VI. "On the Application of the Principle of Reciprocity to
Acoustics." By Lord RAYLEIGH, F.R.S. Received May 27,
1876.

In a memoir published some years ago by Helmholtz (Crelle, Bd. lvii.)
it was proved that if a uniform frictionless gaseous medium be thrown
into vibration by a simple source of sound of given period and intensity,
the variation of pressure is the same at any point B when the source of
sound is at A as it would have been at A had the source of sound been
situated at B, and that this law is not interfered with by the presence
of any number of fixed solid obstacles on which the sound may impinge.

A simple source of sound is a point at which the condition of con-
tinuity of the fluid is broken by an alternate introduction and abstrac-
tion of fluid, given in amount and periodic according to the harmonic
law.

The reciprocal property is capable of generalization so as to apply to
all acoustical systems whatever capable of vibrating about a configuration
of equilibrium, as I proved in the Proceedings of the Mathematical
Society for June 1873, and is not lost even when the systems are subject
to damping, provided that the frictional forces vary as the first power of
the velocity, as must always be the case when the motion is small enough.
Thus Helmholtz's theorem may be extended to the case when the medium
is not uniform, and when the obstacles are of such a character that they
share the vibration.

But although the principle of reciprocity appears to be firmly grounded
on the theoretical side, instances are not uncommon in which a sound
generated in the open air at a point A is heard at a distant point B,
when an equal or even more powerful sound at B fails to make itself
heard at A; and some phenomena of this kind are strongly insisted
upon by Prof. Henry in opposition to Prof. Tyndall's views as to the
importance of "acoustic clouds" in relation to the audibility of fog-
signals. These observations were not, indeed, made with the simple sono-
rous sources of theory; but there is no reason to suppose that the re-
sult would have been different if simple sources could have been used.

In experiments having for their object the comparison of sounds heard
under different circumstances there is one necessary precaution to which
it may not be superfluous to allude, depending on the fact that the au-
dibility of a particular sound depends not only upon the strength of that
sound, but also upon the strength of other sounds which may be heard
along with it. For example, a lady seated in a closed carriage and carry-
ing on a conversation through an open window in a crowded thorough-
fare will hear what is said to her far more easily than she can make herself
heard in return; but this is no failure in the law of reciprocity.

The explanation of his observations given by Henry depends upon the
peculiar action of wind, first explained by Prof. Stokes. According to

this view a sound is ordinarily heard better with the wind than against it, in consequence of a curvature of the rays. With the wind a ray will generally be bent downwards, since the velocity of the air is generally greater overhead than at the surface, and therefore the upper part of the wave-front tends to gain on the lower. The ray which ultimately reaches the observer is one which started in some degree upwards from the source, and has the advantage of being out of the way of obstacles for the greater part of its course. Against the wind, on the other hand, the curvature of the rays is upwards, so that a would-be observer at a considerable distance is in danger of being left in a sound-shadow.

It is very important to remark that this effect depends, not upon the mere existence of a wind, but upon the velocity of the wind being greater overhead than below. A uniform translation of the entire atmosphere would be almost without effect. In particular cases it may happen that the velocity of the wind diminishes with height, and then sound is best transmitted *against* the wind. Prof. Henry shows that several anomalous phenomena relating to the audibility of signals may be explained by various suppositions as to the velocity of the wind at different heights. When the distances concerned are great, comparatively small curvatures of the ray may produce considerable results.

There is a further possible consequence of the action of wind (or variable temperature), which, so far as I know, has not hitherto been remarked. By making the velocity a suitable function of height it would be possible to secure an actual convergence of rays in a vertical plane upon a particular station. The atmosphere would then act like the lens of a lighthouse, and the intensity of sound might be altogether abnormal. This may perhaps be the explanation of the extraordinary distances at which guns have sometimes been heard.

The difference in the propagation of sound against and with the wind is no exception to the general law referred to at the beginning of this communication, for that law applies only to the vibrations of a system about a configuration of equilibrium. A motion of the medium is thus excluded. But the bending of the sound-ray due to a variable temperature, to which attention has been drawn by Prof. Reynolds, does not interfere with the application of the law.

An experiment has, however, been brought forward by Prof. Tyndall, in which there is an apparent failure of reciprocity not referable to any motion of the medium*. The source of sound is a very high-pitched reed mounted in a short tube and blown from a small bellows with which it is connected by rubber tubing. The variation of pressure at the second point is made apparent by means of the sensitive flame, which has been used by Prof. Tyndall with so much success on other occasions. Although the flame itself, when unexcited, is 18 to 24 inches high, it was

* Proceedings of the Royal Institution, January 1875; also Prof. Tyndall's work on Sound, 3rd edition.

proved by a subsidiary experiment that the root of the flame, where it issues from the burner, is the seat of sensitiveness. With this arrangement the effect of a cardboard or glass screen interposed between the reed and the flame was found to be different, according as the screen was close to the flame or close to the reed. In the former case the flame indicated the action of sound, but in the latter remained uninfluenced. Since the motion of the screen is plainly equivalent to an interchange of the reed and flame, there is to all appearance a failure in the law of reciprocity.

At first sight this experiment is difficult to reconcile with theoretical conclusions. It is true that the conditions under which reciprocity is to be expected are not very perfectly realized, since the flame ought not to be moved from one position to the other. Although the seat of sensitiveness may be limited to the root of the flame, the tall column of highly heated gas might not be without effect; and in fact it appeared to me possible that the response of the flame, when close to the screen, might be due to the conduction of sound downwards along it. Not feeling satisfied, however, with this explanation, I determined to repeat the experiment, and wrote to Prof. Tyndall, asking to be allowed to see the apparatus. In reply he very kindly proposed to arrange a repetition of the experiment at the Royal Institution for my benefit, an offer which I gladly accepted.

The effect itself was perfectly distinct, and, as it soon appeared, was not to be explained in the manner just suggested, since the response of the flame when close to the screen continued, even when the upper part of the heated column was protected from the direct action of the source by additional screens interposed. I was more than ever puzzled until Mr. Cottrell showed me another experiment in which, I believe, the key of the difficulty is to be found.

When the axis of the tube containing the reed is directed towards the flame, situated at a moderate distance, there is a distinct and immediate response; but when the axis is turned away from the flame through a comparatively small angle, the effect ceases, although the distance is the same as before, and there are no obstacles interposed. If now a cardboard screen is held in the prolongation of the axis of the reed, and at such an angle as to reflect the vibrations in the direction of the flame, the effect is again produced with the same apparent force as at first.

These results prove conclusively that the reed does not behave as the simple source of theory, even approximately. When the screen is close (about 2 inches distant) the more powerful vibrations issuing along the axis of the instrument impinge directly upon the screen, are reflected back, and take no further part in the experiment. The only vibrations which have a chance of reaching the flame, after diffraction round the screen, are the comparatively feeble ones which issue nearly at right angles with the axis. On the other hand, when the screen is close to

the flame, the efficient vibrations are those which issue at a small angle with the axis, and are therefore much more powerful. Under these circumstances it is not surprising that the flame is affected in the latter case and not in the former.

The concentration of sound in the direction of the axis is greater than would have been anticipated, and is to be explained by the very short wave-length corresponding to the pitch of the reed. If, as is not improbable, the overtones of the note given by the reed are the most efficient part of the sound, the wave-length will be still shorter and the concentration more easy to understand[*].

The reciprocal theorem in its generalized form is not restricted to simple sources, from which (in the absence of obstacles) sound would issue alike in all directions; and the statement for *double sources* will throw light on the subject of this note. A double source may be thus defined :—Conceive two equal and opposite simple sources, situated at a short distance apart, to be acting simultaneously. By calling the two sources opposite, it is meant that they are to be at any moment in opposite phases. At a moderate distance the effects of the two sources are antagonistic and may be made to neutralize one another to any extent by diminishing the distance between the sources. If, however, at the same time that we diminish the interval, we augment the intensity of the single sources, the effect may be kept constant. Pushing this idea to its limit, when the intensity becomes infinite and the interval vanishes, we arrive at the conception of a double source having an axis of symmetry coincident with the line joining the single sources of which it is composed. In an open space the effect of a double source is the same as that communicated to the air by the vibration of a solid sphere whose centre is situated at the double point and whose line of vibration coincides with the axis, and the intensity of sound in directions inclined to the axis varies as the square of the cosine of the obliquity.

The statement of the reciprocal theorem with respect to double sources is then as follows :—If there be equal double sources at two points A and B, having axes A P, B Q respectively, then the *velocity* of the medium at B resolved in the direction B Q due to the source at A is the same as the *velocity* at A resolved in the direction A P due to the source at B. If the waves observed at A and B are sensibly plane, and if the axes A P, B Q are equally inclined to the waves received, we may, in the above statement, replace " velocities " by " pressures," but not otherwise.

Suppose, now, that equal double sources face each other, so that the common axis is A B, and let us examine the effect of interposing a screen near to A. By the reciprocal theorem, whether there be a screen or not, the velocity at A in direction A B due to B is equal to the velocity at B

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[*] July 13.—I have lately observed that the flame in question is extremely sensitive to one of Mr. F. Galton's whistles, which gives notes near the limits of ordinary hearing.

in direction A B due to A.   The waves received at B are approximately
plane and perpendicular to A B, so that the relation between the velocity
and pressure at B is that proper to a plane wave; but it is otherwise in
the case of the sound received at A.   Accordingly the reciprocal theorem
does not lead us to expect an equality between the pressures at A and B,
on which quantities the behaviour of the sensitive flames depends.   On
the contrary, it would appear that the pressure at A corresponding to
the given velocity along A B should be much greater than in the case of
a plane wave, and then the relative advantage of the position A would
be explained.

It will be seen that if the preceding arguments are correct, Prof.
Tyndall's experiment does not bear out the conclusions that he has based
upon it with respect to the observations of the French Commission at
Villejuif and Montlhéry.   No acoustic clouds could explain the failure
of reciprocity then observed; and the more probable hypothesis that the
effect was due to wind is not inconsistent with the observation that the
air (at the surface) was moving in the direction against which the sound
was best heard.

Further experiments on this subject are very desirable.

VII. " On Phosphorus Pentafluoride."   By T. E. THORPE, F.R.S.,
Professor of Chemistry in the Yorkshire College of Science.
Received June 6, 1876.

Arsenic trifluoride acts violently upon phosphorus pentachloride with
the formation of *arsenic trichloride* and the liberation of a heavy fuming
gas, which analysis shows to be *phosphorus pentafluoride.*

The production of the new gas may be represented by the equation

$$5\,AsF_3 + 3\,PCl_5 = 5\,AsCl_3 + 3PF_5.$$
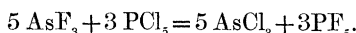
Direct experiments have shown that the amount of arsenic trichloride
actually produced in the reactions agrees closely with that demanded by
this equation.

The accuracy of the formula was further controlled by determinations
of the density of the gas.   Three experiments made by two independent
methods gave the numbers—

$$
\begin{array}{ll}
\text{I.} & 62{\cdot}98 \\
\text{II.} & 63{\cdot}33 \\
\text{III.} & 63{\cdot}39 \\
\end{array}
$$

The number demanded by the formula PF_5 is

$$63{\cdot}0,$$

hydrogen being the unit.