XXVI. *On the Longitudinal Component in Light.* By Prof. George Francis FitzGerald, *M.A., Sc.D., F.R.S., F.T.C.D.**

IN most investigations on the propagation of light attention has been concentrated on the transverse nature of the vibration. Longitudinal motions have been relegated to the case of pressural waves, and investigators have devoted themselves to separating the two as much as possible. In Sir George Stokes's classical paper on Diffraction, and in Lord Kelvin's Baltimore Lectures, the existence of a longitudinal component is mentioned; but it is mentioned only to show that it is very small and that the motion is mostly transverse. Now the longitudinal component is no doubt generally small except in the immediate neighbourhood of a source; but it by no means follows that, as a consequence, the actual direction of motion is transverse at all points in a wave. In every complicated wave there are points and often lines along which the transverse component vanishes, and at all these places the small longitudinal component may be, and often is, of great relative importance, so that the actual motion is largely in the direction of wave propagation at these places.

I. The simplest case is that of a simple oscillator whose theory has been completely worked out by Hertz. There are two kinds of oscillator, an electric and a magnetic one. They are exactly complementary, the magnetic forces in one corresponding exactly with the electric forces in the other.

If the oscillator be taken as an electric one parallel to z, we have for the components of the vector potential

$$\mathrm{F}=\mathrm{G}=0, \quad \mathrm{H}=\mathrm{H}_0\frac{\cos\overline{pt-qr}}{r};$$

and the components of the electric force, which are in general

$$\dot{\mathrm{P}}=\Delta^2\mathrm{F}-\frac{d\mathrm{J}}{dx}, \quad \dot{\mathrm{Q}}=\Delta^2\mathrm{G}-\frac{d\mathrm{J}}{dy}, \quad \dot{\mathrm{R}}=\Delta^2\mathrm{H}-\frac{d\mathrm{J}}{dz},$$

where

$$\mathrm{J}=\frac{d\mathrm{F}}{dx}+\frac{d\mathrm{G}}{dy}+\frac{d\mathrm{H}}{dz},$$

become in this case

$$\dot{\mathrm{P}}=-\frac{d^2\mathrm{H}}{dz\,dx}, \quad \dot{\mathrm{Q}}=-\frac{d^2\mathrm{H}}{dz\,dy}, \quad \dot{\mathrm{R}}=\frac{d^2\mathrm{H}}{dx^2}+\frac{d^2\mathrm{H}}{dy^2}.$$

It is particularly to be observed that $\dot{\mathrm{P}}$ and $\dot{\mathrm{Q}}$ arise entirely from J, which was dismissed by Maxwell as not coming into consideration in cases of wave propagation on account of there being no varying electrification. This is true as regards

* Communicated by the Author.

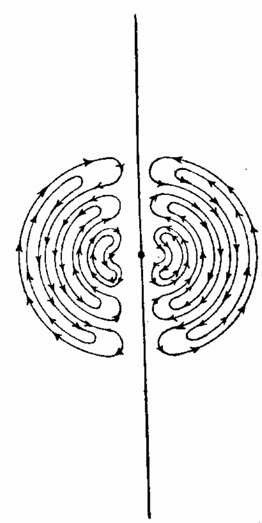
propagation, but not at all as regards origination. In all cases of origination we have to do with conduction, or its equivalent convection, and in most such cases we have changing electrification which brings in the J term.

The longitudinal component at each point is

$$\dot{\sigma} = \frac{x}{r}.\dot{P} + \frac{y}{r}.\dot{Q} + \frac{z}{r}.\dot{R}$$

$$= \frac{2z}{r^3}\left(2q \sin \overline{pt - qr} + \frac{1}{r}\cos \overline{pt - qr}\right).$$

This is no doubt very small at a distance from the oscillator compared with the transverse component which involves $\frac{1}{r}$, and in consequence the motion is transverse at most places. On the axis of z, however, the transverse component, which is proportional to ρ the distance from the axis, vanishes entirely. Hence along the axis there is a beam of purely longitudinal vibration, of no doubt small amplitude, but nevertheless existing necessarily in order that there may be no compressions. This all appears on the face of Hertz's investigation. He carefully studied the forces as represented by the above equations, and has plotted them and shown that they represent a series of whirl rings thrown off from the oscillator and growing gradually thinner and thinner until at a distance the rings become nearly plane waves, and the opposite sides being always a wave-length apart are the two opposite phases of the wave. The accompanying diagram roughly represents this state of affairs. It is evident on the most cursory consideration that these waves must have a longitudinal region. The lines of force in any one wave are up to the axis along any one spherical surface all round; and if there is not to be concentration anywhere, *i. e.* if there is no electrification of the medium, they *must* turn round and be continuous with the return phase of the wave. The reason why they are so feebly concentrated in this return region is because it is so enormously extended. If the wave-length be small compared with the distance from the origin, the flows



up and down along the equator are very close to one another and consequently the force is concentrated ; while this same force which is concentrated within a wave-length has the whole hemisphere to return in, and so the longitudinal concentration is quite small, and that is what is represented by the small value of the longitudinal component at any point. The total quantity of longitudinal component must be, on the whole, equal to the transverse component at the equator.

II. In the case of several simple oscillators oriented in different directions the resultant vector potential can be represented by

$$A = U\frac{\cos(pt-qr)}{r} + V\frac{\sin\overline{pt-qr}}{r},$$

where U and V are vectors at right angles to one another. The effect is the same as if two opposite electrons were moving on opposite sides in an elliptic orbit whose plane was that of U and V and whose axes were these two lines.

It is interesting to observe that this case, coupled with a slow rotation of the ellipse which would be produced by almost any small disturbing force in its plane, has been shown by Dr. Stoney to be a sufficient cause for the double lines in spectra which are so common and which are familiar to everyone in the double sodium line.

If the directions of U and V be taken as those of x and y, and z be taken perpendicular to the plane of this ellipse, we may take

$$F = F_0\frac{\cos(pt-qr)}{r}, \quad G = G_0\frac{\sin(pt-qr)}{r}, \quad H = 0,$$

and we get a sort of corkscrew wave with a longitudinal component which can be represented by

$$\dot{\sigma} = \frac{\sin\phi}{r^2}\{L\cos(pt-qr+l)\};$$

where ϕ is the angle between r and z, and L and l are functions of F_0, G_0, r, θ, and q.

This component vanishes along the axis perpendicular to the plane of the ellipse, and is a maximum in this plane. If $F_0 = G_0$, this simplifies to

$$\dot{\sigma} = \frac{\sin\phi}{r^2}\{2q\sin(pt-qr-\theta) + \frac{1}{r}\cos(pt-qr-\theta)\}.$$

This case is rather interesting, as being the form of magnetic wave that is thrown off into space by the rotation of each of the earth's magnetic poles.

The more complex wave thrown out by the earth with its two magnetic poles comes under the next head; but it is waves of this type which must be thrown off by the planets rotating round the sun, if they are electrified, and by their gravitating property if gravitation be propagated in the same way as electromagnetic disturbances.

III. We can produce any desired combination of complex doublets by operating on a simple doublet with a function of $\left(\frac{d}{dx}, \frac{d}{dy}, \frac{d}{dz}\right)$. The typical term of such a function may be taken as

$$\left(\frac{d}{dx}\right)^{\alpha}\cdot\left(\frac{d}{dy}\right)^{\beta}\cdot\left(\frac{d}{dz}\right)^{\gamma}=\delta.$$

If we write

$$u=\frac{\cos(pt-qr)}{r},$$

we get as a typical case,

$$\mathrm{F}=\delta u, \quad \mathrm{G}=\mathrm{H}=0.$$

Also, remembering that $\Delta^2 u+q^2.u=0$, we have for the electric force corresponding to this typical case of a vector potential,

$$\dot{\mathrm{P}}=-q^2\delta u-\delta\frac{d^2u}{dx^2}, \quad \dot{\mathrm{Q}}=-\delta\,\frac{d^2u}{dx\,dy}, \quad \dot{\mathrm{R}}=-\delta\,\frac{d^2u}{dx\,dz}.$$

Now this operation will introduce all sorts of powers of $\frac{1}{r}$ and of q, and I only want to calculate the principal term in the longitudinal component. In making this approximation we may simplify the calculation by observing that the largest terms are always due to differentiations with respect to the circular part of u, and that differentiation with respect to x, y, z, or r lowers a term by one. We may then leave out all differentiation with respect to coordinates outside the circular part in terms of the second order, and it is well to reduce the differentiations represented by δ so as to produce terms of the form $\delta\,\frac{\cos(pt-qr)}{r}$ and $\delta\,\frac{\sin(pt-qr)}{r}$. Of course it very much simplifies calculation to use the typical form e^{-iqr} for the circular functions.

We thus get for the values of the components of electric force to the second order :—

$$\dot{P}=-q^2.\frac{y^2+z^2}{r^2}.\delta\frac{\cos\overline{pt-qr}}{r}$$

$$-\frac{q}{r}\left\{1-2(\beta+\gamma)-\frac{y^2+z^2}{r^2}(\alpha+\beta+\gamma-3)\right\}\delta\frac{\sin\overline{pt-qr}}{r},$$

$$\dot{Q}=q^2.\frac{xy}{r^2}.\delta\frac{\cos\overline{pt-qr}}{r}$$

$$-\frac{q}{r}\left[\frac{\alpha y}{x}+\frac{\alpha x}{y}-\frac{2}{r^2}\left\{\alpha x^2+\beta y^2+(\gamma-\tfrac{3}{2})xy\right\}\right]\delta\frac{\sin\overline{pt-qr}}{r},$$

$$\dot{R}=q^2.\frac{xz}{r^2}.\delta\frac{\cos\overline{pt-qr}}{r}$$

$$-\frac{q}{r}\left[\frac{\alpha z}{x}+\frac{\alpha x}{z}-\frac{2}{r^2}\left\{\alpha x^2+\gamma z^2+(\beta-\tfrac{3}{2})xz\right\}\right]\delta\frac{\sin\overline{pt-qr}}{r}.$$

In this form it is evident at once that the highest terms vanish in the longitudinal component

$$\dot{\sigma}=\dot{P}\frac{x}{r}+\dot{Q}\frac{y}{r}+\dot{R}\frac{z}{r}$$

$$=\frac{q}{r^4x}\Big[(x^2+y^2+z^2)\left\{x^2(1-\beta-\gamma)+\alpha(y^2+z^2)\right\}-2x\Big\{\alpha x^2(y+z)$$

$$+y^2(\overline{\alpha+\beta+2\gamma-\tfrac{9}{2}}x+\beta y)$$

$$+z^2(\overline{\alpha+2\beta+\gamma-\tfrac{9}{2}}x+\gamma z)\Big\}\Big]\delta\frac{\sin\overline{pt-qr}}{r}.$$

In order to get this we have to observe that when applied to the circular part only

$$\delta=\left(\frac{qx}{r}\right)^{\alpha}.\left(\frac{qy}{r}\right)^{\beta}.\left(\frac{qz}{r}\right)^{\gamma}.$$

Any particular typical term of this order vanishes over the quartic cone the coefficient of $\delta\frac{\sin\overline{pt-qr}}{r}$.

This is the cone of intersections of the systems of spheres

$$x^2+y^2+z^2=2\theta.x$$

with the cubics

$$\theta\left\{x^2(1-\beta-\gamma)+\alpha(y^2+z^2)\right\}=\alpha x^2(y+z)+y^2\left\{(\alpha+\beta+2\gamma-\tfrac{9}{2})x+\beta y\right\}$$
$$+z^2\left\{(\alpha+2\beta+\gamma-\tfrac{9}{2})y+\gamma z\right\}.$$

In the particular case of a series of end-on vibrators for which $\beta=\gamma=0$, this cubic breaks up into the plane $x=0$ and the quadric cone

$$y^2+z^2=\frac{\alpha}{\alpha(\theta-1)+\frac{9}{2}}.x(y+z).$$

In every case the quartic cone intersects every plane perpendicular to the axis of x in a bicircular quartic.

In the case of a complex oscillator whose components are not all parallel to the axis of x, as in the case just studied, the longitudinal component will vanish to this order over a quartic cone so long as we confine ourselves to a typical term δ. This cone is of the form

$$U \equiv (x^2+y^2+z^2)(a_1x^2+b_1y^2+c_1z^2) - \{ax^3(y+z)+by^3(z+x)+cz^3(x+y) + ly^2z^2+mz^2x^2+nx^2y^2 + xyz(px+qy+rz)\}.$$

In general it is quite evident that the motion along the radius does not vanish.

On considering the general case, we may observe that if the differentiations involved in δ are such that for every term $\alpha+\beta+\gamma$ is either even or odd, then there will be a complex surface all over which the normal component will vanish to the second order of small quantities; but that if $\alpha+\beta+\gamma$ be even in some terms and odd in others, we shall have $\dot{\sigma}$ of the form

$$\dot{\sigma} = U \cos \overline{pt-qr} + V \sin \overline{pt-qr},$$

and this will only vanish over the curve of intersection of $U=0$ and $V=0$.

IV. If we now consider the case of diffraction through a narrow aperture, it is simpler to take the case of the electric displacement of the incident wave as parallel to the edges. In this case the electric force is everywhere parallel to the edge, and consequently its longitudinal component everywhere vanishes. On the other hand the magnetic force is perpendicular to the slit and has a longitudinal component everywhere except in the plane through the slit perpendicular to the wave-face. In considering the more complicated case of the electric displacement being perpendicular to the slit, it is necessary to take account of the nature of the edges, whether they are non-conductors or conductors, whether they are crystalline, and so forth, because their electrification &c. must come into consideration. Similarly, in the case of the electric displacement being parallel to the slit the magnetic properties of the edges may be important. In this case, too, their conductivity influences the effective width of the slit, as is evidently the case when we are dealing with wire gratings in the path of the Hertzian radiations. These questions are involved in a complicated way in the whole discussion of the effect of a grating on the plane of polarization of the incident light. I will take the simple case of a slit bounded by obstacles which completely stop all action. Although such do not exist, very close approximations to them do exist.

If we take the slit as parallel to z, and make this axis the centre of the slit, and assume the phase the same all over the slit, we have for the vector potential at a point x_0, y_0, 0, due to any line of the slit at a distance y from its centre,

$$h=2\mathrm{H}_0\int_0^\infty \frac{\cos \overline{pt-qr}}{r}\,dz,$$

where

$$r^2=x_0{}^2+\overline{y_0-y}^2+z^2.$$

Integrating this for the width of the slit, *i. e.* from $+b$ to $-b$, we get for the complete value of the vector potential

$$\mathrm{H}=2\mathrm{H}_0\int_{-b}^{+b}\int_0^\infty \frac{\cos \overline{pt-qr}}{r}\,dz\,dy.$$

When we are dealing with the case of b being a small quantity we may take

$$\int \frac{\cos \overline{pt-qr}}{r}\,dy=f(y_0-y),$$

and we have

$$\int_{-b}^{+b} \frac{\cos \overline{pt-qr}}{r}\,dy=f(y_0+b)-f(y_0-b)$$
$$=2b\left(\frac{df}{dy}\right)_0+\frac{2b^3}{1\,.\,2\,.\,3}\left(\frac{d^3f}{dy^3}\right)_0+\ldots$$

But

$$\left(\frac{df}{dy}\right)_0=\frac{\cos\,(pt-qr)}{r}=u,$$

and when $y=0$ is put in

$$r^2=x_0{}^2+y_0{}^2+z^2=\rho^2+z^2,$$

$$\therefore \qquad \int_{-b}^{+b} u\,dy=2\left(bu+\frac{b^3}{1\,.\,2\,.\,3}\cdot\frac{d^2u}{dy^2}+\ldots\right).$$

If we now integrate with respect to z we get

$$\mathrm{H}=2\mathrm{H}_0\int_0^\infty\int_{-b}^{+b} u\,dy\,dz=4\mathrm{H}_0\left(b\int_0^\infty u\,dz+\frac{b^3}{1.2.3}\cdot\frac{d^2}{dy^2}\int_0^\infty u\,dz+\ldots\right).$$

Now $\int_0^\infty u\,dz$ is a function of ρ only, and is a Bessel function subject to the equation

$$\frac{d^2J}{d\rho^2}+\frac{1}{\rho}\frac{dJ}{d\rho}+q^2J=0,$$

so that we can write

$$H=4H_0\left(J+\frac{b^3}{1.2.3}\frac{d^2J}{dy^2}+\dots\right)$$

$$=4H_0\left[J+\frac{b^3}{1.2.3}\left\{\frac{1}{\rho}\frac{dJ}{d\rho}+\frac{y^2}{\rho^2}\left(\frac{d^2J}{d\rho^2}-\frac{1}{\rho}\frac{dJ}{d\rho}\right)\right\}+\dots\right].$$

By means of the differential equation we may of course express all the differentials of J in terms of J and $\frac{1}{\rho}\frac{dJ}{d\rho}$. We may, however, simplify matters very much in the ordinary case of light by observing that q is generally a very large number, so that terms involving its powers are large. Keeping to these we see that $\frac{d^2J}{d\rho^2}=-q^2J$, and that the highest term in $\frac{d^nJ}{d\rho^n}$ is $\left(\frac{qy}{\rho}\right)^nJ$. Using these terms only we get

$$H=4H_0J\left(b-\frac{b^3}{1.2.3}\cdot\left(\frac{qy}{\rho}\right)^2+\dots\right)$$

$$=4H_0J\cdot\frac{\sin\frac{bqy}{\rho}}{\frac{qy}{\rho}}=4H_0Jb\cdot\frac{\sin\epsilon}{\epsilon}$$

if $\epsilon=\frac{bqy}{\rho}$.

Without going into the question as to the best series to express J by it is evident from its integral form and from the dynamics from which it is derived that it must represent a wave propagation. In fact by integrating by parts it could be expanded in the form

$$J=J_1\cos(pt-q\rho)+J_2\sin(pt-q\rho).$$

In any case we can see that for any constant value of ρ H passes through a series of values giving the alternate lights and darks on a screen illuminated by a narrow slit.

Considering now the magnetic force we have

$$\alpha=\frac{dH}{dy},\quad\beta=-\frac{dH}{dx},\quad\gamma=0,$$

and hence the longitudinal magnetic component

$$m=\frac{d\mathrm{H}}{dy}\cdot\frac{x}{\rho}-\frac{d\mathrm{H}}{dx}\cdot\frac{y}{\rho}.$$

From this it is evident that in every such case $m=0$ so far as H is a function of ρ only. Thus we get

$$m=4\mathrm{H}_0\mathrm{J}\frac{l^2q}{\rho}\cdot\frac{x}{\rho}\frac{\epsilon\cos\epsilon-\sin\epsilon}{\epsilon^2}.$$

This shows that m does not in general vanish but has alternations of value like H. The tangential component has for its most important term

$$\tau=4\mathrm{H}_0\frac{d\mathrm{J}}{d\rho}\frac{\sin\epsilon}{\epsilon}.$$

It is evident that this longitudinal displacement is necessary at the edge of the beam in order to prevent any concentration of the magnetic force. So far as our *à priori* knowledge of pure æther is concerned there seems no sufficient reason for not supposing a concentration of magnetic force just as probable as one of electric force. It would certainly complicate our equations very much to suppose both. If both existed we might have two kinds of pressural waves, one a wave of electric condensation and rarefaction, and the other a wave of magnetic condensation and rarefaction.

It is quite evident from all these cases and from general considerations that the edge of every beam of light is bordered by a region where there are longitudinal vibrations taking place.

V. As a final example I take the case of a series of slits forming an optical grating.

In this case the simplest supposition is to assume that the opacity of the grating varies in a simply periodic manner. This leads to the same sort of equation for H as in the last case except that the intensity in each line is proportional to $(1+\cos ly)$, where $l=\frac{2\pi}{s}$ and s is the interval between the lines.

This leads to the integral

$$\mathrm{H}=2\mathrm{H}_0\int_0^\infty\int_0^\infty\frac{(1+\cos ly)\cos(pt-qr)}{r}\,dy\,dz,$$

where

$$r^2=x_0^2+\overline{y_0-y}^2+z^2.$$

Now from general considerations it is evident that it must be possible to expand this in terms of cos ly by Fourier's theorem, so that

$$H = h_0 + h_1 \cos ly + h_2 \cos 2ly + h_3 \cos 3ly + \ldots$$

Observing then that H being a function of x_0 and y_0 only satisfies the equation

$$\frac{d^2H}{dx_0^2} + \frac{d^2H}{dy_0^2} + q^2H = 0,$$

we get that in general

$$\frac{d^2h_n}{dx_0^2} + (q^2 - n^2l^2)h_n = 0,$$

so that

$$h_n = H_n \cos \sqrt{q^2 - n^2l^2} \,.\, x,$$

so long as nl is $< q$; and when nl is $> q$

$$h_n = H_n e^{-\sqrt{n^2l^2 - q^2}x},$$

as the value cannot increase to infinity.

We thus get the general form for H,

$$\begin{aligned} H = H_0 \cos(pt - qx) &+ H_1 \cos ly \cos(pt - \sqrt{q^2 - l^2}x) + \ldots \\ &+ H_n \cos nly \cos(pt - \sqrt{q^2 - n^2l^2} \,.\, x) + \ldots \\ &+ H_m \cos mly\, e^{-\sqrt{m^2l^2 - q^2}x} \cos pt + \ldots \end{aligned}$$

It would appear from this that at the surface of the grating, where $x=0$ when $t=0$,

$$H = H_0 + H_1 \cos ly + \ldots + H_n \cos nly + \ldots$$

It would consequently seem that this must in general represent the distribution of opacity at the grating, and that in the case of a simply periodic distribution the general form of H would be

$$H = H_0 \cos(pt - qx) + H_1 \cos ly \cos(pt - \sqrt{q^2 - l^2} \,.\, x).$$

We thus get an interesting form for the double integral for H.

The magnetic force to be calculated from this is

$$\alpha = \frac{dH}{dy}, \quad \beta = -\frac{dH}{dx}, \quad \gamma = 0,$$

and consequently

$$\alpha = -lH_1 \sin ly \cos(pt - \sqrt{q^2 - l^2} \,.\, x),$$

$$\beta = qH_0 \sin(pt - qx) + \sqrt{q^2 - l^2}H_1 \cos ly \sin(pt - \sqrt{q^2 - l^2}x).$$

In this α is the longitudinal component of the magnetic force. This represents a series of waves being propagated a way from the grating, together with a series of elliptic whirls whose length is $\frac{2\pi}{l}$ and breadth $\frac{2\pi}{\sqrt{q^2-l^2}}$. The length is the same as the width of the lines of the grating, and the breadth somewhat greater than the length of a wave $=\frac{2\pi}{q}$. It is especially obvious in this case that some longitudinal component exists.

The existence of the terms depending on $e^{-\sqrt{m^2l^2-q^2}x}$ shows that there may be something analogous to total reflexion with its extinction wave in the case of a grating in respect of the spectra that are of a higher order than can be transmitted by the grating. It would seem, then, that the whole energy of the wave might not be distributed over the spectra unless the variation of opacity in each line be judiciously made. This may also be connected with the high absorbing and radiating powers of rough surfaces and with the action of coherers.

It is a matter for consideration whether it would not be worth while manufacturing photographic gratings by causing the two first spectra on each side of the central image, together with this central image, or without it, to interfere on the surface of a sensitive film. We might thereby produce a grating which had such a distribution of opacity as to reproduce only these first order spectra and have all the light that passed through concentrated in them. Similarly we might manufacture a grating which would have the light concentrated in any desired pair of spectra, though this would practically come to the same thing as the first proposal, with the lines closer together. This comes to the same thing as producing gratings by means of the interference of two beams of parallel rays of monochromatic light in the manner that Wiener has shown to be possible.

In all these cases it is quite evident that a longitudinal component of either electric or magnetic force is essential to the existence of waves whose intensity is not constant all over their surface, and that it is a practically universal concomitant of all waves of noncondensational type. That in the case of short waves which vary slowly from point to point, the intensity of the longitudinal component at any place will be in general very small, because the area is very large over which the motion along the surface at one place has at its disposal in which to turn and be continuous with the motion back along the face of the next wave. This does not make it unimportant, however. In a great many cases the total flow along the face of a wave must somewhere flow longitudinally so as to be continuous with the flow back along the other face of the wave. Unless these longitudinal flows are taken into consideration the whole energy of the wave is not accounted for. If the rate of variation of intensity over the surface be comparable with a wave-length, as in the case of fine gratings, the longitudinal component is a large part of the phenomenon, and, in fact, represents a large part of the energy in this case transmitted to the secondary image. This is all quite obvious in the case of gratings from the ordinary theory, for the equations given as a solution of this case represent a series of waves being transmitted in different directions from the grating corresponding to the directions of the secondary spectra.

XXVII. *On the Measurement of very large and very small Alternating Currents.* By ALBERT CAMPBELL, *B.A.**

AIR-core transformers, although quite inefficient for ordinary lighting circuits, are yet much more valuable for testing purposes than most people are aware of. By the help of such transformers it is possible to extend almost indefinitely the ranges of many ordinary measuring instruments. If the secondary of an ironless transformer be kept in open circuit the secondary volts are accurately proportional to the primary P.D. if the frequency is constant, and hence by using an electrostatic voltmeter on the secondary we can transform either up or down, and thus measure voltages above or below the range of the electrostatic instrument. Of course the arrangement would have to be calibrated; this might sometimes be done by taking a reading for which both the primary and secondary voltages lay within the range of the voltmeter used.

The above way of using an air-core transformer was suggested to me some time ago by Mr. Hugh Erat Harrison, of Faraday House. I hear since from Mr. Mather that it has been also used at the Central Technical College†.

If we attempt to measure *current* (in the primary) by observing the voltage on an open-circuit secondary, we find that for a given primary current the readings depend also on the *frequency*. It therefore occurred to me that the secondary

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† Since writing the above I have listened with interest to the paper on the "True Resistance of the Electric Arc," by Messrs. Frith and Rodgers. Their beautiful application of an air-core transformer to measure a small alternating current superimposed on a large direct current might, I think, be also employed to separate a *large* alternating current from a much *smaller* direct current.