II. "On the Theory of Resonance." By the Hon. J. W. Strutt. Communicated by W. Spottiswoode, F.R.S. Received July 2, 1870.

(Abstract.)

An attempt is here made to establish a general theory of a certain class of resonators, including most of those which occur in practice. When a mass of air or other gas is enclosed in a space bounded nearly all round by rigid walls, but communicating with the external air by one or more passages, there are certain natural periods of vibration or resonant notes whose determination is a matter of interest. If the dimension of the air-space is small compared to the wave-length of the vibration, the dynamics of the motion is, in its general character, of remarkable simplicity. It is for the most part under this limitation that the problem is considered in the present paper. The formula determining the resonant note is

$$n=\frac{a}{2\pi}\sqrt{\frac{c}{S}},$$

where n is the number of complete vibrations per second, a the velocity of sound, and S the capacity of the air-space; c is a quantity proved to be identical with the measure of electric *conductivity* between the interior of the vessel and the external space, on the supposition that the air is replaced by a uniform conducting mass of unit specific conducting-power, and the sides of the vessel and passages by insulators. When there is more than one passage the formula is still applicable according to the above definition of c; and when the passages are sufficiently far apart not to interfere with each other, the resultant c is by the electrical law of parallel circuits simply the sum of the separate values for each passage considered by itself. When this condition is not satisfied the value of c, thus found by mere addition, is too great.

The question thus resolves itself into the determination of the conductivity (or the *resistance* which is its reciprocal) for different forms of passages or openings. The case of openings, which are mere holes in the sides of the vessel, has been already treated, although in a very different way, by Helmholtz, who, in his celebrated paper on vibrations in open pipes, compared his theory with the observations of Sondhauss and others on the notes produced, when such resonators are made to speak by a stream of air blown across the mouth. Sondhauss has also given an empirical formula applicable when the connecting passages are of the form of long cylindrical necks. These previous results are in agreement, as far as they go, with the formula here investigated, and which is applicable whatever may be the length of the neck. If L be the length and R the radius, $\frac{1}{c}$ or the electrical resistance $=\dfrac{L+\frac{\pi}{2}R}{\pi R^2}$.

This supposes the neck a circular cylinder. If the section be an approximate circle of area σ, we may put

$$\frac{1}{c}=\frac{\mathrm{L}}{\sigma}+\frac{1}{2}\sqrt{\frac{\pi}{\sigma}}.$$

When the neck is very long the second term may be neglected, and when L is very small the first term becomes insignificant. In the third part experiments are described which were instituted to compare the general formula with observation, and which gave a satisfactory agreement. The value given above for $\frac{1}{c}$ is only approximate. It is proved, however, that the resistance of a finite cylindrical conductor whose plane ends lie in two infinite insulating planes, but join on to conducting masses on the further side, corresponds to a length $\mathrm{L}+\alpha$ of the cylinder, where

$$\alpha<2{\cdot}305\,\mathrm{R}\,\frac{10{\cdot}615-\epsilon^{-8\frac{\mathrm{L}}{\mathrm{R}}}}{14{\cdot}771-\epsilon^{-8\frac{\mathrm{L}}{\mathrm{R}}}}$$

$$>\frac{\pi}{2}\mathrm{R}.$$

As a particular case, it appears that the correction to the length of an organ-pipe, supposed, as in Helmholtz's paper, to be surrounded at the mouth by a wide flange, lies between ·785 R and ·8282 R.

Approximate formulæ are investigated for the resistance of tubes which are not exact circular cylinders. It will be sufficient to particularize here the case of tubes of revolution. The resistance is shown to lie between the two limits

$$\frac{1}{\pi}\int\frac{dx}{y^2}$$

and

$$\frac{1}{\pi}\int\frac{1}{y^2}\left\{1+\frac{1}{2}\left(\frac{dy}{dx}\right)^2\right\}dx,$$

where y denotes the radius of the tube at the point x.

When there is more than one vessel in the vibrating system, there are several independent periods of vibration corresponding to the degrees of freedom. The theory of these vibrations is also considered.

In the experimental part of the investigation the object is to determine with sufficient precision the pitch of the resonant note. This is generally done by causing the resonator to speak. For several reasons, which are detailed, I consider this course unsatisfactory, and have availed myself of other indications to fix the pitch, which are not, indeed, capable of so great an apparent precision, but yet are more to be depended on.