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A Test Theory of Special Relativity: I. Simultaneity and Clock Synchronization

REZA MANSOURI AND ROMAN U. SEXL

Institut für Theoretische Physik der Universität Wien and Institut für Weltraumforschung der Österreichischen Akademie der Wissenschaften, Wien, Austria

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Abstract

The role of convention in various definitions of clock synchronization and simultaneity is investigated. We show that two principal methods of synchronization can be considered: system internal and system external synchronization. Synchronization by the Einstein procedure and by slow clock transport turn out to be equivalent if and only if the time dilatation factor is given by the Einstein result $(1 - v^2)^{1/2}$. An ether theory is constructed that maintains absolute simultaneity and is kinematically equivalent to special relativity.

§(1): *Introduction*

In this series of papers we shall present a systematic approach towards a test theory of special relativity. One generally has the feeling that Einstein's theory is in agreement with experiment to a high degree of accuracy, but it is difficult to express this "high degree" in specific numbers. This is in contrast to the situation in general relativity, where a well developed test theory exists (see, e.g., [1, 2]).

Here we shall investigate the effects of deviations from special relativity on the outcome of various experiments. For this purpose a class of rival theories has to be defined and to be compared with relativity. The results of Frank and Rothe [3] as refined by Berzi and Gorini [4] show that the relativity principle (if assumed to be strictly valid) implies the constancy of one fundamental velocity. There is little reason to doubt that this velocity is to be identified with the velocity of light and the concept of "small deviations" does not seem to be useful here. On the other hand the validity of the relativity principle seems to be less evident now than, say, twenty years ago. The discovery of the cosmic background radiation has shown that cosmologically a preferred system of reference does exist. This system is defined and singled out much more unambiguously to be a candidate for a possible "ether frame" than was the solar rest frame in Einstein's days. It seems worthwhile, therefore, to investigate possible effects of deviations from the relativity principle on local experiments.

§(2): *Historical Survey*

One of the most debated problems in special relativity is the role of convention in the definition of the simultaneity of distant events and the related question of first-order experiments (first order in v/c). Around the turn of the century a number of experiments were designed and in part also carried out that attempted to measure the velocity of light in a system moving relative to the ether and to determine in this way the velocity of the earth in the ether. The assumption in these experiments was at first that the velocity of light is, in the simplest case, $c(v) = c \pm v$, where v is the velocity of the system considered (earth) with respect to the ether. By measuring the one-way velocity of light between two points one could hope to determine v. One soon became aware of the fact that this experiment required the synchronization of clocks at these points. The only accurate method proposed for achieving this was the synchronization with the help of electromagnetic waves. S. Newcom [5] had shown already in 1880 that this procedure compensates the very effect which one seeks to measure and no anisotropy of the one-way velocity of light can be expected if this synchronization method is chosen. This fact was not generally known and acknowledged, as for instance the papers of Schweizer [6] and Wien [7] show. Other methods for synchronizing clocks were discussed by Andrade [8] and Brillouin [9].

In these papers "synchronization" was not discussed as a theoretical problem requiring a definition, but rather as an experimental difficulty to be overcome. H. Poincaré seems to have been the first one to recognize the synchronization of distant clocks to be a fundamental problem. In a number of papers [10] he discussed this problem in detail and analyzed synchronization with the help of optical and other (even gravitational) signals. His considerations culminated in a talk given in 1904 at the International Congress of the Arts and Sciences [10b] at St. Louis, in which he also formulated a relativity principle (see also [10e, f]).

Poincaré never abandoned "absolute time." For him the ether was a reality and the universe consisted of "electrons, ether and nothing else" [10c]. It was but the generally valid principle of relativity which prevented the measurement of absolute time. Poincaré was aware of the fact that clock synchronization by means of light signal leads to Lorentz's concept of local time. He regarded this procedure concerning local time to be a useful mathematical tool which permitted one to express the laws of nature in the simplest form.

Einstein's paper of 1905 [11] gives a different turn to the problem. Einstein defined simultaneity by postulating the constancy of the velocity of light. When clocks are synchronized according to the Einstein procedure the equality of the velocity of light in two opposite directions is trivial and cannot be the subject of an experiment.

A different situation results when we consider the two-way velocity of light (closed light path). Here no synchronization of clocks at different points is required and the constancy of the round trip time for light can be the subject of an experiment.

Einstein's procedure to synchronize clocks at different space points is but one of several possible alternative conventions. Another convention which has been discussed frequently in the recent epistemological literature is the synchronization by (slow) clock transport [12, 13]. This had been considered in some detail by Eddington [14], who noted the conventional character of synchronization. He showed that transport synchronization is equivalent to the Einstein procedure within the framework of special relativity. The definitory character of synchronization has also been emphasized by Mandelshtam [15].

The role of convention in the synchronization of clocks has been advocated especially by Reichenbach [16], Grünbaum [17], and also by Molchanov [18]. Reichenbach has even defined a method of synchronizing clocks by means of light signals which differs from Einstein's.

Ives, who never acknowledged Einstein's role in the creation of relativity, has tried in several papers [19] to define synchronization by means of clocks moving at arbitrarily large velocities.

Møller's [20] discovery in 1957 of the possibility of new first-order experiments (due to improved experimental method) led to a renaissance of interest in these experiments and to new discussions about the role of convention in the formulation of special relativity. This epistemological discussion is summarized in "A panel discussion" [13, 21].

In this paper we shall investigate under which conditions the clock synchronization by slow clock transport is equivalent to the Einstein procedure. The importance of this result is that, if transportation is chosen as the method of synchronization, the one-way light velocity experiments discussed frequently in the recent literature [22] become nontrivial.

Both the Einstein procedure and the transportation-synchronization will be called *system-internal synchronization*. There are other such procedures, such as shaft synchronization [23-26], and the problem to be solved here is the equivalence of the various synchronization procedures. This problem will be solved in part in this paper.

System-internal methods of synchronization are not the only conceivable ones. In Section 3 we shall discuss in detail an alternative procedure belonging to the class of *system-external synchronization* methods. Here one system of reference is singled out ("the ether system") and clocks in all systems are synchronized by comparing them with standard clocks in the preferred system of reference. Infinitely many inequivalent system-external synchronization procedures are possible. Among these, one is of special interest: A convention about clock synchronization can be chosen that does maintain absolute simultaneity. Based on this convention an ether theory can be constructed that is, as far as kinematics is concerned (dynamics will be studied in a later paper in this series), equivalent to special relativity. In this theory measuring rods show the standard Fitzgerald-Lorentz contraction and clocks the standard time dilatation when moving relative to the ether. Such a theory would have been the logical consequence of the development along the lines of Lorentz-Larmor-Poincaré. That the actual development went along different lines was due to the fact that "local time" was introduced at the early stage in considering the covariance of the Maxwell equations.

§(3): *Synchronizing Clocks in One Space Dimension*

In this section we shall study the problem of the synchronization of clocks in one space dimension. Several methods of synchronization will be studied, such as the Einstein procedure s_E (synchronization by light signals) and synchronization by slow clock transport s_T (the method considered in classical physics).

We first restrict the class of theories considered by an assumption concerning the propagation of light:

(L1) The velocity of light is independent of the motion of the source.

Secondly, we assume that a preferred reference frame Σ ("ether frame") exists which is characterized by the following condition:

(Σ1) In Σ the synchronization method s_E and s_T agree.

(Σ1) implies that the velocity of light, as measured by s_T synchronized clocks is isotropic in Σ. We shall set the velocity of light in Σ equal to 1 by an appropriate choice of units. (Σ1) will, in general, not define Σ uniquely. In this case any reference frame that fulfils (Σ1) can be chosen to be the ether system.

We now consider a second reference frame S with a relative velocity $v < c$ with respect to Σ, i.e., the origin of S moves with velocity v when measured with standard rods and clocks in Σ. In S a number of clocks and rods with the same internal constitution as those in Σ are assumed to exist.

We also note explicitly the following assumption (which is usually tacitly made) that two measuring rods of different composition which agree in length in Σ also agree in length in S. Finally we assume that two clocks that have the same period in Σ also do have the same period when brought into system S. These assumptions are by no means trivial.

Note that no procedure for synchronizing clocks in S has been specified yet. One of our aims is to determine the influence of various conventions about synchronization on the transformation (generalized Lorentz transformation) between Σ and S.

Standard arguments (which we shall criticize later) imply that the transformation between the space-time coordinates T, X in Σ and t, x in S has to be linear. The following choice of the coefficients of the transformation will turn out to be suitable for our purposes:

$$\begin{aligned} t &= aT + \epsilon x \\ x &= b(X - vT) \end{aligned} \tag{3.1}$$

here a, b, and ϵ are arbitrary functions of v, which are to be determined either by experiment or by theoretical reasoning. The coefficients $1/a(v)$ and $b(v)$ are the time-dilatation and length contraction factors, which are to be determined either theoretically or by experiments such as the Michelson–Morley, Kennedy–Thorndike, and Ives–Stilwell experiments. We shall adopt the point of view here, however, that these factors have not been determined exactly to be given by their relativistic values

$$\frac{1}{a(v)} = b(v) = \frac{1}{(1 - v^2)^{1/2}} \tag{3.2}$$

because of the limited accuracy of all experiments. We shall explore the consequences of possible deviations of a and b from the values predicted by special relativity.

While a and b can thus be determined by experiment the situation is very different as far as ϵ is concerned. ϵ is determined by the convention about clock synchronization, and arbitrary values of ϵ can be achieved by defining a suitable synchronization mechanism.

If we take, for example, special relativity to be our starting point the values of a, b are given by (3.2) and $\epsilon = -v$. Resynchronizing clocks by some other prescription (for which we shall give explicit examples later) in special relativity amounts to replacing

$$t \to t + f(x, v) \tag{3.3}$$

where $f(x, v)$ characterizes the synchronization procedure chosen. The substitution (3.3) amounts to readjusting clocks in each inertial system at each position x by a specified amount $f(x, v)$. If we start, for example, from the Lorentz transformation

$$\begin{aligned} t &= (1 - v^2)^{1/2}\, T - vx \\ x &= \frac{1}{(1 - v^2)^{1/2}} (X - vT) \end{aligned} \tag{3.4}$$

and choose to readjust our clocks according to

$$f(x, v) = -vx \tag{3.5}$$

we obtain [28]

$$\begin{aligned} t &= (1 - v^2)^{1/2}\, T \\ x &= \frac{1}{(1 - v^2)^{1/2}} (X - vT) \end{aligned} \tag{3.6}$$

This transformation is—as far as the prediction of experimental results is concerned—completely equivalent to (3.4) [i.e., if the result of an experiment can be predicted correctly with the help of (3.4) it can also be predicted correctly using (3.6)]. The only difference is that one inertial system Σ has been singled out arbitrarily to be the ether system (X, T) and clocks have been synchronized in this system only by means of the Einstein synchronization. In all other systems clock synchronization can be thought of as being achieved in two different ways:

(a) First Einstein synchronization is carried out and then clocks are readjusted according to (3.3) and (3.5), where v is the velocity of the system considered with respect to the (arbitrarily chosen) ether system Σ.

(b) According to (3.6) $T = 0$ implies $t = 0$. This means that the synchronization procedure given above can be carried out simply by choosing one system to be the ether system, synchronizing clocks according to the Einstein procedure in this system, and then synchronizing clocks in all other systems moving past Σ by adjusting these clocks to $t = 0$ whenever they fly past a clock in Σ which shows $T = 0$. Such a synchronization procedure, in which clocks in a system S are synchronized by comparing them with clocks in an ether system Σ will be called *external synchronization*. The special external synchronization given by (3.6) will be called s_A (A stands for absolute simultaneity). External synchronization is a possible, but not very meaningful, synchronization procedure in special relativity. By singling out arbitrarily one system Σ to be the ether system one destroys the equivalence of all inertial systems by choosing a different clock synchronization procedure (3.5) in each system.

If we intend to compare special relativity with various ether theories, however (as is our intention here), one has to study all possible conventions about clock synchronization in order to determine the possible empirical equivalence of theories which differ greatly in their basic concepts (theoretical terms).

Before we enter into a detailed discussion of how one has to readjust space-time diagrams if one wishes to work with (3.6) instead of the usual Lorentz transformation we mention one other peculiar fact. Usually it is stated that the transformation connecting two inertial systems has to be linear [4, 27]. The reason for this is, that the world-line of a body undergoing force free motion has to be a straight line in space-time in each inertial system (rectilinear and uniform motion). This presupposes, however, that the uniformity of the motion of the body is determined by comparing the motion with clocks at the various instantaneous positions of the body. In order for this procedure to be possible a number of clocks have to exist that are synchronized, for example, in the way prescribed by Einstein. The existence of such a class of clocks is, however, by no means necessary in order to determine uniform motion. If one clock only is given in an inertial system the uniformity of the motion of a body can also be determined by having it emit a standard signal towards the clock whenever it arrives at a number of equidistant space points. If these signals (e.g., light signals) are received at regular intervals at the clock, the motion of the body is defined to be uniform. (An equivalent way would be to carry along a second clock with the moving body and to emit signals at standard time intervals towards the clock at rest in the inertial system. In this way one determines in fact the uniformity of the motion of a space ship without having to have synchronized clocks all over space.)

§(4): *Relativity without Relativity*

In this section we shall discuss in some detail the much debated question of the equivalence of special relativity and ether theories. We have already seen that the transformation

$$\begin{aligned} t &= (1 - v^2)^{1/2}\, T \\ x &= \frac{1}{(1 - v^2)^{1/2}}\,(X - vT) \end{aligned} \tag{4.1}$$

differs from the Lorentz transformation only insofar as the convention about the synchronization of clocks is concerned. We shall investigate here how the results of various experiments, which are usually considered to be tests of special relativity, can be interpreted using (4.1).

The transformation (4.1) is the very relation one would write down if one has to formulate an ether theory in which rods shrink by a factor $(1 - v^2)^{1/2}$ and clocks are slow by a factor $(1 - v^2)^{1/2}$ when moving with respect to the ether. Note that (4.1) implies the existence of *absolute simultaneity* since $\Delta T = 0$ implies $\Delta t = 0$. We thus arrive at the remarkable result that *a theory maintaining absolute simultaneity is equivalent to special relativity.*

To illustrate this fact let us draw the diagram corresponding to the transformation (4.1) (Figure 1). The main difference between this space-time diagram and the standard disgrams well known from special relativity is that the x axis is not rotated with respect to the X-axis. This expresses the fact that (4.1) maintains the concept of absolute simultaneity. The only difference between Figure 1 and diagrams corresponding to *Galilei transformations* is the position of the unit

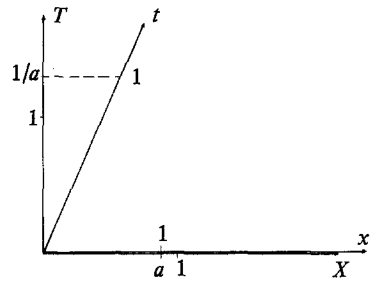

Fig. 1. Space-time diagram corresponding to transformation (4.1).

points on the x, t axis. These are found from (4.1) to be at the positions shown with $a = (1 - v^2)^{1/2}$, whereas for the Galilei transformation we would have $a = 1$.

The position of the unit point on the t axis shows that the rate of clocks moving with respect to the ether system is decreased by the standard factor as compared to clocks in the (T, X) system. Seen from the moving systems the the clocks in the ether system are, however, fast by a factor $T_2 = t/(1 - v^2)^{1/2}$. The relation between the clocks in the moving and the ether system is thus not reciprocal as usual. This is due to the fact that we have singled out one system with the help of the synchronization procedure.

Let us consider the clock paradox from the point of view chosen here (Figure 2). If twin A remains at rest in the ether system for two years and twin B travels into space and returns with velocity v then B will be aged $T_2 = 2(1 - v^2)^{1/2}$ upon return. The reason for this is that in the ether theory clocks moving with respect to the ether are slow. Next consider a clock in B moving uniformly with velocity v with respect to the ether as shown in Figure 2b. Clock A is first at rest in the ether system and thus is fast as seen from the moving clock B (this statement does depend on a definition of simultaneity, e.g., on the convention about synchronization). At the time $s_1 = T = (1 - v^2)^{1/2}$ clock A starts to move uni-

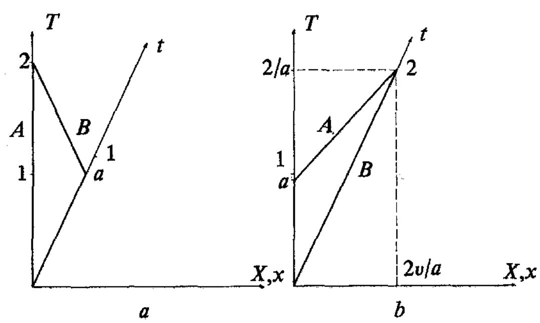

Fig. 2. The twin paradox considered from the point of view of ether theory $[a = (1 - v^2)^{1/2}]$.

formly towards clock B with velocity $w = 2v/a(2/a - a)^{-1}$ such as to reach clock B at time $t = 2$. Using (4.1) one easily finds that the proper time interval (as measured by A) $s_2 = (1 - v^2)^{1/2}$ is spent by A on its way towards twin B. Twin A spends the same (proper) time moving away from B as towards 2, so that the situation is analogous to Figure 2a. Owing to the very fast motion of A on the second part of her trip through space-time, A ages very little on the second part of the trip and thus arrives at the meeting point with B after a proper time interval $s = 2(1 - v^2)^{1/2} < 2$ has elapsed. We thus arrive (not surprisingly) at a result completely equivalent to the standard result of special relativity. Here however we have to distinguish carefully between the case of the twin remaining at rest in the ether system or a twin moving with constant velocity through space-time (Figures 2a and 2b, respectively). The explanation is completely different in the two cases when we use the convention about simultaneity chosen here. From the point of view of the transformation (4.1) the agreement of the final results in both cases appears as a fortunate coincidence without deeper reason. This fact is very reminiscent of the old, prerelativistic theories and shows how the choice of unsuitable conventions can destroy the internal symmetry of a physical theory.

As a second example we consider Lorentz contraction. Figure 1 shows clearly that a measuring rod moving with respect to the ether does show the standard length contraction. A measuring rod at rest in the ether does appear, however, to be elongated from a moving system of reference rather than contracted. This is due to the fact that the definition of length measurement does depend on the definition of simultaneity and thus on the convention on clock synchronization. The Lorentz dilatation obtained here is thus in no contradiction with any experiments but simply reflects our different choice of conventions.

Summarizing these results we may say that the following statement is in perfect agreement with all experimental evidence: A preferred system of reference, the ether system, exists. Clocks are slow when moving with respect to the ether system and measuring rods shrink. As seen from a moving system clocks in the ether system are fast and measuring rods elongated.

As a final example of the influence of a nonstandard clock synchronization on physical laws we discuss the velocity addition theorem. Putting $x = ut$, $X = wT$ in (4.1) we obtain for the velocity w (as measured in the ether system) of an object moving with velocity u in system S

$$w = v + u(1 - v^2) \tag{4.2}$$

One easily sees that this modified velocity addition theorem does not exclude super light velocities and in fact does predict unisotropic light propagation in all frames except Σ. Starting from here we can study the influence of various conventions on experiments attempting to measure the one-way velocity of light. This will be done in the next paper in this series (Paper II, [22]).

§(5): *System Internal Synchronization*

In order to arrive at a test theory of special relativity we shall abandon the assumption that the values of a, b are given by their special relativistic values (3.2).

We start from the transformation

$$\begin{aligned} t &= a(v)\,T + \epsilon(v)\,x \\ x &= b(v)\,(X - vT) \end{aligned} \tag{5.1}$$

where $a(v)$ and $b(v)$ remain unspecified at first.

Our first task will be to determine the values $\epsilon(v)$ corresponding to various conventions about clock synchronization. Since no relativity principle will be assumed to be valid in this section, there will be in general only one preferred ether frame Σ, in which synchronization by slow clock transport and by the Einstein procedure will agree.

Consider a clock in the system S, [S is the (t, x) system] moving with a small velocity u along the x axis. This clock will be used (in the limit of infinitesimal u) to synchronize all other clocks in S. Using $x = ut$ and (5.1) we obtain

$$X = T\left[\frac{au}{b(1-\epsilon u)} + v\right] \simeq T\left(\frac{au}{b} + v\right) =: wT \tag{5.2}$$

if we neglect terms of order u^2 and higher. In (5.2) w is the velocity of the clock relative to Σ. If we consider this clock to be at rest at the origin of the system $S' : (t', x')$ moving with velocity w relative to Σ we obtain for the time t' which the moving clock shows

$$t'|_{x'=0} = a(w)\,T + \epsilon(w)\,x'|_{x'=0} = a(w)\,T \tag{5.3}$$

Synchronizing clocks by slow clock transport means that we require $t'(x) = t(x)$. Therefore we have

$$a(v)\,T + \epsilon(v)\,x = a(w)\,T \tag{5.4}$$

where

$$x = b(v)\,(X - vT) = b(v)\,(w - v)\,T = ua(v)\,T \tag{5.5}$$

Inserting this into (5.4) we obtain

$$\begin{aligned} a(v)\,u\epsilon_T &= a(w) - a(v) \simeq \frac{a(v)\,u}{b(v)}\frac{da}{dv} \\ \epsilon_T &= \frac{1}{b(v)}\frac{da(v)}{dv} \end{aligned} \tag{5.6}$$

The index T indicates that the value of ϵ given by (5.6) refers to transport synchronization of clocks. Assuming the special relativistic values for a, b we obtain the well-known relativistic result $\epsilon = -v$.

Next we shall consider the convention for clock synchronization proposed by Einstein and we shall calculate the factor ϵ_E for this case. Consider two clocks A and B at rest in the system S as shown in Figure 3. At $t = 0$ we send a light signal from A to B, where it arrives at time t_1 and is sent back to A, where it is received at time t_2. According to the Einstein procedure we shall define $t_2 = 2t_1$.

Since the clock A is at rest at the origin of S we have according to (5.1)

$$t_2 = a(v)\, T_2 + \epsilon_E x \big|_{x=0} = a(v)\, T_2 \tag{5.7}$$

and furthermore

$$t_1 = a(v)\, T_1 + \epsilon_E x_1 = \tfrac{1}{2} t_2 = \tfrac{1}{2} a(v)\, T_2 \tag{5.8}$$

where $x_1 = b(v)\,(X_1 - vT_1)$. The isotropy of the propagation of light in Σ implies $X_1 = T_1$ and therefore we obtain from (5.7) and (5.8)

$$a(v)\, T_1 + \epsilon_E b(v)\,(1 - v)\, T_1 = \tfrac{1}{2} a(v)\, T_2 \tag{5.9}$$

Now consider clock A, for which $X_2 = vT_2$. Inserting this and $X_1 - X_2 = T_2 - T_1$ (propagation of light) into (5.9) we obtain

$$\epsilon_E = -\frac{va(v)}{(1 - v^2)b(v)} \tag{5.10}$$

We thus arrive at the important result that the Einstein procedure in general differs from the synchronization by clock transport. The equality of both procedures is neither trivial nor logically cogent.

If we require the equality of ϵ_T and ϵ_E we have

$$a(v) = (1 - v^2)^{1/2} \tag{5.11}$$

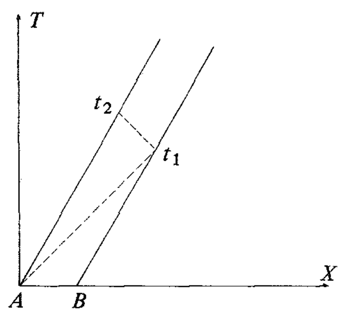

Fig. 3. Einstein synchronization of clocks.

Thus clock synchronization by clock transport and by the Einstein procedure agree if and only if the time dilatation factor is given exactly by the special relativistic value $a(v) = (1 - v^2)^{1/2}$.

§(6): *Synchronization in Three Dimensions*

We now turn to the full three-dimensional case. Here the most general linear transformation between the ether system Σ and a moving system S involves 16 coefficients, which have to be determined by kinematics, convention (synchronization), and physics. It is instructive to present the stages of simplification of the transformation in some detail, since a number of assumptions become transparent that are usually tacitly made. Our starting point is the general linear transformation

$$
\begin{aligned}
t &= aT + \epsilon x + \epsilon_2 y + \epsilon_3 z \\
x &= b_1 T + bX + b_2 Y + b_3 Z \\
y &= d_1 T + d_2 X + dY + d_3 Z \\
z &= e_1 T + e_2 X + e_3 Y + eZ
\end{aligned}
\tag{6.1}
$$

The first kinematical restriction is that the x and X axes slide along one another, i.e.,

(Kin 1) $$\forall T, X:\ y = z = 0 \longrightarrow Y = Z = 0 \tag{6.2}$$

From this we obtain

$$d_1 = d_2 = e_1 = e_2 = 0 \tag{6.3}$$

Secondly, we postulate that the (x, z) and the (X, Z) planes coincide at all times, i.e., the systems Σ and S slide along these planes:

(Kin 2) $$\forall T, X, Z:\ y = 0 \longrightarrow Y = 0 \tag{6.4}$$

whereupon we obtain

$$d_3 = 0.$$

The third requirement is that the origin of S moves with velocity v with respect to Σ:

(Kin 3) $$X = vT, \qquad Y = Z = 0 \longrightarrow x = y = z = 0 \tag{6.6}$$

This leaves us with the transformation

$$
\begin{aligned}
t &= aT + \epsilon x + \epsilon_2 y + \epsilon_3 z \\
x &= b(X - vT) + b_2 Y + b_3 Z \\
y &= dY \\
z &= eZ + e_3 Y
\end{aligned}
\tag{6.7}
$$

This exhausts the possible kinematical conditions and leaves us with a transformation containing ten arbitrary coefficients, three of which we understand to be determined by synchronization requirements (synchronization along three axes).

The other seven coefficients are to be determined either experimentally or by restricting the class of theories considered. We shall assume the following here

(S1) There is no preferred direction in Σ.

The assumptions (S1) and (Σ1) imply in the three-dimensional case that the velocity of light is isotropic in Σ. [(S1) also implies that the only preferred direction, as far as the system S seen from Σ is concerned, is $\mathbf{v}$.]

The importance of these assumptions is illustrated in an example in Figure 4. A possible tilt of the (x, y) plane with respect to the (X, Y) plane is described by $e_3 \neq 0$. No such tilt can exist, because $\mathbf{v}$ is a vector (rather than a pseudovector, needed to construct a rotation) and no other vectors (preferred direction) can be used to construct the transformation. Thus

$$(S1) \longrightarrow e_3 = 0 \tag{6.8}$$

Similarly one shows that also

$$(S1) \longrightarrow b_2 = b_3 = 0, \qquad e = d$$

This leaves us with the transformation

$$\begin{aligned} t &= a(v)\,T + \epsilon x + \epsilon_2 y + \epsilon_3 z \\ x &= b(v)\,(X - vT) \\ y &= d(v)\,Y \\ z &= d(v)Z \end{aligned} \tag{6.10}$$

Here $\epsilon, \epsilon_2, \epsilon_3$ are determined by synchronization procedures and $a(v)$, $b(v)$, $d(v)$ by theory (e.g., relativity) or experiment.

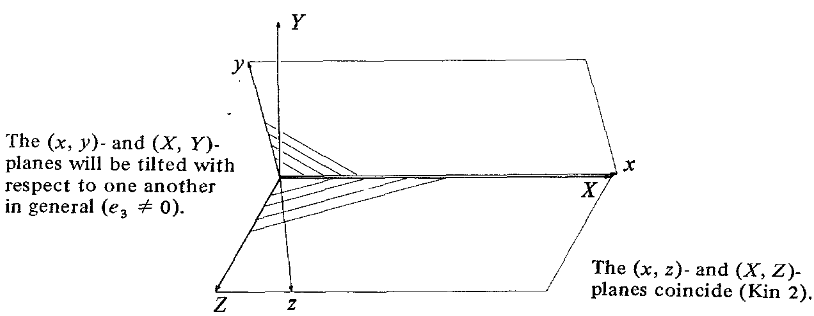

Fig. 4. The meaning of e_3 in equation (6.8).

Calculations analogous to those performed in Section 4 lead to

$$\epsilon_T = a'/b, \qquad \epsilon_{2T} = \epsilon_{3T} = 0 \tag{6.11}$$

for transport synchronization and

$$\epsilon_E = -\frac{av}{b(1-v^2)}, \qquad \epsilon_{2E} = \epsilon_{3E} = 0 \tag{6.12}$$

for the Einstein procedure, while external synchronization with the help of clocks in the ether system leads to

$$\epsilon_{\text{Ext}} = \epsilon_{2\text{Ext}} = \epsilon_{3\text{Ext}} = 0 \tag{6.13}$$

The *arguments given above can be summarized* in an abbreviated manner as follows: If no preferred directions exist in space (only a preferred rest system Σ, the ether system), then the transformation between Σ and S has to be of the general form

$$\begin{aligned} t &= aT + \vec{\epsilon}\vec{\mathbf{x}} \\ \mathbf{x} &= d\mathbf{X} + \frac{b-d}{v^2}\mathbf{v}(\mathbf{v}\mathbf{x}) - b\mathbf{v}T \end{aligned} \tag{6.14}$$

since only $\mathbf{v}$ can be used to construct the transformation coefficient. (6.14) contains the functions $a(v)$, $b(v)$, $d(v)$ (physics) and $\vec{\epsilon}(v) = (\epsilon, \epsilon_2, \epsilon_3)$ (synchronization).

Neither transport synchronization nor Einstein synchronization introduce preferred directions into (6.14) so that in these cases $\vec{\epsilon} \propto \mathbf{v}$, as we have seen before. This leaves us with (6.10), with $\epsilon_2 = \epsilon_3 = 0$.

Finally we turn to the expression for the velocity of light in the system S. The velocity of a light ray propagating in an angle θ with respect to the x axis is given by

$$\begin{aligned} c(\theta) = {} & \{\cos\theta\, \epsilon b\,(1-v^2) + va\cos\theta + \epsilon_2(1-v^2)\, b\sin\theta \\ & - a\,[\cos^2\theta + b^2 d^2(1-v^2)\sin^2\theta]^{1/2}\}\,[\cos^2\theta\,(\epsilon^2 b \\ & - \epsilon^2 bv^2 - a^2/b + 2\epsilon va) + \sin^2\theta\,(\epsilon_2^2 b - v^2\epsilon_2^2 b - d^2 ba^2) \\ & + 2\sin\theta\cos\theta \times \epsilon_2(\epsilon b - v^2\epsilon b + va)]^{-1} \end{aligned} \tag{6.15}$$

This rather lengthy expression is obtained by transforming the light cone $\mathbf{X}^2 - T^2 = 0$ into S [for simplicity we have assumed $\epsilon_2 = \epsilon_3$ in (6.15)]. Various special cases are contained in (6.15):

(A) First order effects result if transport synchronization of clocks is used. In this case we expand

$$a = 1 + \alpha v^2 + \cdots, \qquad b \simeq d \simeq 1, \qquad \epsilon = \epsilon_T = a'/b = 2\alpha v, \qquad \epsilon_2 = 0$$

and obtain

$$c(\theta) = 1 - v(1 + 2\alpha) \cos\theta \tag{6.16}$$

The one-way velocity of light is a measurable quantity in this case and is direction dependent if $\alpha \neq -\frac{1}{2}$.

(B) In the case of Einstein synchronization we obtain (in all orders in v)

$$c(\theta) = \frac{b(1 - v^2)}{a\,[\cos^2\theta + b^2 d^2 (1 - v^2)\sin^2\theta]^{1/2}} \tag{6.17}$$

Here $c(\theta + \pi) = c(\theta)$ so that the velocity of light does not depend on the sense in which the light ray moves. No first-order effects exist in this case and for the second-order effects we obtain

$$1/c(\theta) = 1 + (\beta + \delta - \tfrac{1}{2})v^2 \sin^2\theta + (\alpha - \beta + 1)v^2 \tag{6.18}$$

where β and δ are the coefficients in the expansions

$$b = 1 + \beta v^2 + \cdots, \qquad d = 1 + \delta v^2 + \cdots \tag{6.19}$$

The term $\propto \beta + \delta - \frac{1}{2}$ is tested by the Michelson–Morley experiment, while the Kennedy-Thorndike experiment measures $\alpha - \beta + 1$ (see Paper III, [31]).

§(7): *Conclusion*

In order to arrive at a test theory of special relativity a class of rival theories has to be chosen, with which the theory is to be compared. In this series of papers we consider theories where a privileged system of reference ("ether") exists. This ether system is defined by the requirements that the Einstein and the transport synchronization of clocks agree and that, furthermore, light propagation is isotropic in the ether system.

The synchronization of clocks in other systems will then in general be defined by a synchronization function $f(x, v)$ such that $t = a(v)T + f(x, v)$. While f can in principle be nonlinear in x, only linear functions $f(x, v) = \epsilon(v)x$ turn out to be of practical importance. The three synchronization coefficients $\vec{\epsilon} = (\epsilon, \epsilon_2, \epsilon_3)$ are characteristic for the synchronization method considered.

The possible methods of synchronization can be divided into two classes:

External synchronization sets the clocks in S by comparing $C(S)$ with $C(\Sigma)$, the set of clocks in the ether system Σ.

Internal synchronization does not require that Σ be known and makes use only of operations defined in S. Einstein and transport synchronization are internal synchronization procedures.

Of special interest is the external synchronization S_A (absolute synchronization) for which $f(x, v = -vx$. This is achieved by setting the clocks in S to zero when they pass a clock in Σ showing $T = 0$. A theory maintaining the concept of absolute simultaneity can be obtained in this way which is [when the coefficients $a(v)$ and $b(v)$ are chosen appropriately] empirically equivalent to special relativity, at least as far as kinematics is concerned. Thus the much debated question [29, 30] concerning the empirical equivalence of special relativity and an ether theory taking into account time dilatation and length contraction but maintaining absolute simultaneity can be answered affirmatively.

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