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A Test Theory of Special Relativity: III. Second-Order Tests

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Abstract

Various second-order optical tests of special relativity are discussed within the framework of the test theory developed previously. Owing to the low accuracy of the Kennedy–Thorndike experiment, the Lorentz contraction is known by direct experiments only to an accuracy of a few percent. To improve this accuracy several experiments are suggested.

§(1): *Introduction*

Experiments of second order in v/c have dominated the discussion concerning the experimental tests of special relativity for a long time. Almost every textbook on special relativity discusses the Michelson-Morley experiment and frequently also the Kennedy–Thorndike experiment. Very often one gets the impression that these experiments prove conclusively the correctness of special relativity. Frequently the Michelson–Morley experiment is quoted as a direct proof of constancy of the speed of light [1, 2].

A systematic analysis of the importance of second-order experiments has been given by Robertson [3]. Robertson has derived the Lorentz transformation by using general postulates such as the isotropy and homgeneity of space and time together with three experiments of second order. These experiments are the Michelson–Morley, Kennedy–Thorndike, and Ives–Stillwell experiments [3, 4]. In these papers the role of the problem of synchronization of distant clocks has not been studied in any detail, and thus no attention has been paid to first-order experiments.

In Paper II of our series we have seen that experiments of first order determine the time-dilatation parameter α with great accuracy. In this paper we shall show that the two remaining parameters β and δ are determined by the Michelson-Morley and Kennedy-Thorndike experiments. In the course of our analysis the importance of these experiments for a principle of the constancy of the velocity of light will become clear. Furthermore we shall discuss the Ives-Stillwell experiments within the framework of our test theory. No assumptions concerning synchronization will be made in the analysis of these experiments.

§(2): *The Experiments of Michelson and Morley and Kennedy and Thorndike*

In this section we shall study the experiments of Michelson and Morley and Kennedy and Thorndike. Both experiments are optical experiments of second order using closed light paths. The mean velocity of light along a closed path has been calculated in equation (36) of Paper I[1] and is independent of the synchronization coefficient ϵ. In discussing the experiments we need the inverse velocity of light to second order in v/c:

$$\frac{1}{c(\theta)} = 1 + \left(\beta + \delta - \frac{1}{2}\right) v^2 \sin^2\theta + (\alpha - \beta + 1)\, v^2 \tag{2.1}$$

The Michelson-Morley Experiment. In this experiment two light beams emitted by one source are sent in orthogonal directions. After transversing paths of the length l_1 and l_2, respectively, the difference in the optical path $\tau = \tau_1 - \tau_2$ is measured with the help of interference methods. Then the apparatus is rotated by 90° and any change in the interference pattern is registered. Thus one measures the variation

$$\delta_\theta \tau = \tau(\theta + \tfrac{1}{2}\pi) - \tau(\theta) \tag{2.2}$$

inserting in (2.1) this becomes

$$\delta_\theta \tau = (l_1 + l_2)(2\beta + 2\delta - 1)\, v^2 \cos 2\theta \tag{2.3}$$

An upper limit for $\beta + \delta - \frac{1}{2}$ can thus be derived if no change in the interference pattern is observed. In the original Michelson-Morley experiment of 1887 no fractional shifts of the interference pattern larger than 0.005 were observed [5].[2] This implies

$$\delta_\theta \tau < 0.005\lambda \tag{2.4}$$

where $\lambda = 6.10^{-7}$ m is the wave length of light used. Inserting $l_1 = l_2 = 6.10^{-7}$ m we obtain

$$\beta + \delta = 0.5 \pm 10^{-3} \tag{2.5}$$

[1] Heafter equations from Paper I will be cited as, for example (I.36).
[2] For a discussion of numerous Michelson-Morley experiments, see [15].

Joos repeated the Michelson-Morley experiment in 1930 [6]. He obtained

$$\beta + \delta = 0.5 \pm 3.10^{-5} \tag{2.6}$$

In both cases we have inserted $v = 300$ km/s for the velocity of the earth to the ether, as discussed before.

A modern version of the Michelson-Morley experiment has been performed with the help of two lasers [7]. From the data of this experiment one derives

$$\beta + \delta = 0.5 \pm 10^{-5} \tag{2.7}$$

We can look at the Micheson-Morley experiment also from another point of view, which will make it comparable with an experiment performed by L. Essen [8]. The light ray going both ways in one of the arms of the interferometer can be considered as a clock, the period of which is determined by the return time of the light ray. The Michelson-Morley experiment can thus be considered as the comparison of the frequencies of two clocks. The experiment shows that the relative frequency is not affected by a rotation of the interferometer. In the experiment of Essen a similar comparison between two clocks is made. The frequency of a cylindrical cavity resonator is controlled with the help of a quartz clock, while the resonator is being rotated. No change of relative frequencies was observed within the experimental accuracy. This gives an upper limit for ether drift of 3 km/s or

$$\beta + \delta = 0.5 \pm 10^{-4} \tag{2.8}$$

The importance of Essen's experiment is not so much its accuracy, which is inferior to the best Michelson-Morley experiments, but rather that it checks that the relative frequency of two clocks of very different construction is unaffected by relative rotation. The experiment of Essen could nowadays also be performed in a different way. The distance earth-moon could be used as one arm of a Michelson interferometer. The monthly rotation of the moon around the earth causes the desired rotation of the arm of the interferometer. The round trip time of a laser being reflected by a reflector on the moon can be used as one of the clocks. Comparing this with the time given by an atomic clock on earth one obtains a possible accuracy of

$$\beta + \delta = 0.5 \pm 10^{-4(5)} \tag{2.9}$$

if the travel time can be measured with an accuracy corresponding to a determination of the earth-moon distance to 10(1) cm.

Another experiment similar to the Michelson-Morley experiment has been performed by Fox and Shamir [9]. They repeated the Michelson-Morley experiment in a solid transparent medium. According to these authors this experiment is able to decide between the special theory of relativity and an ether theory incorporating Lorentz contraction and time dilatation. As we have shown quite generally in the first and second parts of this paper such a distinction is impossible in principle. The limit given for the ether drift is 7 km/sec.

Another experiment that determines the isotropy of the velocity of light has been performed by Trimmer et al. [10]. Here unisotropies of the velocity of light that are proportional to $P_1 (\cos\theta)$ and $P_3 (\cos\theta)$ (P_1 and P_3 are Legendre polynomials) are investigated, while the Michelson-Morley experiment measures $P_2 (\cos\theta)$. Anisotropies proportional to these Legendre polynomials cannot occur in the test theory given here. These polynomials could appear only if preferred directions exist in space. The experiment of Trimmer et al. can thus be used to exclude the existence of such preferred directions.

The experiment of Kennedy and Thorndike. In the Kennedy-Thorndike experiment [11] the velocity v of the interferometer in the ether is changed by performing the experiment over a period of several months. Except for the unlikely case that the solar system is exactly at rest in the ether, one obtains an upper limit for the second term in (2.1). Thus this experiment amounts to a measurement of

$$\delta_v \tau = 4(l_1 - l_2)(\alpha - \beta + 1)\, v dv \qquad (2.10)$$

The original measurement of Kennedy and Thorndike leads to

$$2(\alpha - \beta + 1)\, v = 10 \pm 10 \text{ km/s} \qquad (2.11)$$

and inserting $v = 300$ km/s we obtain

$$\alpha - \beta = 1.02 \pm 2.10^{-2} \qquad (2.12)$$

Since α is known with high accuracy from the first-order experiments, we can regard this as a measurement of the Lorentz contraction coefficient β. The experimental accuracy (2.12) is, however, not very good. No direct measurement of the Lorentz contraction exists, therefore, that is better than a few percent. More accurate experiments are thus highly desirable. The experiments of Essen [8] and Jaseja et al. [6] could be performed over a period of several months in order to improve the accuracy of our knowledge of β. If the accuracy of these experiments can be maintained for a period of several months one can expect the following limits:

$$\alpha - \beta = 1 \pm 10^{-3} \qquad \text{(Essen)} \qquad (2.13)$$

$$\alpha - \beta = 1 \pm 5.10^{-5} \qquad \text{(Jaseja et al.)} \qquad (2.14)$$

Alternatively one could use a measurement of the earth-moon distance to improve on the accuracy of the Kennedy-Thorndike experiment. If the accuracy discussed before can be achieved one can expect a limit

$$\alpha - \beta = 1 \pm 10^{-4(5)} \qquad \text{(earth-moon)} \qquad (2.15)$$

§(3): *The Ives-Stillwell Experiment*

In Paper II of this series we have discussed the rotor experiments which determine the time dilatation to high accuracy. In Robertson's classical paper [3] the Ives–Stillwell experiment [12][3] is used to define limits for the time-dilatation parameter α instead of the first-order experiments, which were not available at that time. For the sake of completeness we shall give here the limits derivable from the Ives–Stillwell experiment, which turns out to be inferior, however, to those given by modern rotor experiments.

The Ives–Stillwell experiment was the first historical experiment that measured time dilatation directly. In this experiment the wavelength emitted by a moving radiator is determined. By measuring the radiation emitted in the forward and backward directions simultaneously one can eliminate the first-order Doppler effect. This leaves us with the second-order Doppler effect caused by time dilatation.

In discussing this effect within the framework of our test theory we consider the source of radiation at rest in an inertial system $\bar{S}$ moving with the velocity u in the laboratory system $\bar{S}$. Denoting the velocity of S with respect to the ether by v one easily derives for the relevant change of wavelength

$$\frac{\Delta\lambda}{\lambda} = \frac{a(v)}{a(u)} \frac{1 - \mathbf{uv}}{1 - v^2} - 1 \tag{3.1}$$

since $u \gg v$ we can approximate this by

$$\Delta\lambda/\lambda \simeq -\alpha\mu^2 \tag{3.2}$$

The measurements of Ives and Stillwell imply

$$\alpha = -0.5 \pm 10^{-2} \tag{3.3}$$

§(4): *Conclusions*

Our studies of the experiments of second order in v/c have shown that our knowledge of the three second-order coefficients α, β, and δ is of very different accuracy.

The best-known kinematical effect of special relativity is presently the time dilatation. It is determined to an accuracy better to one part in a million by first-order experiments (rotor experiments). No improvement of the accuracy of these experiments is necessary in the near future.

In contrast to our excellent knowledge of time dilatation, the Lorentz contraction is known only to an accuracy of a few percent. This is due to the fact that the Michelson-Morley experiment determines only the combination $\beta + \delta$, but not the Lorentz contraction directly. It is remarkable indeed that no direct measurements of the Lorentz contraction are possible, and it is very unlikely that any direct experiments can be found in view of the fact that a direct optical observation of the Lorentz contraction is impossible.

The Kennedy–Thorndike experiment, which is our best source of information

[3]For some modern versions of this experiment see [13, 14].

on the coefficient β, is a very difficult and time-consuming experiment to perform. This is probably the reason why no modern high-precision versions of this experiment has been attempted. Our analysis has shown, however, that new versions of this historical experiment are both possible and desirable. Only if one of the experiments suggested in this paper is performed will we have direct experimental evidence for all the second-order coefficients in the Lorentz transformation with an accuracy better to one part in ten thousand. Considering the fundamental role that special relativity plays in our understanding of all of modern physics, it seems well worthwhile to perform such an experiment.

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