment to its zero-energy value for neutral meson theories.

Here x and y are integration parameters, $a=\mu^2/M^2$; $u=q^2(x^2-x)/M^2$. The first term here represents the effective charge of the proton, the second its anomalous magnetic moment. For $u=0$ this term gives just the value for the anomalous moment previously derived by Case.[7]

For the case $u\neq0$ the integrals are very complicated. The integral on y can be performed analytically. The remaining integral on x then depends on the parameters q^2/μ^2 and q^2/M^2. If q^2 is of comparable order of magnitude to μ^2, but much smaller than M^2, a region of considerable interest, the integral may be expanded to first order in the parameter q^2/M^2 and then performed analytically. For larger values of q^2 it must be carried out numerically. The other meson theories require the same type of calculation.

Figures 3, 4, 5, and 6 give the results of these integrations.

Figures 3 and 4 are graphs of $\kappa'e'/\kappa_0e$, the ratio of the effective anomalous magnetic moment to the zero-energy anomalous moment. On Fig. 3 we have also plotted the "classical" formula (3). We have assumed in all calculations that μ, the meson mass, is 276 electron masses, consistent with experimental values for the π-meson. These ratios are independent of the coupling constant, which will determine only the magnitude of the zero-energy moment. It should be noted, however, that the scalar charged and pseudoscalar neutral theories predict the wrong sign for the proton moment.

Figures 5 and 6 are graphs of the effective proton charge. For reasons discussed below, we have plotted $e'/e=e^{-\delta}$, rather than $e'/e=1-\delta$ as obtained directly from (4). As can be seen from (4), δ is directly proportional to g^2. We have plotted $e^{-\delta}$ for those values of g^2 necessary to predict the correct value for the magnitude of the zero-energy proton moment. These values are given in Table I.

To illustrate the use of the graphs, and to show how they may be adapted to symmetrical theory, let us

[7] K. Case, Phys. Rev. 76, 6 (1949).
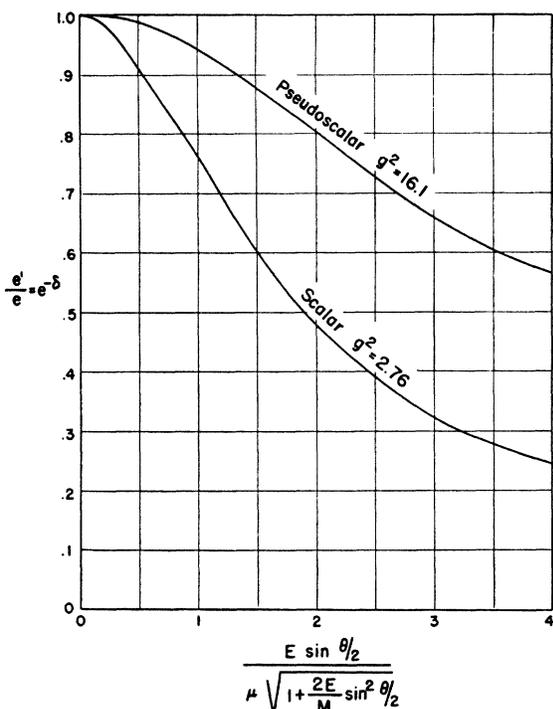
Fig. 5. Effective proton charge for charged meson theories with coupling constants chosen to fit the magnitude of the observed proton anomalous magnetic moment.

calculate the effective charge and magnetic moment for a 500-Mev electron scattered through 90° on the basis of symmetrical pseudoscalar theory with coupling constant 57.2. The abscissa

$$\left(E \sin\frac{\theta}{2} \right) \Big/ \mu\left(1+\frac{2E}{M}\sin^2\frac{\theta}{2} \right)^{\frac{1}{2}} = (-q^2)^{\frac{1}{2}}/2\mu$$

in this case is equal to 2.03. Since the neutral and charged theories give opposite signs for the magnetic moment:

$$(\kappa' e')_s = (\kappa' e')_c - (\kappa' e')_n.$$

Using Table I and Figs. 3 and 4,

$$(\kappa' e'/\kappa_0 e)_s = 0.81(57.2/16.1) - 0.94(57.2/22.4) = 0.48.$$

To obtain the effective charge:

$$\delta_s = \delta_c + \delta_n = 0.22(57.2/16.1) + 0.15(57.2/22.4) = 1.16,$$
$$(e'/e)_s = \exp(-\delta_s) = 0.31.$$

To obtain the final cross section these values for e' and κ', and Schwinger's[3] value for e'' are substituted in (1).

It will be noted that symmetrical theory predicts a rapid dropping off of magnetic moment and charge due to the large coupling constant.

IV. CONCLUSIONS

It will be noted at once that the values of g^2 listed in Table I are so large as to throw grave doubts on the use of second-order perturbation theory. This is especially true for the pseudoscalar case where we expect second-order terms to be small compared to higher order terms. In this connection it may be noted that the values of g^2 listed in Table I do not give the correct neutron moment. Some justification for the perturbation theory procedure may be found in the fact that experimental results on photo-meson production do seem to agree with the qualitative predictions of second-order pseudoscalar perturbation theory.[8] (There has been no effort to measure the absolute cross section so that no experimental value of g^2 is obtained.) It is because of doubt of the adequacy of the second-order theory that we have plotted $e'/e = \exp(-\delta)$, thus considering at least some of the higher order terms.

It will be noted that even though the meson clouds are more tightly bound than a naive picture would predict (see Fig. 3), there is nonetheless a very sizeable decrease in proton charge and magnetic moment to be expected at high energies. This is especially true if we assume the large values of coupling constant necessary to predict the proper proton moment. Even if only the qualitative features of these curves are dependable the experimental results should at least indicate (1) if the proton magnetic moment is really due to the π-meson
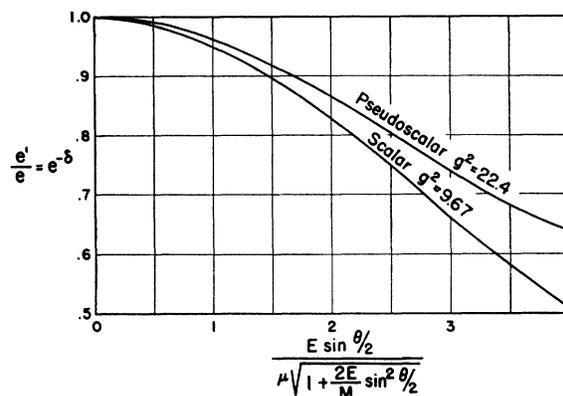
Fig. 6. Effective proton charge for neutral meson theories with coupling constants chosen to fit the magnitude of the observed proton anomalous magnetic moment.

Table I. Coupling constants necessary for correct magnitude of proton magnetic moment.

Theory	Scalar charged	Scalar neutral	Scalar symmetrical	Pseudo-scalar charged	Pseudo-scalar neutral	Pseudo-scalar symmetrical
g^2	2.76*	9.67	3.86*	16.1	22.4*	57.2

* Indicates wrong sign for magnetic moment.

[8] J. Steinberger, experiments performed at Berkeley and not yet published.

field, (2) whether a loose-bound scalar type theory or a tight-bound pseudoscalar-type theory is preferable, and (3) how much faith can be placed in the "quantum electrodynamical radiative corrections" to scattering and in the far more dubious second-order meson corrections to scattering.

I would like to thank Professor L. I. Schiff for many helpful discussions and suggestions. I would like to express my gratitude also to Dr. Ross Thompson of Cornell University who has performed most of these calculations independently, and who very kindly checked, and improved upon one of, my results.

Anomalous Molecular Rotation and the Temperature of the Upper Atmosphere*

LEWIS M. BRANSCOMB**
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts
(Received May 12, 1950)

This experiment verifies the prediction of Oldenberg that the spectroscopically measured rotational temperature of a diatomic gas will be lower than the translational temperature when (1) the pressure is low, (2) the gas is excited by electron impact, and (3) the excited electronic state from which the measured bands are radiated has an equilibrium nuclear separation greater than the internuclear distance in the ground state. For gas temperatures from 400 to 670°K rotational temperatures from the second negative bands of O_2^+ were found in qualitative agreement with the predicted relation $T_{rot} = T_{trans} B'/B''$. Upper atmosphere temperatures derived from band profiles in night sky spectra are consistently lower than temperatures estimated from other data. The possible occurrence of anomalous rotation of the night sky molecules casts some doubt on the meaningfulness of the night sky temperature measurements. A partial rotational analysis in the course of this experiment suggests revisions of the B_0 and α-values for the O_2^+ molecule in the $^2\Pi_u$- and $^2\Pi_g$-states.

I. INTRODUCTION

A KNOWLEDGE of the molecular density and temperature at high altitudes is fundamental to an understanding of the processes occurring in the earth's upper atmosphere. For the region above 80 km the atmospheric temperature is deduced from indirect evidence from various sources. Unfortunately most of the data is only qualitative and, to make matters worse, much of it is contradictory. Evidence for a steadily increasing temperature above 80 km (a gradient of perhaps 4°/km) is found in the relative widths of the ionosphere layers, the apparent scarcity of helium at high altitudes, and the slow decrease in density at very great heights (as is indicated by high altitude auroral rays).[1] On the other hand, the more direct spectroscopic measurements on bands in the spectra of the aurora and the night sky luminescence are interpreted by some authors as conclusive evidence for a constant temperature of about 250°K above 90 km, a result which seems quite incompatible with the other temperature estimates.

These spectroscopic temperature measurements are based on the relationship between the equilibrium temperature of a radiating diatomic gas and the relative intensities of the lines in the rotational fine structure of an emission spectrum. The relative intensities of the rotational lines depend on two quantities: the relative transition probabilities of the lines, and the population of the initial rotational levels. If the temperature dependence of the populations of the initial rotational states is known, one can calculate the transition probabilities from quantum theory and can then calculate the gas temperature from the experimental relative intensities of the lines in a band.

In the measurements on upper atmospheric spectra it was assumed that the excited molecules in the aurora and night sky are in a Boltzmann distribution with respect to rotational energy, at an equilibrium temperature T. The significance of the spectroscopic temperature measurements depends on the validity of this assumption. Laboratory investigations of bands excited by electron impact in a glow discharge have afforded many examples of cases in which the excited radiating molecules are indeed in a Boltzmann rotational energy distribution, for the spectroscopically determined temperatures were quite close to the directly measured gas temperatures.

In 1934 Oldenberg[2] pointed out that although the molecules in a low pressure glow discharge are initially in thermal equilibrium, the assumption of thermal equilibrium in the *excited* rotational states is justified only for molecules whose nuclear separation is the same in the excited and ground states. Although this is usually the case, there are certain molecules in which the excited state has a much larger nuclear separation

---

* This work is described in detail in the author's Ph.D. thesis, Harvard University (1949).
** Now a member of the Society of Fellows, Harvard University.
[1] S. K. Mitra, *The Upper Atmosphere* (the Royal Society of Bengal, Calcutta, 1947); G. P. Kuiper, *The Atmosphere of the Earth and Planets* (University of Chicago Press, Chicago, 1947). See especially chapters by P. Swings and L. Spitzer, Jr.
[2] O. Oldenberg, Phys. Rev. 46, 210 (1934).