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A Test Theory of Special Relativity: II. First Order Tests

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Abstract

First-order tests of special relativity are based on a comparison of clocks synchronized with the help of slow clock transport with those synchronized by the Einstein procedure. This comparison enables the measurement of the one-way velocity of light and is equivalent to a measurement of the time dilatation factor. The accuracy of present measurements is of the order 10^{-7}, yielding an upper limit of 3 cm/sec for the ether drift.

§(1): *Introduction*

Experiments of first order in the "aberration constant" v/c play a special role in the history of ether theories and of special relativity.[1] They have been crucial in developing and comparing the various concepts of the ether that have been proposed during the nineteenth century [2]. Three classes of such ether theories can be distinguished, differing by their assumptions about the dynamics of the ether.

(a) Complete dragging of ether by matter has been proposed by Stokes [3][2] and Hertz [6],[3] among others. Stokes has tried to construct a mechanical model

[1] The experimental situation as it appeared in 1910 has been summarized by Laub [1].

[2] In order to explain abberation on the basis of the complete ether drag theory, Stokes [3] assumed the motion of the ether to be irrotational. Lorentz [4] showed that Stokes' assumptions were contradictory unless some special assumptions about the ether were made. See also Whittaker [5].

[3] Hertz [6] considered the idea of complete dragging of ether by matter to be a preliminary "ansatz," which would later be replaced by a more complete theory of the dynamics of the ether.

of an ether dragged completely by matter. Hertz' electrodynamics was based on this ether concept. In this theory, which is not in conflict with Newtonian mechanics, electrodynamic phenomena depend only on the relative motion of the observer and other bodies. It explains successfully the negative outcome of all attempts to measure the velocity of the earth in ether. It is incapable, however, of predicting the outcome of the aberration experiments [5] and of Fizeau's [6] measurement of the velocity of light in moving water correctly.

(b) Partial dragging of ether by matter was suggested by Fresnel [9][4] in order to explain the aberration experiments.

(c) Ether permanently at rest and not influenced by the motion of matter is the basis of Lorentz' famous "electron theory" [10].

In order to distinguish between these theories–i.e., in order to explore the dynamics of the ether–and to find the motion of the earth through the ether, various experiments were carried out around 1870 [11, 12].[5] These were first-order experiments, based on the assumption that the velocity of light on earth is $c \pm v$, depending on the direction of propagation of light being parallel or antiparallel to the motion of the earth through the ether.

The hope was to detect this anisotropy with the help of indirect experiments using closed light paths. Direct tests of the anisotropy were not considered owing to insurmountable experimental difficulties. Such tests would have made the problem of synchronizing spatially separated clocks immediately obvious. Closed light paths require, however, the use of a single clock only.

During the following years it became clear that only second-order effects could be expected in all such arrangements [13]. Maxwell suggested the use of extraterrestrial experiments in order to circumvent this difficulty [14]. Measurements similar to those carried out by Römer in 1676 were considered. We shall show that these experiments involve synchronization by slow clock transport and permit thereby genuine measurements of one-way light velocities.

The success of Lorentz' electron theory [10] stimulated a period of intense search for the ether. On the basis of his theory Lorentz was able to prove rigorously what was suggested before by Maxwell and also by Veltman and by Potier [13]: No first-order effects can be obtained in any experiment using closed light paths. In spite of this theoretical result several searches for first-order effects were carried out around 1900, using closed light paths [1, 15].

The situation around the turn of the century can be summarized as follows: Extraterrestrial measurements were considered to be too inaccurate in order to measure first-order effects reliably. No first-order effects were expected in terrestrial experiments using closed light paths. Technical difficulties prevented the

[4] See [5], pp. 108–127.

[5] Mascart's conclusion was "the translatory motion of the earth has no appreciable effect at all on the optical phenomena produced with a terrestrial light source or with solar light. These phenomena are incapable of demonstrating the absolute motion of a body. Relative motions are the only ones we can make evident" [12].

execution of terrestrial experiments using nonclosed light paths. First-order tests were therefore discontinued and the emphasis of future experimental work was centered on second-order tests, with the Michelson-Morley experiment being the pioneering study.

This situation changed around 1960, when the advent of masers and lasers and of the Mössbauer effect made new types of experiments technically feasible. Møller in 1957 [16] and Ruderfer in 1960 [17] pointed out these possibilities and gave a preliminary theoretical analysis. This led to a renaissance of interest in first-order experiments. Not all experiments carried out recently can be considered to be genuine first-order experiments, however. Some of these experiments[6] use closed light paths and lead therefore trivially to null results, as far as first-order effects are concerned.

In this paper we shall consider some typical first-order experiments and analyze them from the point of view of the test theory developed before [22]. We shall start with an analysis of Römer's experiment which is the oldest experiment measuring the one-way velocity of light. Modern versions of the Römer experiment using interplanetary spacecraft and the terrestrial clock system will be considered. We then turn to the laboratory experiments that have been performed since Møller's discovery of their feasibility. Our analysis will show that these experiments provide tests of time dilatation with an accuracy of about 10^{-7}. All experiments can be explained either on the basis of special relativity or by an ether theory incorporating time dilatation. This demonstrates again the impossibility of an "experimentum crucis" deciding between ether theories and the special theory of relativity.[7]

All other "first-order experiments" described in historical surveys of the experimental basis of special relativity [1, 13] cannot be discussed with the help of the test theory developed in I. The reason for this is that experiments such as those of Fizeau, Eichenwald, Wilson etc. involve special assumptions about the electrodynamic properties of bodies moving through the ether. On the basis of a kinematical analysis alone—such as the one provided by a parametrization of a generalized Lorentz transformation given in I—no predictions can be made about the outcome of these experiments. Only when additional assumptions (e.g., those made by Fizeau) are added to the theoretical framework can one attempt to analyze the relevance of these experiments. Only when special relativity is assumed to be valid are no such additional assumptions needed. The reason for

[6]Weinberger [18] proposed a new type of interferometer experiment using a closed light path. The authors' claim that first-order effects are to be expected in this experiment has been discussed and corrected by Stedman [19] and by Erlicson [20]. Silvertooth [21] uses an interferometer with a closed light path and two frequency-doubling crystals.

[7]Erlicson [23] suggests that first-order experiments might provide the "experimentum crusis" to decide between ether theories incorporating Lorentz contraction and time dilatation and the special theory of relativity. Strakhouskii and Uspenskii [24] suggest that first-order experiments are crucial tests of the relativity principle and can be used to rule out ether theories.

this is that the symmetry group contained in Einstein's theory restricts the possible forms of electrodynamic and other interactions so strongly that on the basis of Lorentz invariance alone an interaction that is known in one system of reference can be rewritten in any other inertial system. This does not apply when no invariance group exists. The knowledge, for example, of electromagnetic interactions in the ether system is insufficient to predict the electrodynamics of moving bodies. In other words, the symmetry group contained in relativity makes many predictions possible, which have to be derived with the help of additional assumptions in ether theories. These predictions cannot be parametrized in terms of the coefficients of the Lorentz group and can thus not be discussed in the framework of our test theory.

§(2): *The Römer Experiment and its Modern Versions*

The first measurements of the velocity of light have been performed by Olaf Römer in 1676 [25].[8] He determined c from the occultations of the moons of Jupiter. The intervals in which one of the moons of Jupiter enters into the shadow of this planet are constant, as seen from Jupiter. Seen from the earth irregularities appear due to the change in the Earth–Jupiter distance. When this distance increases, light takes longer to reach the Earth. This permits a determination of the velocity of light.

The Römer experiment has been analyzed by Born [26][9] and by Reichenbach [27], who was mainly interested in the role of this experiment in the verification of the principle of the constancy of the speed of light. An incorrect discussion of the Römer experiment has been given by Karlov [28], who states (incorrectly) that Römer's method does not lead to a determination of the one-way velocity of light.

We shall discuss the Römer experiment here from the point of view of the test theory developed in I. Jupiter and its moons can be regarded to be one clock J. This clock emits light signals (occultations) at regular intervals towards the earth where they are timed by a second clock E. To make the discussion as simple and transparent as possible we shall assume the earth to move with uniform velocity v through the ether while Jupiter moves around it in circles. While the clock J is being transported through space it emits light signals towards the earth, the velocity of which depends on their angle θ with respect to v. This velocity has been calculated in I to be [see (I. 6.16)]

$$c(\theta) = 1 - (1 + 2\alpha)v \cos\theta \tag{2.1}$$

[8]This experiment is discussed in [5] p. 22, where remarks on the accuracy of the experimental determination of c can be found.

[9]Born [26] analyzes the relevance of the Römer experiment for the problem of the isotropy of the velocity of light.

(2.1) is the one-way velocity of light as measured in the inertial system S in which the earth is at rest with the help of clocks synchronized by slow clock transport. The anisotropy of the velocity of light leads to irregularities in the occultations (periods of the clock Jupiter) as seen from the earth. Numerically these irregularities are expected to be

$$\Delta t/t = 2(1 + 2\alpha)v \tag{2.2}$$

where the factor 2 is due to the variation of the cosine between -1 and 1 and $t = 3000$ sec is the (average) time in which the light rays reach the earth. Since no irregularities of the occultations of the moons of Jupiter are seen with an accuracy of about $\Delta t \simeq 0.1$ sec, we conclude

$$(1 + 2\alpha)v \lesssim 3.10^{-5} = 10 \text{ km/sec} \tag{2.3}$$

If we put $\alpha = 0$ (Galileian relativity) we obtain the result that the velocity of the solar system through the ether is less than 10 km/sec. If we assume alternatively that the velocity of the solar system in a potential ether is of the order of 300 km/sec, as suggested by observations of the microwave background and other cosmological evidence [29], we obtain

$$\alpha = -0.5 \pm 2.10^{-2} \tag{2.4}$$

The deviations from Galileian relativity and the relativistic prediction for the time dilatation factor are thus verified with an accuracy of about 5% by the Römer experiment.

A modern terrestrial version of the Römer experiment has been suggested by Rapier [30]. He proposed to measure the one-way velocity of light with the help of two atomic clocks which are to be synchronized at one point in space and then separated. This is actually the way in which the world's precision network of clocks is synchronized [31]. The clocks contributing to UTC (Coordinated Universal Time) are synchronized with the help of radio signals, the propagation delays of which are measured with the help of clock transport. No influence of the motion of the earth or the solar system on the comparison between synchronization by slow clock transport and by the Einstein procedure have been found at the 10^{-7}-sec level. Since delay times of the order of 3.10^{-3} sec are involved we obtain

$$(1 + 2\alpha)v \leqslant 3.10^{-5} = 10 \text{ km/sec} \tag{2.5}$$

i.e., an accuracy similar to the one resulting from the Römer experiment.

Actually it is not even necessary to transport clocks on the earth's surface in order to measure anisotropies of the one-way velocity of light. The rotation of the earth carries out the necessary clock transport automatically. All one has to do is to compare two separated atomic clocks, say in Europe and in the United States continuously with the help of radio signals. Two such clocks synchronized, for example, at noon will be out of synchronization by midnight if the anisotropies (2.1) are actually present (since the value of θ is changed continuously by the earth's rotation). In this case light travel times of about 5.10^{-2} sec are involved and no diurnal changes in clock synchronization are observed at the 10^{-6}-sec level [32]. We obtain therefore

$$(1 + 2\alpha)v \leqslant 10^{-5} = 3 \text{ km/sec} \tag{2.6}$$

or, inserting again $v \simeq 300$ km/sec,

$$\alpha = -0.5 \pm 5.10^{-3} \tag{2.7}$$

The synchronization of the terrestrial network of precision clocks is thus equivalent to a measurement of relativistic time dilatation with an accuracy of about 1%!

Many similar experiments can be performed. Atomic clocks in satellites or on the lunar surface could improve the accuracy of the types of measurement described here by about two orders of magnitude. In these cases the moon or the satellite provides the necessary clock transport. Potentially even more accurate experiments can be performed with the help of space probes exploring the solar system. The travel time of signals emitted by atomic clocks aboard such satellites could reach 10^4 sec and if no anisotropies are observed at the 10^{-6} sec level (assuming that this accuracy can be maintained throughout the space flight) this would imply $(1 + 2\alpha)v \leqslant 3$ cm/sec

§(3): *Measurements of the Transversal Doppler Effect*

According to prerelativistic ether theories the Doppler effect depends on the relative velocity **u** of the observer with respect to the source as well as on the observer's velocity **v** with respect to the ether. The frequency ν of the wave received by the observer from the direction **e** is easily derived to be according to classical ideas

$$\nu = \nu_0[1 + \mathbf{e} \cdot \mathbf{u} + (\mathbf{e} \cdot \mathbf{u})^2 + \mathbf{v} \cdot \mathbf{u}] \tag{3.1}$$

ν_0 is the frequency measured in the rest system of the source. This equation has been obtained by Møller [16] by applying a Galilei transformation to a plane wave. Numerous experiments have been performed in order to check (3.1). We shall discuss here rotor experiments where a source and an absorber of radiation are placed at the ends of the arms of a rotor [33-36].[10] Since both the source and the absorber of the radiation rotate in these experiments, we have to modify (3.1). This is done most easily by comparing the frequency ν_1 of the source and the frequency ν received at the absorber with a dummy source placed at the center of the rotor. At the dummy source, which moves uniformly through the

[10] Turner and Hill [36] claim incorrectly that their experiment is equivalent to the Kennedy–Thorndike experiment in its test value for special relativity.

ether with velocity v, the radiation is received from a direction e such that $\mathbf{e} \cdot \mathbf{u} = 0$ (transversal Doppler effect). Thus (3.1) simplifies to

$$\begin{aligned} \nu &= \nu_0(1 + \mathbf{v} \cdot \mathbf{u}) \\ \nu_1 &= \nu_0(1 - \mathbf{v} \cdot \mathbf{u}) \end{aligned} \tag{3.2}$$

since the velocity of the rotation of absorber and source is $\mathbf{u}$ and $-\mathbf{u}$, respectively. Eliminating the frequency ν_0 of the source at the center we have

$$\nu = \nu_1(1 + 2\mathbf{v} \cdot \mathbf{u}) \tag{3.3}$$

In contrast to (3.3) special relativity predicts a null result ($\nu = \nu_1$) for the rotor experiments.

In order to analyze the role of the one-way velocity of light and the convention about synchronization in these experiments we rederive (3.3) from kinematical considerations.

Consider two subsequent wave crests emitted by the source. Crest 1 starts at $t_1 = 0$ and arrives at the absorber at $t_1' = 2R(1 - v \cos\theta)^{-1}$, where R is the radius of the centrifuge and θ is the angle between the instantaneous direction of the rotor and its velocity v relative to the ether. When crest 2 is emitted at $t_2 = 1/\nu_1$ the centrifuge has rotated by the angle $\Delta\theta = u/R\nu$. This wave crest is received at the time $t_2' = 1/\nu_1 + 2R[1 - v\cos(\theta + \Delta\theta)]^{-1}$. The frequency observed at the absorber becomes

$$\nu^{-1} = t_2' - t_1' = \nu_1^{-1}(1 + 2u \cdot v \sin\theta) = \nu_1^{-1}(1 - 2\mathbf{u} \cdot \mathbf{v}) \tag{3.4}$$

in agreement with (3.3).

The following assumptions enter into the two versions of the derivation of (3.3) and (3.4) respectively.

(a) The proper frequency of the source is constant, corresponding to no time dilatation, $a(v) = 0$.

(b) The factor ϵ has been assumed to be zero. This results from the use of the Galilei transformation in the derivation of (3.3) and implies that clocks are not to be synchronized with light.

(c) t_1' and t_2' are times measured by clocks in S that are synchronized with the clock of the source. In determining the frequency ν the time indicated by a moving clock (absorber) has been set equal to the time shown by clocks in S, implying transport synchronization.

In Møller's treatment of the rotor experiments assumptions (a) and (b) have been used, while the kinematical treatment given above is based on (b) and (c). The results agree, showing the consistency of the assumptions (a), (b), and (c). The formal proof for this consistency is contained in Section 5 of I, where (b) has been shown to be a consequence of (a) and (c).

Both treatments given above assume Galileian relativity to be valid exactly. In order to apply the test theory of relativity developed in I, we modify assumption (a) by introducing a time dilatation factor $a(v) \neq 1$. Expanding $a(v) = 1 + \alpha v^2 + \cdots$, where the parameter α is predicted by relativity to be $\alpha = -\frac{1}{2}$, the one-way velocity of light has been derived in I to be (see Section 6 of I)

$$c(\theta) = 1 - v(1 + 2\alpha) \cos \theta \tag{3.5}$$

Here θ is the angle between v and the direction of propagation of the light ray. Repeating the kinematical treatment with this expression for the light velocity we obtain

$$\nu = \nu_1 [1 + 2\mathbf{u} \cdot \mathbf{v}(1 + 2\alpha)] \tag{3.6}$$

The experiment of Champeney et al. [35] showed that $(1 + 2\alpha)v = 1.6 \pm 2.8$ m/sec. Inserting $v = 300$ km/sec we obtain

$$\alpha = -0.5 \pm 10^{-5} \tag{3.7}$$

An even better value for α can be derived from the experiment of Isaak [37], which gives an upper limit $(1 + 2\alpha)v \leqslant 5$ cm/s. This implies

$$\alpha = -0.5 \pm 10^{-7} \tag{3.8}$$

The time dilatation factor predicted by relativity is thus verified with an accuracy of about 1 in 10^7 by first-order experiments.

A different type of first-order experiment has been carried out by Cialdea [38]. Cialdea studies changes in the interference pattern of two lasers that are mounted on an optical bench when this arrangement is rotated. Such changes are expected owing to the direction dependence (3.5) of the velocity of light. His final result is $(1 + 2\alpha)v < 0.9$ m/sec, or

$$\alpha = -0.5 \pm 10^{-6} \tag{3.9}$$

Similar experiments have been proposed by Strakhovskii [24] and by Carnaham [39]. An incorrect criticism of Cialdea's and similar experiments has been published by Tyapkin [40].[11]

§(4): *Conclusions*

The first-order tests of special relativity discussed in this paper are based on the comparison of clocks synchronized with the help of slow clock transport and by means of the Einstein procedure. The coefficient ϵ in the generalized Lorentz transformation

$$t = a(v)T + \epsilon x$$

$$x = b(v)(x - vT)$$

[11] Tyapkin [40] states that Cialdea's [38] results are trivial, since the one-way velocity of light is of purely conventional character.

being fixed by clock transport (I.5.6.) the one-way velocity of light is no longer conventional, but a measurable quantity. Expanding

$$a(v) = 1 + \alpha v^2 + \cdots$$

we have for transport synchronization $\epsilon_T = 2\alpha v$ and the one-way velocity of light becomes

$$c(\theta) = 1 - (1 + 2\alpha)v \cos\theta$$

where θ is the angle between the propagation direction of light and the velocity v of the inertial system considered with respect to the ether.

First-order tests are thus equivalent to measurements of the time dilatation factor $a(v)$, and the present accuracy of these measurements, about 10^{-7}, is far better than direct determinations of time dilatation.

First-order tests cannot be used to distinguish between special relativity and ether theories, as has sometimes been stated. No such "experimentum crucis" is possible in principle, since the two classes of theories can be transformed into one another by a change of conventions about clock synchronization, as has been shown in I.

References

1. Laub, J. (1910). *Jahrb. Radioakt. Elektron.*, **7**, 405.
2. Schaffner, K. F. (1972). *Nineteenth Century Ether Theories*, Pergamon Press, New York.
3. Stokes, G. G. (1845). *Phil. Mag.*, **27**, 9; (1846). **28**, 76; (1846). **29**, 6.
4. Lorentz, H. A. (1896). *Arch. Neerl*, **21**, 103.
5. Whittaker, E. (1951). *A History of the Theories of Aether and Electricity*, Thomas Nelson and Sons Ltd, London, Vol. I, p. 387.
6. Hertz, H. (1890). *Ann. Phys. Leipzig*, **41**, 369.
7. Bradley, J. (1729). *Phil. Trans. R. Soc. London*, **35**, 637.
8. Fizeau, H. (1857). *Ann. Chim. Phys.*, **57**, 385.
9. Fresnel, A. (1818). *Ann. Chim.*, **9**, 57.
10. Lorentz, H. A. (1895). *Versuch einer Theorie der elektrischen und optischen Erscheinungen in bewegten Körpern*, Leiden 1895; (1909). *Theory of Electrons*, Leipzig.
11. Klinkerfues, J. (1870). *Nachr. Kgl. Ges. Wiss. Göttingen*, 226.
12. Mascart, E. (1872). *Ann. Sci. Ecole Norm. Sup.*, **1**, 157; (1874) **3**, 363.
13. Veltmann, W. (1870). *Astron. Nachr.*, **75**, 145; (1870). **76**, 129; (1873). *Poggendorff's Ann.*, **150**, 497; Potier, A. (1874). *J. Phys. (Paris)*, **3**, 201. See Newburgh, R. and Costa de Beauregard, O. (1975). *Am. J. Phys.*, **43**, 528 for an analysis of the importance of these papers.
14. Maxwell, J. C. (1880). *Proc. R. Soc. London*, **30**, 108.
15. Lecat, M. (1924). *Bibliographie de la Relativité*, Brüssel.
16. Møller, C. (1957). *Suppl. Nuovo Cimento*, **6**, 381; (1960). *Proc. R. Soc. A*, **270**, 306.
17. Ruderfer, M. (1960). *Phys. Rev. Lett.*, **5**, 191; (1961). **7**, 361 (erratum); (1960). *Proc. IRE*, **48**, 1661.
18. Weinberger, H., and Mossel, M. (1971). *Am. J. Phys.*, **39**, 606.
19. Stedman, G. E. (1973). *Am. J. Phys.*, **40**, 782; (1973). **41**, 1300.
20. Erlicson, H. (1973). *Am. J. Phys.*, **41**, 1298.
21. Silvertooth, E. W. (1972). *J. Opt. Soc. Am.*, **62**, 1330.
22. Mansouri, R., and Sexl, R. U. (1977). *Gen. Rel. Grav.*, **8**, 497. This paper will be cited as I and equations from it as, for example, (I.1.1).

23. Erlicson, H. (1973). *Am. J. Phys.*, **41**, 1068.
24. Strakhovskii, G. M., and Uspenskii, A. V. (1966). *Sov. Phys. Usp.*, 8, 505.
25. Römer, O. (1666-99). *Mem. Acad.*, **10**, 575.
26. Born, M. (1969). *Die Relativitätstheorie Einsteins*, Berlin.
27. Reichenbach, H. (1925). *Z. Phys.*, **33**, 628.
28. Karlov, L. (1970). *Austral. J. Phys.*, **23**, 243.
29. Conklin, E. K. (1969). *Nature*, **222**, 971.
30. Rapier, P. M. (1961). *Proc. IRE*, 49, 1322.
31. Becker, G. (1975). *Kleinheubacher Berichte*, 18, 45.
32. Becker, G. Physikalisch-Technische Bundesanstalt Braunschweig (private communication).
33. Cedarholm, J. P., Bland, G. F., Havens, B. L., and Townes, C. H. (1958). *Phys. Rev. Lett.*, **1**, 342; Cedarholm, J. P., Townes, C. H. (1959). *Nature*, **189**, 1350.
34. Champeney, D. C., and Moon, P. B. (1961). *Proc. Phys. Soc.*, 77, 350.
35. Champeney, D. C., Isaak, G. R., and Khan, A. M. (1963). *Phys. Lett.*, 7, 241.
36. Turner, K. C., and Hill, H. A. (1964). *Phys. Rev.*, **134**, B252.
37. Isaak, G. R. (1970). *Phys. Bull.*, 255.
38. Cialdea, R. (1972). *Nuovo Cimento Lett.*, **4**, 821.
39. Carnaham, C. W. (1962). *Proc. IRE*, **50**, 1976.
40. Tyapkin, A. A. (1973). *Nuovo Cimento Lett.*, 7, 760.