Now the maximum uncertainties present in the $H\rho$'s are no more than 1/3000, or $\epsilon_3=0.03$ percent, and the maximum uncertainty present in XR is no more than 0.8/100, or $\epsilon_4=0.8$ percent. Thus we may say that v is good to less than 0.9 percent and m/e is accurate to well within 1.0 percent. These are the *maximum* relative errors which can be present (aside from any undetected systematic errors) in the values of v and m/e got in this experiment.

Thus the evidence seems to point conclusively to the fact that the RaB β-particles conform more nearly to the expression derived on the Lorentz theory than to the expression derived on the Abraham theory.

Finally, the writers wish to express their sincere appreciation for the valuable aid given them by Dr. H. A. Wilson throughout the experiment. They wish to thank Mrs. O. S. Moilliet for her help in taking the data.

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Relativistic Magnetic Moment of a Charged Particle

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(Received December 26, 1939)

Formulas are derived for the magnetic moment of a particle moving rapidly in a central field of force. Possible nuclear applications, particularly to the problem of the deuteron, are discussed. In view of the greatly increased accuracy in the measurement of magnetic moments, the relativity effect appears to be of measurable magnitude.

THE fact that the Zeeman splitting of the hydrogen lines is given correctly by the Landé formula, even when the relativistic wave equation is used in the calculation, was demonstrated by Dirac.[1] Since his interest was confined to the slowly moving electron in the hydrogen atom, he neglected terms of order v^2/c^2. Breit[2] has given a formula for the magnetic moment of an electron in a heavy atom. At present there is renewed interest in the Zeeman effect problem because of its intimate relation with nuclear magnetic moments. The precision which has recently been achieved in the measurements of the latter, chiefly by the ingenious magnetic-resonance method of Rabi, makes it appear worth while to inquire how the magnetic moment of a charged particle depends in detail on its velocity. The basis of the computation will be Dirac's equation.

In nonrelativistic theory there are two equivalent ways of calculating the magnetic moment of a particle, both giving the same result. One is to determine the energy change of the particle in a weak field and subsequently to compute $\partial E/\partial H$. The other is to calculate the mean value of the operator $(e/2mc)(L_z+2S_z)$. Relativistically, the two procedures give different answers, and the second is probably not justified. Nevertheless it will be discussed briefly later. We first calculate the magnetic energy of a particle moving in a central field of force.

The magnetic term in the Dirac Hamiltonian is $-e\boldsymbol{\alpha}\cdot\mathbf{A}$, $\boldsymbol{\alpha}$ being the operator for $\mathbf{v}/c$ and $\mathbf{A}$ the vector potential. When the uniform field H is chosen along z this perturbation term takes the form

$$V=\tfrac{1}{2}eH(\alpha_x y-\alpha_y x).$$

Written as a matrix[3] it becomes

$$V=ir\sin\theta\begin{pmatrix} 0 & 0 & 0 & e^{-i\varphi} \\ 0 & 0 & -e^{i\varphi} & 0 \\ 0 & e^{-i\varphi} & 0 & 0 \\ -e^{i\varphi} & 0 & 0 & 0 \end{pmatrix}.$$

We must calculate the diagonal elements of this operator for the two states $j=l+\frac{1}{2}$ and $j=l-\frac{1}{2}$. If the components of a ψ-function are $u_1\cdots u_4$,

[1] P. A. M. Dirac, Proc. Roy. Soc. **A118**, 351 (1928).
[2] G. Breit, Nature **122**, 649 (1928).
[3] The representations for α_x and α_y are those in Dirac, *Principles of Quantum Mechanics.*

the diagonal element is

$$\bar{V}=\tfrac{1}{2}ieH\{(1|\rho e^{-i\varphi}|4)-(2|\rho e^{i\varphi}|3)+(3|\rho e^{-i\varphi}|2)-(4|\rho e^{i\varphi}|1)\},\quad (1)$$

where $\rho=r\sin\theta$ and

$$(1|a|4)=\int u_1{}^*au_4d\tau.$$

The normalized functions $u_1\cdots u_4$ are[4]

$$\left.\begin{aligned}
u_1&=\left(\frac{l-m+\frac{3}{2}}{2l+3}\right)^{\frac{1}{2}} if(r)Y_{l+1,\,m-\frac{1}{2}}\\
u_2&=\left(\frac{l+m+\frac{3}{2}}{2l+3}\right)^{\frac{1}{2}} if\ Y_{l+1,\,m+\frac{1}{2}}\\
u_3&=\left(\frac{l+m+\frac{1}{2}}{2l+1}\right)^{\frac{1}{2}} gY_{l,\,m-\frac{1}{2}}\\
u_4&=-\left(\frac{l-m+\frac{1}{2}}{2l+1}\right)^{\frac{1}{2}} gY_{l,\,m+\frac{1}{2}}
\end{aligned}\right\}\text{ if } j=l+\tfrac{1}{2},$$

$$\left.\begin{aligned}
u_1&=\left(\frac{l+m-\frac{1}{2}}{2l-1}\right)^{\frac{1}{2}} if\ Y_{l-1,\,m-\frac{1}{2}}\\
u_2&=\left(\frac{l-m-\frac{1}{2}}{2l-1}\right)^{\frac{1}{2}} if\ Y_{l-1,\,m+\frac{1}{2}}\\
u_3&=\left(\frac{l-m+\frac{1}{2}}{2l+1}\right)^{\frac{1}{2}} gY_{l,\,m-\frac{1}{2}}\\
u_4&=\left(\frac{l+m+\frac{1}{2}}{2l+1}\right)^{\frac{1}{2}} gY_{l,\,m+\frac{1}{2}}
\end{aligned}\right\}\text{ if } j=l-\tfrac{1}{2}.$$

When they are inserted in (1) and the integrals evaluated, the result is

$$\bar{V}=\begin{cases}-4eH\dfrac{(l+1)m}{(2l+3)(2l+1)}\displaystyle\int fgrd\tau \text{ if } j=l+\tfrac{1}{2}\\[2ex] 4eH\dfrac{lm}{(2l-1)(2l+1)}\displaystyle\int fgrd\tau \text{ if } j=l-\tfrac{1}{2}.\end{cases}$$

Both may be combined to

$$\bar{V}=4eH\frac{\kappa m}{(2\kappa-1)(2\kappa+1)}\int fgrd\tau,\qquad (2)$$

if we follow the convention of putting $\kappa=-(l+1)$ for $j=l+\frac{1}{2}$; $\kappa=l$ for $j=l-\frac{1}{2}$. f and g are the two radial functions occurring in $u_1\cdots u_4$.

The integral in (2) can be transformed with the use of the two differential equations defining f and g:

$$\begin{aligned}(\hbar c)^{-1}(E-U+Mc^2)f&=g'+(1+\kappa)g/r\\ (\hbar c)^{-1}(E-U-Mc^2)g&=-f'+(\kappa-1)f/r.\end{aligned}\qquad (3)$$

On multiplying these by g and f, respectively, and subtracting, there results

$$rfg=(\hbar/2Mc)\{rg'g+rf'f+(\kappa+1)g^2-(\kappa-1)f^2\}.$$

Now

$$\int rg'gd\tau=\int g'gr^3dr=-\tfrac{3}{2}\int g^2r^2dr,$$

and a similar relation holds for $\int rf'fd\tau$. Hence

$$\int rfgd\tau=\frac{\hbar}{2Mc}\left[(\kappa-\tfrac{1}{2})\int g^2d\tau-(\kappa+\tfrac{1}{2})\int f^2d\tau\right]=\frac{\hbar}{2Mc}\left[\kappa-\tfrac{1}{2}-2\kappa\int f^2d\tau\right]$$

because

$$\int g^2d\tau+\int f^2d\tau=1.$$

When this is introduced in Eq. (2), the result is

$$\bar{V}=\frac{e\hbar}{2Mc}Hm\frac{\kappa}{\kappa+\frac{1}{2}}\left[1-\frac{2\kappa}{\kappa-\frac{1}{2}}\int f^2d\tau\right].\qquad (4)$$

The first term of this expression is the ordinary formula for the anomalous Zeeman effect, $\kappa/(\kappa+\frac{1}{2})$ being the Landé g factor (whose value is $2(l+1)/2l+1$ if $j=l+\frac{1}{2}$, $2l/l+1$ if $j=l-\frac{1}{2}$). The second term is the relativistic correction. The magnetic moment is obtained from (4) by putting $m=j$ and dividing $\bar{V}$ by H. Thus

$$\mu=\begin{cases}\mu_0(j+\tfrac{1}{2})\left[1-\dfrac{2j+1}{j+1}\displaystyle\int f^2d\tau\right] \text{ if } j=l+\tfrac{1}{2}\\[2ex] \mu_0\dfrac{j}{j+1}(j+\tfrac{1}{2})\left[1-\dfrac{2j+1}{j}\displaystyle\int f^2d\tau\right] \text{ if } j=l-\tfrac{1}{2}.\end{cases}\qquad (5)$$

Here μ_0 is written for $e\hbar/2Mc$. The remaining integral may be evaluated easily for the two limiting cases of high and low energy particles.

[4] Cf. H. A. Bethe, *Handbuch der Physik*, Vol. 24/1, p. 311 *et seq.*

If $E-U \gg Mc^2$, then

$$\int f^2 d\tau = \int g^2 d\tau,$$

so that each integral equals $\frac{1}{2}$. This follows at once when Eqs. (3) are multiplied by f and g, respectively, and then integrated. Hence, for this case, Eqs. (5) reduce to

$$\mu = \pm \tfrac{1}{2}\mu_0 \frac{j+\frac{1}{2}}{j+1}, \tag{6}$$

the upper sign referring to $j=l+\frac{1}{2}$, the lower to $j=l-\frac{1}{2}$.

To discuss the case where $v/c \ll 1$ we turn to the relation

$$\int (E-U+Mc^2) f^2 d\tau = \int (E-U-Mc^2) g^2 d\tau,$$

which is easily obtainable from (3). If we write ϵ for the ratio of the kinetic energy $(E-Mc^2-U)$ to $2Mc^2$, we have

$$\int (1+\epsilon) f^2 d\tau = \int \epsilon g^2 d\tau.$$

Neglecting the small quantity ϵ on the left against 1 we see that $\int f^2 d\tau$ is small against $\int g^2 d\tau$, so that

$$\int f^2 d\tau \doteq \int \epsilon (g^2+f^2) d\tau = \bar{\epsilon}.$$

Thus the correction term in (5) is proportional, in this approximation, to the mean kinetic energy of the particle.

The present results are applicable strictly only to electrons. In attempting to deal with nuclear particles one is confronted with the difficulty that μ_0 is no longer the Bohr (nuclear) magneton. On the other hand, considerable success has been obtained by treating nuclear magnetic moments as composed of two parts, that due to orbital, and that due to spin angular momentum. This corresponds to the assumption that the magnetic moment operator for a single particle is given by

$$\mu_0 \mathbf{L}\delta + 2\gamma \mathbf{S}, \tag{7}$$

where $\delta=1$ if the particle is a proton, 0 if it is a neutron, and γ is the intrinsic magnetic moment of the free particle. It is perhaps of some interest to compute the mean value of (7) with Dirac wave functions. The most convenient way is to put at once $m=j$ in the u functions and then to calculate the mean value of the z component of (7). This procedure yields

$$\mu = (j-\tfrac{1}{2})\delta + \gamma + \frac{j+\frac{1}{2}}{j+1}(\delta - 2\gamma)\int f^2 d\tau \quad \text{if } j=l+\tfrac{1}{2}$$

$$= \frac{j(j+\frac{3}{2})}{j+1}\delta - \frac{j}{j+1}\gamma - \frac{j+\frac{1}{2}}{j+1}(\delta-2\gamma)\int f^2 d\tau \quad \text{if } j=l-\tfrac{1}{2}.$$

The leading terms of these expressions agree, as they must, with those of (5) when δ and γ are put equal to μ_0. The relativistic terms, however, are quite different, the second even with respect to sign.

The relativity effect here computed seems at present to lie within the accuracy of measurement, and it may be of interest in connection with the magnetic moment of the deuteron. When the question is raised as to the additivity of nuclear moments, the present effect will have to be considered. Thus far experiments[5] indicate that the deuteron moment is the exact sum of proton and neutron magnetic moments. This would not be expected to be the case if the deuteron has a sizeable electric quadrupole moment. For the existence of the latter can only be explained by an admixture of a D state function to the ground state of the deuteron. A ^3D state in the relative coordinates of neutron and proton produces a magnetic moment equal to $\frac{3}{4}-\frac{1}{2}(\mu_n+\mu_p)$, while the S state contributes $(\mu_n+\mu_p)$. If we assume, following Bethe,[6] that the relative weights of S and D states in the deuteron wave function are approximately 93 percent and 7 percent, and that[7,8] $\mu_n+\mu_p = (2.789\pm0.003)-(1.935\pm0.030) = 0.854\pm0.033$ nuclear magneton, strict additivity would yield $\mu(H^2) = 0.93(0.854\pm0.033)+0.07(0.323\pm0.016) = 0.817\pm0.032$ n.m. The measured value is 0.856 ± 0.002 n.m. So far, then, there is perhaps

[5] Kellogg, Rabi, Ramsey and Zacharias, Phys. Rev. **56**, 728 (1939).

[6] H. A. Bethe, Phys. Rev. **55**, 1261 (1939). This assumption is of course independent of the meson theory of nuclear forces.

[7] Values and limits of error were kindly communicated by Professor Rabi.

[8] L. W. Alvarez and F. Bloch, Bull. Am. Phys. Soc. **14**, 13 (1939).

no definite evidence against additivity of nuclear moments in the deuteron, although the ranges of error of the computed and measured values barely overlap. It seems, however, that a slight disagreement is produced if the relativity effect is included.

While the exact relativistic treatment of the deuteron problem is ambiguous, it is possible to determine the sign and the order of magnitude of the correction required in the magnetic moment. If we consider the proton alone and apply the first of formulas (5) with $j=\frac{1}{2}$, the correction is $-4\mu_0\bar{\epsilon}/3$. Now $\bar{\epsilon}$ may be computed from any model of the deuteron. It depends of course upon the type of force chosen for the interaction between proton and neutron, and there is a further uncertainty connected with the mass appearing in Mc^2. A reasonable estimate for $\bar{\epsilon}$ arrived at by the potential hole model, seems to be 0.006. This would make the correction -0.022 n.m. It is difficult to see how to treat the neutron and its negative moment. If its absolute value is also diminished in proportion to its μ_0 the sum of the moments will undergo a correction only about $\frac{1}{3}$ as large as the value stated, but the correction will still be negative. These matters, however, will be of greater interest when the neutron moment is known with greater accuracy.

I express my gratitude to Professor Wigner, whose remarks have stimulated these computations.

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The Gamma-Radiation from Nitrogen Bombarded by Deuterons

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(Received January 8, 1940)

The energies and the relative intensities of the gamma-rays emitted from nitrogen bombarded by deuterons of 700 kev energy have been measured by the positron-electron pairs and recoil electrons ejected from thin laminae placed inside a cloud chamber. The distribution of pairs ejected from a lead lamina 0.026 cm thick reveals two strong components of quantum energy 7.2±0.4 Mev and 5.3±0.4 Mev, and a number of weaker components which may be attributed to radiation of about 4 and 2 Mev. There are also a number of pairs which extend up to 11 Mev. The distribution of recoil electrons from a carbon lamina 0.12 cm thick indicates two strong groups of quantum energy 4.2 and 2.2 Mev. No attempt was made to extend the recoil measurements to higher energies. The 7.2-Mev radiation is attributed to the reaction

$$N^{14}+H^2\rightarrow C^{12}+He^4,$$

because radiation of this energy has been observed in other reactions producing C^{12}. The 5.3-Mev radiation is attributed to an excited state of N^{15} of this energy according to the reaction

$$N^{14}+H^2\rightarrow N^{15}+H^1$$

in good agreement with the value of 5.4 Mev predicted by the range measurements of Cockcroft and Lewis. An attempt is made to correlate the energies and intensities of the gamma-rays produced by excited states in C^{12}, N^{15} and O^{15} according to several reactions.

INTRODUCTION

THE gamma-radiation from nitrogen bombarded by deuterons was first investigated by Crane, Delsasso, Fowler and Lauritsen[1] by measuring the recoil electrons ejected from a thick glass wall of a cloud chamber. They obtained a complex spectrum consisting of a number of components at 1.9, 3.1, 4.0, 5.3 and 7.0 Mev. Employing the method of measuring gamma-ray energies by the positron-electron pairs and recoil electrons ejected from thin laminae placed inside a cloud chamber,[2] we have reinvestigated the radiation, and have obtained results which are not in contradiction with the

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[1] Crane, Delsasso, Fowler and Lauritsen, Phys. Rev. **48**, 100 (1935).
[2] Delsasso, Fowler and Lauritsen, Phys. Rev. **51**, 391 (1937); Fowler, Gaerttner and Lauritsen, Phys. Rev. **53**, 628 (1938).