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LETTERS AND COMMENTS

Reply to 'Comment on "Note on Dewan–Beran–Bell's spaceship problem"'

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Abstract
Peregoudov argues that in the tough variant of the thread-between-spaceships problem, the thread breaks sooner or later. I point out that another conclusion is possible.

In the note [1], I pointed out that the distance between two spaceships B and C (the front end and back end ships, respectively), moving forever with identical constant proper accelerations along the same line and starting simultaneously from rest with respect to an inertial frame S (the lab frame), at the moment τ of B's proper time is given by

$$\Delta'_B(\tau) = \frac{c^2}{a} + h\cosh\left(\frac{a\tau}{c}\right) - \sqrt{\left(\frac{c^2}{a}\right)^2 + h^2\sinh^2\left(\frac{a\tau}{c}\right)}, \qquad (1)$$

as measured in the instantaneous rest frame $S'_{B\tau}$ of B at the moment τ.[1] Since in the limit $\tau \to \infty$ the distance $\Delta'_B(\tau)$ tends to c^2/a, I concluded that the thread connecting ships (for which I assumed in [1] that it in no way affects the motion of ships) may never break. The conclusion was so unexpected, and absent from the literature, that I rushed into print. The conclusion was based, I see it now, also on a tacit assumption that the final outcomes (either real or in the sense of limiting values) of events must be the same with respect to the infinite continuous sequence of instantaneous rest frames of either ship.

In the very recent comment [2], Peregoudov objected to my conclusion on two grounds. First, he pointed out that considering the thread that connects ships from the sequence of the instantaneous rest frames of the front end ship $S'_{B\tau}$ only does not cover the whole motion history of the thread; second, and not restricted to [1], he argued that the length of the thread as measured with respect to the instantaneous rest frame of one of its ends 'has nothing to do with real thread stretch'. The second of Peregoudov's objections seemed to me to be in the

[1] Equation (1) is identical with equation (17) of [1], except that the subscript B on $\Delta'(\tau)$ is added. In the present reply, all notation will be that of [1], with slight modifications where necessary.

nascent state, i.e., it was not elaborated in [2][2]; so in this reply I shall concentrate on the first one.

Equation (1) (i.e. equation (17) of [1]) applies at the instant $t' = 0$, where t' denotes the time coordinate in the $S'_{B\tau}$ frame. It is not difficult to verify that at a later instant $t' = (h/c)\sinh(a\tau/c)$ *in the same frame* $S'_{B\tau}$ (τ is arbitrary but fixed!) ship C comes (instantaneously) at rest with respect to that frame[3]. To that event [C instantaneously at rest with respect to $S'_{B\tau}, t' = (h/c)\sinh(a\tau/c)$] corresponds, of course, C's proper time τ.[4] Now an analysis, analogous to the one that led us to equation (17) of [1], reveals that the distance between C and B as measured in $S'_{B\tau}$ at the instant $t' = (h/c)\sinh(a\tau/c)$, when $S'_{B\tau}$ becomes the instantaneous rest frame of C and C's clock shows time τ, is given by

$$\Delta'_C(\tau) = h\cosh\left(\frac{a\tau}{c}\right) - \frac{c^2}{a} + \sqrt{\left(\frac{c^2}{a}\right)^2 + h^2\sinh^2\left(\frac{a\tau}{c}\right)}. \qquad (2)$$

Equation (2) obviously implies that in the limit $\tau \to \infty$, $\Delta'_C(\tau)$ tends to infinity[5]. In what follows, I shall denote by $S'_{C\tau}$ the instantaneous rest frame of C at the moment τ of C's proper time.

Now things seem to be clear. Considering the thread that connects ships from the infinite continuous sequence of $S'_{B\tau}$ frames covers only the part of the thread's history below the corresponding light cone. On the other hand, considering the thread from the $S'_{C\tau}$ sequence covers its whole motion history, and the thread breaks, since $\Delta'_C(\tau)$ tends to infinity with increasing τ, as Peregoudov pointed out. While the $S'_{B\tau}$ sequence covers the whole history of ship B, it does not cover the whole history of the thread, so an inference about what will happen to the thread based on the partial history would necessarily be wrong. Thus Redžić's surprising conclusion of [1] that the thread may never break is incorrect.

While Peregoudov's [2], and Semay's [4], conclusion that the thread breaks sooner or later is undoubtedly correct, one point should be noted: after all, there is no single answer to the query whether the thread will eventually break in the tough variant of the riddle. It will certainly break with respect to the $S'_{C\tau}$ sequence as well as with respect to the lab frame S. However, it may never break with respect to the $S'_{B\tau}$ sequence and this is reality for the $S'_{B\tau}$ sequence of inertial observers. If the thread never breaks for those observers, its eventual breaking for the $S'_{C\tau}$ observers and for the S observer is totally irrelevant for the motion of ship B. Namely, a real thread would necessarily involve the specific work programmes of the motors of B and C in order to maintain their exact hyperbolic motions (cf [5]) and, as can be seen, for the work programme of B's motor the $S'_{B\tau}$ sequence is all that matters[6].

[2] By the way, nowhere in [1] did I state that there exists a comoving frame common to both spaceships in the tough variant of the problem, nor did I imply the existence of such a frame. A contrary statement is clearly found in footnote 7 of [1] (cf also related footnote 21 of [3]).

[3] Needless to say, B is not at rest with respect to $S'_{B\tau}$ at that instant, as is clear from the corresponding Minkowski diagram.

[4] We obviously assume that B's clock and C's clock as well as the clocks that remained at rest in the lab frame S are identical to one another, and also that $\tau_C = \tau_B = 0$ when $t = 0$, which implies $\tau_C = \tau_B \equiv \tau$. It is clear that events [B instantaneously at rest with respect to $S'_{B\tau}, t' = 0$] and [C instantaneously at rest with respect to $S'_{B\tau}, t' = (h/c)\sinh(a\tau/c)$] happen simultaneously with respect to the lab frame S at the instant $t(\tau) = (c/a)\sinh(a\tau/c)$; cf equation (12) of [1].

[5] Semay obtained the result (2) by another route [4]. Unfortunately, I came across Semay's interesting paper only after the publication of [1]. Note that there is an unhappy misprint in [4]: on the right-hand side of equation (30) of [4], one should replace 1 by c^4/a^2.

[6] It is perhaps worth noting that the motion of a spaceship with constant proper acceleration during an infinite time interval seems to be impossible to realize. Recall that in the case of a self-propelling relativistic rocket that moves rectilinearly with constant proper acceleration, the mass of the rocket is an exponentially decreasing function of its proper time τ (cf, e.g., [6]). The limit $\tau \to \infty$ implies vanishing of the rocket's mass, which is meaningless. This probably means that only the mild variant of the problem is possible.

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