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NOTE

Note on Dewan–Beran–Bell's spaceship problem

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Abstract

The thread-between-spaceships problem is analysed both in its 'mild' variant (after some time the ships' acceleration ceases and they coast at the same constant speed, with respect to the lab frame), and in a special case of its 'tough' variant (the ships' acceleration never ceases). It is pointed out that in the special case of the tough variant the thread connecting spaceships may never break, regardless of how close the ships' speed approaches c.

1. A little special relativistic riddle

Consider a little riddle with pictures suitable to a primer of relativity.

Three small spaceships A, B and C drift freely in a region of space remote from other matter, without rotation and relative motion, with B and C equidistant from A (figure 1). The spaceships are at rest relative to an inertial frame S.

At one moment two identical signals from A are emitted towards B and C. On simultaneous (with respect to S) reception of these signals the motors of B and C are ignited (figure 2) and they accelerate gently along the straight line connecting them.

Let ships B and C be identical, and have identical acceleration programmes. Then each point of B will have at every moment the same velocity as the corresponding point of C, and thus any two corresponding points of the ships will always be at the same distance from one another, all measured in S. Let us suppose that a fragile thread connects two identical projections placed exactly at the midpoints of B and C before the motors were started. If the thread *with no stress* is just long enough to span the initial distance in question, then as the ships accelerate the thread travels with them (figure 3). Assume that the thread does not affect the motion of the ships. Will the thread break when B and C reach a sufficiently high speed?

This fascinating riddle was devised by Dewan and Beran (1959) as an illustration of the reality of the FitzGerald–Lorentz contraction and especially of the reality of stress effects due to artificial prevention of the relativistic length contraction. Dewan and Beran's original formulation of the riddle was corrected by Evett and Wangsness (1960), and was recently criticized by Cornwell (2005). Since the problem has been made famous by the famous

Figure 1. Three small spaceships *A*, *B* and *C* at rest relative to an inertial frame *S*, with *B* and *C* equidistant from *A*.

Figure 2. The motors of ships *B* and *C* are ignited simultaneously relative to *S*, and the ships accelerate identically along the straight line connecting them.

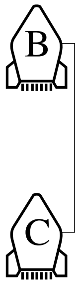

Figure 3. As *B* and *C* accelerate, the thread spanning between the midpoints of the ships before the motors were started travels with them, keeping its initial length.

physicist John Bell (1976), it is now widely known as Bell's problem or Bell's spaceship paradox. In the 'tough' variant of the problem, the acceleration of ships *B* and *C* may never cease and their speed may increase indefinitely approaching *c*, as measured in *S* (Dewan and Beran 1959, Bell 1976, Gershteĭn and Logunov 1998, Flores 2005, 2008). In its 'mild' variant, at an instant of the *S* time the ships' acceleration ceases and they coast with the same constant velocity as measured in *S* (Dewan 1963, Evett 1972, Tartaglia and Ruggiero 2003, Matsuda and Kinoshita 2004, Styer 2007).

According to the testimony of John Bell (1976), a polemic over this old problem that was started once between him and a distinguished experimental physicist in the CERN canteen was eventually passed on to a significantly broader forum for arbitration: the CERN theory division. A clear consensus emerged, testifies Bell, that the thread would not break.

It is now accepted, however, that the answer is wrong. The elementary explication, in Bell's formulation, runs as follows: 'if the thread is just long enough to span the required distance initially, then as the rockets speed up, it will become too short, because of its need to FitzGerald contract, and *must finally break*. It must break when, at a sufficiently high velocity, the artificial prevention of the natural contraction imposes intolerable stress' (Bell 1976, emphasis added).

The purpose of the present note is to point out that the accepted solution to the riddle (the thread will break when the ships reach a sufficiently high speed), is generally wrong. More precisely, while the accepted solution is correct for the mild variant of the problem, it is generally wrong for its tough variant. In our analysis, we in no way question Einstein's special theory of relativity or the reality of Dewan–Beran stresses that develop in a body which is constrained to move in such a manner that its dimensions are fixed with respect to an inertial frame. We simply point out that according to special relativity in some cases the thread will never break, regardless of how close the ships' speed approaches c. The conclusion appears to be new.

The Dewan–Beran–Bell problem with its apparent paradoxes and the rich physics hidden behind it may be used as a vehicle for acquiring special relativistic mentality. So we believe that the present note could be an intriguing reading for the student of relativity at the upper undergraduate level. On the other hand, it is hoped that the reader who is less innocent of relativity might also find some food for thought in it.

2. The mild variant

For the sake of completeness, we shall first discuss the mild variant of the riddle.

Let h denote the distance between any two corresponding points of ships B and C, as measured in S. As is pointed out above, h is the time-independent quantity and equals the initial distance between the two points, all relative to S. Let B and C stop accelerating simultaneously at time $t = t_1$ and eventually, after transient effects have died out, simultaneously reach the same final speed v, again all with respect to S. What will be the distance h'_0 between any two corresponding points of B and C (say between the tips of the projections connected by the thread, shown in figure 3), as measured in the ships' final rest frame S'?

Thus we have to find the relationship connecting the distance, as measured in S', between two material points that are at *permanent rest* relative to S', and that are uniformly moving at the same velocity $\boldsymbol{v}$ along the same straight line relative to S, with the distance between the two (uniformly moving) points as measured in S. Denote the two distances by L'_0 and L_v, respectively. All we have to know, except of course the Lorentz transformation (preferably in its simplest form) and its meaning, is how to measure in an inertial frame the distance between two material points that are uniformly moving with respect to this frame. As is well known, a definition was provided by Einstein more than a century ago, in his first and fundamental paper on special relativity. True, in the relativity paper, Einstein (1905) was speaking about how to measure in the S frame the length of a rod in uniform motion along its length relative to S. However, a little reflection reveals that we do not need the whole rod: its two end (material) points are quite sufficient. According to Einstein's simple and natural definition, we should mark simultaneously (relative to S) instantaneous positions of the two uniformly moving points on the straight material line at rest in S along which they move, and then measure the distance between the marks by a measuring rod at rest relative to S. The sought relationship is well known,

$$L_v = L'_0\sqrt{1 - v^2/c^2}, \tag{1}$$

where, as Einstein pointed out, L'_0 is measured with the measuring rod *already employed* which is now at rest relative to S'.[1]

[1] It should be mentioned that equation (1) *by itself* is not *directly* related to the FitzGerald–Lorentz contraction (or, synonymously, with the relativistic length contraction) of a *free* uniformly moving rod, which requires the additional equation $L_0 = L'_0$, where L_0 denotes the length of the same rod at rest in S, as measured in S. The last equation is an immediate consequence of the principle of relativity, if the rod at rest in S is in the same internal state as that in which the same rod is when at rest in S' (cf Redžić 2008). The meaning of equation (1) is more general than that. It applies whatever is found between the two material points at permanent rest in S': empty space, a free (unstressed or stressed) rod or a stretched (or compressed) string. All that matters for the validity of equation (1) is that the two points are in uniform motion at the same speed v along the same straight line relative to S. So it could be misleading to call equation (1) the 'relativistic length contraction formula' (cf Torretti (2006), and also Redžić (2008)).

The above discussion immediately implies that the sought distance between any two corresponding points of ships B and C, after they reach the persistent state speed v relative to S is given by

$$h'_0 = \frac{h}{\sqrt{1 - v^2/c^2}}, \tag{2}$$

as measured of course in the ships' final rest frame S'. Now we ask a simple question: what would be the length, as measured in S', of the same fragile thread connecting the midpoints of ships B and C, if they were at rest in S' in the same configuration as that depicted in figure 1, and if the thread were again *with no stress* just long enough to span the distance in question? According to the principle of relativity, the length would be h. (In modern parlance, the 'copy-paste' procedure is in perfect agreement with the principle of relativity.) Thus the final stretch of the thread, as measured in S', denoted by δ'_f, is given by

$$\delta'_f = h\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right), \tag{3}$$

in the persistent final state. Obviously, when v increases, δ'_f increases tending to infinity when $v \to c$. Thus, for a sufficiently high speed v the thread reaches its elastic limit and breaks. Recall that S' is the ships' final rest frame, after they reach the constant speed v relative to S. From the corresponding Minkowski diagram it is clear that the ships do not come to rest simultaneously: B stops first; also, B begins to decelerate first, all with respect to S'.

It is interesting to compare the S and S' descriptions of the phenomenon.

In S', ships B and C are initially uniformly moving at the speed v along the same straight line but in the backward, head to tail, direction. The thread connecting the ships' midpoints then has the length $h\sqrt{1 - v^2/c^2}$, according to equation (1). Since, as is assumed above, the thread with no stress has the length h in its rest frame, it follows that the thread uniformly moving along its length at the speed v and having the length $h\sqrt{1 - v^2/c^2}$ is Lorentz contracted, all relative to S'. The thread is perfectly relaxed, with no elastic stress, also with respect to S', since being Lorentz contracted is its natural state when it is in uniform motion along its length with respect to S'. (The thread is *rigidly moving* (cf Rindler 1991)[2].) Then at S' time t'_{iB} the motor of B is ignited, B decelerates and eventually it stops at time t'_{sB}. In the meantime C continues to move uniformly at the speed v until the motor of C is ignited at time $t'_{iC} = t'_{iB} + (vh/c^2)/\sqrt{1 - v^2/c^2}$, then C goes through the deceleration programme identical to that of B, and eventually C stops at time $t'_{sC} = t'_{sB} + (vh/c^2)/\sqrt{1 - v^2/c^2}$. It is clear that during the time interval between t'_{iB} and t'_{sC} the separation between the ships' midpoints, i.e. the length of the thread, continuously increases from $h\sqrt{1 - v^2/c^2}$ to $h'_0 = h/\sqrt{1 - v^2/c^2}$ due to a relative velocity between the ships with respect to S'. It is also clear that the following equation holds:

$$h'_0 = h\sqrt{1 - v^2/c^2} + v\Delta t', \tag{4}$$

[2] By the way, as Dewan and Beran (1959) pointed out, though in a somewhat imperspicuous way, the following distances are not of the same sort: (*a*) the distance between two (unconnected or connected) material points that are constrained to always move at the same instantaneous velocity (which can be time dependent) along the same line with respect to an inertial frame of reference; (*b*) the distance between two ends of a rod *rigidly moving* along its length at a constant speed (which involves the constancy of the rod's *rest* length), with respect to the same frame. Differentiating the two distances is essential in the explanation of the disappearance of the electric field of steady currents in the framework of an elementary but non-trivial model (Zapolsky 1988), and elsewhere (Cavalleri and Tonni 2000).

where $\Delta t' = t'_{iC} - t'_{iB} = t'_{sC} - t'_{sB} = (vh/c^2)/\sqrt{1 - v^2/c^2}$, as Dewan (1963) pointed out[3]. (Note the essential fact that $v\Delta t'$ tends to infinity when $v \to c$, whereas $h\sqrt{1 - v^2/c^2}$ tends to zero in the same limit.) Thus the thread initially uniformly moving with no stress eventually comes to rest being stretched to δ'_f given by equation (3), with the relative stretch equal to $(1 - v^2/c^2)^{-1/2} - 1$ in the persistent final state.

On the other hand, the length of the thread is always h with respect to S, it is a purely spatial, time-independent quantity[4]. As the rockets speed up, the Dewan–Beran stress develops in the thread due to the artificial prevention of its natural need to FitzGerald–Lorentz contract. If the thread *with no stress* were in the uniform motion along its length at the final speed v, its length would be $h\sqrt{1 - v^2/c^2}$, all relative to S. However, if the thread *uniformly* moving along its length at the final speed v is constrained to have the length h, the stretch of the thread with respect to its natural, FitzGerald–Lorentz-contracted length at that speed $h\sqrt{1 - v^2/c^2}$ is given by $h - h\sqrt{1 - v^2/c^2}$, again all relative to S; when $v \to c$, the corresponding relative stretch of the thread $(1 - v^2/c^2)^{-1/2} - 1$ tends to infinity. Thus the thread breaks at a sufficiently high final speed when the stress becomes intolerable.

The above discussion shows that the S and S' frames' *physical realities*[5], corresponding to the same events in the Minkowski world, may be almost comically different. (This of course does not contradict the principle of relativity.) While S blames the breaking of the thread on the artificial prevention of the FitzGerald–Lorentz contraction, S' blames it on a relative velocity between the rockets (due to lack of simultaneity), as Dewan (1963) pointed out in his fine paper. However, the final outcome in the various physical realities is one and the same: the thread breaks at a sufficiently high speed. Also, the S- and S'-explanations of the cause of the breakage are essentially identical: the relative stretch of the thread with respect to its natural, i.e. *free of stress*, length in the corresponding *persistent* final state tends to infinity when the corresponding steady speed of the rockets approaches c, leading to a stress that transcends the thread's elastic limit.

3. The tough variant

It appears, however, that the final outcome need not be fatal in the tough variant of the riddle.

In what follows by B and C we shall denote the tips of the projections at the (initial, relative to S) midpoints of the ships B and C, respectively. Assume that B and C move along the positive x-axis with identical constant proper accelerations a starting from rest at $t = 0$, relative to S.[6] Take that $x_B = x_{B0}$ when $t = 0$ and assume, for the sake of simplicity, that the

[3] Equation (4) can be proved as follows. If the midpoint of ship B traverses a distance s' from the instant t'_{iB} until it stops, then the midpoint of ship C in the time interval between t'_{iB} and its own stopping traverses the distance $v\Delta t' + s'$, due to its uniform motion between t'_{iB} and t'_{iC} and also due to identical deceleration programmes. Taking into account that at t'_{iB} the distance between the ships' midpoints is $h\sqrt{1 - v^2/c^2}$, one obviously has $s' + h'_0 = h\sqrt{1 - v^2/c^2} + v\Delta t' + s'$, where h'_0 denotes of course the distance between the midpoints at t'_{sC} and thereafter.

[4] That a quantity which is purely spatial, time-independent, as measured in one inertial frame, may be time-dependent as measured in another inertial frame is obvious from the corresponding Minkowski diagram. It is, however, somewhat more difficult to imagine that there is such a feature at all, due to our Galilean instincts, inherited from our pre-relativistic ancestors.

[5] By the *physical reality* of an inertial observer we understand what Rindler (1991) calls the *world map* of that observer, i.e. 'a frozen instant in the observer's spatial reference frame'. Note that figure 3 corresponds to a world map and not to a *world picture*.

[6] Note that the motion of a real relativistic rocket in space would necessarily involve a time-dependent rest mass of the rocket (cf, e.g., Henry and Barrabes (1972), and also Rindler (1991, pp 96–7)), contrary to the assumption tacitly made by Gershteĭn and Logunov (1998). The equation of motion of a rocket analysed by the authors would be correct only in the imaginary case when there is solely an external force field acting upon the rocket.

initial distance (for $t \leqslant 0$) between B and C, i.e. the initial length h of the thread with no stress connecting B and C, satisfies the condition

$$\frac{c^2}{a} > h. \tag{5}$$

As is well known, B and C perform identical hyperbolic motions (except for the starting point), their equations of motion being given by

$$x_B = \sqrt{c^2t^2 + \left(\frac{c^2}{a}\right)^2} - \frac{c^2}{a} + x_{B0}, \tag{6}$$

$$x_C = \sqrt{c^2t^2 + \left(\frac{c^2}{a}\right)^2} - \frac{c^2}{a} + x_{B0} - h, \tag{7}$$

respectively, relative to S.

Our analysis simplifies somewhat if we introduce in our inertial frame S a new *spatial* coordinate system such that

$$x^* = x + \frac{c^2}{a} - x_{B0}, \tag{8}$$

leaving other spatial coordinates untouched, $y^* = y, z^* = z$. In what follows by S^* we understand our frame S in which the starred coordinates x^*, y^* and z^* are used as the Cartesian coordinates instead of the original x, y and z. Obviously, in S^* the equations of motion simplify to

$$x_B^* = \sqrt{c^2t^2 + \left(\frac{c^2}{a}\right)^2}, \tag{9}$$

$$x_C^* = \sqrt{c^2t^2 + \left(\frac{c^2}{a}\right)^2} - h. \tag{10}$$

Denoting by τ the proper time of B, taking $\tau = 0$ when $t = 0$, we have (cf, e.g., Rindler 1991, Styer 2007)

$$x_B^*(\tau) \equiv x_{B\tau}^* = \frac{c^2}{a}\cosh\left(\frac{a\tau}{c}\right), \tag{11}$$

$$t(\tau) \equiv t_\tau = \frac{c}{a}\sinh\left(\frac{a\tau}{c}\right) = \frac{x_{B\tau}^*}{c}\tanh\left(\frac{a\tau}{c}\right). \tag{12}$$

A little analysis reveals that the event on C that is simultaneous, with respect to the instantaneous rest frame $S'_{B\tau}$ of B at the moment τ (of course all successive instantaneous rest frames of B are assumed to be in standard configuration with the S^* frame), with the event $(ct_\tau, x_{B\tau}^*)$ satisfies

$$(x_{C\tau}^* + h)^2 - \left(\frac{c^2}{a}\right)^2 = \left[\tanh^2\left(\frac{a\tau}{c}\right)\right]x_{C\tau}^{*2}. \tag{13}$$

The physically acceptable (positive) root of this equation is

$$x_{C\tau}^* = \left[\sqrt{\left(\frac{c^2}{a}\right)^2 - \left(\frac{c^2}{a}\right)^2\tanh^2\left(\frac{a\tau}{c}\right) + h^2\tanh^2\left(\frac{a\tau}{c}\right)} - h\right]\cosh^2\left(\frac{a\tau}{c}\right), \tag{14}$$

with $t_{C\tau}$, lying on the same line of simultaneity, given by

$$t_{C\tau} = \frac{x^*_{C\tau}}{c} \tanh\left(\frac{a\tau}{c}\right). \tag{15}$$

The distance $\Delta'(\tau)$ between B and C, with respect to $S'_{B\tau}$, is equal to[7]

$$\Delta'(\tau) = \sqrt{(x^*_{B\tau} - x^*_{C\tau})^2 - c^2(t_\tau - t_{C\tau})^2} = (x^*_{B\tau} - x^*_{C\tau})\cosh^{-1}\left(\frac{a\tau}{c}\right). \tag{16}$$

From equations (16), (14) and (11) after a somewhat cumbersome but in every step simple calculus we get

$$\Delta'(\tau) = \frac{c^2}{a} + h\cosh\left(\frac{a\tau}{c}\right) - \sqrt{\left(\frac{c^2}{a}\right)^2 + h^2\sinh^2\left(\frac{a\tau}{c}\right)}. \tag{17}$$

Equation (17) obviously implies that the stretch of the thread $\delta'(\tau)$, as measured of course in $S'_{B\tau}$, is given by

$$\delta'(\tau) = \frac{c^2}{a} - h + h\cosh\left(\frac{a\tau}{c}\right) - \sqrt{\left(\frac{c^2}{a}\right)^2 + h^2\sinh^2\left(\frac{a\tau}{c}\right)}. \tag{18}$$

While the results in (17) and (18) are not new (Gleeson (2006) and Styer (2007, equation (29))), the following interesting consequence of equation (18) is not pointed out in the literature, as far as I am aware.

It can be easily verified that the stretch of the thread δ' is an increasing function of τ, since condition (5) is satisfied. In the limit $\tau \to \infty$ we have

$$\delta' \to \delta'_f \equiv \frac{c^2}{a} - h, \tag{19}$$

$$\Delta' \to \Delta'_f \equiv \frac{c^2}{a}, \tag{20}$$

and thus both functions have horizontal asymptotes[8].

Now it seems that for this type of spaceship motion we are in a position to give a somewhat surprising solution to the riddle posed by Dewan and Beran almost half a century ago: will the thread break?

The solution is: it depends.

On the basis of equation (19) we infer that the thread will never break if its critical stretch δ'_c, as measured in the instantaneous rest frame of its front end point, for which it inevitably breaks, is greater than the thread's limiting stretch $\delta'_f \equiv c^2/a - h$. If so, the thread will be under specific, ever increasing stress but unbroken. True, the thread will certainly break sometime due to the fatigue of its material but that is another story.

What about the S frame account of the phenomenon? This is far from being an easy matter. It is clear that a persistent stress that exists in the persistent final state of the thread from the mild variant, must be of a quite different nature than transient (instantaneous) stress corresponding to a transient state of the thread from the tough variant. This circumstance

[7] Note that only the front end B of the thread is instantaneously at rest with respect to $S'_{B\tau}$ at the instant τ; its back end C has a velocity in the direction of decreasing x' at the same instant of the $S'_{B\tau}$ time, moving farther away from B, as is clear from the corresponding Minkowski diagram. (Only in the limit $\tau \to \infty$, the speed of C vanishes.) So it could be misleading to call Δ' the proper distance between B and C since the two material points are not both instantaneously at rest relative to $S'_{B\tau}$.

[8] Recall that the quantity c^2/a may have peculiar meanings in special relativity (Taylor and French (1983), Rindler (1991, p 38)).

seems to be of crucial importance in distinguishing between the two variants, as considered in the S frame. Otherwise, one could argue that the (instantaneous) relative stretch of the thread tends to infinity when $v \to c$ in the tough variant too. As far as I can see, the only way out of this predicament seems to be that the concept of the instantaneous relative stretch, *as considered in S*, should be dismissed as meaningless in its actual form and replaced by a different concept in the tough variant.

4. A comparison of the two variants

Some apparently simple questions may come to mind in relation to the above analyses. In the first place, one may ask what is the difference between the mild and the tough variant of the riddle. The obvious difference lies in the final stage of the motion: in the mild variant the rockets eventually turn their engines off simultaneously and move uniformly, in the tough variant they accelerate all the time, all relative to S. It seems somewhat odd that there may appear a limiting stretch with respect to the instantaneously co-moving frame of the front rocket in the tough variant, with ever accelerating rockets, whereas in the mild variant the stretch may be arbitrarily great, despite the fact that there is a finite final speed of the rockets. So it would be perhaps helpful to briefly analyse the mild variant in the framework of the tough variant. In what follows we attempt an explanation of how the gigantic stretch may appear in the mild restriction of the tough variant discussed above.

Assume that both rockets B and C turn their engines off simultaneously relative to S at the instant t_τ given by equation (12). Thus the instantaneous rest frame $S'_{B\tau}$ of the front rocket B at the moment τ of B's proper time from our tough variant becomes the S' frame from the mild variant discussed in section 2. Let V_τ denote the speed of $S'_{B\tau}$ (i.e. of S') relative to S; from equations (6), (9) and (12) we obtain

$$\frac{V_\tau}{c} = \tanh\left(\frac{a\tau}{c}\right), \tag{21}$$

and consequently

$$\gamma(V_\tau) \equiv \left(1 - V_\tau{}^2/c^2\right)^{-1/2} = \cosh\left(\frac{a\tau}{c}\right). \tag{22}$$

Since the S' and S^* frames are in standard configuration, using equations (12) and (21) we now have

$$t'_{sB} = 0, \tag{23}$$

$$t'_{sC} = \frac{V_\tau h}{c^2}\gamma(V_\tau) = \frac{h}{c}\sinh\left(\frac{a\tau}{c}\right) \tag{24}$$

and

$$\Delta t' = t'_{iC} - t'_{iB} = t'_{sC} - t'_{sB} = \frac{h}{c}\sinh\left(\frac{a\tau}{c}\right). \tag{25}$$

The final length of the thread in S' is given by

$$h'_0 = \frac{h}{\sqrt{1 - V_\tau{}^2/c^2}} = h\cosh\left(\frac{a\tau}{c}\right). \tag{26}$$

From equations (26) and (17) it follows that in the time interval between t'_{sB} and t'_{sC} the thread stretches for the length

$$h'_0 - \Delta'(\tau) = \sqrt{\left(\frac{c^2}{a}\right)^2 + h^2\sinh^2\left(\frac{a\tau}{c}\right)} - \frac{c^2}{a}. \tag{27}$$

On the other hand, from equations (21) and (25) it follows that the length $V_\tau \Delta t'$ that the midpoint of ship C traverses in uniform motion between t'_{iB} and t'_{iC} equals

$$V_\tau \Delta t' = h\left[\sinh\left(\frac{a\tau}{c}\right)\right]\tanh\left(\frac{a\tau}{c}\right). \tag{28}$$

Obviously, in the limit $\tau \to \infty$ the right-hand sides of equations (27) and (28) both tend to infinity as $h\sinh(a\tau/c)$. This implies that the stretching $h'_0 - \Delta'(\tau)$ during the time interval between stopping of B and stopping of C, originating from a segment of the deceleration stage of C's motion, in the limit $\tau \to \infty$ approaches $V_\tau \Delta t'$ and thus h'_0, as is clear from equations (4) and (21).

It is perhaps worthwhile to point out that in the tough variant one has an endless sequence of transient states of the thread, whereas in the mild variant a sequence of transient states ends in a persistent state. Also, the part played in the processes by the 'magic length' c^2/a remains rather intriguing.

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