electro-conducting power, which permits the formation of currents in it by inductive forces, that cannot produce the same in a corresponding degree in bismuth, and of course not at all in heavy glass.

2340. Any ordinary magnetism due to metals by virtue of their inherent power, or the presence of small portions of the magnetic metals in them, must oppose the development of the results I have been describing: and hence metals not of absolute purity cannot be compared with each other in this respect. I have, nevertheless, observed the same phænomena in other metals; and as far as regards the sluggishness of rotatory motion, traced it even into bismuth. The following are the metals which have presented the phænomena in a greater or smaller degree:—

| | |
|---|---|
| Copper. | Mercury. |
| Silver. | Platinum. |
| Gold. | Palladium. |
| Zinc. | Lead. |
| Cadmium. | Antimony. |
| Tin. | Bismuth. |

2341. The accordance of these phænomena with the beautiful discovery of Arago*, with the results of the experiments of Herschel and Babbage†, and with my own former inquiries (81.)‡, are very evident. Whether the effect obtained by Ampère, with his copper cylinder and a helix§, was of this nature, I cannot judge, inasmuch as the circumstances of the experiment and the energy of the apparatus are not sufficiently stated; but it probably may have been.

2342. As, because of other duties, three or four weeks may elapse before I shall be able to complete the verification of certain experiments and conclusions, I submit at once these results to the attention of the Royal Society, and will shortly embody the account of the action of magnets on magnetic metals, their action on gases and vapours, and the general considerations in another series of these Researches.

Royal Institution, Nov. 27, 1845.

* *Annales de Chimie*, xxvii. 363; xxviii. 325; xxxii. 213. I am very glad to refer here to the *Comptes Rendus* of June 9, 1845, where it appears that it was M. Arago who first obtained his peculiar results by the use of electro- as well as common magnets.

† Philosophical Transactions, 1825, p. 467.

‡ Ibid. 1832. p. 146.

§ *Bibliothèque Universelle*, xxi. p. 48.



## LXXVII. *On the Equations applying to Light under the action of Magnetism.* By G. B. Airy, *Esq., Astronomer Royal.*

*To the Editors of the Philosophical Magazine and Journal.*

Gentlemen,

By the indulgence of Dr. Faraday, I have been able to observe in the most satisfactory way the phænomena of the rotation of the plane of polarization of light passing through boracic glass and other media under the action of magnetic currents passing nearly in the direction of the light. And in particular I have verified the very remarkable fact that, upon passing the light successively in opposite directions while the magnetic adjustments remain the same, the plane of polarization undergoes the same change of position in regard to space, or undergoes opposite changes of position in regard to the expression of "rotation to the right," or "rotation to the left," as referred to the eye of the observer.

On reflecting upon the important fact that this change is not produced except there be an intermediate diaphanous body, it seems impossible not to conceive that the effect on the light is produced mediately by the action of the magnetic forces on the diaphanous body. The object of this communication is to point out what, as I conceive, must be the form of the mathematical equations existing among the movements of the particles of the glass, &c. or its contained æther, in order to explain the phænomena on mechanical laws.

In order to justify my intruding upon you with a suggestion which is exceedingly imperfect, I think it right to state to you my opinion upon the present condition of the optical theory, and upon several steps which, though leading to nothing conclusive, have nevertheless contributed to the real intellectual progress of the science.

On the truth of the undulatory theory, as regards the geometrical representation of light by undulations based upon transversal vibrations, the resolution of which into vibrations at right angles to each other constitutes polarization, I have not the shadow of a doubt. These undulations, whatever may be the way in which they may have been originally created, I conceive to be propagated by mechanical laws applying to the attractive or repulsive forces of the particles of the medium, the assumed æther, or the medium and the æther combined. But I have seen no mechanical theory to which I attach much importance or any unqualified belief. Nevertheless I think that the investigation and publication of these mechanical theories have been advantageous to the science, the same mechanical equations *referred to the same directions in absolute space* must apply to all these displacements.

In ordinary crystals or fluids possessing the property of causing rotation of the plane of polarization in the same direction as referred to the eye of the observer, whether the ray be incident on one side or on the other, mechanical equations are to be sought which will produce the result, that in both cases the velocity of Ray No. I. is greater than that of Ray No. II. (or *vice versâ*); so that if $v'_1$ is greater than $v''_1$, $v'_2$ will also be greater than $v''_2$. But in the glass affected by magnetism, if in the first experiment the velocity of Ray No. I. is greater than that of Ray No. II., then in the second experiment the velocity of Ray No. I. must be less than that of Ray No. II.; or if $v'_1$ is greater than $v''_1$, $v'_2$ must be less than $v''_2$.

Now the equation which is deduced from every mechanical supposition that accounts for the propagation of undulations, is of the form

$$\frac{d^2\,Y}{d\,t^2} = A \,.\, \frac{d^2\,Y}{d\,x^2},$$

$$\frac{d^2\,Z}{d\,t^2} = A \,.\, \frac{d^2\,Z}{d\,x^2}.$$

And it seems probable that these equations, with the addition to each of a small term, may explain the difference of velocities of the Rays No. I. and No. II.

It was pointed out by Prof. MacCullagh, that the equations

$$\frac{d^2\,Y}{d\,t^2} = A \,.\, \frac{d^2\,Y}{d\,x^2} + B \,.\, \frac{d^3\,Z}{d\,x^3},$$

$$\frac{d^2\,Z}{d\,t^2} = A \,.\, \frac{d^2\,Z}{d\,x^2} - B \,.\, \frac{d^3\,Y}{d\,x^3}$$

would explain this difference. I may remark here, that in the last term of the second side of each equation, any differential coefficient of an odd order would have sufficed to explain the general fact of difference of velocity; but the third order was adopted by Prof. MacCullagh in order to reconcile the expression for difference of velocity in differently-coloured rays with the fact established by experiment.

It is however necessary to inquire whether, if this assumption makes $v'_1$ greater than $v''_1$, it will make $v'_2$ greater than $v''_2$. For this purpose we must convert the various expressions into expressions referred to the same co-ordinates.

Let $\quad x_1=x,\ x_2=-x;\ y_1=y,\ y_2=-y$:

in the first experiment let

$$Y'_1=Y',\ \ Y''_1=Y'';\ \ Z'_1=Z',\ \ Z''_1=Z'':$$

in the second experiment let

$$Y'_2=-Y',\ \ Y''_2=-Y'',\ \ Z'_2=Z',\ \ Z''_2=Z''.$$

Then,

in the first experiment, for Ray No. I.,

$$Y' = a \,.\cos \frac{2\pi}{\tau}\left(t - \frac{x}{v'_1}\right),$$

$$Z' = a \,.\sin \frac{2\pi}{\tau}\left(t - \frac{x}{v'_1}\right);$$

and Prof. MacCullagh's equations become

$$-\frac{4\pi^2}{\tau^2} a \,.\cos \frac{2\pi}{\tau}\left(t - \frac{x}{v'_1}\right) = -A\frac{4\pi^2}{\tau^2}\left(\frac{1}{v'_1}\right)^2 a \,.\cos \frac{2\pi}{\tau}\left(t - \frac{x}{v'_1}\right)$$
$$+B\frac{8\pi^3}{\tau^3}\left(\frac{1}{v'_1}\right)^3 a \,.\cos \frac{2\pi}{\tau}\left(t - \frac{x}{v'_1}\right),$$

$$-\frac{4\pi^2}{\tau^2} a \,.\sin \frac{2\pi}{\tau}\left(t - \frac{x}{v'_1}\right) = -A \,.\frac{4\pi^2}{\tau^2}.\left(\frac{1}{v'_1}\right)^2 a \,.\sin \frac{2\pi}{\tau}\left(t - \frac{x}{v'_1}\right)$$
$$+B \,.\frac{8\pi^3}{\tau^3}.\left(\frac{1}{v'_1}\right)^3 a \,.\sin \frac{2\pi}{\tau}\left(t - \frac{x}{v'_1}\right),$$

which agree in giving

$$(v'_1)^2 = \frac{A}{1 + B\frac{2\pi}{\tau}\left(\frac{1}{v'_1}\right)^3}.$$

For Ray No. II.,

$$Y'' = b \,.\cos \frac{2\pi}{\tau}\left(t - \frac{x}{v''_1}\right),$$

$$Z'' = -\,b \,.\sin \frac{2\pi}{\tau}\left(t - \frac{x}{v''_1}\right).$$

The equations become

$$-\frac{4\pi^2}{\tau^2} b \,.\cos \frac{2\pi}{\tau}\left(t - \frac{x}{v''_1}\right) = -A \,.\frac{4\pi^2}{\tau^2}\left(\frac{1}{v''_1}\right)^2 b \,.\cos \frac{2\pi}{\tau}\left(t - \frac{x}{v''_1}\right)$$
$$-B \,.\frac{8\pi^3}{\tau^3}\left(\frac{1}{v''_1}\right)^3 b \,.\cos \frac{2\pi}{\tau}\left(t - \frac{x}{v'_1}\right),$$

$$+\frac{4\pi^2}{\tau^2} b \,.\sin \frac{2\pi}{\tau}\left(t - \frac{x}{v''_1}\right) = +A \,.\frac{4\pi^2}{\tau^2}\left(\frac{1}{v''_1}\right)^2 b \,.\sin \frac{2\pi}{\tau}\left(t - \frac{x}{v''_1}\right)$$
$$+B \,.\frac{8\pi^3}{\tau^3}\left(\frac{1}{v''_1}\right)^3 b \,.\sin \frac{2\pi}{\tau}\left(t - \frac{x}{v''_1}\right),$$

which agree in giving

$$(v''_1)^2 = \frac{A}{1 - B\frac{2\pi}{\tau}\left(\frac{1}{v''_1}\right)^3}.$$

Hence $v'_1$ is less than $v''_1$.

In the second experiment, for Ray No. I.,

$$Y' = -a \cdot \cos\frac{2\pi}{\tau}\left(t + \frac{x}{v'_2}\right),$$

$$Z' = a \cdot \sin\frac{2\pi}{\tau}\left(t + \frac{x}{v'_2}\right).$$

The equations become

$$+\frac{4\pi^2}{\tau^2}a \cdot \cos\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right) = +A\frac{4\pi^2}{\tau^2} \cdot \left(\frac{1}{v'_2}\right)^2 a \cdot \cos\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right)$$
$$-B \cdot \frac{8\pi^3}{\tau^3} \cdot \left(\frac{1}{v'_2}\right)^3 a \cdot \cos\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right),$$

$$-\frac{4\pi^2}{\tau^2}a \cdot \sin\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right) = -A \cdot \frac{4\pi^2}{\tau^2}\left(\frac{1}{v'_2}\right)^2 a \cdot \sin\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right)$$
$$+B \cdot \frac{8\pi^3}{\tau^3} \cdot \left(\frac{1}{v'_2}\right)^3 a \cdot \sin\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right),$$

which agree in giving

$$(v'_2)^2 = \frac{A}{1 + B\frac{2\pi}{\tau}\left(\frac{1}{v''_1}\right)^3}.$$

And similarly, for Ray No. II.,

$$(v''_2)^2 = \frac{A}{1 - B\frac{2\pi}{\tau}\left(\frac{1}{v''_1}\right)^3}.$$

Hence $v'_2$ is less than $v''_2$.

Thus in both experiments (that is, whether the light passes from one side or from the other side) the Ray No. II. travels more quickly than the Ray No. I. And therefore, if in each experiment there is incident a plane-polarized ray, consisting of the combination of a Ray No. I. and a Ray No. II., the plane-polarized ray which is formed by their union after emergence will have its plane of polarization turned from the original plane of polarization, in both experiments in the same direction as the hands of a watch, or in both experiments in the direction opposite to that of the hands of a watch, as referred to the eye of a person looking in the direction of the path of the light.

This result agrees with the phænomena of quartz, turpentine, &c.; and therefore Prof. MacCullagh's equations apply



to the explanation of crystalline rotation of the plane of polarization. But it does not agree with the phænomena of glass, &c. under magnetic action; and for this case new equations must be sought.

The equations which I offer as competent to represent this case are,

$$\frac{d^2Y}{dt^2} = A \cdot \frac{d^2Y}{dx^2} + C \cdot \frac{dZ}{dt},$$

$$\frac{d^2Z}{dt^2} = A \cdot \frac{d^2Z}{dx^2} - C \cdot \frac{dY}{dt},$$

which are to be verified in the same manner as those applying to the phænomena of quartz, &c.

Thus, in the first experiment, for Ray No. I.,

$$Y' = a \cdot \cos\frac{2\pi}{\tau}\left(t - \frac{x}{v'_1}\right),$$

$$Z' = a \cdot \sin\frac{2\pi}{\tau}\left(t - \frac{x}{v_1}\right).$$

The equations become

$$-\frac{4\pi^2}{\tau^2} \cdot a \cdot \cos\frac{2\pi}{\tau}\left(t-\frac{x}{v'_1}\right) = -A\frac{4\pi^2}{\tau^2}\left(\frac{1}{v'_1}\right)^2 a \cdot \cos\frac{2\pi}{\tau}\left(t-\frac{x}{v'_1}\right)$$
$$+C\frac{2\pi}{\tau} \cdot a \cdot \cos\frac{2\pi}{\tau}\left(t-\frac{x}{v'_1}\right),$$

$$-\frac{4\pi^2}{\tau^2}a \cdot \sin\frac{2\pi}{\tau}\left(t-\frac{x}{v'_1}\right) = -A\frac{4\pi^2}{\tau^2}\left(\frac{1}{v'_1}\right)^2 a \cdot \sin\frac{2\pi}{\tau}\left(t-\frac{x}{v'_1}\right)$$
$$+C \cdot \frac{2\pi}{\tau} \cdot a \cdot \sin\frac{2\pi}{\tau}\left(t-\frac{x}{v'_1}\right),$$

which agree in giving

$$(v'_1)^2 = \frac{A}{1 + \frac{\tau}{2\pi}C}.$$

For Ray No. II.,

$$Y'' = b \cdot \cos\frac{2\pi}{\tau}\left(t - \frac{x}{v''_1}\right),$$

$$Z'' = -b \cdot \sin\frac{2\pi}{\tau}\left(t - \frac{x}{v''_1}\right);$$

and the equations become

$$-\frac{4\pi^2}{\tau^2} \cdot b \cdot \cos\frac{2\pi}{\tau}\left(t-\frac{x}{v''_1}\right) = -A\frac{4\pi^2}{\tau^2}\left(\frac{1}{v''_1}\right)^2 b \cdot \cos\frac{2\pi}{\tau}\left(t-\frac{x}{v''_1}\right)$$
$$-C\frac{2\pi}{\tau}b \cdot \cos\frac{2\pi}{\tau}\left(t-\frac{x}{v''_1}\right),$$

$$+\frac{4\pi^2}{\tau^2}b.\sin\frac{2\pi}{\tau}\left(t-\frac{x}{v''_1}\right)=+A\frac{4\pi^2}{\tau^2}\left(\frac{1}{v''_1}\right)^2 b.\sin\frac{2\pi}{\tau}\left(t-\frac{x}{v''_1}\right)$$
$$+C\frac{2\pi}{\tau}b.\sin\frac{2\pi}{\tau}\left(t-\frac{x}{v''_1}\right),$$

which agree in giving

$$(v'_{\prime\prime})^2=\frac{A}{1-\frac{\tau}{2\pi}C}.$$

Hence $v'_1$ is less than $v'_{\prime\prime}$.

In the second experiment, for Ray No. I.,

$$Y''=-a.\cos\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right),$$
$$Z''=a.\sin\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right);$$

and the equations become

$$+\frac{4\pi^2}{\tau^2}a.\cos\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right)=+A\frac{4\pi^2}{\tau^2}\left(\frac{1}{v'_2}\right)^2 a.\cos\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right)$$
$$+C\frac{2\pi}{\tau}a.\cos\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right),$$
$$-\frac{4\pi^2}{\tau^2}a.\sin\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right)=-A\frac{4\pi^2}{\tau^2}\left(\frac{1}{v'_2}\right)^2 a.\sin\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right)$$
$$-C\frac{2\pi}{\tau}a.\sin\frac{2\pi}{\tau}\left(t+\frac{x}{v'_2}\right),$$

which agree in giving

$$(v'_2)^2=\frac{A}{1-\frac{\tau}{2\pi}C}.$$

Similarly,

$$(v''_2)^2=\frac{A}{1+\frac{\tau}{2\pi}C}.$$

Hence $v'_2$ is greater than $v''_2$.

Thus if in one experiment the Ray No. II. travels more quickly than the Ray No. I., in the other experiment the Ray No. II. travels more slowly than the Ray No. I. And therefore if in each experiment there is incident a plane-polarized ray consisting of the combination of a Ray No. I. and a Ray No. II., the plane-polarized ray which is formed by their



union after emergence will have its plane of polarization turned from the original plane of polarization, in one experiment in the same direction as the hands of a watch, and in the other experiment in the opposite direction, as referred to the eye of a person looking in the direction of the path of the light.

This result agrees with the phænomena of boracic glass, &c. under the action of magnetic forces.

Instead of making the second term on the right-hand side of the equation depend on $\frac{dZ}{dt}$, we might with equal success have adopted $\frac{d^3Z}{dt^3}$, $\frac{d^3Z}{dx^2.dt}$, or any other differential coefficient of an odd order in which the number of differentiations with respect to $t$ is odd. Different powers of $\tau$ and $v$ will be introduced by different selections. In order to determine which of these selections is best adapted to represent the phænomena, it will be necessary to determine the deviation of the plane of polarization for light of different colours.

If $\frac{dZ}{dt}$ be adopted, the equations suggested by me will amount to this:—"The force upon any particle in the direction of one ordinate depends in part upon its velocity in the direction of the other ordinate." There is no insurmountable difficulty in conceiving that this may be true, although we have at present no mechanical reason *à priori* for believing that it is true.

To remove the possibility of misunderstanding, I will repeat that I offer these equations with the same intention with which Prof. MacCullagh's equations were offered; not as giving a mechanical explanation of the phænomena, but as showing that the phænomena may be explained by equations, which equations appear to be such as might possibly be deduced from some plausible mechanical assumption, although no such assumption has yet been made.

I am, Gentlemen,
Your obedient Servant,
G. B. Airy.

Royal Observatory, Greenwich,
May 7, 1846.