

HAS THE LORENTZ-COARIANT ELECTROMAGNETIC FORCE LAW BEEN DIRECTLY TESTED EXPERIMENTALLY?

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So far, no convincing experimental evidence for the Lorentz-covariant electromagnetic force law has been reported. This paper utilizes an available experiment, which is often claimed as one of the most precise experiments in modern physics, to analyze the Lorentz-covariant electromagnetic force law. The Lorentz-covariant electromagnetic force law does not receive convincing confirmation by that experiment; rather, its analysis suggests that the Lorentz-covariant electromagnetic force law might be only approximately true. A clean experiment is proposed to test the Lorentz-covariant electromagnetic force law, avoiding ambiguities in the physical meaning of measures in the experiment and complications by other theoretical bones of contention than Einstein's special relativity itself. The proposed experiment can be performed with high precision within present technology.

1. INTRODUCTION

Einstein's special relativity [1,2] and quantum theory are twin pillars of twentieth-century physics. Einstein's special relativity modifies Newton's theory of classical motion for particles whose speeds are comparable with the speed of light. There are many experiments claiming to have corroborated Einstein's special relativity, for example:

the Michelson-Morley experiment [3], the transverse (relativistic) Doppler shift [4-6], the velocity-dependence of relativistic mass [7-11], and the relativistic time-dilation [12-14].

The Lorentz-covariant electromagnetic force law is one of the most important laws in Einstein's special relativity. The Lorentz-covariant electromagnetic force law has been successfully applied in high energy physics; the Lorentz-covariant electromagnetic force law is employed to design synchrotron accelerators and it is also utilized to estimate the relativistic energy-momentum of elementary particles in high energy experiments. So far, no inconsistencies have been found in the implications of the Lorentz-covariant electromagnetic force law. Hence, it is frequently claimed that the successful operation of synchrotron accelerators in high energy physics gives an accurate confirmation of the Lorentz-covariant electromagnetic force law. Nonetheless, no *specific* experimental evidence has been *reported* for the Lorentz-covariant electromagnetic force law in the region of *relativistic* particles [15,16]. In that case, any *direct* experimental evidence for the Lorentz-covariant electromagnetic force law in the relativistic region would be of value.

Recently, experiments to test the electromagnetic force law for particles with speeds comparable with the light-speed c have been suggested by Waldron [17]: "Make systematic observations of the track curvature of a charged particle in a magnetic field, and simultaneously measure the velocity of the *same* particle by a time-of-flight method. Carry out for a range of particle velocities from $c/3$ to nearly c . This would tell us whether the velocity deduced from the Lorentz force law is the same as the velocity as normally understood." Here, the time-of-flight method is a method of measuring the particle velocity by timing its flight over a known distance. We think there are practical obstacles to carrying out the proposed test. Performance of simultaneous measurements of the track curvature as well as the velocity of a relativistic particle by the time-of-flight method in a uniform magnetic field inside a practically accepted region, for example, a cloud chamber, seems to face insuperable technical difficulties.

Another proposed test is: "Accelerate a particle through a high DC voltage (not RF because of the uncertainty of phase) and measure its velocity by a time-of-flight method. Carry out for different voltages, and compare the velocity-voltage relationship with that predicted by various theories." [17] An experiment, which is close to the proposed test, was performed about three decades ago by Bertozzi [18]. Bertozzi's experiment was originally designed to test the velocity-dependence of

kinetic energy, rather than to test the Lorentz force law. In Bertozzi's experiment, the speeds of electrons are indeed determined by measuring the time required for the electrons to traverse a given distance. The kinetic energies of electrons are determined by a thermal technique, *calorimetry*. The measured kinetic energies agree with the expected energies with the experimental error about 10%. That is, the experimental results show that the dependence of the speed of a particle on its kinetic energy follows the prediction of special relativity, but with the experimental uncertainty about 10%. The experimental results also show that there exists a universal limiting speed equal to the speed of light; thus Newtonian mechanics is indeed not valid in the relativistic region. However, due to the large experimental uncertainty, the experiment is not decisive evidence for special relativity being the only correct theory modifying Newtonian mechanics in the relativistic region. It is impossible for that experiment to distinguish special relativity from the other competitive theories [19,20]; the precision of experiments should be, at least, at the order $(v/c)^2$ in order to distinguish the minute differences in predictions between special relativity and the competitive theories. We consider the experiment of Bertozzi as having little significance, owing to its large experimental uncertainty. The relativistic mass experiments [7-11] are also utilized to distinguish the minute differences in predictions between the Lorentz force law in special relativity and various electromagnetic force laws in the competitive theories [21]. However, the precision of these relativistic mass experiments is not beyond the order of $(v/c)^2$ [22-24]. Therefore, it is also impossible for these experiments to distinguish special relativity from the competitive theories. Furthermore, it should be noted that in these experiments the speeds of particles are not directly determined by *the time-of-flight method*. For example, the speeds of particles are either inferred from the Lorentz force law or the formula of relativistic energy-momentum, or determined from measurements of the angle of the Cerenkov radiation. Regarded as tests of the Lorentz force law, these experiments involve either logic circularity or complications by experimental methods whose validity in the relativistic region is very questionable. Therefore, these experiments cannot be convincingly reinterpreted as a *direct* test of the Lorentz force law.

To the best of our knowledge [15-17], the Lorentz force law has not been *directly* tested for particles in the *relativistic* region; and there exist no direct experimental data with respect to the degree of accuracy in testing this fundamental law. Thus, we are unable to formulate a

precise idea as to how the Lorentz force law has supposedly been confirmed by experiments. Is the Lorentz force law for relativistic particles true or only approximately true? Does the Lorentz force law need to be modified? The decisive way to answer these questions is to experimentally test the Lorentz force law in the relativistic region with great precision and then to see whether or not deviations of the predictions of the theory, when compared with experimental results, are within the allowed uncertainties of measurements.

In the present paper we propose a clean experiment testing the Lorentz force law for relativistic particles in which *speeds get measured by the time-of-flight method*. In addition, the proposed experiment can be performed with existing apparatus taken from modern experiments in high energy physics, for example, Bailey *et al.*'s experiments [12-14]. The experiments of Bailey *et al.* are the $(g - 2)$ experiments for muons (g is the gyromagnetic ratio of spin). Those experiments are also claimed to give convincing confirmation of the relativistic time-dilation predicted by Einstein's special relativity. Our paper is organized as follows. First, we utilize their own supposedly positive experimental results to question whether or not the Lorentz force law is true in the relativistic region. Next, we utilize their experiments to suggest how a clean and crucial experiment is to be performed. Finally, we conclude by critiquing the presumption of the veracity of the Lorentz force law and by making a plea for close experimental attention to its status.

2. IS THE LORENTZ-COVRTANT ELECTROMAGNETIC FORCE LAW CONFIRMED EXPERIMENTALLY?

According to Einstein's special relativity, the Lorentz force law is [25]

$$d(\gamma m \mathbf{v})/dt = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \quad (1)$$

for a charged particle of mass m and charge q moving in given electric and magnetic fields, \mathbf{E} and \mathbf{B} , respectively, where $\gamma = (1 - (v/c)^2)^{-1/2}$ and \mathbf{v} is the velocity of the particle. Considering the simplest case of a charged particle in uniform circular motion in a uniform magnetic field \mathbf{B} , we have

$$r = (mv/qB)(1 - (v/c)^2)^{-1/2}, \quad (2)$$

where r is the radius of circular motion. Also, we have, as the period of circular motion of the particle,

$$T = 2\pi r/v. \quad (3)$$

Let us first utilize the experimental results of Bailey *et al.* [12,13] to examine the Lorentz force law. For muons in uniform circular motion in uniform magnetic fields, from Eqs. 2 and 3, we obtain the predictions of v/c versus magnetic field shown in Fig. 1 and the predictions of period versus magnetic field shown in Fig. 2. Here, we use the mass of muon $m = 1.8835326 \times 10^{-28}$ kg, the charge $q = 1.60217738 \times 10^{-19}$ C, and the speed of light $c = 2.99792458 \times 10^8$ m/sec [26], as well as the mean radius of circular motion $r = 7.0059(7)$ m which is estimated from Bailey *et al.*'s experimental data as shown in Table 4 in the paper [13].

Figure 2 in the paper of Bailey *et al.* [12] (or Fig. 18 in the paper [13]) gives the experimental results showing the intensity structure of a muonic bunch in uniform circular motion inside the storage ring. From this relationship of the intensity of the circulating muonic bunch versus the time of flight, the mean period of the circular motion is estimated as 147 ns (ns = 10^{-9} sec) by Bailey *et al.* (The effect on the circular motion of muons by radiation due to acceleration is negligible.) The more precise value of the mean period is 146.910(19) ns, which is deduced from the mean rotation frequency 6.8069(9) MHz as given in the paper [27]. The magnetic field inside the storage ring in Bailey *et al.*'s experiment is about 1.472 tesla. In addition, the magnetic field inside the storage ring is stable and reproducible to a few ppm (part per million) as claimed by Bailey *et al.* [13]. That is, the uncertainty in the magnetic field is only a few micro-tesla. From the theoretical predictions as shown in Fig. 2, at a magnetic field of 1.472 tesla the predicted value of the period is 146.918 ns; at a magnetic field of 1.55 tesla the predicted value of the period is 146.910 ns. Could the Lorentz force law be considered to be confirmed by Bailey *et al.*'s experiment?

Critically reexamining these experimental results, Fig. 2 in the paper [12] (or Fig. 18 in the paper [13]), we see that the muonic bunch circulates ≈ 27 turns (slightly less than 27 turns) during the time interval from the time $6 \mu\text{s}$ to the time $10 \mu\text{s}$ ($\mu\text{s} = 10^{-6}$ sec). That is, the muonic bunch circulates ≈ 27 turns in a time interval $4 \mu\text{s}$. Therefore, the mean period of the circular motion should be about 148.15 ns. Also, the quoted bin width in measuring the muonic bunches

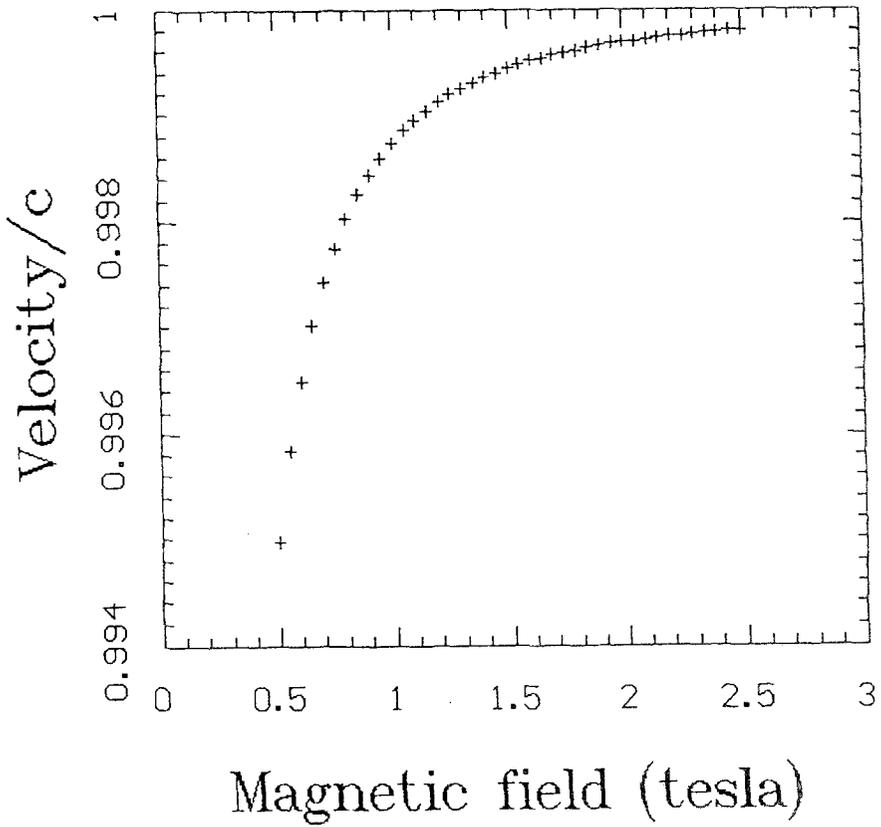


Fig. 1. Theoretical data of v/c vs. magnetic field for muons in uniform circular motion predicted by the Lorentz-covariant electromagnetic force law. The radius of circular motion is 7.0059 m.

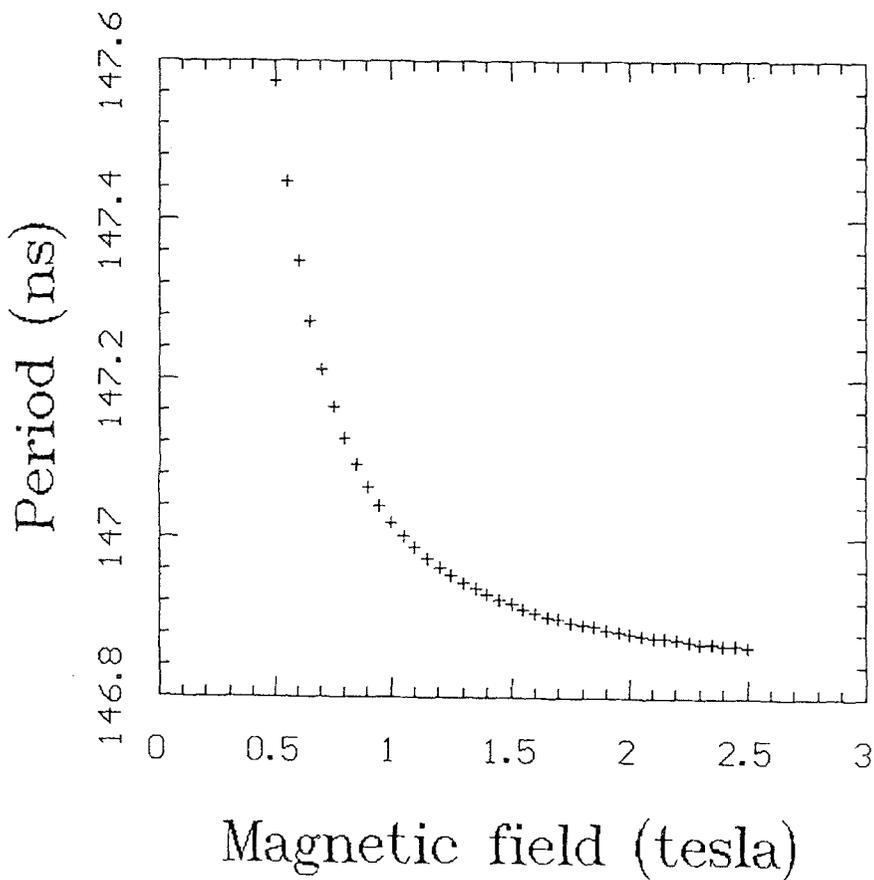


Fig. 2. Theoretical data of period vs. magnetic field for muons in uniform circular motion predicted by the Lorentz-covariant electromagnetic force law. The radius of circular motion is 7.0059 m.

in Fig. 2 in the paper [12] (or Fig. 18 in the paper [13]) is 10 ns. Owing to at most 1 bin width uncertainty in determining the position of either the first or the last peak, the uncertainty of the mean period is estimated as, at most, $0.74 = (2 \times 10)/27$ ns. The experimental value of the mean period should be 148.15 ± 0.74 ns. However, according to the theoretical predictions, the values of period should be in the range between 146.919 ns and 146.917 ns, as the magnetic field varies from 1.47 tesla to 1.48 tesla. These theoretical values are clearly different from the experimental values 147.41 - 148.89 ns.

The deviation in the period between the experimental observation and the theoretical prediction is about 1.23 ns (148.15 - 146.92 ns). Is the deviation due to small uncertainties in measurements in the experiment? The statistical error on the mean radius of the circulating muons in Bailey *et al.*'s experiment is typically 0.1 - 0.2 mm. [12,13] The overall uncertainty in the mean radius is estimated as 0.7 mm, according to the experimental data shown in Table 4 in the paper [13]. Let us allow an uncertainty of 3 mm in the mean radius, which is much larger than the experimental uncertainty 0.7 mm. In this case, the corresponding deviation in the period is only by ≈ 0.06 ns. As mentioned above, the uncertainty in the magnetic field is only a few micro-tesla. Let us also allow a much larger uncertainty in the magnetic field, for example, 0.01 tesla. Corresponding to this uncertainty, the deviation in the period is only by ≈ 0.0012 ns. The deviation in the period found in Bailey *et al.*'s experiment, 1.23 ns, cannot be simply attributed to the uncertainty in measurements of the mean radius and the magnetic field.

Is the found discrepancy due to some errors in the experimental results given in Fig. 2 in the paper [12] (or Fig. 18 in the paper [13])? Two methods of analyzing the radial distribution of the stored muons are mentioned in Bailey *et al.*'s experiment [13]. "In the first, which is applicable only to non-overlapping bunches, the centroid in time of each bunch is calculated. By fitting a straight line to these values the mean rotation frequency f_{rot} is obtained. The mean radius is then deduced from the average relativistic γ -parameter $\bar{\gamma}$," In the second method, "More detailed information on the radial distribution can be obtained by making a least-squares fit to the shape of the bunches as a function of storage time." The mean rotation frequency is obtained by a fit to the experimental results, Fig. 2 in the paper [12] (or Fig. 18 in the paper [13]). The mean period is then deduced from the mean rotation frequency. However, no detailed information, is presented in Bailey *et*

al.'s papers, on how the value of the mean rotation frequency is obtained by a fit to the experimental results. Most physicists would be apt to assume that the mean value of the period determined by a fit to the experimental results shown in Fig. 2 in the paper [12] (or Fig. 18 in the paper [13]) is more accurate than the value estimated by inspecting those figures only. However, those figures clearly indicate that the mean value of the period should be at least 148.15 ns; it is very unlikely that the mean value of the period could be changed so considerably as from 148.15 ns to 146.910 ns just by a method of fitting only.

In addition, it should be stressed that in the experiment of Bailey *et al.* the mean radius of circular motion is inferred from the mean rotation frequency. Therefore, the experiment of Bailey *et al.* cannot provide *direct* evidence for the Lorentz force law without logical circularity. We speculate that the Lorentz force law for relativistic particles might be only approximately true and needs modification. *To say the least, the Lorentz force law cannot be considered to be confirmed by Bailey et al.'s experiment.* Alternately, we feel it is not completely convincing to examine the Lorentz force law *via* just one particular experimental result. More systematic experimental tests of the Lorentz force law for particles with various relativistic speeds are necessary.

3. A CLEAN EXPERIMENT PROPOSED TO TEST THE LORENTZ-COVARIANT ELECTROMAGNETIC FORCE LAW

Though the experiment of Bailey *et al.* does not convincingly confirm the Lorentz force law in the relativistic region, it does indicate that such an experiment can be performed to test this fundamental law with relativistic particles whose speeds are measured by the noncontroversial time-of-flight method. We propose a similar experiment utilizing a *circular* storage ring to constrain *protons* in the uniform circular motion. The predictions of the Lorentz force law for protons are shown in Fig. 3 and Fig. 4, by using the following data: the mass of proton $m = 1.6726230 \times 10^{-27}$ kg and the charge $q = 1.60217738 \times 10^{-19}$ C [26], as well as the mean radius of circular motion $r = 7$ m, which is the mean radius of the storage ring at CERN, as mentioned in Bailey *et al.*'s experiment [12,13]. (In addition, similar predictions of the Lorentz force law for *electrons* are shown in Fig. 5 and Fig. 6; here we

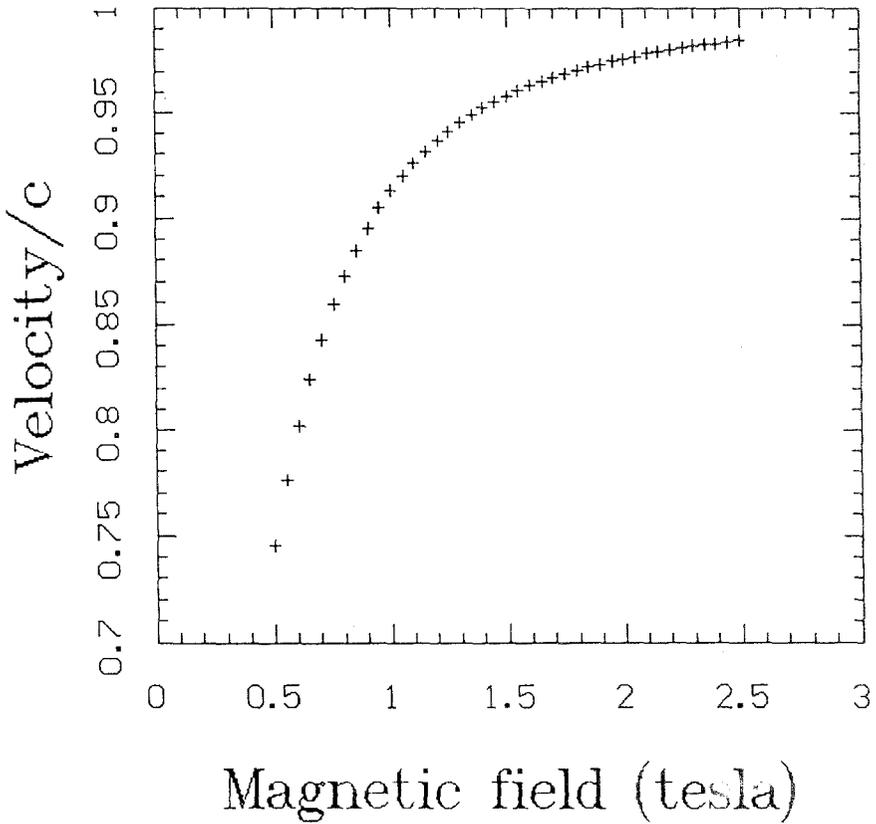


Fig. 3. Theoretical data of v/c vs. magnetic field for protons in uniform circular motion predicted by the Lorentz-covariant electromagnetic force law. The radius of circular motion is 7 m.

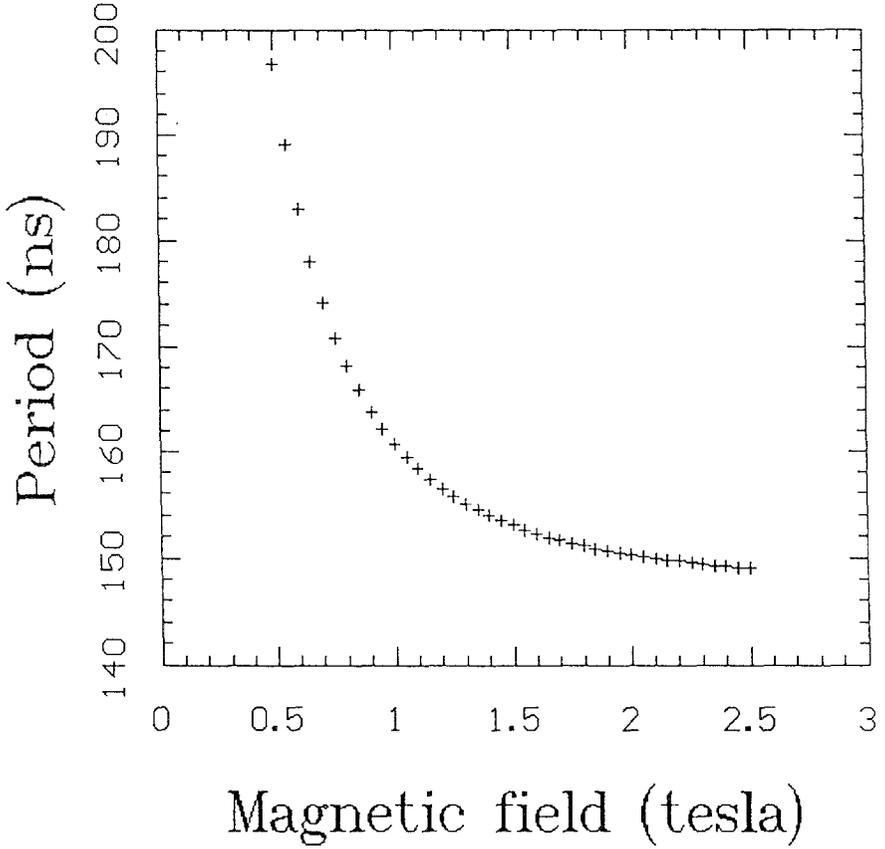


Fig. 4. Theoretical data of period vs. magnetic field for protons in uniform circular motion predicted by the Lorentz-covariant electromagnetic force law. The radius of circular motion is 7 m.

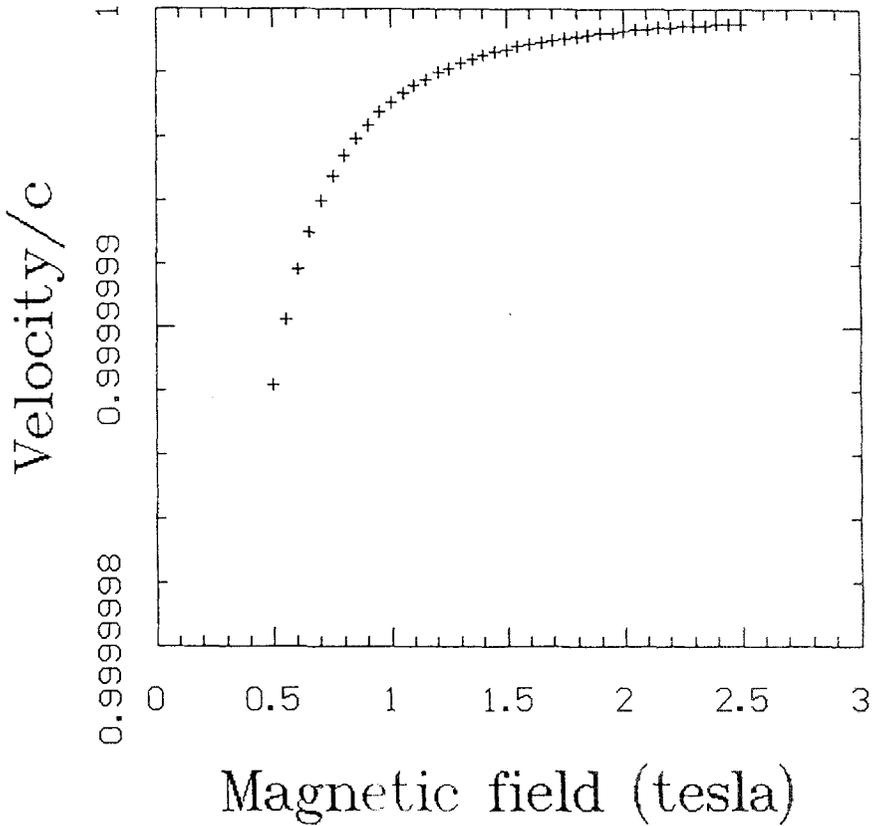


Fig. 5. Theoretical data of v/c vs. magnetic field for electrons in uniform circular motion predicted by the Lorentz-covariant electromagnetic force law. The radius of circular motion is 7 m.

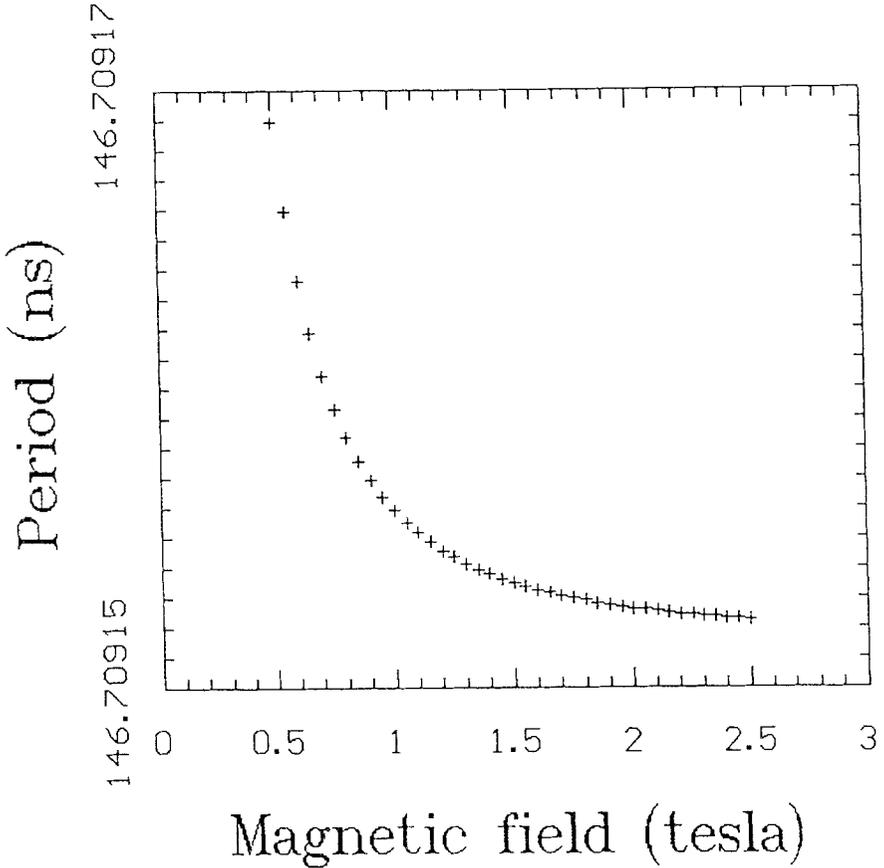


Fig. 6. Theoretical data of period vs. magnetic field for electrons in uniform circular motion predicted by the Lorentz-covariant electromagnetic force law. The radius of circular motion is 7 m.

use the same data as that in the predictions for protons, except the mass of electron $m = 9.1093897 \times 10^{-31}$ kg.)

Since the proposed experiment is similar to the experiments of Bailey *et al.*, the proposed experiment can be instrumented as in Bailey *et al.*'s experiments. Furthermore, the apparatus needed for performing the proposed experiment were analyzed, in detail, by Bailey *et al.* [12,13]. Therefore, in the following we only highlight how the proposed experiment should be performed:

- (1) Set up a uniform magnetic field inside the storage ring and measure the value of that magnetic field. (Such measurements of magnetic field are done in the nondisputed *nonrelativistic* region.)
- (2) Accelerate a proton bunch by a synchrotron accelerator to that speed where, in accordance with the Lorentz force law, it should perform uniform circular motion inside the storage ring, with the setup uniform magnetic field and mean radius of the storage ring.
- (3) Inject the proton bunch into the storage ring to see whether the proton bunch actually performs that uniform circular motion inside the storage ring.
- (4) Measure the radius and the period of that uniform circular motion.
- (5) Repeat the above steps (1) to (4) for different values of magnetic field.

By *directly* measuring the magnetic field inside the storage ring, as well as the period and the mean radius of the circulating proton bunch, we can systematically test the predictions of the Lorentz force law (for particle velocities from $\approx 3c/4$ to nearly c) as shown in Figs. 3 and 4. Since the proposed experiment is based on direct measures in the laboratory frame, there are no ambiguities in the physical meaning of the measurements of the mean radius and the mean period.

There are advantages to using *protons* instead of *muons* in the proposed experiment: In Bailey *et al.*'s experiment, the muons are produced in $\pi - \mu$ decay; the pions are first produced by injecting high-momentum protons onto a target in the ring. The above technical difficulties can be alleviated by using protons instead of muons in the proposed experiment. It is easier to obtain a variety of particle velocities (by a synchrotron accelerator) for protons than for muons, because

protons are simpler to prepare in experiments and because protons do not decay. The decay of muons might also affect the stability of the muonic bunch in closed orbits inside the storage ring. The radiative effect on the motion due to centripetal accelerations is even more negligible for protons than for muons, because protons are more massive. (The radiative effect on the motion of electrons may not be totally negligible, as that for protons, because electrons are much less massive than protons. To avoid possible controversy on the radiative effect on motion, we do not recommend performing the proposed experiment on electrons.) Furthermore, the variation of the period for protons in the proposed experiment is about 50 ns (see Fig. 4), whereas it is only about 0.8 ns for muons (see Fig. 2), as the magnetic field varies from 0.5 tesla to 2.5 tesla. (The even less massive electrons would have an even narrower time range 0.00002 ns for the magnetic field varying from 0.5 tesla to 2.5 tesla, and thus would be inappropriate to use; see Fig. 6.) Suppose that the experimental errors in the mean radius and the magnetic field are allowed to be 2 mm and 10 gauss, respectively; the presumed errors are well within the limits of present technology. In that case, the uncertainty in the period is 0.04-0.18 ns out of 50 ns for protons, compared to about 0.042 ns out of 0.8 ns for muons, for the magnetic field varying in the range between 0.5 tesla and 2.5 tesla. Therefore, it is considerably easier to systematically test the Lorentz force law by using protons instead of muons.

The proposed experiment can systematically test the electromagnetic force law for relativistic particles in which *the speeds are measured noncontroversially by the time-of-flight method*. The proposed experiment involves neither ambiguities in the physical meaning of relativistic spacetime nor complications by theoretical bones of contention other than Einstein's special relativity itself. In addition, the proposed experiment can be performed with high accuracy within present technology.

4. CONCLUSIONS

The majority of physicists will probably be content to assume that the proposed experiment is in fact performed daily in high energy experiments utilizing storage rings, whose radius is kilometers, known to millimeters, and whose period is known with high precision from the tuning of the RF accelerating cavities. The Lorentz force law has been

confirmed with high precision every day without anyone remarking on this fact. Hence, the Lorentz force law is considered to be *absolutely* true; it needs no further examination.

However, in Sec. 2 we demonstrated that the Lorentz force law is not convincingly confirmed by Bailey *et al.*'s experiment, which is frequently considered as one of the most precise experiments in high energy physics [28]. We speculate that the Lorentz force law might be only approximately true. Since the Lorentz force law is one of the most fundamental laws in modern physics, even a slightest discrepancy between the theoretical formula and the *actual* force law would present a serious problem for modern physics.

Are those "daily" performed experiments more reliable than the experiment of Bailey *et al.* in evaluating the Lorentz force law? The storage rings utilized in daily performed experiments are generally not *circular*, and the deflecting magnets of the storage rings do not completely cover the entire path of the circulating particles. In addition, the RF accelerating cavities do accelerate the circulating particles and alter their speeds. Thus, in those experiments the circulating particles inside the storage ring are not in uniform circular motion. Consequently, analyses of those experiments are rendered difficult by the complex motion of particles inside the storage ring. Owing to the considerable uncertainties related to the nonuniform fringe field around the edges of the deflecting magnets and the complicated accelerations of particles by the RF accelerating cavities, analyses of those experiments may result in a failure to convincingly detect a small discrepancy between the Lorentz force law and the actual electromagnetic force law. We doubt that those experiments would be more convincing than the experiment of Bailey *et al.* employed by us in evaluating the Lorentz force law.

Finally, we hope that this paper will intrigue some physicists to search for whatever available experimental evidence there is for the Lorentz force law. We sincerely welcome any citation of *specific* experimental evidence for the Lorentz force law which is more reliable than the experiment of Bailey *et al.* Without convincing experimental evidence for the Lorentz force law, it is highly desirable to directly test this fundamental law with great precision. If the experimental results turn out to verify the Lorentz force law with great precision, then the experiment will be one of the most convincing and important pieces of evidence for Einstein's special relativity.

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